Turning Trash into Treasure: Accelerating Inference of Large Language Models with Token Recycling

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Abstract

Massive parameters of LLMs have made inference latency a fundamental bottleneck. Speculative decoding represents a lossless approach to accelerate inference through a guess-andverify paradigm. Some methods rely on additional architectures to guess draft tokens, which need extra training before use. Alternatively, retrieval-based train-free techniques 009 build libraries from pre-existing corpora or by n-gram generation. However, they face challenges like large storage requirements, time-011 consuming retrieval, and limited adaptability. 012 Observing that candidate tokens generated during the decoding process are likely to reoccur in 014 future sequences, we propose Token Recycling. It stores candidate tokens in an adjacency matrix and employs a breadth-first-search (BFS)like algorithm to construct a draft tree, which is then validated through tree attention. New candidate tokens from the decoding process are 021 then used to update the matrix. Token Recycling requires <2MB of additional storage and achieves approximately 2x speedup across all sizes of LLMs. It significantly outperforms existing train-free methods by 30% and even a 026 widely recognized training method by 25%.

1 Introduction

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Large Language Models (LLMs) (Brown et al., 2020; Gemini Team et al., 2023; Touvron et al., 2023; Meta, 2024) have becoming the foundation of numerous applications such as chatbots, code assistants, and agents (OpenAI, 2023; Chen et al., 2021; Wang et al., 2024a). However, due to the *auto-regressive* decoding strategy, LLMs can only generate a single token at each decoding step, leading to high inference latency (Brown et al., 2020). The latency mainly comes from transferring billions of parameters from high bandwidth memory to the accelerator cache at each decoding step, rather than arithmetic computations (Kim et al., 2024; Shazeer, 2019; Cai et al., 2024).

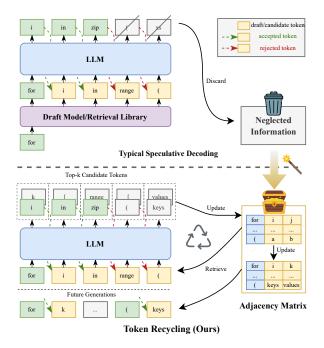


Figure 1: A comparison of typical speculative decoding and Token Recycling (TR). Typical methods draft some tokens and verify them in parallel in one decoding step. Unlike other methods that discard candidate tokens, TR stores them in an adjacency matrix. In future generations, draft tokens are retrieved from the matrix which is updated with new candidate tokens. TR effectively recycles tokens in the decoding process.

Many approaches (Xu et al., 2024; Frantar and Alistarh, 2023; Dao, 2024; DeepSeek-AI, 2024) seek to reduce the latency, with *speculative decoding* as a key lossless technique. This approach employs a *guess and verify* process to obtain multiple tokens during a single decoding step (Chen et al., 2023; Leviathan et al., 2023; Miao et al., 2024; Xia et al., 2023). It first speculates several subsequent draft tokens and then verifies them using the original LLMs. The time cost of verification on multiple tokens is comparable to that of generating one token due to the high parallelism of accelerators. Once some draft tokens are correct, the decoding

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steps is significantly shortened without sacrificing quality. To fully utilize the parallelism of accelerators, *tree attention* slightly adjust the attention mask to verify multiple token continuations in one model forward (Cai et al., 2024; Miao et al., 2024).

Speculative decoding aims not only to maintain quality but also to minimize the cost of speculation. Additional model architectures are constructed to guess the draft tokens, including small draft models (Leviathan et al., 2023; Chen et al., 2023) and parameter-efficient structures (Cai et al., 2024; Lin et al., 2024). However, these approaches require resources for additional training on each LLM. The typical approach to achieve train-free speculative decoding is retrieve-based. In this case, a retrieval library is pre-defined to obtain tokens following the suffix of current content as draft tokens. Several methods have been proposed in this category, each with its trade-offs: (i) REST (He et al., 2023) transforms existing corpora into a retrieval library, but the storage is large, retrieval is time-consuming, and the library lacks flexibility as it's static to any queries. (ii) PLD (Saxena, 2023) only retrieves the previous content with minimal cost. However, it can not predict new tokens or new token combinations. (iii) Lookhead (Fu et al., 2024) construct and update an n-gram library by decoding n times with LLMs. However, LLMs have to generate n-grams while in inference, causing low efficiency.

Furthermore, all speculative decoding approaches fail to fully utilize candidate tokens, which are multiple possible next tokens generated by LLMs at each decoding step. In greedy decoding, only the top-1 candidate token of accepted tokens is selected as the output, while other candidate tokens, including all candidate tokens from rejected tokens, are discarded, such as 'k' and 'keys' in Figure 1. However, we observe that when current input tokens reappear in future generations, the following tokens could be candidate tokens generated several steps prior. Based on the observation, we propose **Token Recycling** (**TR**), which utilizes candidate tokens as draft tokens. It stores candidate tokens in an adjacency matrix. Before each decoding step, a BFS-like approach retrieves a draft tree from the matrix, which is then verified using tree attention. Once verified, the newly generated candidate tokens update the matrix. (i) The matrix provides a flexible retrieval library that is tailored to each query and offers low retrieval costs due to its small size (<2MB). (ii) Compared to using the previous content solely, candidate tokens naturally

include more tokens, providing **many possible continuations**. (iii) The construction and update of our library (matrix) utilize the 'trash' tokens without requiring any additional generation. 107

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We conduct comprehensive experiments on general benchmark SpecBench (Xia et al., 2024), and specialized dataset on code domain, MBPP (Chen et al., 2021) with Vicuna (Zheng et al., 2023) and Code Llama (Roziere et al., 2023) . The results show that TR greatly exceeds previous train-free approaches, and improves more than 31% on all sizes (7b, 13b, 33b/34b). The speed-up ratio even exceeds the widely used training approach–Medusa, demonstrating its high efficiency.

Our contributions are summarized below:

- Based on the observation that candidate tokens can be reused as draft tokens in subsequent sequences, we propose a train-free speculative decoding method, Token Recycling.
- TR requires minimal storage space (<2MB) with a low retrieval cost and covers many new tokens. Continuously updating provides a dynamic retrieval space.
- TR achieves approximately 2x speedup on all sizes of LLMs. It achieves a new SOTA with an improvement greater than 31% compared to previous train-free approaches and even exceeding a training approach.

2 Background

In this section, we overview the speculative decoding. We first define auto-regressive (AR) decoding formally, then discuss speculative decoding, focusing on two key strategies: guess-and-verify and tree attention.

2.1 Auto-Regressive Decoding

AR is the default decoding strategy of LLMs. At each step t, LLMs calculate the probability distribution of the next token given the current content $s = (x_0, x_1, \dots, x_t)$ which $x_i \in \mathcal{V}$:

$$p_{t+1} = P(x|s;\theta).$$

Here \mathcal{V} is the vocabulary and θ denotes LLM parameters. The next token is selected from p_{t+1} based on the sampling method. Followed Kou et al. (2024), we focus on greedy decoding in this paper, where the next token is:

$$x_{t+1} = \operatorname{argmax} p_{t+1}.$$

Candidate tokens are the top-k tokens with the highest probabilities

$$(x_{t+1}^0, x_{t+1}^1, \dots, x_{t+1}^{k-1}) = \operatorname{argtop} k(p_{t+1})$$

where k is the number of candidate tokens, and 142 $\operatorname{argtop} k(\cdot)$ returns the indices of the top-k highest 143 values in p_{t+1} . 144

2.2 Speculative Decoding

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Guess and Verify Speculative decoding effectively utilizes the parallel capability of accelerators. Given s, it first guesses n subsequent draft tokens $(\tilde{x}_{t+1}, \cdots, \tilde{x}_{t+n})$. The combination $(s, \tilde{x}_{t+1}, \cdots, \tilde{x}_{t+n})$ is then sent to LLMs for **one** forward pass, resulting in:

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$$p_{t+1} = P(x \mid s; \theta),$$

153 $\tilde{p}_{t+i} = P(x \mid s, \tilde{x}_{t+1}, \dots, \tilde{x}_{t+i-1}; \theta), i = 2, \dots, n.$

 p_{t+1} is the same as AR decoding so the ground truth x_{t+1} is determinable. If the draft token \tilde{x}_{t+1} matches x_{t+1} , then \tilde{p}_{t+2} is assumed to identical to p_{t+2} . Thus, the next ground truth is selected: $x_{t+2} = \operatorname{argmax} \tilde{p}_{t+2}$. This verification process continues until the draft token does not match the ground truth, indicated by:

$$x_{t+j} = \operatorname{argmax} \tilde{p}_{t+j} \neq \tilde{x}_{t+j}$$

Ultimately, j new tokens are confirmed in one for-154 ward pass. The time cost of one forward pass with 155 $(s, \tilde{x}_{t+1}, \cdots, \tilde{x}_{t+n})$ is nearly the same as with s 156 due to the high parallel performance of accelerators. Figure 1 shows an example. The draft tokens 158 are ['i', 'in', 'range', '('] and the output tokens are ['i', 'in', 'zip', '(', 'xs'] after the forward pass. 160 Though 'zip' fails to match 'range', three tokens ['i', 'in', 'zip'] are confirmed in one forward pass. 162

Tree Attention Traditional causal attention 163 masks are designed for linear sequences, which 164 restricts speculative decoding to verifying one se-165 quence at a time. However, as the sequence lengthens during draft token generation, the number of 167 potential continuations increases. For example, in 168 the draft tree in Figure 2, the token following 'guest' could be 'speaker' or 'speak'. Tree attention mod-171 ifies the attention mask to verify multiple draft sequences simultaneously. It compresses multiple 172 sequences into a single merged sequence, such as 173 ['guest', 'speaker', 'speak'], while preserving the 174 tree structure through tree attention mask. Each 175

child node attends only to its parent nodes, preventing sibling tokens from interfering with each other. After the LLM processes the merged sequence, all possible sequences such as 'guest speaker' and 'guest speak', along with their corresponding output tokens are extracted based on the tree structure and verified in parallel. The longest correct sequence is selected as the final output. In rare cases, when tokens have identical probabilities, tree attention and AR decoding may select different tokens, but this affects the response quality minimally. The detailed explanation is in Appendix A.1.

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In summary, speculative decoding, through guess and verify and tree attention, improves the inference latency robustly and efficiently.

3 Methodology

Figure 2 provides an overview of Token Recycling (TR). It leverages a hot-start adjacency matrix to store candidate tokens and employs a BFS-like algorithm to construct a draft tree. It utilizes tree attention to verify draft sequences and continuously updates the matrix with new candidate tokens generated during the decoding process.

3.1 Adjacency Matrix Initialization

The adjacency matrix \mathcal{M} is a key component in TR, used to store top-k candidate tokens for each token in the vocabulary:

$$\mathcal{M} \in \mathcal{V}^{|\mathcal{V}| imes k}$$
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where k is a user-defined hyperparameter. Each element $\mathcal{M}[i, j]$ indicates that the token $V_{\mathcal{M}[i, j]}$ is the *j*-th candidate token associated with V_i . The use of matrix format, as opposed to other structures like tries, enables efficient parallel processing of candidate tokens, which is crucial for reducing retrieval and update times.

Initially, all elements are set to zero, meaning that a token must appear in draft tokens before it has valid candidate tokens. This initialization leads to the matrix starting with limited predictive capability, potentially causing inefficiencies during the early stages of inference. To mitigate this limitation, we implement a hot start strategy. This involves continuing to use the existing matrix, thereby leveraging prior knowledge. Even if queries differ in the domain, candidate tokens often include common expressions and patterns that frequently appear across various queries. Consequently, hot start ensures that the matrix has a

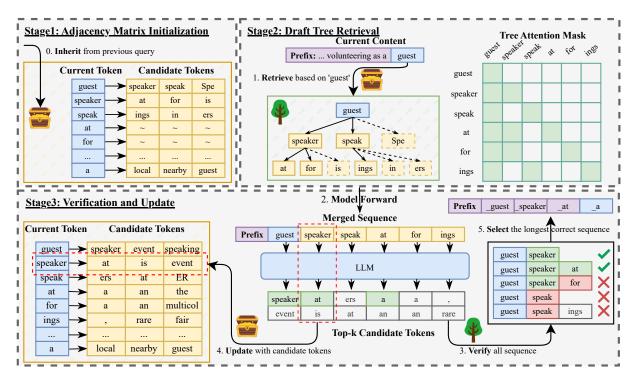


Figure 2: An overview of Token Recycling (TR). The adjacency matrix, initialized by inheriting from the previous query, stores candidate tokens. TR first retrieves a draft tree from the matrix which is then verified through tree attention. After add the longest correct sequence to the content, the new top-k candidate tokens update the matrix.

broader starting point, covering a wide range of potential continuations.

3.2 Draft Tree Retrieval

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The adjacency matrix \mathcal{M} stores candidate tokens, which can be used as draft tokens when their corresponding tokens appear later. Directly using the matrix could only determine the immediate next token, such as finding 'speaker' following 'guest' (see Figure 2). Even if 'speaker' is correct, it only slightly improves upon AR decoding, adding just one additional token. In fact, the matrix also holds possible continuations for these candidate tokens, suggesting subsequent tokens like 'at' following 'speaker'. Extending the sequence step by step allows for longer draft sequences. Furthermore, by storing top-k candidate tokens, multiple potential continuations can be explored in parallel for each token, such as 'at' and 'for' following 'speaker'. This BFS process enables the construction of a draft tree with only the adjacency matrix, which can be directly applied to tree attention.

Unlike a complete BFS, we use heuristic rules to define a static and imbalanced tree structure. This tree structure and its construction process are detailed in the Appendix A.2. **Static**: The number of children for each node remains constant across all decoding steps, which facilitates preprocessing and enables efficient parallel operations during layer traversal. Avoiding the need to traverse each node individually significantly reduces retrieval time. **Imbalance**: Nodes positioned earlier in each layer have more children and extend deeper. This allocates computational resources to the most probable continuations since candidate tokens are ordered by probabilities in the matrix. 250

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The BFS-like approach for retrieving the draft tree begins with the matrix \mathcal{M} and the tree structure Tree. The root is the last token of current content, like 'guest' in Figure 2. As the root forms the first layer, all candidate tokens for 'guest' are extracted from \mathcal{M} , resulting in ['speaker', 'speak', 'Spe']. According to Tree, the first layer allows each token to have two children, Therefore, 'speaker' and 'speak', which have the top-2 probabilities, are added to the second layer. The process then proceeds to expand a new layer. All candidate tokens of the second layer are retrieved in parallel, resulting in ['at', 'for', 'is'] and ['ings', 'in', 'ers']. Tree specifies that the first node ('speaker') can have two children, while the subsequent node ('speak') can only have one child. Consequently, the new layer tokens are ['at', 'for'], and ['ings']. This process repeats until the specified depth is reached. The

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detailed Algorithm 1 is provided in Appendix A.2.

This retrieval method constructs a draft tree effectively and efficiently with the desired length and variety, which can later be verified by tree attention.

3.3 Verification and Update

The verification of the draft tree aligns with Section 2.2. Merged sequence S is constructed through traversing the draft tree by layers. All potential draft sequences are then verified and the longest correct sequence is selected.

Following verification, the adjacency matrix \mathcal{M} is updated in parallel based on the output distributions \tilde{p}_{i+1} of each draft token $x_i \in S$:

$$\mathcal{M}[\tilde{x}_i] = \operatorname{argtop} k(\tilde{p}_{i+1}).$$

Since multiple preceding tokens may have the same candidate token, duplicates may appear in S, and their output distributions are likely to differ. When performing updates in parallel, CUDA operations may merge these updates, leading to variations in the final result. For example, if x_i appears twice and has two different top-2 output tokens, $[y_0, y_1], [z_0, z_1]$, then $\mathcal{M}[x_i]$ could be updated to exactly one of the following results: $[y_0, z_1], [y_0, y_1], [z_0, z_1]$ or $[z_0, y_1]$. We do not resolve this merging, as adding controls reduces overall performance, as discussed later in Section 5.2.

The update process directly overwrites the previous candidate tokens and leverages the new ones as draft tokens for subsequent decoding steps. This allows the retrieval space to dynamically adapt to the current content, focusing on the most relevant and probable continuations. It also eliminates the necessity for extra operations beyond the standard decoding to update the retrieval space.

In summary, TR capitalizes on the 'trash' present in speculative decoding by implementing a cycling process between candidate and draft tokens. It accelerates inference without the need for additional model structures or training, making it highly adaptable and seamlessly integrated with any architecture or model size.

4 Experiment

4.1 Experimental Setup

Align with previous work (Kou et al., 2024), we focus on common computational redundancy scenarios, specifically greedy decoding with a batch size of one. The following evaluation metrics are used: **Mean Accepted Token (MAT)** (Xia et al., 2024) represents the average number of tokens confirmed in a single decoding step; **Tokens per Second** (**Ts/s**) measures the number of tokens processed per second; **Speed**up ratio compares the performance relative to HuggingFace's implementation of AR decoding. We set k = 8 for \mathcal{M} (<2MB storage in sum) and the draft tree structure is shown in Appendix A.2. All experiments are conducted using Pytorch 2.3 with a single A100-80GB GPU and 128 CPUs under CUDA 12.2.

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Datasets and LLMs We conduct experiments on SpecBench (Xia et al., 2024) and MBPP (Austin et al., 2021). SpecBench is a comprehensive benchmark encompassing diverse scenarios including Multi-turn Conversation (MT), Translation (Trans), Summarization (Sum), Question Answering (QA), Mathematical Reasoning (Math), and Retrieval-Augmented Generation (RAG). MBPP is a widely used dataset in code generation, which has a growing demand for efficient generation. These datasets enable a comparative analysis with prior work across both general and specialized domains. We follow the standard practice of utilizing Vicuna (Chiang et al., 2023) for SpecBench and Code Llama (Roziere et al., 2023) for MBPP across three different scales: 7B, 13B, and 33B¹.

Baseline We compare TR with three train-free retrieval-based methods. Lookahead (Lade) constructs an n-gram retrieval library through additional n-gram generation during decoding, consuming significant computational resources. PLD treats previous content as the retrieval library, which is constrained and cannot introduce new tokens or new token combinations. REST builds the retrieval library from existing training datasets, requiring large storage and considerable retrieval time. The static nature of the library also prevents it from adapting to individual queries. Furthermore, we also include a train-need baseline for border comparison. Medusa adds multiple additional LM heads in the final layer to predict draft tokens. We focus on losses Medusa-1 since Medusa-2 is lossy. All baselines use their default hyperparameters.

4.2 Main Results

Table 1 shows the performance of TR compared to other methods. On SpecBench, it achieves more than a 2x speedup on the 7B model, nearly 30% higher than the previous train-free methods. Even

¹The largest model of Code Llama is 34B, for consistency and convenience in our comparisons, we refer to it as 33B.

#Para	Method	SpecBench							MBPP				
		MT	Trans	Sum	QA	Math	RAG	MAT	Ts/s	Speed	MAT	Ts/s	Speed
	AR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	54.30	1.00	1.00	56.15	1.00
	Lade	1.42	1.12	1.21	1.21	1.52	1.13	1.64	69.03	1.27	1.66	79.16	1.41
7D	PLD	1.53	0.98	2.36	1.10	1.50	1.74	1.75	83.30	1.53	1.39	66.65	1.19
7B	REST	1.37	1.05	1.12	1.42	1.06	1.30	1.84	66.29	1.22	2.08	87.08	1.55
	Medusa	1.90	1.57	1.48	1.58	1.87	1.45	2.31	89.41	1.65	-	-	-
	TR	2.17	1.90	1.94	1.95	2.40	1.78	2.70	110.06	2.03	2.93	131.20	2.34
	AR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	39.41	1.00	1.00	41.31	1.00
120	Lade	1.29	1.06	1.16	1.12	1.48	1.09	1.63	47.50	1.21	1.73	56.87	1.38
	PLD	1.45	1.01	2.10	1.02	1.55	1.65	1.67	57.01	1.45	1.48	52.20	1.26
13B	REST	1.51	1.14	1.31	1.50	1.17	1.50	1.82	53.34	1.35	2.05	70.13	1.70
	Medusa	1.94	1.66	1.57	1.62	1.98	1.53	2.39	67.92	1.72	-	-	-
	TR	1.98	1.77	1.89	1.75	2.21	1.73	2.72	74.57	1.89	3.08	93.42	2.26
33B	AR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	18.44	1.00	1.00	19.44	1.00
	Lade	1.32	1.09	1.20	1.17	1.55	1.14	1.61	23.03	1.25	1.70	29.22	1.50
	PLD	1.43	1.06	1.94	1.08	1.55	1.41	1.55	25.89	1.40	1.41	25.89	1.33
	REST	1.63	1.27	1.42	1.61	1.29	1.57	1.81	26.99	1.46	2.10	36.85	1.90
	Medusa	1.98	1.75	1.63	1.68	2.09	1.61	2.32	33.11	1.80	-	-	-
	TR	1.95	1.75	1.92	1.77	2.24	1.78	2.63	35.16	1.91	3.05	45.43	2.34

Table 1: Performance of different methods on SpecBench (Vicuna) and on MBPP (Code Llama) across all parameter sizes. Speed is the displayed metric for Categories of SpecBench. MBPP results exclude Medusa as it lacks a Code Llama variant. Medusa involves training while others are training-free. **Bold** represents the highest performance.

Method	Memory (MB)	Speed
Lade	105	1.27
PLD	0	1.53
REST	465	1.22
Medusa	>800	1.65
TR	1.95	2.03

Table 2: The additional memory costs for all methods. Medusa adds extra LM heads to the model, so the memory usage depends on the model size and the precision. 800MB is based on a 7B LLM and fp16 precision.

compared to tuning Medusa, it shows an improvement of almost 25%. For the 13B and 33B models, it consistently provides nearly 2x speedup, maintaining the 30% acceleration advantage. These results demonstrate that TR is the most effective train-free method on SpecBench, offering substantial and consistent speedup across all model sizes.

Notably, TR achieves the best speedup across most sub-tasks as well, except it slightly trails PLD on Sum. This may be due to this task often involves many repetitions of previous content. However, the performance gap between TR and PLD narrows as the model size increases, reaching only a 1% difference with the 33B model. This is due to larger models tending to generate new tokens rather than repeat previous content. In other tasks such as MT, Trans, QA, and Math, TR shows a significant improvement of about 40%~70% for the 7B model. This demonstrates the strong generalization of our method across various scenarios. Although the improvement on RAG is less than 3% for the 7B model, it increases with model size, exceeding 10% for the 33B one. This improvement is consistent with the preference of larger models for new tokens. Compared to the general domain, all methods achieve greater acceleration on the code domain due to its higher content redundancy. TR provides approximately 2.3x speedup across all model scales, achieving the SOTA performance.

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Furthermore, performances on Trans show the advantages of our method compared to PLD and REST. While PLD shows negligible speedup (close to 1x) and REST achieves its lowest speedup across tasks, TR consistently delivers over 1.75x speedup across all model sizes. Notably, on the 7B model, PLD results in a slowdown, and REST achieves just 1.05x, whereas TR reaches 1.9x. Trans requires generating new tokens continuously, involving minimal repetition of previous content. Additionally, it is highly context-sensitive, making it challenging to find exact matches from any preexisting database. These pose challenges for PLD and REST. In contrast, the adaptive and diverse retrieval space of TR leads to superior performance. In addition to Speed, TR achieves the highest MAT across both benchmarks. This is attributed to its shorter retrieval times and the avoidance of additional generations like Lade. This allows for deeper and wider draft trees, enabling more tokens to be

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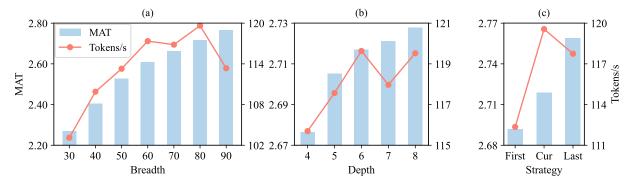


Figure 3: Effects of tree breadth, depth and updating strategies on MAT and Tokens/s are in (a), (b), and (c).

accepted in a single decoding step.

	Tokens/s	Speed
AR	54.98	1.00
Random	95.07	1.73
Zero	102.68	1.87
Fixed	117.43	2.12
Shuffle	118.78	2.16
TR	119.56	2.17

Table 3: The impact of different initialization strategies of the adjacency matrix. Random means randomly selected from the vocabulary, Zero means all set to zero, Fixed means inherited from a fixed matrix and Shuffle means shuffle the test set.

Table 2 summarizes the GPU memory requirement for all methods. Compared to REST and Lade, TR achieves higher speedup with far less memory. While PLD requires no additional memory, its speedup is limited. Unlike Medusa, our approach is training-free, requires minimal memory, and still achieves superior performance.

TR demonstrates significant improvements across all scenarios, highlighting its efficiency and broad applicability. Importantly, **TR is train-free and self-drafting, allowing for an approximate 2x speedup that can be seamlessly applied as a 'free lunch' to any existing LLM**.

5 Analysis

5.1 Tree Structure

As previously outlined in Section 3.2, our tree struc-438 ture is static and imbalanced. The tree size is a 439 crucial factor to accelerate. A larger tree allows 440 more tokens confirmed in one decoding step but 441 442 also introduces more computational overhead, increasing the time required for each decoding step. 443 To investigate the impact of tree size, specifically 444 its depth and breadth, experiments are conducted 445 on MT-Bench using Vicuna-7B. 446

Breadth Increasing the breadth of the tree allows for covering more possibilities. In Figure 3(a), the breadth is expanded by adding nodes while keeping the depth fixed at six layers. This leads to a consistent improvement in MAT. However, when the breadth exceeds 80, Tokens/s begins to decrease. The additional computational overhead eventually outweighs the benefits of a higher MAT.

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Depth Increasing the depth of the tree allows for accepting longer sequences during decoding. In Figure 3(b), with the number of nodes fixed at 80, the depth is gradually increased. MAT initially rises rapidly but eventually shows minimal improvement, while Tokens/s noticeably fluctuates. Because the matrix stores candidate tokens for only adjacent steps, longer sequences weaken the connections between distant tokens. This limitation reduces the effectiveness of increased depth, causing Tokens/s to fluctuate.

5.2 Ablation Study

Hot Start In TR, the adjacency matrix inherits from the previous one. In Table 3, we explore the impact of different initialization strategies. Random means randomly selecting tokens from the vocabulary, while Zero sets all matrix elements to zero. Fixed selects 100 queries from AlpacaEval (Li et al., 2023) (unrelated to the test set), executes them, and stores the resulting matrix. This matrix is then used to initialize each query in the test set. Shuffle refers to shuffling the test set. Compared to the Zero, the irrelevant noise introduced by Random leads to a sharp decrease in performance. Fixed, Shuffle and TR show significant improvements over Zero, suggesting that the prior matrix may capture common patterns that effectively assist subsequent queries. The relatively small difference among them indicates that these patterns are generalizable and not tied to specific tasks or content.

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	SpecBench	MBPP
Only Accepted	1.63	1.99
All Draft	2.69	2.93

Table 4: Mean Accepted Token (MAT) for updating candidate tokens from only accepted or all draft tokens.

Update Strategies Section 3.1 discuss duplicate 485 tokens in the merged sequence during matrix up-486 487 dates. We compare three updating strategies: using candidate tokens from the first occurrence, from 488 the last occurrence, and the current method (merg-489 ing via parallel CUDA operations). Figure 3(c) 490 indicates that using the last occurrence yields the 491 highest MAT, which may benefit from more contex-492 tual information. However, the differences among 493 different strategies in MAT are minimal. In terms of 494 495 Tokens/s, the current approach significantly outperforms the other two, as it avoids the additional pro-496 cessing required to manage token positions, thereby 497 reducing delays. Speculative decoding is highly 498 sensitive to latency, any extra operation must pro-499 vide substantial benefits to outweigh its time cost.

Effect of Rejected Tokens During the update, we refresh the candidate tokens for all draft tokens, 502 503 including both accepted and rejected tokens. To further illustrate the significant effect of trash tokens, we compare two settings: updating only the candi-505 dates of accepted tokens versus of all draft tokens. As shown in Table 4, including candidates of rejected tokens significantly improve the MAT. This indicates that rejected tokens also carry valuable information necessary for subsequent decoding. 510

Related Work 6

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Efficient inference is crucial for real-time appli-512 cations and low-resource scenarios. Many strate-513 gies have been developed to reduce latency (Zhou 514 et al., 2024b). Among these, speculative decod-515 ing (Chen et al., 2023; Leviathan et al., 2023; Miao 516 et al., 2024; Xia et al., 2023) is a losses tech-517 nique that predicts multiple possible continuations simultaneously. It reduces the number of decod-519 ing steps needed without compromising accuracy. Some speculative decoding methods rely on addi-521 tional draft models to guess draft tokens. These 523 typically involve using smaller models from the same series (Zhao et al., 2024; Spector and Re; 524 Sun et al., 2023; Liu et al., 2024b; Yuan et al., 525 2024; Gong et al., 2024) or training new models with a shared vocabulary (Leviathan et al., 2023; 527

Chen et al., 2023; Zhou et al., 2024a; Li et al., 2024). It is worth noting that Zhao et al. (2024) also uses rejected tokens but does not include candidate tokens. Additionally, Kou et al. (2024); Wang et al. (2024b) propose training the original LLMs to enable non-aggressive decoding. While effective, these approaches require managing or training multiple models, which can be non-trivial and resource-intensive. Other methods focus on parameter-efficient structures. These approaches minimize the need for complete retraining but still require model-specific training and adaptation, limiting their scalability and general applicability (Lin et al., 2024; Liu et al., 2024a).

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Train-free methods construct retrieval libraries to obtain draft tokens (Yang et al., 2023). Lookahead (Fu et al., 2024) generates n-grams through multiple decodings, building a retrieval library that can hit multiple tokens in one step. However, it requires the LLM to generate n-grams while responding to queries, which reduces efficiency. PLD (Saxena, 2023) retrieves only from previous content, resulting in minimal overhead and significant speedup in high-redundancy tasks like summarization. However, it provides little acceleration for tasks requiring the generation of new content, like translation. REST (He et al., 2023) constructs retrieval libraries using existing corpora and performs well in common scenarios. However, this approach requires large storage, time-consuming retrieval, and cannot adapt to each query.

Token Recycling is a train-free, retrieval-based method. It requires no additional generation, covers a broader range of possible continuations, and demands minimal storage with low retrieval costs. The update process ensures the retrieval space remains adaptable.

7 Conclusion

In this work, we introduce Token Recycling, a speculative decoding method for accelerating the inference of LLMs. It utilizes an adjacency matrix to store candidate tokens and retrieve a draft tree, which is then verified with tree attention. The matrix is updated with new candidate tokens generated during decoding. Token Recycling could be integrated seamlessly with existing LLMs and various tasks. As a train-free approach, it achieves a speedup of approximately 2x with <2MB of additional storage, improving over 31% compared to previous train-free approaches.

578 Limitations

Our study is comprehensive, but has certain limita-579 tions that we plan to address in future research. In 580 constructing the draft tree, we use a static tree struc-581 ture. However, a dynamic tree could be employed 582 instead. While dynamic trees introduce additional complexity, they allow for better adaptation to each 584 decoding step, potentially improving performance 585 by tailoring the tree structure to the specific require-586 ments of each query. 587

8 Ethical Considerations

The data for the proposed methods is drawn solely from publicly accessible project resources on reputable websites, ensuring that no sensitive information is included. Moreover, all datasets and baseline models used in our experiments are also available to the public. We have taken care to acknowledge the original authors by properly citing their work.

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A.1 Identical Probability Tokens

Appendix

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Method	MT-Bench	GSM8K
AR Decoding	6.17	35.2
Tree Attention	6.23	35.2

Table 5: Quality/Accuracy comparison of AR-Decoding and Tree Attention on MT-Bench and GSM8K. MT-Bench results are taken from Cai et al. (2024). It shows that Tree Attention has minimal impact on both answer accuracy and quality.

Floating-point representation in the computer has precision errors, commonly known as 'floatingpoint rounding errors'. Specifically, the precision of floating-point numbers is determined by the number of bits in the mantissa. In the IEEE 754 standard, the float32 type has a 23-bit mantissa, meaning the smallest representable difference is 2^{-23} , approximately 1.19×10^{-7} . The float16 type, with a 10-bit mantissa, can represent differences as small as 2^{-10} , or about 9.77×10^{-4} . If the difference between two token probabilities is smaller than the precision limit of floating-point representation, these two probabilities will be rounded to the same value, and these tokens will be treated as having identical probabilities during sampling.

AR Decoding uses 'torch.argmax' to return the token with the highest probability. When the probabilities are the same, 'torch.argmax' defaults to returning the one with the smallest index. In Tree Attention, the number of mask tokens is increased compared to AR Decoding, and the attention score of the mask tokens after the softmax operation is not strictly zero, but rather a very small value close to zero. These tiny non-zero values perturb the hidden representations, causing tokens that originally had identical probabilities to now differ slightly, resulting in a different argmax outcome compared to AR Decoding.

Nevertheless, as shown in Table 5, due to the extremely rare occurrence of this issue and the affected probabilities being so close to each other, the impact on experimental accuracy and model performance is negligible.

A.2 Draft Tree Algorithm and Structure

Utilizing tree attention (Miao et al., 2024) to extend the path in the verification phase has become a widely adopted strategy for speculative decoding

Algorithm 1 Static Tree Based BFS

Require:	Adjacency matrix \mathcal{M} , Static tree struc-
ture T	Tree, the last prompt token x_t

- **Ensure:** Merged Sequence S
- 1: Initialize $S \leftarrow \emptyset$
- 2: Initialize $root \leftarrow x_t$
- 3: Initialize the current layer $L \leftarrow (root)$
- 4: Initialize the current depth $d \leftarrow 0$
- 5: while d < Tree.depth do
- 6: Initialize next layer $L_{\text{next}} \leftarrow \emptyset$
- 7: Get all candidate tokens of L from \mathcal{M} in parallel
- 8: xs = M[L]
- 9: Extract next layer tokens from xs with Tree

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10: $L_{next} = xs[Tree[d].index]$ 11: Concatenate *S* and *L* 12: $S \leftarrow (S; L)$ 13: $L \leftarrow L_{next}$ 14: end while 15: return *S*

methods.

In Token Recycling, we also use a heuristically constructed token tree to perform the verification. As shown in Figure 5, we construct a static and unbalanced tree inspired by Cai et al. (2024). The number k on a node indicates that it is the k-th candidate token for its parent node. The construction process is below. We begin with a fully balanced 10-branch tree and use an independent validation set to identify the top k nodes that most frequently yield correct tokens. These top k nodes and their children are retained to form a new tree, and the process is repeated to identify the next set of top knodes. This iterative process continues until performance no longer shows significant improvement. The final tree is determined, and the k is set to consider the maximum number of children across all nodes and the memory requirement. While empirical, this iterative approach has proven to be effective. Further details on tuning the n are provided in Section 5.1. Overall, the tree we construct contains 80 nodes (including the root node) in 6 layers. This means that each forward requires an additional draft input of 79 tokens with a maximum acceptance length of 6.

Building on the tree structure described above, we construct a draft tree for the current content by a BFS-like algorithm in the inference phase. As described in Algorithm 1, we infill the child nodes 907of each layer in turn according to the matrix. At908last, the merged sequence S is returned and sent to909tree attention with Tree.

A.3 Time Allocation

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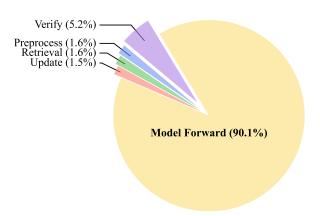


Figure 4: Time allocation for each operation when LLMs respond to a query.

For speculative decoding to be effective, it is es-911 sential to maintain a high hit rate while minimizing 912 the time spent on additional operations. We divide 913 each decoding step into several components: pre-914 processing, retrieving draft tokens, model forward 915 pass, verifying draft sequences, and updating the 916 matrix, input tokens, and key-value cache. The 917 average time spent on each component is shown in 918 Figure 4. The results indicate that the majority of 919 the time is consumed by the model forward pass. 920 921 The verification process also takes a significant amount of time due to the need to extract and ver-922 ify all feasible paths. Retrieving draft tokens and 923 updating operations take roughly the same amount 924 of time. 925

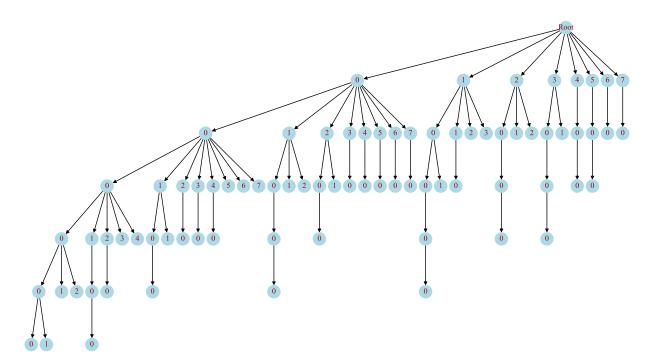


Figure 5: The static tree used in Token Recycling.