Adaptive Tracking Control of 2-DoF Helicopter under State and Input Constraints with Verifiable Feasibility

Poulomee Ghosh and Shubhendu Bhasin

***Abstract*— Ensuring constraint satisfaction of aerial systems under uncertainties and disturbances remains a fundamental challenge in safety-critical control. This work develops an adaptive tracking controller for a 2-DoF helicopter that guarantees the plant states and control inputs remain within user-defined safe sets, even in the presence of parametric uncertainties and bounded external disturbances. State constraints are enforced using a Barrier Lyapunov Function (BLF), while a saturated control law ensures that the input adheres to prescribed limits. Furthermore, an offline verifiable feasibility condition is established to certify the existence of a feasible control policy. The efficacy of the proposed controller is demonstrated through real-time experiments on a Quanser 2-DoF helicopter platform.**

I. Introduction

Safety-critical control has gained significant importance in recent times, with safety often regarded as a primary design objective, sometimes even more important than stability. In many applications, safety requirements typically translate into constraints on the system states and inputs, which is challenging to handle especially in presence of under parametric uncertainties and external disturbances. Classical adaptive methods [1] guarantee bounded trajectories but do not impose user-defined limits on state or input, and enforcing such constraints while ensuring stability, robustness, and feasibility remains nontrivial.

Existing approaches such as model predictive control (MPC) [2] or control barrier function (CBF)-based methods [3], typically rely on full model knowledge or online optimization, making them computationally demanding for real-time use. BLFs [4] offer an optimization-free solution to enforce state constraints, but they can drive the control input toward saturation near the boundary of the safe set. Saturated control techniques [5] and anti-windup strategies [6] handle input limits, yet they do not address state constraints and uncertainties together. In [7], prescribed performance of output error is achieved by modifying the reference online, which precludes direct imposition of state constraints.

Motivated by these research gaps and building on our earlier research [8], this work develops an adaptive controller for 2-DoF helicopter model that ensures tracking while respecting user-defined state and input limits in presence of parametric uncertainties and bounded disturbances. A key contribution is to establish an offline verifiable feasibility condition that certifies when a constraint-compliant control policy exists. The design blends a BLF-based adaptive law with a saturated feedback controller, while σ-modification ensures robustness. The resulting controller is approximation-free, optimization-free and offers a practical and efficient solution to enforce user-defined safety constraints in uncertain Euler–Lagrange (E-L) systems.

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II. Problem Formulation

Consider simplified E-L dynamics of a 2-DoF helicopter

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G_r(q) = \tau + d \tag{1}$$

where $M(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $V_m(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the centripetal-Coriolis matrix, and $G_r(q) \in \mathbb{R}^n$ denotes the gravity vector. The plant states are $q(t) = \begin{bmatrix}\theta(t) & \psi(t)\end{bmatrix}^\top$, representing the pitch and yaw angles, respectively, with $\dot{q}(t) \in \mathbb{R}^{\kappa}$ their corresponding angular velocities. The disturbance $d(t) \in \mathbb{R}^n$ is bounded as $\|d(t)\| < \bar{d}$, where $\bar{d} > 0$ is known. The motor voltages map to the generalized

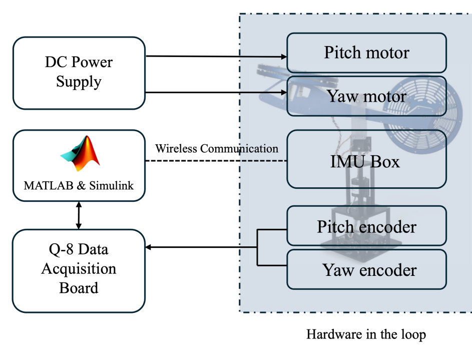

Fig. 1: Quanser 2-DoF helicpter model

torques via $\tau = T(q)V$, where $T(q)$ is non-singular. The matrices in (1) are given by

$$M(q) = \begin{bmatrix} J_p & 0 \\ 0 & J_y + ml^2\cos^2\theta \end{bmatrix} \quad V_m(q,\dot{q}) = \begin{bmatrix} 0 & \rho\dot{\psi} \\ -\rho\dot{\psi} & -\rho\dot{\theta} \end{bmatrix}$$

$$G_r(q) = \begin{bmatrix} -m_p g l_p \cos\theta + m_h g l_h \sin\theta \\ 0 \end{bmatrix} \quad T(q) = \begin{bmatrix} K_{pp} & K_{py} \\ K_{yp} & K_{yy} \end{bmatrix} \tag{2}$$

where, $\rho = \frac{1}{2}ml^2\sin(2\theta)$. The E–L dynamics satisfy the standard structural properties [9]. Let $q_d(t) \in \mathbb{R}^n$ denote the desired trajectory. The control objective is to design a feasible input $\tau(t)$ ensuring that the tracking errors $e(t) = q(t) - q_d(t)$, $\dot{e}(t) = \dot{q}(t) - \dot{q}_d(t)$ remain bounded while the states and inputs remain within the user-defined safe sets, i.e., $q(t), \dot{q}(t) \in \Omega_q$ and $\tau(t) \in \Omega_\tau$ for all $t \geq 0$, where $\Omega_q := \{q, \dot{q} \in \mathbb{R}^n : \|q(t)\| < \bar{Q}, \|\dot{q}(t)\| < \bar{\mathcal{V}}\}$, $\Omega_\tau := \{\tau \in \mathbb{R}^n : \|\tau(t)\| \leq \bar{\tau}\}$, and $\bar{Q}, \bar{\mathcal{V}}, \bar{\tau} > 0$ are user-specified constants.

III. Proposed Methodology

A. Constraint Conversion

Define the filtered tracking error $r \triangleq \dot{e} + \alpha e$, with $\alpha > 0$.

Assumption 1: Reference trajectory and its derivatives are bounded $\|q_d(t)\| \leq \bar{Q}_d < \bar{Q}$, $\|\dot{q}_d(t)\| \leq \bar{\mathcal{V}}_d < \bar{\mathcal{V}}$, $\|\ddot{q}_d(t)\| \leq \bar{\mathcal{A}}_d$, $\forall t \geq 0$ where $\bar{Q}_d, \bar{\mathcal{V}}_d, \bar{\mathcal{A}}_d$ are known positive constants. Provided, Assumption 1, state constraints can be transformed to error constraints, $\|e(t)\| < \mathcal{E}_Q$, $\|\dot{e}(t)\| < \mathcal{E}_\mathcal{V}$ $\forall t \geq 0$, where $\mathcal{E}_Q = \bar{Q} - \bar{Q}_d$, $\mathcal{E}_\mathcal{V} = \bar{\mathcal{V}} - \bar{\mathcal{V}}_d$ are known constants.

Assumption 2: The initial errors satisfy $\|e(0)\| < \xi \leq \mathcal{E}_Q$, $\|r(0)\| < \kappa$, where κ, ξ are known positive constants which satisfies $\kappa < \min(\alpha\xi, \mathcal{E}_\mathcal{V} - \alpha\xi)$ and α is chosen s.t. $\alpha < \frac{\mathcal{E}_\mathcal{V}}{\xi}$.

Lemma 1: Provided Assumption 1-2 hold, if the filtered tracking error satisfies $\|r(t)\| < \kappa$, $t \geq 0$, the error constraints and consequently, the state constraints are satisfied.

Assumption 3: The E-L dynamics satisfy standard bounds, i.e., $\|M(q)\| \leq k_m$, $\|V_m(q, \dot{q})\| \leq k_v\|\dot{q}\|$, $\|G_r(q)\| \leq k_g$, where k_m, k_v, k_g are known constants.

B. Input and State Constraint Satisfaction

Consider the saturated feedback controller

$$\tau_i(t) = \begin{cases} u_i(t), & \|u\| \leq \bar{\tau}, \\ \frac{\bar{\tau}}{\|u\|} u_i(t), & \|u\| > \bar{\tau}, \end{cases} \quad i = 1, \ldots, n \tag{3}$$

where, $u = -Y\hat{\theta} - Kr$ is the auxiliary input and $K > 0$ is user-defined constant. The filtered error dynamics is given by $M\dot{r} = Y\tilde{\theta} - Kr - V_m r + \Delta\tau$, where, $Y \in \mathbb{R}^{n \times m}$ is known regressor matrix, $\theta \in \mathbb{R}^m$ is unknown parameter vector, $Y\theta = M(\alpha\dot{e} - \ddot{q}_d) + V_m(r - \dot{q}) - G_r$, $\|\theta\| < \bar{\theta}$, where $\bar{\theta}$ is assumed to be known. $\tilde{\theta}(t) \triangleq \theta(t) - \hat{\theta}(t) \in \mathbb{R}^n$ is the parameter estimation error and $\Delta\tau(t) \triangleq \tau(t) - u(t)$ represents the saturation error, which can be considered as bounded disturbance. State constraints are enforced using the BLF

$$V_1(r) = \frac{1}{2}\log\left(\frac{\kappa_r^2}{\kappa_m^2 - \bar{m}\|r\|^2}\right) \tag{4}$$

defined on $\Omega_r' = \{r : \bar{m}\|r\|^2 < \kappa_r^2\}$, with $\kappa_r = \kappa\sqrt{\bar{m}}$. The adaptive update law is

$$\dot{\hat{\theta}} = \text{proj}_{\Omega_\theta}\left(\frac{\Gamma Y^T r}{\kappa_m^2 - \bar{m}\|r\|^2} - \sigma\Gamma\hat{\theta}\right) \tag{5}$$

where $\Gamma \in \mathbb{R}^{n \times n}$ is a positive definite matrix, $\sigma > 0$ is a constant and $\Omega_\theta = \{\hat{\theta} \in \mathbb{R}^m : \|\hat{\theta}\| \leq \bar{\theta}\}$, with projection ensuring bounded parameter estimates.

C. Main Result

Theorem 1: For the E-L system (1), given Assumptions 1-3 hold, the controller (3) and adaptive law (5) ensure that the trajectory tracking errors remain bounded while satisfying state and input constraints, i.e., $q(t), \dot{q}(t) \in \Omega_q$, $\tau(t) \in \Omega_\tau$, provided the following feasibility condition is satisfied.

$$\bar{\tau} > (\Psi_1\kappa + \Psi_2 - \lambda_{min}\{K\})\kappa + \Psi_3 \tag{6}$$

where $\Psi_1 = 6\bar{\theta}k_v$, $\Psi_2 = \bar{\theta}(2\alpha\bar{m} + 5k_v(\alpha\xi + \bar{\mathcal{V}}_d)) + \lambda_{max}\{K\}$, and $\Psi_3 = \bar{\theta}(\bar{m}(\alpha^2\xi + \bar{\mathcal{A}}_d) + k_v(\alpha\xi + \bar{\mathcal{V}}_d)^2 + k_g + \bar{\mathcal{V}}_d + \bar{d}$ are known positive constants.

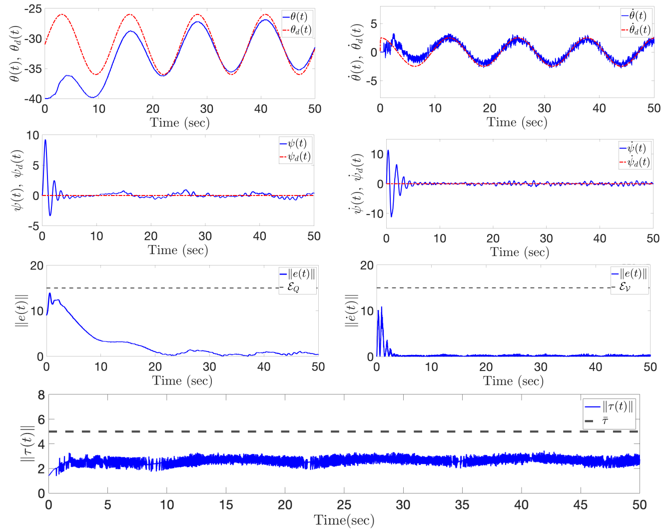

Fig. 2: Tracking performance, tracking error and control input of the 2-DoF helicopter using the proposed controller.

IV. Experimental Result

The proposed controller is implemented on a Quanser 2-DoF helicopter (Fig. 1). The true parameters are considered as $J_p = 0.0384$ kg·m^2, $J_y = 0.0432$ kg·m^2, $m = 1.38$ kg, $m_p = 0.459$ kg, $m_h = 0.295$ kg, $l = 0.1857$ m, $l_p = 0.19685$ m, $l_h = 0.0349$ m, $K_{pp} = 0.2041$ N·m/V, $K_{py} = 0.0068$ N·m/V, $K_{yp} = 0.0219$ N·m/V, $K_{yy} = 0.0720$ N·m/V, and $g = 9.81$ m/s^2. The parameters used for simulation are:$K = 5\mathbb{I}_2$, $\Gamma = 0.05\mathbb{I}_5$, $\bar{d} = 2$, $q_d(t) = -31 + 5\sin(0.5t)$(deg), $\bar{\theta} = 0.12$, $\Psi_1 = 0.0242$, $\Psi_2 = 5.30$, $\Psi_3 = 3.34$, $\alpha = 0.8$, $\kappa = 3$. Fig. 2 demonstrates that the proposed controller ensures state and input constraint satisfaction, i.e., $\|e(t)\| < \mathcal{E}_Q = 15$ (deg), $\|\dot{e}(t)\| < \mathcal{E}_\mathcal{V} = 15$ (deg), $\|\tau(t)\| \leq \bar{\tau} = 5$(N.m) $\forall t \geq 0$.

References

[1] W. M. Haddad and V. Chellaboina, *Nonlinear dynamical systems and control: a Lyapunov-based approach*. Princeton university press, 2008.

[2] A. Bemporad, F. Borrelli, M. Morari, *et al.*, "Model predictive control based on linear programming˜ the explicit solution," *IEEE Transactions on Automatic Control*, vol. 47, no. 12, pp. 1974–1985, 2002.

[3] A. D. Ames, X. Xu, J. W. Grizzle, and P. Tabuada, "Control barrier function based quadratic programs for safety critical systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 3861–3876, 2017.

[4] K. P. Tee, S. S. Ge, and E. H. Tay, "Barrier lyapunov functions for the control of output-constrained nonlinear systems," *Automatica*, vol. 45, no. 4, pp. 918–927, 2009.

[5] S. P. Karason and A. M. Annaswamy, "Adaptive control in the presence of input constraints," in *American Control Conference*, pp. 1370–1374, 1993.

[6] F. Morabito, A. Teel, and L. Zaccarian, "Nonlinear antiwindup applied to euler-lagrange systems," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 3, pp. 526–537, 2004.

[7] F. Fotiadis and G. A. Rovithakis, "Input-constrained prescribed performance control for high-order mimo uncertain nonlinear systems via reference modification," *IEEE Transactions on Automatic Control*, 2023.

[8] P. Ghosh and S. Bhasin, "Adaptive tracking control of uncertain euler-lagrange systems with state and input constraints," in *2023 American Control Conference (ACC)*, pp. 4229–4234, 2023.

[9] M. W. Spong and M. Vidyasagar, *Robot dynamics and control*. John Wiley & Sons, 2008.