# Can LLMs Learn from Previous Mistakes? Investigating LLMs' Errors to Boost for Reasoning

Anonymous ACL submission

#### Abstract

 Large language models (LLMs) have demon- strated striking reasoning capability. Recent works have shown the benefits to LLMs from fine-tuning golden-standard CoT rationales or using them as correct examples in few-shot prompting. While humans can indeed imitate correct examples, learning from our mistakes is another vital aspect of human cognition. Hence, a question naturally arises: *can LLMs learn and benefit from their mistakes, especially for their reasoning?* This study investigates this prob- lem from both the prompting and model-tuning perspectives. We begin by introducing COTER-014 RORSET, a new benchmark with 609,432 ques- tions, each designed with both correct and error references, and demonstrating the types and rea- sons for making such mistakes. To explore the effectiveness of those mistakes, we design two 019 methods: (1) Self-rethinking prompting guides LLMs to rethink whether they have made simi- lar previous mistakes; and (2) Mistake tuning involves finetuning models in both correct and incorrect reasoning domains, rather than only tuning models to learn ground truth in tradi- tional methodology. We conduct a series of experiments to prove LLMs can obtain benefits from mistakes. Both of our two methods serve as potential low-cost solutions to utilize mis- takes to improve reasoning abilities compared with the high cost of making hand-crafted refer- ences. We ultimately make a thorough analysis of the reasons behind LLMs' errors, which pro- vides directions that future research needs to overcome. COTERRORSET will be published soon on Anonymity Link.

## **036 1 Introduction**

 LLMs have recently proven strong capabilities [a](#page-9-0)cross various reasoning tasks [\(Huang,](#page-8-0) [2022;](#page-8-0) [Ko-](#page-9-0) [jima et al.,](#page-9-0) [2022\)](#page-9-0). [\(Wei et al.,](#page-10-0) [2022\)](#page-10-0) proposed CoT prompting, guiding LLMs to think step by step, which becomes a new paradigm to align LLMs' reasoning with the human thinking process. Unfor-



Figure 1: Our two proposed methods to utilize incorrect CoT rationales: self-rethinking prompting and mistake tuning. Our experiments demonstrate LLMs can consistently benefit from incorrect rationales.

tunately, few studies have focused on fully under- **043** standing what kinds of intermediate errors occur **044** in making CoT procedures and whether LLMs can **045** learn from those mistakes like humans. **046**

Indeed, recognizing and correcting previous mis- **047** takes serves as a critical component for better learn- **048** ing and reasoning abilities for our humans [\(Mercer,](#page-9-1) **049** [2008;](#page-9-1) [Reich et al.,](#page-10-1) [2023\)](#page-10-1). In order to thoroughly **050** explore whether LLMs have similar capabilities **051** of utilizing their errors in LLMs' reasoning, we **052** systematically collect a vast dataset of LLMs' rea- **053** soning outputs and built COTERRORSET, which **054** consists of 609,432 questions collected from 1060 **055** tasks across various domains. Each is designed **056** with both the hand-crafted correct reference and  $057$ PaLM2-540B's incorrect rationales. Additionally, **058** we provide the LLMs with the correct reference **059** and annotate the type of each error made and the **060** potential reasons behind them. **061**

In this study, we introduce two possible solutions **062** to investigate the potential benefits of mistakes: **063** self-rethinking prompting and mistake tuning. For **064** self-rethinking, we first provide LLMs with corre- **065** sponding 8 incorrect rationales randomly selected 066 from COTERRORSET as the in-domain knowledge **067**

 for all tasks in arithmetic reasoning. Similarly, for commonsense reasoning, another distinct set of 8 incorrect rationales in the same domain is em- ployed for all questions in commonsense reasoning. Then, after each subsequent reasoning, we guide LLMs to self-rethink whether they make similar mistakes. If they recognize such errors in their out- put, they are then instructed to correct their reason- ing based on the provided domain knowledge. To prevent excessive computational expenditure and avoid loops, we set a threshold to limit the num- ber of times the model can perform self-rethink and corrections. We conduct a series of experi- ments and prove that LLMs can utilize mistakes through self-rethinking, which vastly and consis- tently outperforms self-consistency under the same computational costs.

 We propose mistake tuning to offer another per- spective investigating the potential benefits of uti- lizing mistakes to make LLMs better capable of reasoning. Mistake tuning incorporates the combi- nations of both correct references and incorrect ra- tionales. We finetune two different sizes of Flan-T5 and reveal that tuning on the two domains leads to consistent improvement across various tasks com- pared with only tuning on the correct CoT rationale. Introducing prefixes [CORRECT RATIONALE] and [INCORRECT RATIONALE] before each cor- responding rationale enables Flan-T5 to differenti- ate and contribute to making correct rationales. Our results have proven that learning from mistakes is beneficial to LLMs' reasoning in both prompting and finetuning.

 We conduct comprehensive experiments and studies to prove the effectiveness of utilizing mis- takes in LLMs' reasoning. Aligning self-rethinking to guide the PaLM2-540B model, there have been significant improvements with fewer comput- ing resources observed in areas such as GSM8K (+6.75%) and LogiQA (+5.69%) compared to self- consistency. Finetuning Flan-T5-large (780M) with our principles of mistake tuning, there are notable improvements compared to finetuning on 111 only correct rationales, such as GSM8K(+4.08%) and MathQA(+6.16%).

 Overall, this study introduces the COTER- RORSET, a comprehensive dataset of 609,432 ques- tions with both correct and incorrect rationales across various domains. We underscore the benefits of learning from mistakes to further improve LLMs' reasoning abilities at low cost by proposing two strategies: self-rethinking prompting and mistake

tuning, both of which have demonstrated remark- **120** able improvements. These findings validate the util- **121** ity of the COTERRORSET as a promising direction **122** to advance LLMs' reasoning performance. Further- **123** more, we delve into COTERRORSET and conduct a **124** comprehensive analysis of the errors made by CoT, **125** as well as potential categorizations and reasons for **126** those mistakes. **127**

## 2 Related Work **<sup>128</sup>**

Model Tuning. In the evolving landscape of **129** Natural Language Processing (NLP), the concept **130** of mistake tuning has emerged as a significant **131** advancement in the instruction tuning of Large **132** Language Models (LLMs). Our experiments with **133** Flan-T5 demonstrate that finetuning models on a **134** blend of correct and incorrect rationales, rather **135** than solely on correct Chain-of-Thought (CoT) ra- **136** tionales, yields consistent improvements across **137** a range of tasks. This approach marks a depar- **138** ture from traditional methods that mainly leverage **139** human-crowdsourced tasks from sources like T0 **140** [\(Sanh et al.,](#page-10-2) [2021\)](#page-10-2), FLAN [\(Wei et al.,](#page-10-0) [2022\)](#page-10-0), and **141** NaturalInstructions [\(Mishra et al.,](#page-9-2) [2021\)](#page-9-2), or model- **142** generated tasks. While human-crowdsourced tasks **143** guarantee high quality, they are often limited in **144** scope and require significant human labor. In con- **145** trast, model-generated tasks, which utilize the capa- **146** bilities of advanced language models like PaLM2- **147** 540B, create extensive sets of instructions, inputs, **148** and outputs from initial seed sets [\(Wang et al.,](#page-10-3) **149** [2022;](#page-10-3) [Peng et al.,](#page-10-4) [2023\)](#page-10-4). Our approach, integrating **150** insights from the COTERRORSET, applies mistake **151** tuning to PaLM2-540B, aiming to improve the qual- **152** ity and expand the scope of instruction-following **153** data by incorporating a nuanced understanding of **154** both correct and incorrect reasoning processes. Al- **155** though concurrent work [\(An et al.,](#page-8-1) [2023\)](#page-8-1) has done **156** similar experiments on instruction tuning LLMs to **157** learn why those rationales are incorrect with our **158** mistake tuning, they make GPT4 as a teacher model **159** to finetune the explanations for incorrect answers to **160** downstream LLMs, which demands computational **161** resources and associated costs. **162**

CoT Rationales. In the realm of LLMs, the **163** CoT prompting technique, particularly in zero-shot **164** scenarios, has revolutionized complex reasoning by **165** generating intermediary reasoning steps [\(Wei et al.,](#page-10-0) **166** [2022\)](#page-10-0). This approach, initiated by simple prompts **167** like "Let's think step by step", has been promis- **168** ing in enhancing the reasoning abilities of models **169**

 like PaLM2-540B [\(Zhou et al.,](#page-10-5) [2022b\)](#page-10-5). Following this trend, [Zelikman et al.](#page-10-6) [\(2022\)](#page-10-6) employed GPT-J [\(Wang and Komatsuzaki,](#page-10-7) [2021\)](#page-10-7) to produce ratio- nales, selecting the most effective ones. Our study advances this concept using PaLM2-540B, focus- ing on complex logical reasoning and incorporating insights from the COTERRORSET to understand and correct reasoning errors.

 LLMs with CoT Reasoning. The research con- ducted by [\(Wei et al.,](#page-10-0) [2022\)](#page-10-0) on the emergence of CoT reasoning in large models like PaLM2-540B has catalyzed new research directions. These ca- pabilities have been explored across logical rea- soning [\(Creswell et al.,](#page-8-2) [2022;](#page-8-2) [Pan et al.,](#page-10-8) [2023;](#page-10-8) [Lei et al.,](#page-9-3) [2023\)](#page-9-3), commonsense reasoning [\(Talmor](#page-10-9) [et al.,](#page-10-9) [2018;](#page-10-9) [Geva et al.,](#page-8-3) [2021;](#page-8-3) [Ahn et al.,](#page-8-4) [2022\)](#page-8-4), and mathematical reasoning [\(Miao et al.,](#page-9-4) [2021;](#page-9-4) [Koncel-Kedziorski et al.,](#page-9-5) [2016;](#page-9-5) [Patel et al.,](#page-10-10) [2021;](#page-10-10) [Cobbe et al.,](#page-8-5) [2021;](#page-8-5) [Hendrycks et al.,](#page-8-6) [2021\)](#page-8-6). The exceptional performance of models like PaLM2- 540B has motivated further exploration into opti- [m](#page-10-11)izing CoT reasoning [\(Wang et al.,](#page-10-3) [2022;](#page-10-3) [Zhou](#page-10-11) [et al.,](#page-10-11) [2022a;](#page-10-11) [Creswell and Shanahan,](#page-8-7) [2022;](#page-8-7) [Li](#page-9-6) [et al.,](#page-9-6) [2023b;](#page-9-6) [Lightman et al.,](#page-9-7) [2023\)](#page-9-7), particularly with a focus on error analysis and learning as em-phasized by our COTERRORSET initiative.

 Mathematical Reasoning. Considerable re- search efforts have been directed toward enhancing the capabilities of Large Language Models (LLMs) in solving mathematical problems. This enhance- ment has been approached from various innova- tive perspectives. Some studies have focused on employing voting or verification methods that uti- lize multiple reasoning paths to improve accuracy [a](#page-9-6)nd reliability in solutions [\(Wang et al.,](#page-10-3) [2022;](#page-10-3) [Li](#page-9-6) [et al.,](#page-9-6) [2023b;](#page-9-6) [Lightman et al.,](#page-9-7) [2023\)](#page-9-7). Another di- rection has involved the generation of executable programs or the integration of plug-in tools to en- able the execution of external APIs during the rea- soning process, thereby augmenting the LLMs' problem-solving capabilities [\(Jie and Lu,](#page-8-8) [2023;](#page-8-8) [Wang et al.,](#page-10-12) [2023a;](#page-10-12) [Gou et al.,](#page-8-9) [2023\)](#page-8-9). Additionally, there has been a significant focus on data augmen- tation strategies. These include methods to expand the training datasets and provide external annota- tions, which enrich the LLMs' understanding and approach towards complex mathematical problems [\(Magister et al.,](#page-9-8) [2022;](#page-9-8) [Huang et al.,](#page-8-10) [2022;](#page-8-10) [Ho et al.,](#page-8-11) [2022;](#page-8-11) [Li et al.,](#page-9-9) [2022;](#page-9-9) [Yuan et al.,](#page-10-13) [2023;](#page-10-13) [Li et al.,](#page-9-10) [2023a;](#page-9-10) [Luo et al.,](#page-9-11) [2023;](#page-9-11) [Yu et al.,](#page-10-14) [2023;](#page-10-14) [Liang](#page-9-12) [et al.,](#page-9-12) [2023\)](#page-9-12). Our work, in particular, leverages the MathQA benchmark of multiple-choice math

problems [\(Amini et al.,](#page-8-12) [2019\)](#page-8-12). This benchmark **222** provides a comprehensive and challenging set of **223** mathematical problems, which serves as an excel- **224** lent platform for refining and testing the enhanced **225** problem-solving capabilities of LLMs through CoT **226** reasoning and mistake tuning methods. **227**

Logical Reasoning. Logical reasoning is a **228** fundamental element in both human cognition and **229** AI systems. Various methodologies have been pur- **230** sued to enhance this capability in AI, including 231 [r](#page-9-13)ule-based and symbolic systems [\(MacCartney and](#page-9-13) **232** [Manning,](#page-9-13) [2007\)](#page-9-13), the finetuning of large language **233** models [\(Wang et al.,](#page-10-15) [2018\)](#page-10-15), and a combination of **234** neural and symbolic strategies [\(Li and Srikumar,](#page-9-14) **235** [2019\)](#page-9-14). This intricate, multi-step nature of logical **236** reasoning tasks makes them suitable for CoT in- **237** struction tuning. Our work is novel in applying this **238** technique to logical reasoning with PaLM2-540B, **239** using a comprehensive dataset of reasoning chains **240** from COTERRORSET to refine the model's reason- **241** ing abilities, thereby improving its performance in **242** logical reasoning tasks. **243**

## 3 A Novel Benchmark: COTERRORSET **<sup>244</sup>**

In order to investigate whether incorrect ratio- **245** nales can also contribute to LLMs' reasoning **246** performance, we introduce COTERRORSET, a **247** novel benchmark based on the source of COT- **248** COLLECTION [\(Kim et al.,](#page-9-15) [2023\)](#page-9-15), built upon various **249** domains, including multiple-choice QA, extractive **250** QA, closed-book QA, formal logic, natural lan- **251** guage inference, and arithmetic reasoning. Those **252** public available datasets are QASC [\(Khot et al.,](#page-9-16) **253** [2020\)](#page-9-16), AQuA [\(Ling et al.,](#page-9-17) [2017\)](#page-9-17), GSM8K [\(Cobbe](#page-8-5) **254** [et al.,](#page-8-5) [2021\)](#page-8-5), QED [\(Lamm et al.,](#page-9-18) [2021\)](#page-9-18), Strate- **255** [g](#page-10-16)yQA [\(Geva et al.,](#page-8-3) [2021\)](#page-8-3), SenseMaking [\(Wang](#page-10-16) **256** [et al.,](#page-10-16) [2019\)](#page-10-16), CREAK [\(Onoe et al.,](#page-9-19) [2021\)](#page-9-19), e- **257** [S](#page-8-14)NLI [\(Camburu et al.,](#page-8-13) [2018\)](#page-8-13) and ECQA [\(Aggarwal](#page-8-14) **258** [et al.,](#page-8-14) [2021\)](#page-8-14). Each task within this collection is sys- **259** tematically organized to include a question and **260** instruction section, followed by an answer paired **261** with its golden rationale reference. 262

COTERRORSET diverges from traditional CoT **263** datasets by employing PaLM2-540B's mistakes. **264** We utilized PaLM2 to generate rationales for each **265** question in the dataset, focusing specifically on **266** collecting incorrect rationales. Then we provide **267** PaLM2 with both correct references and its incor- **268** rect answers to demonstrate and reflect and demon- **269** strate why it makes such mistakes. This systematic **270** collection of incorrect rationales can make COTER- **271**

# **272** RORSET a promising benchmark in providing fu-**273** ture improvements from a different perspective.

# **<sup>274</sup>** 4 Our Methodology

# **275** 4.1 Self-rethinking Prompting

 Self-rethinking introduces a unique strategy for prompting LLMs to rethink whether they have made the same previous mistakes. This process begins by presenting LLMs with several incorrect rationales and deducing the reasons for making such errors. The primary objective of this stage is to enable the LLM to introspectively deduce and categorize the nature of mistakes. For example, PaLM2-540B can recognize specific errors they made in GSM8K: application of percentage or ra-tio, making assumptions without basis, etc.

 This explicit demonstration of errors, coupled with the question, golden reference, and incorrect rationales, is instrumental in enabling the LLM to recognize specific types of mistakes it tends to make. Following this, the LLM enters the forward reasoning phase, where it employs a CoT reason- ing approach. Here, it logically progresses step- by-step to solve the problem, actively engaging in the reasoning process. The core of self-rethinking lies in the backward-checking stage. In this phase, the LLM reviews its reasoning chain, but with a specific focus on the error types it previously iden- tified. This targeted review helps the LLM to not just correct random errors but to consciously avoid repeating the same types of mistakes it has made in the past. The process includes a loop for error correction and confirmation. If the LLM finds that it has repeated any of the previously identified mis- takes, it revisits the reasoning process to correct **306** them.

 However, the iterative checking process should have a crucial repeating boundary, denoted as 'k' iterations. If the LLM's error-checking and correc- tion cycles surpass this predefined threshold and errors still persist, the process concludes under the assumption that the issue at hand or the error de- tection might exceed the LLM's current capabili- ties. This constraint prevents the LLM from being caught in an endless loop of self-rethinking, ensur- ing the efficiency and practicality of the reasoning process. In this work, we set k equal to 1 in or- der to trade between the accuracy and computing resources.



## **4.2 Mistake Tuning 320 320 320 320 4.2 Mistake Tuning 320 320 320 320 4.3 minutes 320 320 4.4 minutes 320 320 4.4 minutes 320 320 4.4 minutes 320 4.4 minutes**

In order to fully investigate the potential utiliza- **321** tion of incorrect rationales in COTERRORSET, we **322** propose mistake tuning, instructing LLMs to mem- **323** orize common mistakes, which can further improve **324** their abilities to output correct rationales. By sim- **325** ply appending prefixes [CORRECT RATIONALE] **326** and [INCORRECT RATIONALE] before corre- **327** sponding rationales, mistake tuning is built upon **328** the foundational conclusions of self-rethinking, **329** where LLMs can differentiate the implicit reasons  $330$ and types of mistakes they made in CoT reasoning. **331** This process can be formulated as: **332**

$$
p = [Q \oplus S \oplus R], \tag{1}
$$

$$
\mathcal{L} = -\sum_{t=1}^{|p w o d|} log P(p_t|p_{< t}), \tag{2}
$$

Where Q, S and R represent the given question, **335** special prefix and corresponding rationale respec- **336** tively. ⊕ represents the operation of concatenation. **337**

Mistake tuning presents a cost-effective, straight- **338** forward, and efficient alternative. **339**

## **<sup>340</sup>** 5 Experiments

 In this section, we conducted a series of exper- iments to compare the proposed self-rethinking methods with the existing approach on both arith-metic and commonsense reasoning benchmarks.

### **345** 5.1 Experiment Setup

**346** We conduct comparisons between self-rethinking **347** and several other baselines on multiple bench-**348** marks.

**349** Benchmarks: We consider the following ex-**350** isting math problems benchmarks designed with **351** human rationale reference.

- **352** GSM8K benchmark of math word prob-**353** lems [\(Cobbe et al.,](#page-8-5) [2021\)](#page-8-5).
- **354** AQuA dataset of algebraic math prob-**355** lems [\(Ling et al.,](#page-9-17) [2017\)](#page-9-17).
- **356** MathQA benchmark of multiple-choice math **357** problems [\(Amini et al.,](#page-8-12) [2019\)](#page-8-12).
- **358** Openbook benchmark modeled after open **359** book exams for assessing human understand-**360** ing of a subject [\(Mihaylov et al.,](#page-9-20) [2018\)](#page-9-20).
- **361** LogiQA dataset sourced from expert-written **362** questions for testing human logical reason-**363** ing [\(Liu et al.,](#page-9-21) [2020\)](#page-9-21).
- **364** Critical Reasoning in MARB benchmark of **365** several graduate admission tests, highlighting **366** the reasoning to assumptions, conclusions and **367** paradoxes in arguments [\(Tong et al.,](#page-10-17) [2023\)](#page-10-17).

 Models: In order to evaluate self-rethinking's effects, we choose PaLM2-540B [\(Anil et al.,](#page-8-15) [2023\)](#page-8-15) and GPT4 [\(OpenAI,](#page-9-22) [2023\)](#page-9-22) as the baseline model. PaLM2-540B is a dense left-to-right, decoder-only language model with 540 billion parameters. It is pre-trained on a high-quality corpus of 780 billion tokens with filtered webpages, books, Wikipedia, news articles, source code, and social media conver- sations. GPT4 is a large-scale multimodal SOTA model that exhibits human-level performance on various tasks.

 For mistake tuning, we choose two different- sized Flan T5 [\(Chung et al.,](#page-8-16) [2022\)](#page-8-16), which are specifically designed for instruction tuning strate- gies. This model excels in understanding and gen- erating human-like text, demonstrating remarkable performance across a wide range of natural lan- guage processing tasks. We choose the common settings(random seed=42, learning rate=1e-4) and

finetune using the AdamW optimizer. Consider- **387** ing the vast number of data in AQuA, we only **388** randomly select 10,000 of them to represent the **389** differences in tuning on two different domains. **390**

### 5.2 Self-rethinking Results **391**

Table [1](#page-5-0) presents PaLM2-540B's evaluation re- **392** sults on chosen benchmarks. The self-rethinking **393** method shows superior performance with signifi- **394** cant improvements, especially in GSM8K, AQuA, **395** MathQA, and LogiQA, clearly outperforming self- **396** consistency within the same computing budget. **397** However, while our method surpasses CoT in per- **398** formance on the OpenbookQA dataset, it falls **399** short of achieving self-consistency results. This 400 can be attributed to the nature of the tasks in this **401** dataset, which are less focused on logical difficulty **402** and more on assessing commonsense knowledge. **403** Unlike the other datasets where logical reason- **404** ing and mathematical skills are paramount, Open- **405** bookQA requires a strong understanding of gen- **406** eral knowledge. Table [3](#page-5-1) compares GPT4's per- **407** formance of CoT and self-rethinking. The results **408** demonstrate a notable improvement when using **409** the self-rethinking method over CoT. These find- **410** ings suggest that self-rethinking is a more effective **411** approach for enhancing GPT-4's performance. **412**

Table [2](#page-5-2) presents the 8-shot examples of CoT 413 and self-rethinking, using the PaLM2-540B model **414** across four different tasks: GSM8K, AQuA, **415** MathQA, and LogiQA. The experiment was de- **416** signed with a common setting, employing a ran- **417** dom seed of 42 and selecting 8-shot examples **418** from the respective training sets. A key part of **419** the process involved collecting PaLM2-540B's in- **420** correct rationales for these examples, which were **421** then used as learning demonstrations to rethink. **422** The results show a clear advantage of the self- **423** rethinking method over the standard 8-shot CoT **424** approach. These results highlight the efficacy of the **425** self-rethinking method in improving accuracy in **426** few-shot learning scenarios for complex problem- **427** solving tasks. 428

In conclusion, our self-rethinking method **429** achieved remarkable accuracy improvements in **430** most tests, particularly in scenarios that demand **431** high logical rigor and offer the opportunity to learn **432** from errors by identifying fixed logical patterns, **433** especially in arithmetic reasoning tasks. It indi- **434** cates self-rethinking effectiveness in tasks requir- **435** ing strong logic and prone to minor errors. Addi- **436** tionally, the self-rethinking method proves partic- **437**

<span id="page-5-0"></span>

<b>Methods</b>	<b>GSM8K</b>	AOuA	<b>MathOA</b>	<b>OpenbookOA</b>	LogiOA	<b>CR</b>
Standard (Kojima et al., 2022)	17.06	22.40	27.57	80.92	41.21	24.45
$CoT$ (Wei et al., 2022)	56.29	32.11	30.89	82.66	41.05	51.98
Self-consistency (Wang et al., 2022)	58.38	42.80	41.37	87.61	42.88	22.58
Self-rethinking (Ours)	65.13	44.72	43.95	85.71	49.12	54.53

Table 1: PaLM2-540B's accuracy on Standard Prompting(Standard) [\(Kojima et al.,](#page-9-0) [2022\)](#page-9-0), Chain-of-Thought Prompting(CoT) [\(Wei et al.,](#page-10-0) [2022\)](#page-10-0), self-consistency [\(Wang et al.,](#page-10-3) [2022\)](#page-10-3) and our methods, self-rethinking prompting. In this experiment, we set the times of inference in self-consistency to 3, aligning the computing budget with our method. Our approach involves an initial zero-shot CoT inference, then rethinking whether this rationale has made similar errors. This leads to the final answer if no errors are found. If inaccuracies are detected, it combines a demonstration and the previously suspected erroneous answer for a third inference to arrive at the final answer. Hence, the overall inference times in our methods are between 2 and 3 times per question, which is still lower than self-consistency here.

<span id="page-5-2"></span>

Table 2: PaLM2-540B's accuracy results on fewshot Chain-of-Thought(CoT) and our methods, selfrethinking. We select 8-shot examples from the corresponding trainset. Then we collect PaLM2-540B's incorrect rationales of those 8 examples. The part of the original correct reference is CoT's demonstrations. Those generated incorrect rationales serve as demonstrations for the rethink stage.

<span id="page-5-1"></span>

<b>Methods</b>	<b>GSM8K</b>	AOuA	<b>OpenbookOA</b>	CR
CoT	97.93	88.98	93.21	78.92
self-rethinking	98.02	91.03	95.07	81.37

Table 3: GPT4' results on zero-shot Chain-of-Thought(CoT) and our methods, self-rethinking.

 ularly beneficial in assisting LLMs in identifying and rectifying low-level mistakes or misunderstand- ings that are within the model's capabilities but have been previously overlooked. This capability indicates that self-rethinking can serve as a valu- able tool in refining the accuracy and reliability of responses in LLMs, especially in complex problem-solving contexts.

#### **446** 5.3 Mistake Tuning Results

 Table [4](#page-5-3) showcases the performance of Flan-T5 models in the context of mistake tuning, highlight- ing the impact of combining correct and incorrect rationales. The data presented in Table [4](#page-5-3) reveals significant insights into the performance of Flan- T5 models under mistake tuning, which involves integrating both correct and incorrect rationales. This approach is evident across different model scales, whether it's the smaller 780M version or the larger 3B variant. Notably, in the MathQA domain, Flan-T5-large(780M) tuned by our methods demon-strates superior performance compared to PaLM2-

<span id="page-5-3"></span>

Models	Methods	<b>GSM8K</b>	MathOA	AOuA
Flan-T5-large	<b>Standard Finetuning</b>	14.28	42.79	13.10
(780M)	Mistake Tuning	18.36	48.95	18.07
$Flan-T5-xl$	<b>Standard Finetuning</b>	23.81	47.24	17.81
(3B)	Mistake Tuning	24.29	52.22	20.99

Table 4: Accuracy of Standard Finetuning models (with only correct rationales) vs. our methods, mistake tuning (combined correct and incorrect rationales). Mistake tuning shows consistent and superior performance compared with only fine-tuned correct CoT rationales.

540B, achieving an accuracy of 48.95% versus **459** 41.37%. This phenomenon suggests that LLMs **460** can benefit from engaging with incorrect reason- **461** ing, thereby enhancing their problem-solving and **462** reasoning capabilities. It extends beyond merely **463** bolstering the model's grasp of correct CoT, to also **464** encompassing the ability to identify and learn from **465** incorrect rationales. **466**

Furthermore, the expense of obtaining ground 467 truth or hand-crafted references is significantly **468** higher compared to generating and collecting incorrect rationales. This cost disparity underscores **470** the practical value of our approach, offering a more **471** cost-effective solution without compromising the **472** quality of training data for machine learning mod- **473** els. All mentioned provides a direction for further **474** work of reasoning, which involves not only en- **475** hancing the model's understanding and learning **476** of correct CoT but also the ability to identify and **477** learn from incorrect rationales. **478**

## 6 Further Studies **<sup>479</sup>**

# 6.1 Hyperparameter Analysis of Rethinking **480 Iteration Times** 481

In this section, we conduct experiments to assess **482** the impact of different rethinking iterations, de- **483** noted as k, on the performance of our framework. **484** We evaluate it on two mainstream benchmarks in **485**

 the field of mathematics and commonsense rea- soning, GSM8K and LogiQA. Figure [2](#page-6-0) represents the detailed trend under varying re-thinking times. Notably, as k increases from 1 to 24, GSM8K rep- resents a growth of 8.11% and 12.37% in LogiQA. It is evident as k increases, both LLMs' arithmetic and commonsense reasoning accuracy exhibit an upward trend. This trend suggests a positive corre- lation between the number of rethinking iterations and the overall reasoning abilities. These observa- tions indicate self-thinking's potential benefits with more inference time.

<span id="page-6-0"></span>

Figure 2: Accuracy of different re-thinking iterations(k). As the value of k increases, the overall prediction accuracy improves.

## **498** 6.2 Ablation Study on Rethinking Process

 In this ablation study, we examined the impact of various component combinations in promptings to guide LLMs to self-rethink. Table [5](#page-6-1) shows the performance of different components. The results indicate that the inclusion or exclusion of different components has varying effects on PaLM2-540B's accuracy in domains of GSM8K and LogiQA. How- ever, the overall performance across various com- ponents is relatively similar. It performs similarly well regardless of the specific combination of com- ponents, indicating good generalizability of the method. This study suggests our method's flexibil-ity and stability in future usage.

## **<sup>512</sup>** 7 Unveiling LLM's Reasoning Errors

 In this section, we delve into the detailed types and underlying reasons that lead to mistakes in LLMs's inference process. We sample mistake examples from GSM8K and LogiQA to conduct an in-depth analysis of both arithmetic and commonsense rea-soning. We put some examples in Appendix [A.](#page-10-18)

**519** For commonsense reasoning, we find errors **520** like the misinterpretation of facts or concepts usu-

<span id="page-6-1"></span>

<b>CAT</b>	<b>DEM</b>	COR.	INC.	<b>GSM8K</b>	LogiQA
				64.30	50.21
				62.70	48.57
				65.70	51.01
				65.13	49.21

Table 5: Impact of Component Combinations. CAT. stands for the previous mistakes' type name, DEM. are the reasons for making such mistakes, and COR. and INC. mean corresponding correct and incorrect rationale examples. All components here are generated by LLM itself before reasoning.

<span id="page-6-3"></span>

(a) Commonsense Reasnoing (b) Arithmetic Reasoning

Figure 3: PaLM2's error type distribution in the commonsense and arithmetic reasoning task.

ally arise due to the model's limitations in under- **521** standing and applying context accurately. This **522** reveals current LLMs may still fall short of consis- **523** tently recalling precise factual knowledge within **524** a given context. Consequently, this underscores **525** the imperative to advance toward the develop- **526** ment of Retrieval-Augmented Generation(RAG) **527** systems [\(Guu et al.,](#page-8-17) [2020;](#page-8-17) [Mallen et al.,](#page-9-23) [2022\)](#page-9-23), **528** as they hold the promise of yielding more faithful **529** and contextually aligned results. Additionally, er- **530** rors stemming from logical fallacies or incorrect **531** inferences reveal LLMs' reliance on pattern recog- **532** nition over logical reasoning, sometimes leading **533** them to make logically inconsistent or unsupported **534** connections by the given facts. **535**

<span id="page-6-2"></span>

Figure 4: Our pipeline for clustering PaLM2-540B's mistakes.

Concerning arithmetic reasoning, we observe **536** that the error types identified by LLMs are notably **537** more intricate and diverse compared to those asso- **538** ciated with commonsense reasoning. The complex- **539** ity presents challenges in conducting a thorough **540** analysis. Unfortunately, little previous work has **541**

 explored what kinds of mistakes LLMs make dur- ing inference. In order to tackle this issue and gain a more overarching understanding of LLMs' error types, we utilize an LLM-based clustering mecha- nism shown in Figure [4](#page-6-2) to match diverse error types into more general categories. To be specific, we be- gin by extracting the specific error type names from each output of LLM. Subsequently, we input all the extracted names into the LLMs and prompt them to generate more general categories that encompass the entire spectrum of error names. Following this automated clustering process, we meticulously re- view each cluster, making necessary adjustments to refine the matching results. Finally, we distill the diverse error types into several abstract categories, such as calculation error, numeric error, and logical error in domains of arithmetic reasoning. Detailed

 content is shown in Appendix [B.](#page-10-19) As shown in Figure [3,](#page-6-3) the most errors made by LLMs in arithmetic reasoning are about calculation. This can be attributed to the different nature of LLMs compared to other tools like calculators. To address this issue, [Chen et al.](#page-8-18) [\(2022\)](#page-8-18)'s suggestion using Program-of-Thought (PoT) is a promising approach to instruct LLMs to generate a segment of code to solve the given problem, resulting in more accurate calculation results. Furthermore, it's important to note that logical error is also a type of error that LLMs always suffer from. Com- pared with calculation errors and numeric errors, the causes of logical errors are more complicated and nuanced. For instance, errors like misinterpret- ing given data or misapplying arithmetic operations reveal a lack of depth in understanding mathemati- cal relationships. This can result from the model's limitations in comprehending the nuances of math- ematical concepts or its inability to correctly infer the needed function from the context of the ques- tion. In the future, more fine-grained analysis and methods are needed to address such complex logi-cal errors in arithmetic reasoning.

 For commonsense reasoning, results in Figure [3](#page-6-3) demonstrate that context understanding is the most problematic issue. This problem is intricate and dif- ficult to tackle since this can result from the model's limitations in comprehending the nuances of con- text information and the context's relationship with its own knowledge. [Wang et al.](#page-10-20) [\(2023b\)](#page-10-20) aligns reward models to enhance LLMs' understanding of context information and successfully enhances the model's reasoning abilities, which is consistent with our findings. Table [6](#page-7-0) demonstrates one example where our methods guide PaLM2-540B to **594** change their understanding of the context based on **595** provided mistake examples. **596**

<span id="page-7-0"></span>

Table 6: An example where self-rethinking guides PaLM2-540B to learn from mistakes to better understand the context.

## 8 Conclusions and Future Work **<sup>597</sup>**

In this work, we explore whether LLMs can learn **598** from their mistakes. In order to investigate LLMs' **599** abilities to differentiate and learn from mistakes, **600** we introduce COTERRORSET, a novel benchmark **601** collecting both correct and incorrect CoT rationales **602** across various domains and designed with demon- **603** strations for making errors. We propose two possi- 604 ble solutions to expose the effects of mistakes from **605** different perspectives: self-rethinking and mistake **606** tuning. Both of them have achieved consistent and **607** significant improvements, which demonstrates the **608** potential benefits of learning from reasoning er- **609** rors. In the last, we conduct a comprehensive and **610** detailed analysis of LLMs' common mistakes in **611** both arithmetic and commonsense reasoning. The **612** findings will provide a clear direction for future **613** improvements. **614**

For future work, we envision proposing corre- **615** sponding algorithms or loss functions to learn im- **616** plicit information from mistakes. The primary in- **617** tent of this work is to provide a new paradigm **618** so there are still a lot of improvements that can **619** be down following this work. For example, in- **620** corporating contrastive learning to differentiate **621** correct references and errors is intuitive to make **622** more improvements. Also, some memorization and **623** retrieval-augmented skills can help models benefit **624** from mistakes similar to each question. **625**

**<sup>626</sup>** Limitations

 It is a pity that we can not do very large-scale mistake tunings considering the computational re- sources. As the scaling effects proposed in existing previous work, models over 100B tend to have leap effects after instruction tuning in the reason- ing domain. Moreover, we surprisingly find that most LLMs' end-end unsupervised clustering abil- ities, especially API-based LLMs are still under- explored. This work still lacks large-scale clus- tering for all errors in COTERRORSET to broadly investigate and analyze error types.

### **<sup>638</sup>** References

- <span id="page-8-14"></span>**639** Shourya Aggarwal, Divyanshu Mandowara, Vishwajeet **640** Agrawal, Dinesh Khandelwal, Parag Singla, and Di-**641** nesh Garg. 2021. Explanations for commonsenseqa: **642** New dataset and models. In *Proceedings of the 59th* **643** *Annual Meeting of the Association for Computational* **644** *Linguistics and the 11th International Joint Confer-***645** *ence on Natural Language Processing (Volume 1:* **646** *Long Papers)*, pages 3050–3065.
- <span id="page-8-9"></span><span id="page-8-4"></span>**647** Michael Ahn, Anthony Brohan, Noah Brown, Yevgen **648** Chebotar, Omar Cortes, Byron David, Chelsea Finn, **649** Chuyuan Fu, Keerthana Gopalakrishnan, Karol Haus-**650** man, et al. 2022. Do as i can, not as i say: Ground-**651** ing language in robotic affordances. *arXiv preprint* **652** *arXiv:2204.01691*.
- <span id="page-8-17"></span><span id="page-8-12"></span>**653** Aida Amini, Saadia Gabriel, Peter Lin, Rik Koncel-**654** Kedziorski, Yejin Choi, and Hannaneh Hajishirzi. **655** 2019. Mathqa: Towards interpretable math word **656** problem solving with operation-based formalisms. **657** *arXiv preprint arXiv:1905.13319*.
- <span id="page-8-6"></span><span id="page-8-1"></span>**658** Shengnan An, Zexiong Ma, Zeqi Lin, Nanning Zheng, **659** Jian-Guang Lou, and Weizhu Chen. 2023. Learn-**660** ing from mistakes makes llm better reasoner. *arXiv* **661** *preprint arXiv:2310.20689*.
- <span id="page-8-15"></span><span id="page-8-11"></span><span id="page-8-10"></span>**662** Rohan Anil, Andrew M Dai, Orhan Firat, Melvin John-**663** son, Dmitry Lepikhin, Alexandre Passos, Siamak **664** Shakeri, Emanuel Taropa, Paige Bailey, Zhifeng **665** Chen, et al. 2023. Palm 2 technical report. *arXiv* **666** *preprint arXiv:2305.10403*.
- <span id="page-8-13"></span><span id="page-8-0"></span>**667** Oana-Maria Camburu, Tim Rocktäschel, Thomas **668** Lukasiewicz, and Phil Blunsom. 2018. e-snli: Natu-**669** ral language inference with natural language expla-**670** nations. *Advances in Neural Information Processing* **671** *Systems*, 31.
- <span id="page-8-18"></span><span id="page-8-8"></span>**672** Wenhu Chen, Xueguang Ma, Xinyi Wang, and **673** William W Cohen. 2022. Program of thoughts **674** prompting: Disentangling computation from reason-**675** ing for numerical reasoning tasks. *arXiv preprint* **676** *arXiv:2211.12588*.
- <span id="page-8-16"></span><span id="page-8-7"></span><span id="page-8-5"></span><span id="page-8-3"></span><span id="page-8-2"></span>Hyung Won Chung, Le Hou, Shayne Longpre, Barret **677** Zoph, Yi Tay, William Fedus, Yunxuan Li, Xuezhi **678** Wang, Mostafa Dehghani, Siddhartha Brahma, et al. **679** 2022. Scaling instruction-finetuned language models. **680** *arXiv preprint arXiv:2210.11416*. **681** Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, **682** Mark Chen, Heewoo Jun, Lukasz Kaiser, Matthias **683** Plappert, Jerry Tworek, Jacob Hilton, Reiichiro **684** Nakano, et al. 2021. Training verifiers to solve math **685** word problems. *arXiv preprint arXiv:2110.14168*. **686** Antonia Creswell and Murray Shanahan. 2022. Faith- **687** ful reasoning using large language models. *arXiv* **688** *preprint arXiv:2208.14271*. **689** Antonia Creswell, Murray Shanahan, and Irina Higgins. **690** 2022. Selection-inference: Exploiting large language **691** models for interpretable logical reasoning. *arXiv* **692** *preprint arXiv:2205.09712*. **693** Mor Geva, Daniel Khashabi, Elad Segal, Tushar Khot, **694** Dan Roth, and Jonathan Berant. 2021. Did aristotle **695** use a laptop? a question answering benchmark with **696** implicit reasoning strategies. *Transactions of the* **697** *Association for Computational Linguistics*, 9:346– **698** 361. **699** Zhibin Gou, Zhihong Shao, Yeyun Gong, Yujiu Yang, **700** Minlie Huang, Nan Duan, Weizhu Chen, et al. **701** 2023. Tora: A tool-integrated reasoning agent **702** for mathematical problem solving. *arXiv preprint* **703** *arXiv:2309.17452*. **704** Kelvin Guu, Kenton Lee, Zora Tung, Panupong Pasu- **705** pat, and Mingwei Chang. 2020. Retrieval augmented **706** language model pre-training. In *International confer-* **707** *ence on machine learning*, pages 3929–3938. PMLR. **708** Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul **709** Arora, Steven Basart, Eric Tang, Dawn Song, and Ja- **710** cob Steinhardt. 2021. Measuring mathematical prob- **711** lem solving with the math dataset. *arXiv preprint* **712** *arXiv:2103.03874*. **713** Namgyu Ho, Laura Schmid, and Se-Young Yun. 2022. **714** Large language models are reasoning teachers. *arXiv* **715** *preprint arXiv:2212.10071*. **716** Jiaxin Huang, Shixiang Shane Gu, Le Hou, Yuexin Wu, **717** Xuezhi Wang, Hongkun Yu, and Jiawei Han. 2022. **718** Large language models can self-improve. *arXiv* **719** *preprint arXiv:2210.11610*. **720** [P](https://aclanthology.org/2022.wiesp-1.10)o-Wei Huang. 2022. [Domain specific augmentations](https://aclanthology.org/2022.wiesp-1.10) **721** [as low cost teachers for large students.](https://aclanthology.org/2022.wiesp-1.10) In *Proceed-* **722** *ings of the first Workshop on Information Extraction* **723** *from Scientific Publications*, pages 84–90, Online. **724** Association for Computational Linguistics. **725** Zhanming Jie and Wei Lu. 2023. Leveraging training **726** data in few-shot prompting for numerical reasoning. **727** *arXiv preprint arXiv:2305.18170*. **728**
- <span id="page-9-16"></span>**729** Tushar Khot, Peter Clark, Michal Guerquin, Peter **730** Jansen, and Ashish Sabharwal. 2020. Qasc: A **731** dataset for question answering via sentence compo-**732** sition. In *Proceedings of the AAAI Conference on* **733** *Artificial Intelligence*, volume 34, pages 8082–8090.
- <span id="page-9-15"></span>**734** Seungone Kim, Se June Joo, Doyoung Kim, Joel Jang, **735** Seonghyeon Ye, Jamin Shin, and Minjoon Seo. 2023. **736** The cot collection: Improving zero-shot and few-shot **737** learning of language models via chain-of-thought **738** fine-tuning. *arXiv preprint arXiv:2305.14045*.
- <span id="page-9-0"></span>**739** Takeshi Kojima, Shixiang Shane Gu, Machel Reid, Yu-**740** taka Matsuo, and Yusuke Iwasawa. 2022. Large lan-**741** guage models are zero-shot reasoners. *Advances in* **742** *neural information processing systems*, 35:22199– **743** 22213.
- <span id="page-9-5"></span>**744** Rik Koncel-Kedziorski, Subhro Roy, Aida Amini, Nate **745** Kushman, and Hannaneh Hajishirzi. 2016. Mawps: **746** A math word problem repository. In *Proceedings of* **747** *the 2016 conference of the north american chapter of* **748** *the association for computational linguistics: human* **749** *language technologies*, pages 1152–1157.
- <span id="page-9-18"></span>**750** Matthew Lamm, Jennimaria Palomaki, Chris Alberti, **751** Daniel Andor, Eunsol Choi, Livio Baldini Soares, **752** and Michael Collins. 2021. Qed: A framework **753** and dataset for explanations in question answering. **754** *Transactions of the Association for computational* **755** *Linguistics*, 9:790–806.
- <span id="page-9-3"></span>**756** Bin Lei, Chunhua Liao, Caiwen Ding, et al. 2023. **757** Boosting logical reasoning in large language mod-**758** els through a new framework: The graph of thought. **759** *arXiv preprint arXiv:2308.08614*.
- <span id="page-9-10"></span>**760** Chengpeng Li, Zheng Yuan, Guanting Dong, Keming **761** Lu, Jiancan Wu, Chuanqi Tan, Xiang Wang, and **762** Chang Zhou. 2023a. Query and response augmenta-**763** tion cannot help out-of-domain math reasoning gen-**764** eralization. *arXiv preprint arXiv:2310.05506*.
- <span id="page-9-9"></span>**765** Shiyang Li, Jianshu Chen, Yelong Shen, Zhiyu Chen, **766** Xinlu Zhang, Zekun Li, Hong Wang, Jing Qian, **767** Baolin Peng, Yi Mao, et al. 2022. Explanations from **768** large language models make small reasoners better. **769** *arXiv preprint arXiv:2210.06726*.
- <span id="page-9-14"></span>**770** Tao Li and Vivek Srikumar. 2019. Augmenting neu-**771** ral networks with first-order logic. *arXiv preprint* **772** *arXiv:1906.06298*.
- <span id="page-9-6"></span>**773** Yifei Li, Zeqi Lin, Shizhuo Zhang, Qiang Fu, Bei Chen, **774** Jian-Guang Lou, and Weizhu Chen. 2023b. Making **775** language models better reasoners with step-aware **776** verifier. In *Proceedings of the 61st Annual Meet-***777** *ing of the Association for Computational Linguistics* **778** *(Volume 1: Long Papers)*, pages 5315–5333.
- <span id="page-9-12"></span>**779** Zhenwen Liang, Wenhao Yu, Tanmay Rajpurohit, Peter **780** Clark, Xiangliang Zhang, and Ashwin Kaylan. 2023. **781** Let gpt be a math tutor: Teaching math word problem **782** solvers with customized exercise generation. *arXiv* **783** *preprint arXiv:2305.14386*.
- <span id="page-9-7"></span>Hunter Lightman, Vineet Kosaraju, Yura Burda, Harri **784** Edwards, Bowen Baker, Teddy Lee, Jan Leike, **785** John Schulman, Ilya Sutskever, and Karl Cobbe. **786** 2023. Let's verify step by step. *arXiv preprint* **787** *arXiv:2305.20050*. **788**
- <span id="page-9-17"></span>Wang Ling, Dani Yogatama, Chris Dyer, and Phil Blun- **789** som. 2017. Program induction by rationale genera- **790** tion: Learning to solve and explain algebraic word **791** problems. *arXiv preprint arXiv:1705.04146*. **792**
- <span id="page-9-21"></span>Jian Liu, Leyang Cui, Hanmeng Liu, Dandan Huang, **793** Yile Wang, and Yue Zhang. 2020. Logiqa: A **794** challenge dataset for machine reading compre- **795** hension with logical reasoning. *arXiv preprint* **796** *arXiv:2007.08124*. **797**
- <span id="page-9-11"></span>Haipeng Luo, Qingfeng Sun, Can Xu, Pu Zhao, Jian- **798** guang Lou, Chongyang Tao, Xiubo Geng, Qingwei **799** Lin, Shifeng Chen, and Dongmei Zhang. 2023. Wiz- **800** ardmath: Empowering mathematical reasoning for **801** large language models via reinforced evol-instruct. **802** *arXiv preprint arXiv:2308.09583*. **803**
- <span id="page-9-13"></span>Bill MacCartney and Christopher D Manning. 2007. 804 Natural logic for textual inference. In *Proceedings of* **805** *the ACL-PASCAL Workshop on Textual Entailment* **806** *and Paraphrasing*, pages 193–200.
- <span id="page-9-8"></span>Lucie Charlotte Magister, Jonathan Mallinson, Jakub **808** Adamek, Eric Malmi, and Aliaksei Severyn. 2022. 809 Teaching small language models to reason. *arXiv* **810** *preprint arXiv:2212.08410*. **811**
- <span id="page-9-23"></span>Alex Mallen, Akari Asai, Victor Zhong, Rajarshi **812** Das, Hannaneh Hajishirzi, and Daniel Khashabi. **813** 2022. When not to trust language models: Inves- **814** tigating effectiveness and limitations of paramet- **815** ric and non-parametric memories. *arXiv preprint* **816** *arXiv:2212.10511*. **817**
- <span id="page-9-1"></span>Neil Mercer. 2008. Talk and the development of rea- **818** soning and understanding. *Human development*, **819** 51(1):90–100. **820**
- <span id="page-9-4"></span>Shen-Yun Miao, Chao-Chun Liang, and Keh-Yih Su. **821** 2021. A diverse corpus for evaluating and developing **822** english math word problem solvers. *arXiv preprint* **823** *arXiv:2106.15772*. **824**
- <span id="page-9-20"></span>Todor Mihaylov, Peter Clark, Tushar Khot, and Ashish **825** Sabharwal. 2018. Can a suit of armor conduct elec- **826** tricity? a new dataset for open book question answer- **827** ing. *arXiv preprint arXiv:1809.02789*. **828**
- <span id="page-9-2"></span>Swaroop Mishra, Daniel Khashabi, Chitta Baral, and **829** Hannaneh Hajishirzi. 2021. Cross-task generaliza- **830** tion via natural language crowdsourcing instructions. **831** *arXiv preprint arXiv:2104.08773*. **832**
- <span id="page-9-19"></span>Yasumasa Onoe, Michael JQ Zhang, Eunsol Choi, and **833** Greg Durrett. 2021. Creak: A dataset for com-<br>monsense reasoning over entity knowledge. *arXiv* 835 monsense reasoning over entity knowledge. *arXiv* **835** *preprint arXiv:2109.01653*. **836**
- <span id="page-9-22"></span>OpenAI. 2023. [Gpt-4 technical report.](http://arxiv.org/abs/2303.08774) **837**

- <span id="page-10-8"></span>
- 
- <span id="page-10-10"></span>
- 
- 
- 
- 
- 
- 
- 
- 
- 
- 
- 
- 
- 
- 
- 
- 

- 
- 
- 
- 
- Liangming Pan, Alon Albalak, Xinyi Wang, and William Yang Wang. 2023. Logic-lm: Empower- ing large language models with symbolic solvers for faithful logical reasoning. *arXiv preprint arXiv:2305.12295*.
- Arkil Patel, Satwik Bhattamishra, and Navin Goyal. 2021. Are nlp models really able to solve simple math word problems? *arXiv preprint arXiv:2103.07191*.
- <span id="page-10-4"></span> Baolin Peng, Chunyuan Li, Pengcheng He, Michel Gal- ley, and Jianfeng Gao. 2023. Instruction tuning with gpt-4. *arXiv preprint arXiv:2304.03277*.
- <span id="page-10-1"></span> Taly Reich, Alex Kaju, and Sam J Maglio. 2023. How to overcome algorithm aversion: Learning from mis- takes. *Journal of Consumer Psychology*, 33(2):285– 302.
- <span id="page-10-2"></span> Victor Sanh, Albert Webson, Colin Raffel, Stephen H Bach, Lintang Sutawika, Zaid Alyafeai, Antoine Chaffin, Arnaud Stiegler, Teven Le Scao, Arun Raja, et al. 2021. Multitask prompted training en- ables zero-shot task generalization. *arXiv preprint arXiv:2110.08207*.
- <span id="page-10-9"></span> Alon Talmor, Jonathan Herzig, Nicholas Lourie, and Jonathan Berant. 2018. Commonsenseqa: A question answering challenge targeting commonsense knowl-edge. *arXiv preprint arXiv:1811.00937*.
- <span id="page-10-17"></span> Yongqi Tong, Yifan Wang, Dawei Li, Sizhe Wang, Zi Lin, Simeng Han, and Jingbo Shang. 2023. Elimi- nating reasoning via inferring with planning: A new framework to guide llms' non-linear thinking. *arXiv preprint arXiv:2310.12342*.
- <span id="page-10-15"></span> Alex Wang, Amanpreet Singh, Julian Michael, Felix Hill, Omer Levy, and Samuel R Bowman. 2018. Glue: A multi-task benchmark and analysis platform for natural language understanding. *arXiv preprint arXiv:1804.07461*.
- <span id="page-10-7"></span> Ben Wang and Aran Komatsuzaki. 2021. Gpt- j6b: A 6 billion parameter autoregressive lan- guage model. [https://github.com/kingoflolz/](https://github.com/kingoflolz/mesh-transformer-jax) [mesh-transformer-jax](https://github.com/kingoflolz/mesh-transformer-jax).
- <span id="page-10-16"></span> Cunxiang Wang, Shuailong Liang, Yue Zhang, Xiaonan Li, and Tian Gao. 2019. Does it make sense? and why? a pilot study for sense making and explanation. *arXiv preprint arXiv:1906.00363*.
- <span id="page-10-12"></span> Ke Wang, Houxing Ren, Aojun Zhou, Zimu Lu, Sichun Luo, Weikang Shi, Renrui Zhang, Linqi Song, Mingjie Zhan, and Hongsheng Li. 2023a. Math- coder: Seamless code integration in llms for en- hanced mathematical reasoning. *arXiv preprint arXiv:2310.03731*.
- <span id="page-10-3"></span> Xuezhi Wang, Jason Wei, Dale Schuurmans, Quoc Le, Ed Chi, Sharan Narang, Aakanksha Chowdhery, and Denny Zhou. 2022. Self-consistency improves chain of thought reasoning in language models. *arXiv preprint arXiv:2203.11171*.
- <span id="page-10-20"></span>Yikun Wang, Rui Zheng, Haoming Li, Qi Zhang, Tao **893** Gui, and Fei Liu. 2023b. Rrescue: Ranking llm **894** responses to enhance reasoning over context. *arXiv* **895** *preprint arXiv:2311.09136*. **896**
- <span id="page-10-0"></span>Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten **897** Bosma, Fei Xia, Ed Chi, Quoc V Le, Denny Zhou, **898** et al. 2022. Chain-of-thought prompting elicits rea- **899** soning in large language models. *Advances in Neural* **900** *Information Processing Systems*, 35:24824–24837. **901**
- <span id="page-10-14"></span>Longhui Yu, Weisen Jiang, Han Shi, Jincheng Yu, **902** Zhengying Liu, Yu Zhang, James T Kwok, Zhen- **903** guo Li, Adrian Weller, and Weiyang Liu. 2023. **904** Metamath: Bootstrap your own mathematical ques- **905** tions for large language models. *arXiv preprint* **906** *arXiv:2309.12284*. **907**
- <span id="page-10-13"></span>Zheng Yuan, Hongyi Yuan, Chengpeng Li, Guanting **908** Dong, Chuanqi Tan, and Chang Zhou. 2023. Scal- **909** ing relationship on learning mathematical reason- **910** ing with large language models. *arXiv preprint* **911** *arXiv:2308.01825*. **912**
- <span id="page-10-6"></span>Eric Zelikman, Yuhuai Wu, Jesse Mu, and Noah Good- **913** man. 2022. Star: Bootstrapping reasoning with rea- **914** soning. *Advances in Neural Information Processing* **915** *Systems*, 35:15476–15488. **916**
- <span id="page-10-11"></span>Denny Zhou, Nathanael Schärli, Le Hou, Jason Wei, **917** Nathan Scales, Xuezhi Wang, Dale Schuurmans, **918** Claire Cui, Olivier Bousquet, Quoc Le, et al. 2022a. **919** Least-to-most prompting enables complex reason- **920** ing in large language models. *arXiv preprint* **921** *arXiv:2205.10625*. **922**
- <span id="page-10-5"></span>Yongchao Zhou, Andrei Ioan Muresanu, Ziwen Han, **923** Keiran Paster, Silviu Pitis, Harris Chan, and Jimmy **924** Ba. 2022b. Large language models are human-level **925** prompt engineers. *arXiv preprint arXiv:2211.01910*. **926**
- <span id="page-10-18"></span>A Reasoning Mistake Examples **<sup>927</sup>**
- <span id="page-10-19"></span>**B** Definition for Error Categories **928**

Table 7: Examples of Error Types in Arithmetic Reasoning. All contents are generated by PaLM2-540B itself.

# Error name: Misinterpretation of Given Data Error type: Logical

- *Example:* Natalia sold clips to 48 of her friends in April, and then she sold half as many clips in May. How many clips did Natalia sell altogether in April and May?
- *Correct Answer:* Natalia sold 48/2 = 24 clips in May. Natalia sold 48+24 = 72 clips altogether in April and May.
- *Incorrect Rationale:* Natalia sold 48 \* 2 = 96 clips in May. Natalia sold 48+96 = 144 clips altogether in April and May.
- *Demonstration:* Mistaking multiplication for division led to a significant overestimate of the total clips sold.

# Error name: Misapplication of Arithmetic Operation Error type: Calculation

- *Example:* Weng earns \$12 an hour for babysitting. Yesterday, she just did 50 minutes of babysitting. How much did she earn?
- *Correct Answer:* Weng earns 12/60 = \$0.2 per minute. Working 50 minutes, she earned 0.2 x 50 = \$10.
- *Incorrect Rationale:* Weng earns 12/60 = \$2 per minute. Working 50 minutes, she earned 2 x 50 = \$100.
- *Demonstration:* Confusing the rate per hour with the rate per minute led to a substantial overestimation of earnings.

# Error name: Numerical Error type: Numeric

- *Example:* The chicken crossed the road to get to the other side twice for the thrill of it. The first time, it had to dodge 23 speeding cars. The second time, a person tried to catch it and accidentally pulled out twice as many feathers as the number of cars the chicken had dodged. The chicken had 5263 feathers before its thrill-seeking road crossings. How many feathers did it have afterward?
- *Correct Answer:* The chicken lost 23 \* 2 = «23\*2=46»46 feathers on its second road crossing., it had  $5263 - 46 = \frac{5263 - 46}{5217}$  feathers after crossing the road twice.
- *Incorrect Rationale:* The chicken lost 23 \* 2 = «23\*2=46»46 feathers on its second road crossing., it had  $5263 - 46 = \frac{5263 - 52}{5211 \cdot 5211}$  feathers after crossing the road twice.
- *Demonstration:* 1. The correct answer is 5217, while your answer is 5211. 2. Your answer is wrong because you subtracted 52 instead of 46. 3. The type name of the incorrect answer is numerical.

#### Error name: Logical Fallacy or Incorrect Inference Error type: Logical

- *Example:* "When standing miles away from Mount Rushmore"
- *Correct Rationale:* Objects appear smaller when viewed from a greater distance.
- *Incorrect Rationale:* "The mountains do not look smaller when standing miles away from Mount Rushmore. They look larger." (Logical fallacy)
- *Demonstration:* 1. The correct rationale is that objects appear smaller when viewed from a greater distance, whereas the incorrect rationale states the opposite. 2. This is a logical fallacy as it contradicts a basic principle of perception. 3. The type name of the incorrect rationale is logical.

#### Error name: Incorrect Assumptions or Generalizations Error type: Logical

- *Example:* "Poison causes harm to which of the following?"
- *Correct Rationale:* Poison affects living organisms.
- *Incorrect Rationale:* "Robot do not get hurt by poison." (Incorrect generalization about the effects of poison)
- *Demonstration:* 1. The correct rationale is that poison affects living organisms, but the incorrect rationale generalizes that robots are immune to poison. 2. This is an incorrect generalization because robots, being non-living entities, are not subject to biological effects. 3. The type name of the incorrect rationale is logical.

#### Error name: Misunderstanding Literal vs. Metaphorical Language Error type: Linguistics

- *Example:* "When food is reduced in the stomach"
- *Correct Rationale:* Digestion involves the breakdown of food by stomach acid.
- *Incorrect Rationale:* "Choice D is incorrect because it is not a fact." (Misunderstanding metaphorical language)
- *Demonstration:* 1. The correct rationale is about the literal process of digestion, whereas the incorrect rationale misinterprets the metaphorical language. 2. This demonstrates a misunderstanding of metaphorical language. 3. The type name of the incorrect rationale is linguistics.

#### Error name: Incorrect Application of Knowledge Error type: Commonsense

- *Example:* "Stars are"
- *Correct Rationale:* Stars are massive celestial bodies made of gases.
- *Incorrect Rationale:* "Stars are not made of warm lights that float." (Incorrectly applying knowledge about stars)
- *Demonstration:* 1. The correct rationale states that stars are massive celestial bodies made of gases, but the incorrect rationale describes them as warm lights that float. 2. This is an incorrect application of knowledge, as it fails to accurately describe the nature of stars. 3. The type name of the incorrect rationale is commonsense.

#### Error name: Factual Inaccuracy Error type: Commonsense

- *Example:* "You can make a telescope with a"
- *Correct Rationale:* A telescope requires specific optical elements to function.
- *Incorrect Rationale:* "A telescope needs a lens and a magnifying glass is a lens, so glass is a good choice." (Factually inaccurate about how telescopes are made)
- *Demonstration:* 1. The correct rationale is that a telescope requires specific optical elements, whereas the incorrect rationale assumes any lens, like a magnifying glass, can make a telescope. 2. This shows a factual inaccuracy in understanding how telescopes are constructed. 3. The type name of the incorrect rationale is commonsense.

#### Error type: Misunderstanding Context or Relevance Error type: Context

- *Example:* "an inherited characteristic found on all mammals is"
- *Correct Rationale:* Inherited characteristics in mammals include features like fur.
- *Incorrect Rationale:* "Shoes are not found on all mammals" (Misunderstanding the context of biological characteristics) – *Demonstration:* 1. The correct rationale focuses on relevant inherited physical traits like fur. 2. This error illustrates a
- clear lack of understanding of the context. 3. The type name of the incorrect rationale should be context.

Table 9: PaLM2-540B's Understanding and Definitions for Error Types. All contents are generated by itself after providing its mistakes and corresponding golden-standard references.

