Iterative reconstruction of bandlimited signals from nonuniform samples by sliding periodization of nonuniformity

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***Abstract*—The present article proposes to reconstruct a bandlimited signal from nonuniform samples by a sliding periodization of the nonuniformity and successive approximations. When the nonuniformity consists of bounded deviations of the sampling instants from a uniform grid, the reconstruction can be made arbitrarily accurate either by increasing the period of the periodizations or by increasing the number of successive approximations.**

I. Introduction

While event-based sampling is attracting increasing attention due to a number of advantages on the encoding side, including higher precisions of signal acquisition and lower power consumptions, one is faced with the difficult topic of signal reconstruction from nonuniform generalized samples on the decoding side. This research started all the way from the 50's [1], [2] for the basic case of point sampling, with later developments in [3], [4], [5]. Iterative reconstruction algorithms were more recently introduced in [6], [7] for integration-based sampling that has proved particularly suitable for time encoding. Meanwhile, non-iterative methods for the bandlimited interpolation of nonuniform point samples have been developed in a number of papers including [8], [9], [10]. For that same context of point sampling, we focus in this article on iterative signal reconstructions which aim at successive approximations with simplified implementations. We propose a new principle of iteration that mixes the frame algorithm of [1] with special Lagrange interpolators from [2].

For the reconstruction of a signal $x(t)$ in a given space of bandlimited functions $\mathcal{B}$ from nonuniform samples $(x(t_n))_{n\in\mathbb{Z}}$, the method of [1] can be interpreted as finding functions $(f_n(t))_{n\in\mathbb{Z}}$ of $\mathcal{B}$ that depend on $(t_n)_{n\in\mathbb{Z}}$ and such that the linear transformation

$$Fu(t) := \sum_{n\in\mathbb{Z}} u(t_n)\, f_n(t) \tag{1}$$

is close enough to the identity operator I on $\mathcal{B}$. The requirement is more specifically that $\|I-F\|<1$, where $\|\cdot\|$ designates the operator norm on $\mathcal{B}$. Then the iteration of

$$u^{(k+1)}(t) = u^{(k)}(t) + F\big(x(t) - u^{(k)}(t)\big), \quad k \geq 0. \tag{2}$$

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is guaranteed to converge to $x(t)$. While the authors of [1] chose $f_n(t)$ to be a sinc function shifted at t_n with some constant scaling factor, we propose in this paper to take for each n the ideal reconstruction function that would be obtained if the sampling nonuniformity was N-periodic while matching exactly the N closest sampling instants to t_n. The *explicit* expression of this function is indeed available and given in [2]. In the case where the instants $(t_n)_{n\in\mathbb{Z}}$ are deviations from the Nyquist instants by not more than half the Nyquist period, we show numerically that the rate of convergence of (2) is increasingly fast with increasing N. We also compare our results with the iterative methods of [1], [11] and [9].

II. Method of successive approximations

We start by explaining in more details the convergence properties of (2).

A. *Successive approximations by contraction*

The iteration of (2) can be formulated as

$$u^{(k+1)}(t) := M_x u^{(k)}(t) \tag{3}$$

$$\text{where} \quad M_x u(t) = (I-F)u(t) + Fx(t), \quad u(t) \in \mathcal{B}.$$

The norm $\|A\|$ of an operator A on $\mathcal{B}$ is by definition the supremum value of $\|Au(t)\|/\|u(t)\|$ over $u(t)\in\mathcal{B}\backslash\{0\}$, where $\|\cdot\|$ is here the L^2-norm. Then M_x is a *contraction* of $\mathcal{B}$ whenever $\|I-F\|<1$. In this case, it is known that M_x has a unique fixed point [12, §1.2], and the iterates of (3) systematically tend to this point. Since $M_x x(t) = x(t)$, we conclude that $u^{(k)}(t)$ tends to $x(t)$, thus achieving perfect reconstruction. Moreover, as $M_x u(t) - x(t) = (I-F)(u(t) - x(t))$,

$$\begin{aligned}\|u^{(k)}(t) - x(t)\| &\leq \|(I-F)^k\|\, \|u^{(0)}(t) - x(t)\| \\ &\leq \|I-F\|^k\, \|u^{(0)}(t) - x(t)\|.\end{aligned} \tag{4}$$

Thus, $\|I-F\|$ gives an upper bound on the rate of convergence.

B. *Tight condition of convergence*

We will see in certain cases that (3) converges even when $\|I-F\|>1$. As can be seen in (4), it is in fact sufficient that $\|(I-F)^k\|$ be less than 1 for some $k\geq 1$ for (3) to converge. Mathematically, this happens exactly when the spectral radius $\rho(I-F)$ of $I-F$ is less than 1 due to the relation $\rho(A)=\lim_{k\to\infty}\|A^k\|^{1/k}$ [13, §4.2]. In finite dimension, note that $\|I-F\|$ is the largest singular value of $I-F$ while $\rho(I-F)$ is the largest magnitude of its eigenvalues. The rate of convergence of (3) is asymptotically governed by $\rho(I-F)$ as $\|(I-F)^k\|\leq(\rho(I-F)+\epsilon)^k$ for any arbitrarily small $\epsilon>0$ and large enough k.

C. Discrete-time iteration

The iteration of (2) takes place in the space $\mathcal{B}$ of continuous-time bandlimited signals. In common signal processing culture, one implicitly thinks of discretizing (2) by sampling its functions at the Nyquist rate (or at the instants $n\in\mathbb{Z}$ in the present signal setting). This however makes $Fu(t)$ in (1) a double sum since $u(t_n)$ is itself a sum in terms of the Nyquist samples of $u(t)$, and such a double summation would need to be repeated at each iteration of (2). This can be avoided as follows. With the initial iterate $u^{(0)}(t)=0$ (which actually leads to $u^{(1)}(t)=Fx(t)$), it is easy to see from (2) and (1) that $u^{(k)}(t)$ remains in the linear span of $(f_n(t))_{n\in\mathbb{Z}}$. So for each $k\geq 0$, $u^{(k)}(t)$ is of the form

$$u^{(k)}(t)=\sum_{n\in\mathbb{Z}}\mathrm{c}_n^{(k)}f_n(t) \tag{5}$$

for some coefficients $(\mathrm{c}_n^{(k)})_{n\in\mathbb{Z}}$. Suppose we recursively construct these coefficients by

$$\forall n\in\mathbb{Z},\quad \mathrm{c}_n^{(k+1)}:=\mathrm{c}_n^{(k)}+x(t_n)-\sum_{m\in\mathbb{Z}}\mathrm{c}_m^{(k)}f_m(t_n) \tag{6}$$

for $k\geq 0$ starting from $\mathrm{c}_n^{(0)}:=0$. After noticing from (5) that the last term of (6) is $u^{(k)}(t_n)$, we have $\mathrm{c}_n^{(k+1)}:=\mathrm{c}_n^{(k)}+x(t_n)-u^{(k)}(t_n)$. Then, by injecting this into (5) at $k+1$, one retrieves the recursive relation of (2). Aiming at the estimate $u^{(\ell)}(t)$, one then just needs to iterate ℓ times the discrete-time operation of (6) (with only a single summation), before getting the continuous-time function $u^{(\ell)}(t)$ from (5) at $k=\ell$.

III. Ideal bandlimited interpolation

To find functions $(f_n(t))_{n\in\mathbb{Z}}$ that make the operator F of (1) close to I, it is natural to approximate those that make F equal to I, There are two cases where the latter functions are known at least theoretically. We will assume that $\mathcal{B}$ is a space of bandlimited signals of some maximum frequency $\omega_0<\pi$. The Nyquist period of these signals is therefore larger than 1.

A. General nonuniform sampling

A well known theoretical case of ideal bandlimited interpolation is when the instants $(t_n)_{n\in\mathbb{Z}}$ such that $|t_n-n|$ is bounded and $|t_n-t_m|\geq\epsilon>0$ for all distinct $n,m\in\mathbb{Z}$. It was shown in [14] that

$$\forall x(t)\in\mathcal{B},\qquad x(t)=\sum_{n\in\mathbb{Z}}x(t_n)\,g_n(t) \tag{7}$$

where

$$g_n(t)=\frac{G(t)}{G'(t)(t-t_n)}\quad\text{and}\quad G(t):=(t-t_0)\prod_{n\in\mathbb{Z}\setminus\{0\}}\big(1-\frac{t}{t_n}\big)$$

(under the assumption that $t_n\neq 0$ for $n\neq 0$). It can be easily verified that

$$g_n(t_k)=\begin{cases}1, & k=n\\ 0, & k\neq n\end{cases}. \tag{8}$$

An initial attempt would then be to take $f_n(t)=g_n(t)$ to obtain $F=I$. A slight theoretical difficulty is that $g_n(t)$ is not in $\mathcal{B}$ as its maximum frequency is $\pi>\omega_0$. But (7) remains true when replacing $g_n(t)$ by $\tilde{g}_n(t)$, where we denote by $\tilde{u}(t)$ the lowpass filtered version of $u(t)$ at cutoff frequency ω_0. So $F=I$ is rigorously obtained with $f_n(t)=\tilde{g}_n(t)$, However, the functions $(g_n(t))_{n\in\mathbb{Z}}$ are in general not accessible in practice.

B. Ideal interpolation under periodic nonuniform sampling

There is a case where $g_n(t)$ is explicitly known. It is when there exists an integer $N\geq 1$, i.e.,

$$t_{n+N}=t_n+N,\qquad \forall n\in\mathbb{Z}. \tag{9}$$

Qualitatively speaking, the sampling nonuniformity is N-periodic. Assuming that $(t_n)_{n\in\mathbb{Z}}$ is an increasing sequence, the assumptions of the previous section are automatically satisfied in this case. In this situation, the functions $(g_n(t))_{n\in\mathbb{Z}}$ were explicitly derived in [15]. By an adaptation of eq. (18) of this work for $N=2M+1$, $g_n(t)$ can be presented as equal to the functions

$$h_n(t):=\frac{p_n(t)}{p_n(t_n)}\,\mathrm{sinc}\big(\frac{t-t_n}{N}\big) \tag{10}$$
$$\text{where}\quad p_n(t):=\prod_{0<|k|\leq M}\sin\big(\pi\frac{t-t_{n+k}}{N}\big),\qquad n\in\mathbb{Z}$$

and $\mathrm{sinc}(t):=\sin(\pi t)/(\pi t)$. With (9), one easily sees that $h_{n+N}(t)=h_n(t-N)$ for all $n\in\mathbb{Z}$. In the case $N=1$ (or $M=0$), which corresponds to a uniform sampling of period 1, one retrieves the standard solution $h_n(t)=\mathrm{sinc}(t-t_n)$.

IV. Local periodization of sampling nonuniformity

In this section, we describe our proposed construction of functions $(f_n(t))_{n\in\mathbb{Z}}$ with the goal to make the operator F of (1) arbitrarily close to I.

A. Method description and intuition

Assuming the following condition

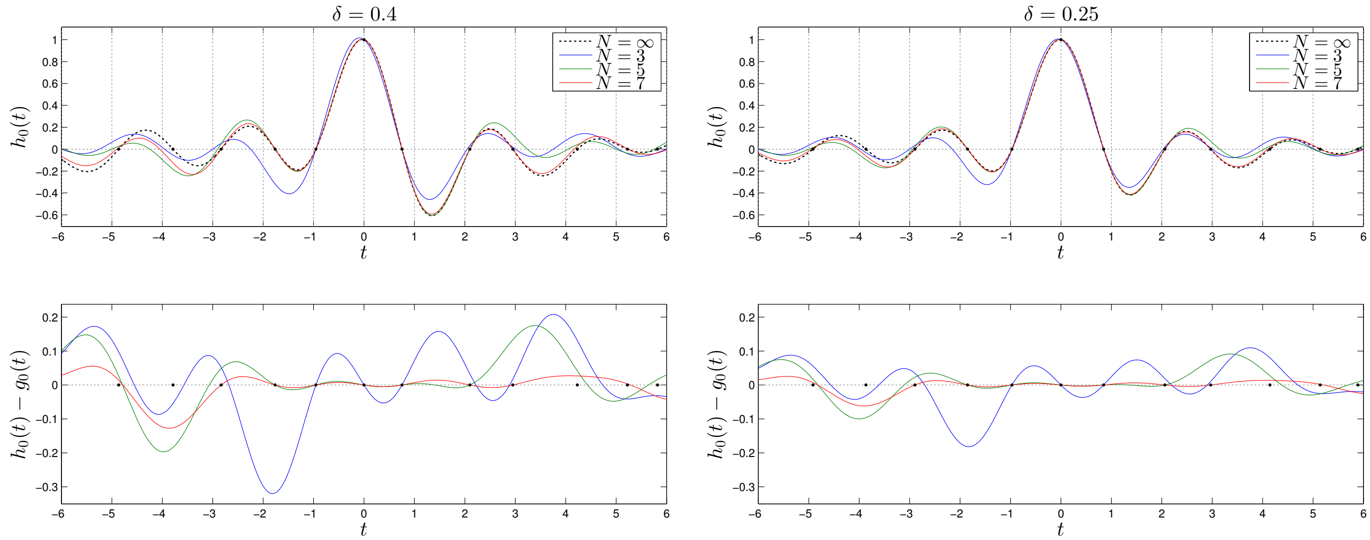
$$t_n=n+\delta_n\quad\text{where}\quad|\delta_n|\leq\delta\quad\text{for all }n\in\mathbb{Z} \tag{11}$$

and some positive constant $\delta<\frac{1}{2}$, we propose to take in (1)

$$f_n(t):=\tilde{h}_n(t),\qquad n\in\mathbb{Z} \tag{12}$$

where $h_n(t)$ is *still* defined by (10) for some choice of $N=2M+1$ in (10), even though the nonuniformity of $(t_n)_{n\in\mathbb{Z}}$ may not be periodic.. We call this estimation scheme the approximation by sliding periodization of the sampling nonuniformity (SPSNU).

Because (9) is not assumed, F has no reason to be identity. But it is intuitive that the larger N is, the closer $h_n(t)$ will be to $g_n(t)$. This can be observed in Fig. 1 where $h_0(t)$ is compared with $g_0(t)$ for some randomly drawn sequence



(a) (b)

Fig. 1. Observation of the function $h_0(t)$ of (10) for given nonuniform instants $(t_n)_{n\in\mathbb{Z}}$ indicated by the time location of the black dots, for various values of N ($h_0(t) = g_0(t)$ with $N = \infty$): (a) $\delta = 0.4$; (b) $\delta = 0.2$.

$(t_n)_{n\in\mathbb{Z}}$ satisfying (11) with $t_0 = 0$, and some various values of N and δ. In the top parts of the figure, $g_0(t)$ is represented by a black dotted line which is labeled as $N = \infty$. The black dots represent the samples of $g_0(t)$ at $(t_n)_{n\in\mathbb{Z}}$, which satisfy (8). By construction, the samples of $h_0(t)$ coincide with those of $g_0(t)$ only for $|n| \leq M$. In the bottom parts, we plot $h_0(t) - g_0(t)$ to highlight the difference between these two functions. Meanwhile, it is also intuitive that for any given N, $h_n(t)$ gets closer to $g_n(t)$ when δ is closer to 0. This is can be observed as well by comparing parts (a) and (b) of Fig. 1 . But an objective confirmation of this intuition will be from the resulting values of $\|I-F\|$ and $\rho(I-F)$, which are evaluated next.

B. Numerical analysis of $I - F$

For the experimental analysis of $I-F$ with (12), we assume that $\mathcal{B}$ is the space of bandlimited signals of Nyquist period 1 (note that this corresponds to $\omega_0 = \pi$) that are periodic of period $L \simeq 300$. This allows us to work in finite dimension without boundary effects. We then form sampling instants of the type (11) where $(\delta_n)_{0\leq n<L}$ is a sequence of random variables that are uniformly distributed in $[-\delta, \delta]$ for various values of δ that are indicated in the figure. For each chosen value of δ, we plot $\|I - F\|$ in Fig. 2(a) in terms of selected values of N in log-log representation. We have chosen values of N that are powers of two minus 1, to obtain odd values that are close to a geometric progression. For each value of N and δ, we have reported in ordinate the quadratic average of $\|I - F\|$ obtained from 100 randomly drawn sequences $(\delta_n)_n$. The figure confirms the decrease of $\|I - F\|$ either with increasing N starting from $N = 3$ for a fixed amplitude δ of nonuniformity, or with decreasing δ for a given N. A possible disappointment is the relatively large resulting values of $\|I-F\|$ and their slow decay with increasing N. In practice however, it may be more relevant to observe the quadratic average of $\|x(t)-Fx(t)\|/\|x(t)\|$ over all $x(t) \in \mathcal{B}$, instead of the worst case. We evaluate this by taking the Frobenius norm of $I-F$ divided by $\sqrt{L}$, which we denote by $\|I-F\|_{\rm ave}$ and is equivalently the quadratic average of the singular values of $I-F$. As can be seen in Fig. 2(b), $\|I-F\|_{\rm ave}$ yields more clearly the sought dependence with N and δ, together with significantly smaller values. We plot in Fig. 2(c) yet another magnitude measure of $I - F$, which is its spectral radius $\rho(I - F)$. In finite dimension, this amounts to measuring the maximum eigenvalue of $I - F$ in magnitude, instead of its maximum singular value as is the case of $\|I - F\|$. While $\rho(I - F)$ yields larger values than $\|I - F\|_{\rm ave}$, this function has some fundamental contribution to approximation, as seen in the next section.

C. Successive approximations

We now consider the iteration of (3) where F is the operator of SPSNU approximation as defined by (12) and (10) for some choice of period N. We show in curves (e)-(i) of Fig. 3 the relative errors $\|u^{(k)}(t)-x(t)\|/\|x(t)\|$ obtained from this iteration under the experimental conditions of Section IV-B for the specific maximum nonuniformity deviation of $\delta = 0.4$. Except for the case $N = 1$, we observe that these errors tend to 0 with increasing k, although $\|I - F\|$ appears to be larger than in 1 in Fig. 2(a) for $\delta = 0.4$ and $N \leq 15$. In fact, $\|I - F\| < 1$ is too strong a condition of convergence. As can be seen in (4), it is sufficient to have $\|(I - F)^k\| < 1$ for some $k \geq 1$. It does appear in Fig. 2(c) that $\rho(I - F) < 1$ for $\delta = 0.4$ and all N, except precisely $N = 1$. The rates of convergence of $\|u^{(k)}(t)-x(t)\|/\|x(t)\|$ to 0 in Fig. 3 is also well correlated to the values of $\rho(I - F)$ in Fig. 2(c) for each N and $\delta = 0.4$.

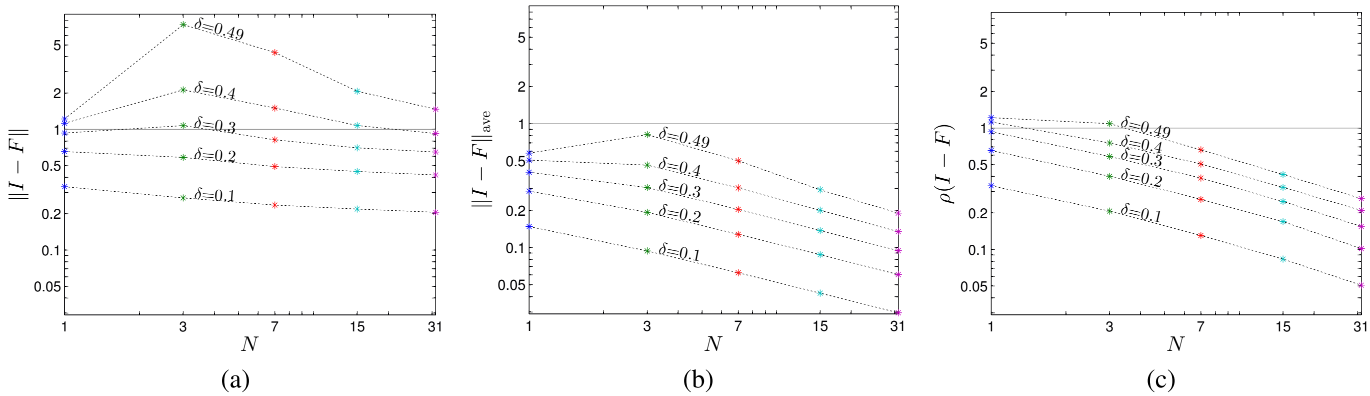


Fig. 2. Log-log plots of $\|I-F\|$, $\|I-F\|_{\text{ave}}$ and $\rho(I-F)$ in terms of N with SPSNU approximation under experimental conditions of Section IV-B.

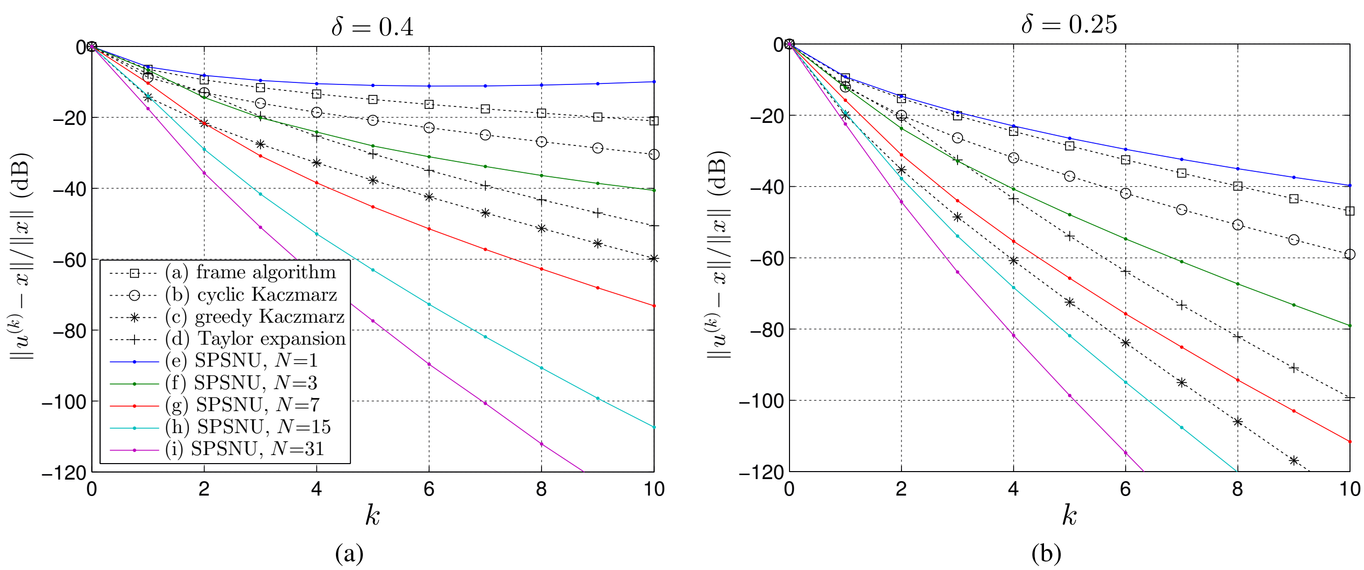


Fig. 3. Relative estimation error of various iterative methods of signal reconstruction methods versus iteration number for two values of δ.

V. Comparison with other iterative methods

For comparison, we plot in curves (a)-(d) of Fig. 3 the results of a number of other methods, which we have selected for their contribution either as historical or theoretical references. The description of these methods is as follows.

(a) Frame algorithm: This is precisely the algorithm introduced in [1, §3]. It follows the iteration of (2) with

$$f_n(t) := \lambda\,\text{sinc}(t-t_n), \qquad n \in \mathbb{Z} \tag{13}$$

where $\lambda := 2/(A+B)$,

$$A := \inf_{\substack{u\in\mathcal{B}\\ \|u\|=1}} \sum_{n\in\mathbb{Z}} |u(t_n)|^2 \quad \text{and} \quad B := \sup_{\substack{u\in\mathcal{B}\\ \|u\|=1}} \sum_{n\in\mathbb{Z}} |u(t_n)|^2.$$

Note that taking $\lambda := 1$ in (13) leads to the SPSNU method with $N = 1$, which was seen to diverge in Fig. 3. With $\lambda := 2/(A+B)$, it was shown in [1] that $\|I-F\| = (B-A)/(A+B)$, which is less than 1 when $A > 0$. A drawback however is that the coefficients A and B are not accessible in practice.

(b) Cyclic Kaczmarz method: This algorithm introduced in [11] is the first practical method of bandlimited interpolation of nonuniform samples, Assuming L sampling instants $t_0, \cdots, t_{L-1}$, it consists in iterating

$$v^{(k+1)}(t) := P_{n_k} v^{(k)}(t), \quad k \geq 0 \tag{14}$$

$$\text{where} \quad P_n u(t) := u(t) + \big(x(t_n) - u(t_n)\big)\,\text{sinc}(t-t_n) \tag{15}$$

and $n_k := k \mod L$. As all samples are visited only after one full cycle of projections, we report in curve (b) of Fig. 3 the error of the estimates $u^{(k)}(t) := v^{(kL)}(t)$. The figure shows how these estimates outperform those of the frame algorithm. Each iteration of (14) has the complexity of one term in the summation of (3). However, the derivation of $u^{(k+1)}(t)$ from $u^{(k)}(t)$ requires L successive transformations of $v^{(\ell)}(t)$ for $\ell = kL, \cdots, kL+L-1$, while it is a direct function of $u^{(k)}(t)$ in the frame algorithm. This implies substantial delays and complications of implementation. Meanwhile, this Kaczmarz method is outperformed in accuracy by the SPSNU method with $N = 3$. Note from (10) that $h_n(t)$ is in this case only a sinc function multiplied by by a product of two sinusoids.

(c) Greedy Kaczmarz method: In this variant of the Kaczmarz method, the indices n_k in (14) are chosen as

$$n_k := \operatorname*{argmax}_{0\leq n<L} \left|x(t_n) - v^{(k)}(t_n)\right|, \qquad k \geq 0.$$

This naturally converges faster than (14) as the nonuniform samples of $u^{(k)}(t)$ with highest errors are treated in priority. The figure shows a drastic improvement over the cyclic variant. Our experiments also show it slightly outperforms the SPSNU method with $N = 5$ (not reported in Fig. 3). However, this greedy variant implies an overhead of computation that makes it unrealistic for real-time processing. It is mostly of interest as a theoretical reference.

(d) Taylor expansion: This method introduced in [9] consists in finding bandlimited estimates $u^{(k)}(t)$ whose kth order Taylor expansion at every instant t_n with respect to $n \in \mathbb{Z}$ tends to $x(t_n)$ when k goes to ∞. After some transformation, the Nyquist samples $u^{(k)}(n)$ of $u^{(k)}(t)$ can be presented as recursively defined by

$$u^{(k+1)}(n) = x(t_n) - \sum_{m\in\mathbb{Z}} u^{(k)}(m)\, f_m^{(k)}(n) \tag{16}$$

where
$$f_m^{(k)}(n) := \sum_{i=1}^{k} \frac{\delta_n^i}{i!}\, d^{(i)}(n-m), \tag{17}$$

$\delta_n := t_n - n$, $d^{(i)}(n)$ is the discrete-time sequence such that

$$\forall u(t) \in \mathcal{B}, \qquad d^{(i)}(n) * u(n) = \frac{\mathrm{d}^i u}{\mathrm{d}t^i}(n), \quad n \in \mathbb{Z}$$

and $*$ denotes discrete-time convolution. It can be seen that (16) is equivalent to

$$u^{(k+1)}(n) = x(t_n) - \sum_{i=1}^{k} \frac{\delta_n^i}{i!} \frac{\mathrm{d}^i u^{(k)}}{\mathrm{d}t^i}(n).$$

Assuming that $u^{(k)}(t)$ tends to a limit $u^{(\infty)}(t)$, this implies at the limit of k towards ∞ that

$$x(t_n) = u^{(\infty)}(n) + \sum_{i=1}^{\infty} \frac{\delta_n^i}{i!} \frac{\mathrm{d}^i u^{(\infty)}}{\mathrm{d}t^i}(n) = u^{(\infty)}(t_n)$$

by Taylor expansion since $t_n = n + \delta_n$. As $u^{(\infty)}(t_n) = x(t_n)$ for all $n \in \mathbb{Z}$, the uniqueness of reconstruction from (7) implies that $u^{(\infty)}(t) = x(t)$. Curve (d) of Fig. 3 not only confirms that $u^{(k)}(t)$ tends to $x(t)$ with increasing k, but it also indicates the decay rate of the estimation error. It outperforms the SPSNU method with $N = 3$. This is however at the price of a tremendous increase of computation with the order k as seen in the sequences $h_m^{(k)}(n)$ of (17). This method is also of interest at least as a theoretical reference.

VI. Summary

The difficulty in nonuniform sampling is the complexity of the nonuniformity for signal reconstruction. It turns out that bandlimited interpolation has an explicit formula when the nonuniformity is periodic [15], the Shannon interpolation formula being the particular case of a period 1. Approximating signal reconstruction by sliding periodization of the sampling nonuniformity enables us to use this explicit formula. At least when the nonuniformity amounts to bounded deviations of a uniform grid of at least Nyquist rate density, this technique leads to approximations of increasing accuracy with increasing periods of periodization. These approximations can also be iterated until perfect reconstruction. The performance of a number of high accuracy iterative methods of bandlimited interpolation can be achieved at substantially lower computation complexities. A crucial task in future research will be to compare these iterative methods with advanced non-iterative methods such as in [10] in terms of total computation complexity for given reconstruction accuracies.

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