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Cooperation in social dilemmas on higher-order networks

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Extended Abstract

The pervasiveness of cooperation has long puzzled researchers [1]. In a Darwinian world driven by self-interest, altruistic behaviours seem counterintuitive, since cooperators bear costs while defectors free-ride. Such situations are known as *social dilemmas*: cooperation benefits the group at individual cost, while defection exploits collective benefits. Thus, although cooperation is socially optimal, defection is individually rational [2]. Evolutionary game theory, particularly the Prisoner's Dilemma (PD), provides a framework to study these dynamics. Structured populations are often modelled as networks, where links capture pairwise interactions. Network properties such as reciprocity, degree heterogeneity, and clustering can promote cooperation [3]. However, networks remain limited, as they only represent pairwise relations, while many real systems involve group interactions of higher order [4].

To address this, we model the population as a hypergraph where nodes are players and hyperedges represent interactions. A payoff tensor of dimension n is assigned to each hyperedge of size n, combining n-body games with the representational power of higher-order networks. We focus on hypergraphs with edges (2-hyperedges) and triangles (3-hyperedges), corresponding to pairwise games (2-games) and three-player games (3-games). Players adopt one of two strategies, cooperation (C) or defection (D), yielding four payoffs for 2-games and six for 3-games. While 2-game payoffs form a 2×2 matrix, 3-games require a $2\times 2\times 2$ cube. The complete payoff structure is shown in Fig. 1(a).

As is customary, mutual cooperation yields the *reward* $R=1$, mutual defection the *penalty* $P=0$, unilateral defection the *temptation* payoff T, and unilateral cooperation the *sucker's* payoff S. The values of T and S classify pairwise games into four types with distinct Nash Equilibria: PD, Chicken, Stag Hunt, and Harmony. Extending to 3-games introduces two additional payoffs: W (defection against a cooperator and a defector) and G (cooperation against a cooperator and a defector), which capture effects absent in pairwise settings.

We analyse a well-mixed population where each interaction is a 3-game with probability δ and a 2-game with probability $1-\delta$. The fraction ρ of cooperators evolves under the Replicator Equation (RE): $\dot{\rho}=\rho(1-\rho)\left[\pi_C-\pi_D\right]$, where π_C and π_D are expected payoffs of cooperators and defectors. Besides the absorbing states $\rho_D^*=0$ and $\rho_C^*=1$, the RE admits two non-trivial stationary states $\rho_\pm^*$:

$$\rho_\pm^* = \frac{c\delta - b - 2S \pm \sqrt{(c\delta-b)^2+4S(b+S)}}{2c\delta}, \tag{1}$$

with $a=2(G-W)$, $b=T-S-1$, $c=a+b$. Real solutions exist when

$$\delta \geq \delta_1^{\text{th}} = \frac{b+\sqrt{-4S(b+S)}}{c}. \tag{2}$$

Stability analysis shows that $\rho_D^*=0$ and ρ_+^* are stable, while ρ_-^* and $\rho_C^*=1$ are unstable. Thus, if $\delta<\delta_1^{\text{th}}$, defection dominates as in the pairwise PD. If $\delta>\delta_1^{\text{th}}$, an explosive transition occurs to a bistable regime where both $\rho_D^* = 0$ and $0 < \rho_+^* < 1$ are stable. Fig. 1(b) confirms analytical predictions with numerical simulations. Temporal dynamics (Fig. 1(c)) further reveal that, beyond the threshold of higher-order interactions, a critical initial fraction of cooperators is also required to reach majority cooperation.

Our main result shows that cooperation can persist even in the PD, where pairwise interactions alone lead to full defection. Higher-order interactions enable an explosive transition to stable cooperation once their fraction exceeds a threshold. However, bistability implies that cooperation survives only if a critical mass of cooperators is initially present. We also find that larger overlap between interactions of different orders and multi-dimensional strategies further promote cooperation. Overall, higher-order interactions provide a new pathway for sustaining cooperation in competitive settings, offering a potential resolution to social dilemmas.

References

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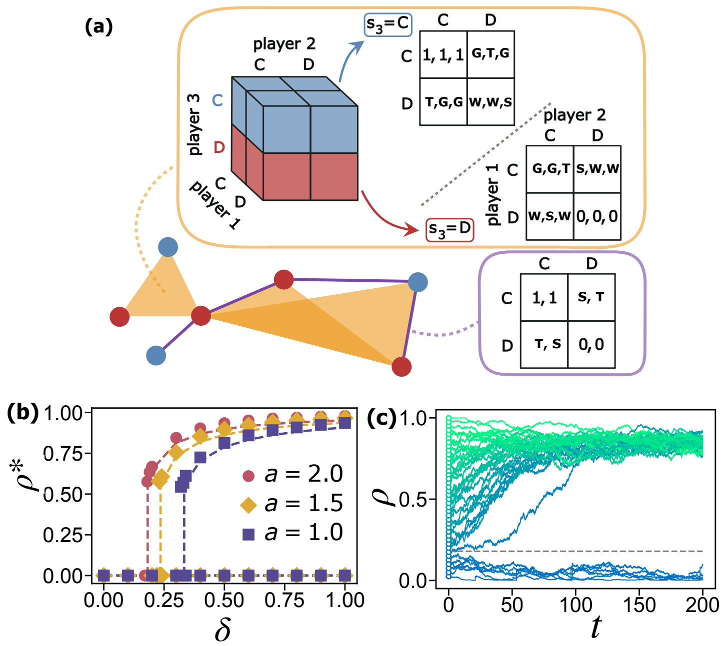

Figure 1: **(a)** Hypergraph with 2- and 3-player games and their payoffs. **(b)** Equilibrium cooperator fraction in the PD as a function of δ, the share of 3-player interactions, for different a, with simulations (symbols) matching analytics (dotted). **(c)** Temporal evolution of cooperators showing the critical initial mass for cooperation.