# **Eigenspectrum Analysis of Neural Networks without Aspect Ratio Bias**

Yuanzhe Hu<sup>1</sup> Kinshuk Goel<sup>23</sup> Vlad Killiakov<sup>4</sup> Yaoqing Yang<sup>2</sup>

# Abstract

Diagnosing deep neural networks (DNNs) through the eigenspectrum of weight matrices has been an active area of research in recent years. At a high level, eigenspectrum analysis of DNNs involves measuring the heavytailness of the empirical spectral densities (ESD) of weight matrices. It provides insight into how well a model is trained and can guide decisions on assigning better layer-wise training hyperparameters. In this paper, we address a challenge associated with such eigenspectrum methods: the impact of the aspect ratio of weight matrices on estimated heavytailness metrics. We demonstrate that matrices of varying sizes (and aspect ratios) introduce a nonnegligible bias in estimating heavytailness metrics, leading to inaccurate model diagnosis and layer-wise hyperparameter assignment. To overcome this challenge, we propose FARMS (Fixed-Aspect-Ratio Matrix Subsampling), a method that normalizes the weight matrices by subsampling submatrices with a fixed aspect ratio. Instead of measuring the heavytailness of the original ESD, we measure the average ESD of these subsampled submatrices. We show that measuring the heavytailness of these submatrices with the fixed aspect ratio can effectively mitigate the aspect ratio bias. We validate our approach across various optimization techniques and application domains that involve eigenspectrum analysis of weights, including image classification in computer vision (CV) models, scientific machine learning (SciML) model training, and large language model (LLM) pruning. Our results show that despite its simplicity, FARMS uniformly improves the accuracy

of eigenspectrum analysis while enabling more effective layer-wise hyperparameter assignment in these application domains. In one of the LLM pruning experiments, FARMS reduces the perplexity of the LLaMA-7B model by 17.3% when compared with the state-of-the-art method.

## 1. Introduction

Random Matrix Theory (RMT) has long been used as a foundational tool in multiple research domains (Guhr et al., 1998; Tulino & Verdú, 2004; Bai & Silverstein, 2010; Tao, 2012; Nadakuditi & Newman, 2012) and has now been applied to providing precise characterizations of neural networks (NNs) (Pennington & Worah, 2017; Dobriban & Wager, 2018; Adlam & Pennington, 2020; Mei & Montanari, 2022; Hastie et al., 2022; Hu & Lu, 2022; Ba et al., 2022; Couillet & Liao, 2022; Derezinski et al., 2020). Among several emerging topics in this field, Heavy-Tailed Self-Regularization (HT-SR) (Martin & Mahoney, 2021; Mahoney & Martin, 2019; Martin & Mahoney, 2019; Martin et al., 2021) Theory has been gaining significant attention. Unlike conventional RMT approaches, HT-SR Theory focuses on studying weight matrices of DNNs with strongly correlated elements, a typical characteristic of well-trained, practical DNNs. The essence of HT-SR theory is that the weight matrices of well-trained DNNs are strongly correlated, and that leads to heavy-tailed (HTed) ESDs (Martin & Mahoney, 2021). Analyzing these HT ESDs provides valuable insights into the quality of trained models, which provides methods and techniques for model diagnostics and model training. For example, Martin & Mahoney (2021) show that one can locate undertrained layers by finding weight matrices with less HTed ESDs, which provides useful metrics for model ranking, comparison and selection (Martin et al., 2021; Yang et al., 2023). Furthermore, one can use a larger learning rate on the undertrained layers (Zhou et al., 2024), which provides an empirically successful training method to balance these layers with other, more well-trained ones. This technique has been developed into a general approach to improve test accuracy, training efficiency, and model interpretability in various applications, such as image classification (Zhou et al., 2024), LLM model pruning (Lu et al., 2024), and SciML model training and fine-tuning (Liu

<sup>&</sup>lt;sup>1</sup>Department of Computer Science and Engineering, University of California, San Diego, CA, USA <sup>2</sup>Department of Computer Science, Dartmouth College, NH, USA <sup>3</sup>Department of Computer Science & Engineering, SRM Institute of Science & Technology, India <sup>4</sup>Independent Researcher, University of California, Berkeley, CA, USA. Correspondence to: Yuanzhe Hu <yuanzhe.hu@dartmouth.edu>.

Proceedings of the  $42^{nd}$  International Conference on Machine Learning, Vancouver, Canada. PMLR 267, 2025. Copyright 2025 by the author(s).



Figure 1. Comparing FARMS with common HT-SR methods in analyzing the weight matrices of a DNN. We use the weight matrix  $\mathbf{W}_i$  of a well-trained layer with a large aspect ratio as an example (i.e.,  $\gamma = m/n$  is much larger than 1). Due to the influence of the aspect ratio  $\gamma = m/n \gg 1$ , the ESD is more concentrated and less HTed than other layers. As a result, previous methods for fitting a power-law distribution (shown as "PL Fitting") tend to overestimate the heavytailness metric PL\_Alpha\_Hill and incorrectly suggest that the layer is poorly trained. Due to this aspect ratio bias, some optimization methods (such as TempBalance) cannot accurately assign per-layer hyperparameters, leading to suboptimal training. In contrast, our method, FARMS, which conducts ESD analysis with a fixed aspect ratio, accurately measures the training quality of the layer. See a concrete numerical example in Figure 7 in Appendix A.

#### et al., 2024; 2025b).

The core of the HT-SR theory lies in the measurement of the heavytailness of ESDs. In this context, we identified an often-overlooked issue in prior studies: weight matrices of different sizes are analyzed using the same HT measuring procedure, even though their spectral distributions can differ in theory. Specifically, algorithms inspired by the HT-SR theory often begin by measuring the ESD of a weight matrix W, that is, the empirical distribution of eigenvalues of the correlation matrix  $\mathbf{W}^{\top}\mathbf{W}$ . The ESD will then be analyzed and used for downstream applications such as model selection, training, and compression (Mahoney & Martin, 2019; Martin & Mahoney, 2019; 2021; Zhou et al., 2024; Lu et al., 2024; Liu et al., 2024). For weight matrices initialized using the i.i.d. random Gaussian elements, it is well-known that the ESD converges to the Marchenko-Pastur (MP) distribution as the dimensions of the matrix grow infinitely large in the proportional limit, i.e., when the number of rows m and the number of columns n increase but the ratio m/n stays fixed. The MP distribution depends explicitly on the aspect ratio  $\gamma = m/n$ , since this aspect ratio determines the limiting bulk range,  $[(1 - \sqrt{\gamma})^2, (1 + \sqrt{\gamma})^2]$ , and it influences the overall shape of the ESD. As a result, weight matrices of different aspect ratios exhibit distinct shapes in their ESDs, especially when they transition from the i.i.d. initialization to being correlated. Comparing ESDs with different aspect ratios  $\gamma$  directly, as is commonly done in prior work, overlooks this dependency on aspect ratio and can lead to inaccurate conclusions. Therefore, a careful consideration of the aspect ratio is critical when analyzing or comparing weight matrices, especially in eigenspectrum analysis.

Considering these challenges, we introduce FARMS (Fixed-Aspect-Ratio Matrix Subsampling), a new method for measuring HT characteristics in HT-SR weight analysis. FARMS analyzes the training quality of layers by partitioning each weight matrix into (overlapping) submatrices of fixed aspect ratios and then do an eigenspectrum analysis on the average ESD of these submatrices. This approach enables accurate computation of HT-SR metrics regardless of variations in matrix aspect ratios, ensuring a robust evaluation of layer quality across diverse matrix sizes that one can have in a DNN.

To validate the effectiveness of FARMS, we conduct experiments across various application domains, including CV, SciML, and LLM pruning. Additionally, these experiments are conducted across different parameter settings and model architectures. We compare FARMS with several prior HT-SR approaches that use weight eigenspectrum analysis for hyperparameter scheduling (Zhou et al., 2024; Lu et al., 2024; Liu et al., 2024). We also do weight analysis to measure various post-training and pruned models, making FARMS useful for efficient, compressed models. Our findings demonstrate that models optimized using FARMS exhibit lower mean and variation in HT-SR metrics across layers, a sign of goodquality training as reported in prior work (Martin et al., 2021; Liu et al., 2024). Our code is available here<sup>1</sup>. Our key contributions are summarized below.

- Mitigating Aspect Ratio Bias in Eigenspectrum Analysis. FARMS addresses the aspect ratio bias of existing HT-SR eigenspectrum analysis, and it enables better computation of HT metrics on various DNN models. This improvement is achieved through a subsampling-based HT estimation method independent of the aspect ratio of weight matrices. In particular, we use a numerical example in Figure 6 and multiple real-data experiments in Section 4.3 to demonstrate that FARMS mitigates the aspect ratio bias in training.
- Improved Layer-wise Hyperparameter Tuning. Since HT-SR Theory has been recently applied to various layer-wise hyperparameter tuning methods, FARMS thus improves these methods by offering a more accurate evaluation of HT metrics. In particular, we conduct experiments on DNN training and pruning across various model architectures. In CV models like ResNet and VGG, integrating FARMS into learning rate assignments yields improved accuracy compared to TempBalance, a recently proposed layer-wise learning rate scheduling method (Zhou et al., 2024). In LLMs pruning, FARMS improves AlphaPruning (Lu et al., 2024), the SOTA method in assigning layer-wise pruning ratios. Specifically, FARMS can reduce the perplexity of the LLaMA-13B model from 2029.20 to 413.76 using the magnitude pruning method at a 0.7 sparsity ratio. As another example in LLaMA-7B, it reduces the perplexity from 96.02 to 79.42 using the SparseGPT pruning method (Frantar & Alistarh, 2023) at a 0.8 sparsity ratio. In SciML, FARMS helps scientific models achieve up to a 5.66% error reduction during fine-tuning compared to the HT-SR based method TB\_Sigmoid (Zhou et al., 2024), even at relatively low L2 relative error (L2RE) levels.

# 2. Related Work

### 2.1. Prior Work on HT-SR Theory

In this section, we provide an overview of prior work on HT-SR theory, a framework derived from RMT and statistical physics that is relevant to understanding modern, practical DNNs. HT-SR theory (Martin & Mahoney, 2021; 2019) originates from RMT but extends well beyond it. The theory was proposed based on the observation that welltrained, state-of-the-art DNNs often exhibit HTed structures in the ESD of each layer. In the meantime, several rigorous theories in SGD relating HT phenomena to generalization performance were established, providing further theoretical support for HT-SR theory (Hodgkinson & Mahoney, 2021; Hodgkinson et al., 2022; Gurbuzbalaban et al., 2021; Simsekli et al., 2019; 2020). Building on the theoretical foundation of HT-SR, a model analysis tool called Weight-Watcher (Martin & Mahoney, 2021) was developed. Without accessing any training or test data, methods based on HT-SR Theory can be used to assess the performance of models across various domains, such as CV and NLP (Martin & Mahoney, 2021; Yang et al., 2023; Liu et al., 2025b).

### 2.2. Optimization Methods Based on Eigenspectrum Analysis

Our paper is mainly motivated by HT-SR theory, focusing on the eigenspectrum analysis of weight matrices. Our paper connects to several DNN optimization methods that use eigenspectrum analysis to improve model performance. For example, spectral norm regularization has been applied to improve generalizability of trained models (Yoshida & Miyato, 2017; Miyato et al., 2018; Farnia et al., 2018). Stable rank normalization (SRN) (Sanyal et al., 2019), a normalization technique, scales matrices using their stable rank to enhance training stability in GANs and generalization in DNNs. TE-NAS (Chen et al., 2021) is a training-free neural architecture search framework that identifies highperforming networks by analyzing the neural tangent kernel spectrum and input space linear regions.

Although the aforementioned optimization methods based on eigenspectrum analysis have achieved a certain degree of model improvement, they do not provide fine-grained layer-wise optimization within DNNs. However, eigenspectrum analysis based on HT-SR theory offers a novel perspective by analyzing the shape of the HT ESDs, enabling a more precise estimate of the training quality for each layer. TempBalance (Zhou et al., 2024) performs eigenspectrum analysis on ESDs of the weight matrices of DNNs to assess the training progress of each layer. It enables a balanced adjustment of the learning rate across layers during training-an important parameter that functions as a "temperature-like" parameter within the language of statistical mechanics of learning. AlphaPruning (Lu et al., 2024), on the other hand, uses a similar weight analysis method to evaluate the training quality of each layer in LLMs. It then uses this information to strategically allocate layer-wise sparsity ratios to maintain model performance after pruning.

<sup>&</sup>lt;sup>1</sup>https://github.com/HUST-AI-HYZ/FARMS

# 3. Method

In this section, we first discuss the motivation behind our method FARMS, which is to address the aspect ratio bias of typical HT-SR methods. Then, we describe FARMS in detail and show how FARMS can reduce the aspect ratio bias.

### 3.1. Typical HT-SR Analysis

Before introducing FARMS, we first review the commonly adopted procedures in HT-SR weight analysis. HT-SR analysis relies on estimating the layer quality based on the HT characteristic of the layer ESDs, which is quantified by HT metric PL\_Alpha. The ESD of a weight matrix is the histogram of eigenvalues. The ESD of weight matrices evolves during training, transitioning from a bulk-dominated regime to an HTed regime (Martin et al., 2021). The HTed portion can be modeled by a power-law (PL) distribution within an interval ( $\lambda_{\min}$ ,  $\lambda_{\max}$ ):

$$p(\lambda) \propto \lambda^{-\alpha}, \lambda_{min} < \lambda < \lambda_{max},$$
 (1)

where  $\alpha$ , the PL exponent, is a critical metric for analyzing training quality. To fit a PL distribution to the ESD, methods in HT-SR often use the Hill Estimator (Hill, 1975; Zhou et al., 2024; Lu et al., 2024; Liu et al., 2024)<sup>2</sup>. For the *i*-th layer, suppose the weight matrix is  $\mathbf{W}_i$  and the correlation matrix  $\mathbf{W}_i^{\mathsf{T}}\mathbf{W}_i$  has ascending eigenvalues  $\{\lambda_i\}_{i=1}^n$ . The Hill estimator calculates PL\_Alpha\_Hill as:

$$PL\_Alpha\_Hill = 1 + \frac{k}{\left(\sum_{i=1}^{k} ln \frac{\lambda_{n-i+1}}{\lambda_{n-k}}\right)}, \quad (2)$$

where k is an adjustable parameter. Changing k essentially changes the lower eigenvalue threshold  $\lambda_{\min}$  for (truncated) PL estimation. Various metrics have been proposed to analyze the properties of ESDs, among which shape metrics—those that characterize the distributional shape of ESDs—have been shown to effectively predict the training quality of individual layers. The PL\_Alpha\_Hill metric is one such shape metrics.

There are several metrics used to quantify the structure of ESDs in HT-SR theory. In this work, we mainly consider the PL\_Alpha\_Hill, which is empirically shown to be effective for training tasks (Zhou et al., 2024; Lu et al., 2024; Liu et al., 2024; Qing et al., 2024). In general, undertrained layers in DNNs tend to exhibit larger PL\_Alpha\_Hill values, whereas well-trained or over-trained layers typically have smaller PL\_Alpha\_Hill values. This metric can

also be used for a comprehensive analysis across a series of DNN models with similar architectures to find the best model. Well-trained models tend to exhibit a lower average PL\_Alpha\_Hill across all layers. Moreover, in finetuning tasks, a larger amount of data generally results in a lower standard deviation (STD) of PL\_Alpha\_Hill across the model, indicating a more balanced training progression across different layers (Liu et al., 2024).

#### **3.2. Rationale of FARMS: Why Typical HT-SR Methods** Are Insufficient

Here, we explain the rationale behind our method FARMS. In Section 3.1, we mentioned that layers exhibiting more HTed ESDs tend to be more well-trained. This observation is the key to using HT-SR methods to measure model quality. However, beyond the training quality of weights, the aspect ratio of a weight matrix also influences the heavytailness of its ESD. Specifically, in RMT, the Marchenko-Pastur (MP) distribution describes the limiting behavior of singular values in large rectangular random matrices. According to the MP distribution, the ESD of the correlation matrix  $W^{+}W$ of an i.i.d. Gaussian random matrix  $\mathbf{W}_{m \times n}$  becomes more concentrated as the aspect ratio m/n deviates from 1. An example is presented in Figure 2. Consequently, weight matrices with different aspect ratios naturally exhibit varying ESD shapes, which can interfere with the quality estimation of model layers. We refer to this issue as the aspect ratio bias in HT-SR methods.



Figure 2. ESD shapes can be biased by the aspect ratio m/n. The left ESD is more HTed, while the right ESD is more concentrated. However, this change in the shape of the ESD is entirely due to the different aspect ratios of these two matrices and is not caused by differences in training quality. See detailed settings and supplementary results in Appendix B Figure 8.

As another example, consider a well-trained layer with a large aspect ratio, such as the final layer of a ResNet 18 model trained on the CIFAR 100 dataset. This weight matrix has size  $512 \times 100$  and a large aspect ratio of 512/100. Thus, the ESD may not exhibit an obvious enough HT structure (See a related example in Figure 7 in Appendix A). If previous HT-SR methods are used, such layers may be quantified as poorly trained, but that conclusion results from the aspect ratio bias and is an inaccurate assessments of their

<sup>&</sup>lt;sup>2</sup>One important point to note is that estimating PLs is inherently a challenging task (Clauset et al., 2009). Previous research suggests using the maximum likelihood estimate (Alstott et al., 2014; Martin & Mahoney, 2021). However, empirical evidence indicates that the Hill estimator performs more reliably in DNN optimization applications (Zhou et al., 2024; Lu et al., 2024; Liu et al., 2024).

training quality. Such misjudgment not only interferes with the evaluation of model performance but also leads to inaccurate tuning of hyperparameters in layer-wise optimization methods (Zhou et al., 2024; Lu et al., 2024), as these methods all assign different layer-wise hyperparameters based on the HT estimates. Therefore, eliminating such aspect ratio bias is crucial for improving HT-SR eigenspectrum analysis.

#### 3.3. Analyzing ESDs Using FARMS

To mitigate the aspect ratio bias in analyzing ESDs, we use a block-wise sampling method when processing weight matrices. We partition each weight matrix into overlapping sub-matrices with a fixed aspect ratio (across all layers) following a predefined scheme described below.

Consider the weight matrix  $\mathbf{W}_i$  of the *i*-th layer, which has a shape of  $m \times n$ . Without loss of generality, consider the case when  $m \ge n$ . Note that the shape  $m \times n$  changes across layers. To process this weight matrix, we apply a sliding window approach to partition it into several equally sized submatrices (potentially with overlap), denoted as  $\mathbf{W}_{i1}, \mathbf{W}_{i2}, \ldots, \mathbf{W}_{il}$ , where *l* represents the number of submatrices. Each submatrix has a shape of  $m' \times n'$  and satisfies a fixed aspect ratio Q = m'/n'. The parameters m', n', l, and Q are tunable hyperparameters (on which we will provide ablation studies in Section 4.6), allowing flexibility in the partitioning strategy. The key requirement is that even if the aspect ratio of the whole weight matrix m/n changes for different layer index *i*, that of the submatrices Q = m'/n'is fixed across all layers.

Subsequently, for 2D Linear layers we compute the eigenvalues of the correlation matrices  $\mathbf{X}_{ij} = \mathbf{W}_{ij}^{\top} \mathbf{W}_{ij}$  for each submatrix in the *i*-th layer. We then average the ESDs of  $\mathbf{W}_{i1}, \mathbf{W}_{i2}, \ldots, \mathbf{W}_{il}$  (by concatenating their corresponding eigenvalue series) and measure the HT metrics of the averaged ESD instead. We concatenate the eigenvalue series because that's equivalent to averaging the empirical densities. The average of the ESDs leads to a less variant estimate of HT characteristics.

For CNN layers, since they have 4D tensor weight matrices with four dimensions  $[C_1, C_2, k_H, k_W]$  (which represent input channels, output channels, height, and width, respectively), we employ a slightly different method for subsampling and measuring. We first flatten the two dimensions representing the convolution kernel size,  $[k_H, k_W]$ , in the 4D tensor into a single dimension. This results in  $l' = k_H \times k_W$  two-dimensional matrices of shape  $[C_1, C_2]$ . We assume here that  $C_1 \ge C_2$  for convenience. For a 3D tensor of shape  $l' \times C_1 \times C_2$ , we perform subsampling of size  $m' \times n'$  on each  $C_1 \times C_2$  matrix corresponding to each l'. This results in  $[\frac{C_1}{m'}] \times [\frac{C_2}{n'}] \times l'$  submatrices, each of size  $m' \times n'$ . Next, we average the ESDs by concatenating the

rooted singular values for each submatrix along the l' dimension, and then measure the HT metrics of the ESDs. In this way, we obtain  $\left[\frac{C_1}{m'}\right] \times \left[\frac{C_2}{n'}\right]$  HT metrics values. Finally, we take the average of these metrics values to obtain the HT metrics corresponding to the 2D CNN. We also considered concatenating all the rooted singular values into a single list and calculating the corresponding ESD to measure the degree of heavy-tailedness. However, our experimental results in Table 7 in Appendix C.2 show that averaging the results separately leads to better model performance. The detailed procedures of FARMS are shown in Figure 1 and Figure 3.



Figure 3. Main Steps in FARMS.

# 4. Empirical Results

In this section, we apply FARMS to measure HT metrics. To demonstrate the effectiveness of the new approach, we use these metrics across various optimization methods and tasks from different machine learning subfields.

In Section 4.1, we give full details of the experimental setup. In Section 4.2, we applied FARMS to LLM layerwise pruning. In Section 4.3, we employ FARMS on training VGGs and ResNets on image classification tasks. In Section 4.4, we validate that FARMS achieves better performance in SciML fine-tuning experiments. In Section 4.5, we apply FARMS to analyze model quality, such as the balance of different layers. Finally, we perform ablation studies in Section 4.6.

#### 4.1. Experimental Setup

**Datasets.** For image classification, we consider the CI-FAR100 dataset (Krizhevsky et al., 2009). CIFAR100 consists of 50K pictures for the training set and 10K pictures for the testing set with 100 categories. For evaluating LLM pruning methods, we calculate model perplex-

Table 1. Wiki fext validation perplexity for pruned LLaMA-7B and LLaMA-13B models at different sparsity settings. Our method is
compared to AlphaPruning, each paired with magnitude based pruning, Wanda, and SparseGPT. Lower perplexity indicates improved
model performance. For calculating the standard deviation(STD) and demonstrating the stability of our method, we sample different
calibration sets with 128 samples using six different seeds [0, 1, 2, 3, 4, 5] in Wanda and SparseGPT.

amendal LaMA 7D and LLAMA 12D and data at different an arithmetic setting a form weather different

Sparsity Ratio	Layer-wise	LLaMA-7B					
	Method	Magnitude	Wanda	SparseGPT	Magnitude	Wanda	SparseGPT
0.7	AlphaPruning	231.76	$24.30 {\pm}~0.25$	$18.66 {\pm}~ 0.49$	2029.20	$14.47 {\pm}~0.08$	$13.29 {\pm}~ 0.17$
	Ours	173.49	$22.61 {\pm}~0.18$	$18.53 {\pm}~0.40$	413.76	$14.20 \pm 0.09$	$13.06 \pm 0.14$
0.75	AlphaPruning	2046.22	$104.53 \pm 4.49$	$36.52{\scriptstyle\pm1.13}$	2710.49	$32.18 \pm 0.31$	$22.26{\scriptstyle\pm0.59}$
	Ours	1704.56	$71.67 \pm 1.71$	$35.47{\scriptstyle\pm1.01}$	2634.82	29.56±0.33	$20.80{\scriptstyle\pm0.43}$
0.8	AlphaPruning	28865.67	772.20±78.70	96.02±1.59	5399.87	$160.59{\scriptstyle\pm4.05}$	$47.57{\scriptstyle\pm2.64}$
	Ours	12799.58	$504.58{\scriptstyle\pm23.05}$	$79.42{\scriptstyle\pm3.86}$	5026.86	$127.49{\scriptstyle\pm2.12}$	$41.44{\scriptstyle \pm 1.58}$
0.85	AlphaPruning	71710.96	4609.70±978.39	$272.84 \pm 30.84$	38140.95	3144.01±597.79	122.38±8.88
	Ours	66808.51	$3595.54{\scriptstyle\pm810.54}$	$234.46{\scriptstyle\pm16.42}$	37453.06	$2847.85{\scriptstyle \pm 368.10}$	$101.06{\scriptstyle\pm4.48}$

ity on the held-out WikiText (Merity et al., 2016) validation set and use seven tasks, including BoolQ (Clark et al., 2019), RTE (Wang, 2018), HellaSwag (Zellers et al., 2019), WinoGrande (Sakaguchi et al., 2021), ARC Easy and Challenge (Clark et al., 2018) and OpenbookQA (Mihaylov et al., 2018) for downstream zero-shot evaluation (Gao et al., 2021). For SciML, we fine-tune the models on simulated solutions of time-dependent PDE dataset 2D Compressible Navier-Stokes (CFD) <sup>3</sup> from PDEBench (Takamoto et al., 2022). All datasets considered in this paper are standard and widely studied.

**Models.** We consider different types of NNs in various research fields: VGG (Simonyan & Zisserman, 2014), ResNet (He et al., 2016), OPT (Zhang et al., 2022), LLaMA (Touvron et al., 2023a;b; Grattafiori et al., 2024) and DPOT (Hao et al., 2024). For VGG series, we consider VGG16 and VGG19. For ResNet series, we consider ResNet18 and ResNet34. For OPT series, we consider four different model size: OPT-125M/350M/1.3B/6.7B. For LLaMA series, we consider six different models: LLaMA-7B/13B, LLaMA-V2-7B/13B and LLaMA-V3-3B/8B. For DPOT series, we consider DPOT-Tiny and DPOT-Small.

**Baseline.** We considered several model diagnostic and layerwise hyperparameter scheduling methods as baselines for comparison. TempBalance (Zhou et al., 2024) is a layerwise learning rate allocation algorithm based on HT-SR theory, designed to analyze the training progress of each layer and allocate learning rates accordingly. TempBalance measures the HT metrics of all layers, and it assigns a larger learning rate to weight matrices with a more lighted-tailed ESD. AlphaPruning (Lu et al., 2024), on the other hand, is a layer-wise pruning method for LLMs based on HT-SR. It assigns a larger pruning ratio to Transformer weight matrices with a more lighted-tailed ESD. Additionally, we compared our method with the TB\_Sigmoid (Liu et al., 2024) approach, which was applied to experiments on SciML models. All three optimization methods adopt ESD analysis techniques, which are susceptible to aspect ratio bias because the HT metrics are measured on the whole weight matrix. Therefore, one can replace the HT measurement procedures in these methods with FARMS. The goal is to evaluate the improvement of FARMS compared to the existing HT-SR analysis used in these optimization methods.

### 4.2. Improving LLM Pruning with FARMS

To explore the effectiveness of FARMS, we conduct experiments in the field of LLM pruning. As we have mentioned, we can replace the HT measurement procedures in AlphaPruning with FARMS. Therefore, we can compare it with FARMS and without. We make the comparison with different sparsity ratios in the range {0.7, 0.75, 0.8, 0.85} and different LLM pruning methods, including Magnitudebased (Han et al., 2015), SparseGPT (Frantar & Alistarh, 2023) and Wanda (Sun et al., 2023). We use different LLM pruning methods because AlphaPruning assigns layerwise ratios but not precise pruning locations. We followed the experimental setup from Lu et al. (2024) and our hyperparameter ranges are reported in Table 14 of Appendix F.

Language Modeling. The results in Table 1 illustrate that using FARMS helps AlphaPruning achieve better performance under different sparsity ratios and LLM pruning methods. Specifically, when using the Magnitude method, our approach reduces the perplexity of LLaMA-7B from 231.76 to 173.49 at a sparsity ratio of 0.7. Similarly, when employing more advanced pruning methods such as Wanda and SparseGPT, our approach further reduces the perplexity of LLaMA-7B from 96.02 to 79.42 and LLaMA-13B from

<sup>&</sup>lt;sup>3</sup>CFD means compressible fluid dynamics or, equivalently, the compressible Navier-Stokes equations.

Sparsity Ratio	Layer-wise	LLaMA-7B			LLaMA-13B			
	Method	Magnitude	Wanda	SparseGPT	Magnitude	Wanda	SparseGPT	
0.7	AlphaPruning	35.67	43.67	44.79	38.23	47.46	49.07	
	Ours	35.96	44.26	44.84	38.87	47.75	49.61	
0.75	AlphaPruning	34.59	37.99	40.89	38.16	42.73	44.17	
	Ours	35.88	39.66	40.93	37.68	42.66	44.47	
0.8	AlphaPruning	33.79	34.06	36.63	35.59	38.42	39.07	
	Ours	34.33	35.76	37.50	36.94	39.23	40.18	
0.85	AlphaPruning	33.29	31.69	34.86	33.11	32.05	36.73	
	Ours	34.27	33.09	35.25	33.29	32.41	37.04	

Table 2. Comparison of mean zero-shot accuracies (%) for pruned LLaMA-7B and LLaMA-13B models at different sparsity settings. We evaluate our method against AlphaPruning, each integrated with magnitude-based pruning, Wanda, and SparseGPT. Higher accuracy values indicate better zero-shot ability.

47.57 to 41.44 at a sparsity ratio of 0.8. One notable advantage of AlphaPruning is that it allows pruning LLMs to a sparsity ratio of 0.8 without significantly impacting perplexity. With FARMS, this performance can be further improved. We can also find that FARMS achieves lower STD in most settings. In Table 8 and Table 9 in Appendix C, we provide additional experiment results on OPT series models and more recent LLaMA models.

Zero-Shot Tasks. We test the zero-shot ability of pruned LLMs on seven zero-shot downstream tasks mentioned in Section 4.1 with prompting. Results are shown in Table 2, where we report the mean zero-shot accuracy on seven zero-shot tasks of pruned LLaMA-7B and LLaMA-13B models. FARMS can outperform AlphaPruning in improving accuracy across most settings. For example, although AlphaPruning achieved significant improvements over the uniform method, FARMS further improved accuracy by values of 1.11% in SparseGPT pruning over AlphaPruning at a sparsity ratio of 0.8. We report the detailed performance for each individual task in Table 10 and Table 11 in Appendix C.

#### 4.3. Improving Image Classification with FARMS

In this subsection, we compare FARMS to TempBalance on image classification tasks. As we mentioned, TempBalance is a layer-wise learning rate assignment method using HT metrics. In Figure 4, we report the evaluation results of training the ResNet and VGG series models under different optimizing settings. Again, we compare TempBalance with or without FARMS. "Ours" means replacing the weight analysis method previously employed in the TempBalance with FARMS.

An important configuration when using the TempBalance method is the exclusion of certain layers from the learning rate assignment process. Instead of receiving layerwise learning rate adjustments, these layers are assigned a global learning rate that decays according to cosine annealing. These excluded layers have "tall-and-skinny matrices," which are matrices with a large aspect ratio and a relatively small number of eigenvalues. For example, the final layer of the ResNet 18 model has dimensions of  $512 \times 100$ , resulting in an aspect ratio of 512/100. This kind of extreme size makes it difficult for previous methods to accurately estimate the heavytailness of the ESDs. This issue further exacerbates the inability of the previous TempBalance method to accurately allocate per-layer learning rates, resulting in certain layers remaining at either excessively high or extremely low learning rates. We illustrate this phenomenon in more detail in Figure 12 of Appendix C. Consequently, this imbalance leads to suboptimal model performance after training.

Therefore, according to the experimental setup described in Zhou et al. (2024), the first and last layers of ResNet and VGG models, along with certain layers whose correlation matrix of weight matrix contains a relatively small number of eigenvalues, will be excluded from the layerwise learning rate scheduling. To ensure a comprehensive study, we compare FARMS and TempBalance with and without this layer selection (LS) heuristic. See Figure 4, when the LS method is not applied (and all layers will be included in the adjustable learning rate scheduling), the performance of the original TempBalance method (shown as "TB (no LS)") deteriorates significantly. For example, compared to the LS-enabled setup (shown as "LS+TB"), the test accuracy of the VGG 19 model trained on CIFAR100 without LS using the TempBalance method drops from 74.19% to 73.22%, while the standard deviation increases from 0.159 to 0.277. In contrast, when training models using the TempBalance method with FARMS, performance remains stable or even improves when LS is disabled. This observation demonstrates that our method is more robust to weight matrices of extreme sizes, and the reason is that our

Eigenspectrum Analysis of Neural Networks without Aspect Ratio Bias



Figure 4. Comparing our method to TempBalance at different layer usage settings in training ResNet and VGG series models on CIFAR100. Higher test accuracy values indicate better model performance. See Table 13 in Appendix F for the details in all hyperparameters.

method provides a more accurate assessment of the training progress independent of the matrix size.

In TempBalance algorithm, the layer-wise adjustable learning rate is scaled in the range of  $(s_1\eta_t, s_2\eta_t)$ , where  $\eta_t$  is the global learning rate at time t, and  $s_1$  and  $s_2$  are two tunable lower and upper limits of learning rates. To demonstrate that FARMS can more accurately evaluate the training quality of each model layer, we present the test performance of models trained using the different learning rate scaling ratios  $(s_1, s_2)$ . in which we consider five different settings for (s1, s2): [(0.1, 1.9), (0.2, 1.8), (0.3, 1.7), (0.4, (0.5, 1.5)]. Additionally, we consider that these five scaling ratios are generally close to the optimal scaling ratio. We run tasks on CIFAR100 with four VGG and ResNet architectures. Our results in Table 3 show that FARMS can improve the test accuracy of models among the five scaling ratios and help models achieve a lower overall standard deviation compared to TempBalance. Detailed results can be found in Figure 9, Appendix C.

Table 3. Comparing our method to TempBalance among the five scaling ratios. All results are reported as the mean test accuracy and standard deviation obtained across five different scaling ratios. In this experiment, we allowed TempBalance baseline to use the LS heuristic to achieve slightly better test accuracy.

Method	ResNet 18	ResNet 34	VGG 16	VGG 19
TB	79.03±0.169	$79.81{\scriptstyle \pm 0.145}$	$74.87{\scriptstyle\pm0.214}$	$73.89{\scriptstyle \pm 0.199}$
Ours	$79.35{\scriptstyle\pm0.126}$	$80.07{\scriptstyle\pm0.097}$	$75.16{\scriptstyle \pm 0.212}$	$74.03{\scriptstyle\pm0.163}$

#### 4.4. Improving SciML Models with FARMS

To explore the potential applications of FARMS in multiple domains of machine learning research, we also performed SciML fine-tuning experiments using the 2DCFD dataset with DPOT-Tiny and DPOT-Small models and compared FARMS with TB\_Sigmoid method (Liu et al., 2024). We followed the experimental setup from Liu et al. (2024), and our hyperparameter ranges are detailed in Table 14

Subsampling			
Ratio	Method	DPOT-Tiny	DPOT-Small
5%	FT	1.863e-2	1.546e-2
	TB_Sig	1.856e-2	1.539e-2
	Ours	1.842e-2	1.536e-2
10%	FT	1.747e-2	1.426e-2
	TB_Sig	1.730e-2	1.415e-2
	Ours	1.706e-2	1.407e-2
25%	FT	1.543e-2	1.226e-2
	TB_Sig	1.517e-2	1.203e-2
	Ours	1.499e-2	1.189e-2
50%	FT	1.309e-2	1.025e-2
	TB_Sig	1.283e-2	1.005e-2
	Ours	1.242e-2	9.822e-3
100%	FT	1.096e-2	8.400e-3
	TB_Sig	1.078e-2	8.193e-3
	Ours	1.017e-2	7.949e-3

*Table 4.* FARMS achieves lower L2RE(\$\$) on the test dataset than TB\_Sigmoid and baseline fine-tuning method on SciML tasks.

of Appendix F. In Table 4, we show the results of comparing FARMS with the TB\_Sigmoid (shown as "TB\_Sig") with layer selection and the baseline fine-tuning (shown as "FT") with a uniform learning rate for each layer in different data subsampling ratios in the range {5%, 10%, 25%, 50%, 100%}. Our method consistently outperformed TB\_Sigmoid across all subsampling ratios. For instance, with a full dataset (100% of the data), our approach reduced the model's L2RE to 1.017e-2. Comparing to the TB\_Sigmoid, the L2RE decreased by 5.66%.

#### 4.5. HT Metrics Analysis

In this subsection, we demonstrate that FARMS can effectively control the shape of ESDs, resulting in a more balanced distribution of PL\_Alpha\_Hill values among the layers of NNs compared to the previous methods. We as-



Figure 5. Comparing the distribution of PL\_Alpha\_Hill of ResNet and VGG series models. The top blue-shaded section in each subplot of the experiments (shown as "CAL") represents models trained without employing layer-wise optimization methods. The middle brown-shaded section in each subplot indicates models trained using TempBalance; Lastly, the bottom orange-shaded section in each subplot corresponds to the results obtained using FARMS. Figures 5a, 5b, 5c and 5d present the averaged results obtained from multiple experiments within the optimal parameter range.

sess the models presented in Section 4.3, which include 2D Convolution and 2D Linear layers of ResNets and VGG networks. Results in Figure 5 show the distribution of PL\_Alpha\_Hill of models trained using baseline cosine annealing (CAL), TempBalance, and TempBalance using our method FARMS. It can be seen that FARMS generally leads to a more concentrated distribution and, in most cases, reduces the average PL\_Alpha\_Hill value. These experimental results suggest that our method FARMS enables a more balanced training progression across different layers of the model. These observations align well with the findings reported by Martin et al. (2021); Liu et al. (2024), which suggest that a decrease in both average PL\_Alpha\_Hill values and the variance across layers indicate better training quality.

#### 4.6. Ablation Study

**Different Aspect Ratios of Weight Matrix.** Here, we study the effect of the aspect ratio of the subsampled matrices in FARMS. We consider image classification tasks and use FARMS to replace the HT metrics in TempBalance for assigning layer-wise learning rates. We consider five different aspect ratios in the range {0.5, 0.75, 1.0, 1.5, 2.0} and keep other hyperparameters optimal. We again use ResNet18 and VGG16 as our architectures and show results on CIFAR100. Results in Table 5 show that FARMS achieves a higher test accuracy when the aspect ratio is 1.0.

Table 5. Comparing the different Aspect ratios settings for submatrices on ResNet 18 and VGG 16 models trained on CIFAR100 Dataset.

Q Ratio	0.5	0.75	1.0	1.5	2.0
ResNet 18	79.13±0.158	$79.33{\scriptstyle \pm 0.122}$	$79.53{\scriptstyle \pm 0.177}$	$79.13{\scriptstyle \pm 0.282}$	$79.31{\scriptstyle \pm 0.046}$
VGG 16	75.05±0.285	$75.19{\scriptstyle \pm 0.517}$	$75.36{\scriptstyle\pm0.118}$	75.21±0.345	$75.10{\scriptstyle \pm 0.360}$

**Subsampling Window Sizes and Sampling Steps.** Here, we study different subsampling window sizes and sampling steps (i.e., the number of subsampled submatrices) in the task of LLaMA-7B pruning. The aspect ratio Q is fixed to be 1.0. For all weight matrices in LLaMA-7B, the smallest dimension is 4096. Therefore, when using a window size of 4096, sliding window downsampling is applied only along the larger dimension. This process generates multiple  $4096 \times 4096$  submatrices. We report our results in Table 6. We find that using an appropriately sized sampling window, such as  $2000 \times 2000$ , the model achieves a lower perplexity. As a comparison, baseline perplexity achieved by AlphaPruning without using FARMS is 96.02.

Table 6. Different Window Size and Sampling Steps for pruning the LLaMA-7B at 0.8 sparsity ratio. The pruning method is SparseGPT. The model is evaluated on WikiText dataset with perplexity  $(\downarrow)$ .

		Sampling Steps									
Window size	5	10	15	20							
500	96.34±9.15	$95.93{\scriptstyle\pm2.80}$	$99.23{\scriptstyle\pm3.53}$	$88.08{\scriptstyle\pm4.23}$							
1000	84.71±4.97	81.96±4.22	$89.20{\scriptstyle\pm3.91}$	$84.04{\scriptstyle\pm2.48}$							
2000	82.33±3.18	$79.42{\scriptstyle\pm3.86}$	$80.14{\scriptstyle\pm2.32}$	$86.54{\scriptstyle\pm3.28}$							
4096	82.63±2.45	89.61±5.25	$84.19{\scriptstyle\pm6.05}$	$84.07{\scriptstyle\pm 5.64}$							

### 5. Conclusion

We propose a subsampling strategy to address the measurement bias introduced by varying aspect ratios of weight matrices in HT-SR eigenspectrum analysis. The main idea of our method is to extract submatrices of the original matrix with a fixed aspect ratio, followed by averaging the ESDs of these submatrices to accurately assess the training quality of each weight matrix. Our extensive experiments demonstrate that our method can precisely estimate layer-wise training quality, improve the performance of layer-wise optimization algorithms, and contribute to more balanced training and efficient pruning. Furthermore, results on visual analytics confirm the effectiveness of our approach, showing that it successfully eliminates the measurement bias caused by aspect ratio variations.

# **Impact Statement**

This paper presents research aimed at advancing the fields of Machine Learning and Deep Learning, particularly in training using information from eigenspectrum analysis. While our work has various potential societal implications, we do not find it necessary to highlight any specific ones here.

### Acknowledgments

This work is supported by DOE under Award Number DE-SC0025584 and Dartmouth College.

### References

- Adlam, B. and Pennington, J. The neural tangent kernel in high dimensions: Triple descent and a multi-scale theory of generalization. In *International Conference on Machine Learning*, pp. 74–84. PMLR, 2020.
- Alstott, J., Bullmore, E., and Plenz, D. powerlaw: a python package for analysis of heavy-tailed distributions. *PloS one*, 9(1):e85777, 2014.
- Ba, J., Erdogdu, M. A., Suzuki, T., Wang, Z., Wu, D., and Yang, G. High-dimensional asymptotics of feature learning: How one gradient step improves the representation. *Advances in Neural Information Processing Systems*, 35: 37932–37946, 2022.
- Bai, Z. and Silverstein, J. W. Spectral analysis of large dimensional random matrices, volume 20. Springer, 2010.
- Chen, W., Gong, X., and Wang, Z. Neural architecture search on imagenet in four gpu hours: A theoretically inspired perspective. *arXiv preprint arXiv:2102.11535*, 2021.
- Chen, X.-H., Wang, Z., Du, Y., Jiang, S., Fang, M., Yu, Y., and Wang, J. Policy learning from tutorial books via understanding, rehearsing and introspecting. In *Advances in Neural Information Processing Systems*, volume 37, 2024.
- Clark, C., Lee, K., Chang, M.-W., Kwiatkowski, T., Collins, M., and Toutanova, K. Boolq: Exploring the surprising difficulty of natural yes/no questions. *arXiv preprint arXiv:1905.10044*, 2019.
- Clark, P., Cowhey, I., Etzioni, O., Khot, T., Sabharwal, A., Schoenick, C., and Tafjord, O. Think you have solved question answering? try arc, the ai2 reasoning challenge. *arXiv preprint arXiv:1803.05457*, 2018.
- Clauset, A., Shalizi, C. R., and Newman, M. E. Powerlaw distributions in empirical data. *SIAM review*, 51(4): 661–703, 2009.

- Couillet, R. and Liao, Z. *Random Matrix Methods for Machine Learning*. Cambridge University Press, 2022.
- Derezinski, M., Liang, F. T., and Mahoney, M. W. Exact expressions for double descent and implicit regularization via surrogate random design. *Advances in neural information processing systems*, 33:5152–5164, 2020.
- Dobriban, E. and Wager, S. High-dimensional asymptotics of prediction: Ridge regression and classification. *The Annals of Statistics*, 46(1):247–279, 2018.
- Farnia, F., Zhang, J. M., and Tse, D. Generalizable adversarial training via spectral normalization. arXiv preprint arXiv:1811.07457, 2018.
- Frantar, E. and Alistarh, D. Sparsegpt: Massive language models can be accurately pruned in one-shot. In *International Conference on Machine Learning*, pp. 10323– 10337. PMLR, 2023.
- Gao, L., Tow, J., Biderman, S., Black, S., DiPofi, A., Foster, C., Golding, L., Hsu, J., McDonell, K., Muennighoff, N., et al. A framework for few-shot language model evaluation. *Version v0. 0.1. Sept*, 10:8–9, 2021.
- Grattafiori, A., Dubey, A., Jauhri, A., Pandey, A., Kadian, A., Al-Dahle, A., Letman, A., Mathur, A., Schelten, A., Vaughan, A., et al. The llama 3 herd of models. *arXiv* preprint arXiv:2407.21783, 2024.
- Guhr, T., Müller–Groeling, A., and Weidenmüller, H. A. Random-matrix theories in quantum physics: common concepts. *Physics Reports*, 299(4):189–425, June 1998.
  ISSN 0370-1573. doi: 10.1016/S0370-1573(97)00088-4.
  URL https://www.sciencedirect.com/science/article/pii/S0370157397000884.
- Gurbuzbalaban, M., Simsekli, U., and Zhu, L. The heavytail phenomenon in sgd. In *International Conference on Machine Learning*, pp. 3964–3975. PMLR, 2021.
- Han, S., Pool, J., Tran, J., and Dally, W. Learning both weights and connections for efficient neural network. In Cortes, C., Lawrence, N., Lee, D., Sugiyama, M., and Garnett, R. (eds.), *Advances in Neural Information Processing Systems*, volume 28. Curran Associates, Inc., 2015. URL https://proceedings.neurips.cc/paper\_files/paper/2015/file/ae0eb3eed39d2bcef4622b2499a05fe6-Paper.pdf.
- Hao, Z., Su, C., Liu, S., Berner, J., Ying, C., Su, H., Anandkumar, A., Song, J., and Zhu, J. Dpot: Auto-regressive denoising operator transformer for large-scale pde pretraining. arXiv preprint arXiv:2403.03542, 2024.

- Hastie, T., Montanari, A., Rosset, S., and Tibshirani, R. J. Surprises in high-dimensional ridgeless least squares interpolation. *Annals of statistics*, 50(2):949, 2022.
- He, K., Zhang, X., Ren, S., and Sun, J. Delving deep into rectifiers: Surpassing human-level performance on imagenet classification. In *Proceedings of the IEEE international conference on computer vision*, pp. 1026–1034, 2015.
- He, K., Zhang, X., Ren, S., and Sun, J. Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 770–778, 2016.
- Hill, B. M. A simple general approach to inference about the tail of a distribution. *The annals of statistics*, pp. 1163–1174, 1975.
- Hodgkinson, L. and Mahoney, M. Multiplicative noise and heavy tails in stochastic optimization. In *International Conference on Machine Learning*, pp. 4262–4274. PMLR, 2021.
- Hodgkinson, L., Simsekli, U., Khanna, R., and Mahoney, M. Generalization bounds using lower tail exponents in stochastic optimizers. In *International Conference on Machine Learning*, pp. 8774–8795. PMLR, 2022.
- Hu, H. and Lu, Y. M. Universality laws for high-dimensional learning with random features. *IEEE Transactions on Information Theory*, 69(3):1932–1964, 2022.
- Kothapalli, V., Pang, T., Deng, S., Liu, Z., and Yang, Y. Crafting heavy-tails in weight matrix spectrum without gradient noise, 2024. URL https://arxiv.org/ abs/2406.04657.
- Krizhevsky, A., Hinton, G., et al. Learning multiple layers of features from tiny images. 2009.
- Langley, P. Crafting papers on machine learning. In Langley, P. (ed.), Proceedings of the 17th International Conference on Machine Learning (ICML 2000), pp. 1207–1216, Stanford, CA, 2000. Morgan Kaufmann.
- Liu, L., Zhao, X., Li, G., Li, D., Wang, Han, Y., Li, X., and Wang, Y. BaWA: Automatic optimizing pruning metric for large language models with balanced weight and activation. In *Proceedings of the 42nd International Conference on Machine Learning*, ICML '25. PMLR, 2025a. URL https://icml.cc/virtual/2025/ poster/44892.
- Liu, Z., Hu, Y., Pang, T., Zhou, Y., Ren, P., and Yang, Y. Model balancing helps low-data training and fine-tuning. arXiv preprint arXiv:2410.12178, 2024.

- Liu, Z., Pang, T., Balabanov, O., Yang, C., Huang, T., Yin, L., Yang, Y., and Liu, S. Lift the veil for the truth: Principal weights emerge after rank reduction for reasoningfocused supervised fine-tuning. In *Proceedings of the 42nd International Conference on Machine Learning (ICML)*, Vancouver, Canada, July 2025b. URL https: //icml.cc/virtual/2025/poster/44967.
- Lu, H., Zhou, Y., Liu, S., Wang, Z., Mahoney, M. W., and Yang, Y. Alphapruning: Using heavy-tailed self regularization theory for improved layer-wise pruning of large language models. *arXiv preprint arXiv:2410.10912*, 2024.
- Mahoney, M. and Martin, C. Traditional and heavy tailed self regularization in neural network models. In Chaudhuri, K. and Salakhutdinov, R. (eds.), Proceedings of the 36th International Conference on Machine Learning, volume 97 of Proceedings of Machine Learning Research, pp. 4284–4293. PMLR, 09–15 Jun 2019. URL https://proceedings.mlr.press/v97/ mahoney19a.html.
- Martin, C. H. and Mahoney, M. W. Traditional and heavytailed self regularization in neural network models. *arXiv* preprint arXiv:1901.08276, 2019.
- Martin, C. H. and Mahoney, M. W. Implicit selfregularization in deep neural networks: Evidence from random matrix theory and implications for learning. *Journal of Machine Learning Research*, 22(165):1–73, 2021.
- Martin, C. H., Peng, T., and Mahoney, M. W. Predicting trends in the quality of state-of-the-art neural networks without access to training or testing data. *Nature Communications*, 12(1), July 2021. ISSN 2041-1723. doi: 10.1038/s41467-021-24025-8. URL http://dx.doi.org/10.1038/s41467-021-24025-8.
- Mei, S. and Montanari, A. The generalization error of random features regression: Precise asymptotics and the double descent curve. *Communications on Pure and Applied Mathematics*, 75(4):667–766, 2022.
- Merity, S., Xiong, C., Bradbury, J., and Socher, R. Pointer sentinel mixture models. *arXiv preprint arXiv:1609.07843*, 2016.
- Mihaylov, T., Clark, P., Khot, T., and Sabharwal, A. Can a suit of armor conduct electricity? a new dataset for open book question answering. *arXiv preprint arXiv:1809.02789*, 2018.
- Miyato, T., Kataoka, T., Koyama, M., and Yoshida, Y. Spectral normalization for generative adversarial networks, 2018. URL https://arxiv.org/abs/ 1802.05957.

- Nadakuditi, R. R. and Newman, M. E. Graph spectra and the detectability of community structure in networks. *Physical review letters*, 108(18):188701, 2012.
- Pennington, J. and Worah, P. Nonlinear random matrix theory for deep learning. In Advances in Neural Information Processing Systems, volume 30. Curran Associates, Inc., 2017. URL https://proceedings. neurips.cc/paper/2017/hash/ 0f3d014eead934bbdbacb62a01dc4831-Abstract.tion Theory, 1(1):1-182, June 2004. html. 2023b. Tulino, A. M. and Verdú, S. Randon ory and Wireless Communications and Trends® in Communications 2023b. Tulino, A. M. and Verdú, S. Randon ory and Wireless Communications and Trends® in Communications 2023b.
- Qing, P., Gao, C., Zhou, Y., Diao, X., Yang, Y., and Vosoughi, S. Alphalora: Assigning lora experts based on layer training quality. *arXiv preprint arXiv:2410.10054*, 2024.
- Sakaguchi, K., Bras, R. L., Bhagavatula, C., and Choi, Y. Winogrande: An adversarial winograd schema challenge at scale. *Communications of the ACM*, 64(9):99–106, 2021.
- Sanyal, A., Torr, P. H., and Dokania, P. K. Stable rank normalization for improved generalization in neural networks and gans. arXiv preprint arXiv:1906.04659, 2019.
- Simonyan, K. and Zisserman, A. Very deep convolutional networks for large-scale image recognition. *arXiv preprint arXiv:1409.1556*, 2014.
- Simsekli, U., Sagun, L., and Gurbuzbalaban, M. A tail-index analysis of stochastic gradient noise in deep neural networks. In *International Conference on Machine Learning*, pp. 5827–5837. PMLR, 2019.
- Simsekli, U., Sener, O., Deligiannidis, G., and Erdogdu, M. A. Hausdorff dimension, heavy tails, and generalization in neural networks. *Advances in Neural Information Processing Systems*, 33:5138–5151, 2020.
- Sun, M., Liu, Z., Bair, A., and Kolter, J. Z. A simple and effective pruning approach for large language models. *arXiv preprint arXiv:2306.11695*, 2023.
- Takamoto, M., Praditia, T., Leiteritz, R., MacKinlay, D., Alesiani, F., Pflüger, D., and Niepert, M. Pdebench: An extensive benchmark for scientific machine learning. *Advances in Neural Information Processing Systems*, 35: 1596–1611, 2022.
- Tao, T. *Topics in random matrix theory*, volume 132. American Mathematical Soc., 2012.
- Touvron, H., Lavril, T., Izacard, G., Martinet, X., Lachaux, M.-A., Lacroix, T., Rozière, B., Goyal, N., Hambro, E., Azhar, F., et al. Llama: Open and efficient foundation language models. arXiv preprint arXiv:2302.13971, 2023a.

- Touvron, H., Martin, L., Stone, K., Albert, P., Almahairi, A., Babaei, Y., Bashlykov, N., Batra, S., Bhargava, P., Bhosale, S., et al. Llama 2: Open foundation and finetuned chat models. arXiv preprint arXiv:2307.09288, 2023b.
- Tulino, A. M. and Verdú, S. Random Matrix Theory and Wireless Communications. Foundations and Trends® in Communications and Informa-ISSN 1567-1567-2328. doi: 10.1561/010000001. 2190, URL https://www.nowpublishers.com/ article/Details/CIT-001. Publisher: Now Publishers, Inc.
- Wang, A. Glue: A multi-task benchmark and analysis platform for natural language understanding. arXiv preprint arXiv:1804.07461, 2018.
- Wang, Z., Engel, A., Sarwate, A., Dumitriu, I., and Chiang, T. Spectral Evolution and Invariance in Linear-width Neural Networks, November 2023. URL http://arxiv. org/abs/2211.06506. arXiv:2211.06506 [cs].
- Wang, Z., Fang, M., Tomilin, T., Fang, F., and Du, Y. Safe multi-agent reinforcement learning with natural language constraints. arXiv preprint arXiv:2405.20018, 2024.
- Wang, Z., Zhang, Z., Fang, F., and Du, Y. M3hf: Multiagent reinforcement learning from multi-phase human feedback of mixed quality. In *The Twelfth International Conference on Learning Representations*, 2025.
- Yang, G. and Hu, E. J. Tensor programs iv: Feature learning in infinite-width neural networks. In *International Conference on Machine Learning*, pp. 11727–11737. PMLR, 2021.
- Yang, G., Hu, E. J., Babuschkin, I., Sidor, S., Liu, X., Farhi, D., Ryder, N., Pachocki, J., Chen, W., and Gao, J. Tensor programs v: Tuning large neural networks via zero-shot hyperparameter transfer. *arXiv preprint arXiv:2203.03466*, 2022.
- Yang, J., Tang, A., Zhu, D., Chen, Z., Shen, L., and Wu, F. Mitigating the backdoor effect for multi-task model merging via safety-aware subspace. *arXiv preprint arXiv:2410.13910*, 2024.
- Yang, S., Luo, S., and Han, S. C. Multimodal commonsense knowledge distillation for visual question answering (student abstract). In *Proceedings of the AAAI conference on artificial intelligence*, pp. 29545–29547, 2025a.
- Yang, S., Luo, S., Han, S. C., and Hovy, E. Magic-vqa: Multimodal and grounded inference with commonsense knowledge for visual question answering. *arXiv preprint arXiv:2503.18491*, 2025b.

- Yang, Y., Theisen, R., Hodgkinson, L., Gonzalez, J. E., Ramchandran, K., Martin, C. H., and Mahoney, M. W. Test accuracy vs. generalization gap: Model selection in nlp without accessing training or testing data. KDD '23, pp. 3011–3021, New York, NY, USA, 2023. Association for Computing Machinery. ISBN 9798400701030. doi: 10.1145/3580305.3599518. URL https://doi. org/10.1145/3580305.3599518.
- Yin, L., Wu, Y., Zhang, Z., Hsieh, C.-Y., Wang, Y., Jia, Y., Li, G., Jaiswal, A., Pechenizkiy, M., Liang, Y., et al. Outlier weighed layerwise sparsity (owl): A missing secret sauce for pruning llms to high sparsity. arXiv preprint arXiv:2310.05175, 2023.
- Yoshida, Y. and Miyato, T. Spectral norm regularization for improving the generalizability of deep learning, 2017. URL https://arxiv.org/abs/1705.10941.
- Zellers, R., Holtzman, A., Bisk, Y., Farhadi, A., and Choi, Y. Hellaswag: Can a machine really finish your sentence? *arXiv preprint arXiv:1905.07830*, 2019.
- Zhang, S., Roller, S., Goyal, N., Artetxe, M., Chen, M., Chen, S., Dewan, C., Diab, M., Li, X., Lin, X. V., et al. Opt: Open pre-trained transformer language models. *arXiv preprint arXiv:2205.01068*, 2022.
- Zhou, Y., Pang, T., Liu, K., Mahoney, M. W., Yang, Y., et al. Temperature balancing, layer-wise weight analysis, and neural network training. *Advances in Neural Information Processing Systems*, 36, 2024.
- Zhu, D., Song, Y., Shen, T., Zhao, Z., Yang, J., Zhang, M., and Wu, C. Remedy: Recipe merging dynamics in large vision-language models. In *The Thirteenth International Conference on Learning Representations*, 2025.

# Appendix

# **A. Detailed Matrix Analysis**

### A.1. Random Initialization

In this section, we do weight analysis for ResNet 34 and VGG 19 models that are randomly initialized using methods proposed in He et al. (2015). The weights w are drawn from a normal distribution with mean 0 and variance  $2/n_{in}$ , where  $n_{in}$  represents the number of input neurons. This is represented as:

$$w \sim \mathcal{N}\left(0, \frac{2}{n_{in}}\right)$$

We present our results of comparing FARMS with the previous HT estimation methods in measuring randomly initialized models in Figure 6. In Figure 6a and Figure 6c, all layers in these models should be similarly under-trained (because they are all just initialized). However, previous methods tend to report significantly higher values for PL\_Alpha\_Hill in certain layers, suggesting that these layers are much more under-trained. In contrast, FARMS produces results where the training quality of all initialized layers is nearly identical, aligning more closely with the expected result. Similarly, in Figure 6b and Figure 6d, for randomly initialized ResNet-34 and VGG-19 models with varying widths, the previous method measures PL\_Alpha\_Hill values that increase as model width increases. In contrast, FARMS shows PL\_Alpha\_Hill values that remain unaffected by the aspect ratio bias introduced by model width variations.



Figure 6. Comparing FARMS and previous HT-SR methods for measuring the randomly initialized ResNet 34 and VGG 19 weights. Figure 6a and Figure 6c show the PL\_Alpha\_Hill values for each layer in models (widen factor is 1.0) by using different methods. Figure 6b and Figure 6d show the measured PL\_Alpha\_Hill values of the final linear layer across different model width factors. As the width increases, the aspect ratio of the weight matrix also becomes larger, leading to bias in previous HT-SR methods.

### A.2. Mitigating Aspect Ratio Bias in Specific Layers

In this section, we do weight analysis for the last layer from the ResNet 34 and VGG 16 models trained on CIFAR100. These models are trained with hyperparameters according to the Appendix F. The weight matrices of the final layers in both models have dimensions of  $512 \times 100$  (aspect ratio = 5.12). This means that the previous weight matrix analysis method will be significantly affected by aspect ratio bias, leading to inaccurate assessments of the layer's training quality.

In Figure 7, the presented experimental results validate this observation. Figure 7a and Figure 7c show the ESD fitting results obtained using the previous weight analysis method, where the measured PL\_Alpha\_Hill values are larger than those typically observed in well-trained layers. This leads to the erroneous classification of these layers as poorly trained, ultimately affecting both model performance evaluation and optimization. In contrast, Figure 7b and Figure 7d present the ESD fitting results obtained using FARMS. Our method produces measurements that align more closely with the expected ESD of well-trained layers, providing a more accurate assessment of training quality.

## **B. Marchenko–Pastur Distribution**

In RMT, the Marchenko-Pastur distribution, also known as the Marchenko-Pastur law, characterizes the asymptotic behavior of singular values of large rectangular random matrices.

Let  $X_{ij}$ ,  $1 \le i \le m$ ,  $1 \le j \le n$ , be independent random variables with  $\mathbb{E}X_{ij} = 0$  and  $\mathbb{E}X_{ij}^2 = 1$ , and  $\mathbf{X}_m = 0$ 



Figure 7. Comparing FARMS and the previous method for measuring the ESD of final layers in ResNet 34 and VGG 16 trained on CIFAR 100.



Figure 8. The Marchenko-Pastur (MP) Law for different values of n. The ESD (blue histogram) of the eigenvalues of the sample covariance matrix is compared with the theoretical Marchenko-Pastur probability density function (red curve) for various aspect ratios  $\gamma = m/n$ , where m = 4000 is fixed, and n varies from 4000 to 16000.

 $(X_{ij})_{1 \le i \le m, 1 \le j \le n}$ . Denote by  $\lambda_1 \le \cdots \le \lambda_m$  the eigenvalues of the symmetric matrix

$$\mathbf{W} := \mathbf{W}_m := \frac{1}{n} \mathbf{X}_m \mathbf{X}_m^\top$$

and defined its empirical distribution by

$$F_m(x) = \frac{1}{m} \sum_{k=1}^m I_{\{\lambda_k \le x\}},$$

where  $I_{\{B\}}$  denotes the indicator of an event B. One often investigates the rate of convergence of the expected spectral distribution  $\mathbb{E}F_m(x)$  as well as  $F_m(x)$  to the Marchenko–Pastur distribution function  $F_y(x)$  with density

$$f_y(x) = \frac{1}{2xy\pi}\sqrt{(b-x)(x-a)}I_{\{[a,b]\}}(x) + I_{\{[1,\infty)\}}(y)(1-y^{-1})\delta(x),$$

where y = m/n,  $y \in (0, \infty)$  and  $a = (1 - \sqrt{y})^2$ ,  $b = (1 + \sqrt{y})^2$ . Here we denote by  $\delta(x)$  the Dirac delta function and by  $I_{\{[a,b]\}}(x)$  the indicator function of the interval [a, b].

We visualize the MP distribution and the ESD of the weight matrices in Figure 8. In this figure, we can observe that the empirical distribution converges to the theoretical MP distribution. Additionally, as n increases, the ESD distribution exhibits

an increasingly concentrated shape with a reduced degree of HT. Therefore, ignoring the impact of aspect ratio and directly measuring the HT degree of the ESD to estimate a layer's training quality may lead to inaccurate results.

# **C. Additional Experiment Results**

#### C.1. Detailed Performance on Different Scaling Ratios

We provide detailed results of a hyperparameter study on learning rate scaling ratio  $(s_1, s_2)$ . We set other hyperparameters (including layer selection, initial learning rate, and so on) as the optimal value for each training set. The results in Figure 9 show that across all tested architectures and most scaling ratios, FARMS consistently outperforms TempBalance. This demonstrates that FARMS provides a more accurate measurement of each layer's training quality. As a result, it helps the model achieve better training performance when using different learning rate scaling ratios.



Figure 9. Comparison of test accuracy across different architectures and learning rate scaling ranges. The figure presents the test accuracy (%) of TempBalance (blue, dotted) and FARMS (orange, hatched). These two methods are evaluated on the CIFAR 100 dataset using ResNet and VGG series architectures. The x-axis represents different learning rate scaling ranges  $(s_1, s_2)$ , while the y-axis indicates test accuracy. Error bars denote standard deviations across multiple runs.

#### C.2. Comparison of Different CNN Processing Methods

We compared two different methods for processing the ESD of CNNs. The first method concatenates the rooted singular values of all submatrices and then measures the HT metrics of the resulting ESD. The second method measures the HT metrics of the ESD formed by concatenating the rooted singular values of each of the  $C_1 \times C_2$  submatrices separately, and then averages these HT metrics from each group. In Table 7, we provide a performance comparison of training ResNet-18 and VGG-16 models using these two methods. For each experiment, all other hyperparameters are set to their optimal values. We find that although the performance difference between the two methods is not very large, the second method brings more performance improvement.

Table	7.	Comparing	Two	Stratgy	in	CNN	Processing	with	FARMS
-------	----	-----------	-----	---------	----	-----	------------	------	-------

Method	ResNet 18	VGG 16
Calculating the Overall ESD	79.36±0.101	$75.27{\scriptstyle\pm0.201}$
Averaging ESDs Computed from Subsets	79.53±0.177	$75.36{\scriptstyle \pm 0.118}$

#### C.3. Experiment Results for Additional Models

We prune the OPT models (Zhang et al., 2022) and LLaMA-V2/V3 models (Touvron et al., 2023b; Grattafiori et al., 2024) with AlphaPruning and compare the model performance with FARMS or not. These models and their corresponding variants are widely used as benchmark models in LLM research (Yang et al., 2025b;a). We report the additional experiment results on LLM pruning in Table 8 and Table 9. For the OPT series models, we use four sparsity ratios: {0.6, 0.7, 0.75, 0.8}. For the LLaMA series models, we select {0.7, 0.75, 0.8, 0.85} as the sparsity ratios. Experimental results show that our method can help AlphaPruning achieve better model performance on models such as OPT.

Sparsity Ratio	Layer-wise	OP	T-125M	OP	Т-350М	OF	PT-1.3B	OF	PT-6.7B
	Method	Wanda	SparseGPT	Wanda	SparseGPT	Wanda	SparseGPT	Wanda	SparseGPT
0.6	AlphaPruning	67.12	59.18	87.95	50.94	27.09	22.76	15.6	13.71
	Ours	66.71	58.86	82.55	50.52	27.15	22.69	15.59	13.67
0.7	AlphaPruning	263.71	196.64	595.26	147.20	101.13	50.30	44.92	20.76
	Ours	261.84	188.31	489.75	136.80	100.74	50.01	42.03	20.76
0.75	AlphaPruning	718.69	515.55	1453.21	330.11	613.78	142.68	245.16	35.27
	Ours	721.52	491.98	1298.22	303.07	601.24	122.01	181.92	34.88
0.8	AlphaPruning	1713.45	1525.25	2869.8	921.17	2763.33	511.66	5781.0	98.3
	Ours	1609.36	1412.59	2551.46	798.25	2502.60	499.82	5212.51	90.88

Table 8. WikiText validation perplexity for pruned OPT models at different sparsity settings. Our method is compared to AlphaPruning, each paired with Wanda and SparseGPT. Lower perplexity indicates improved model performance.

Table 9. WikiText validation perplexity for pruned LLaMA-V2 and LLaMA-V3 models at different sparsity settings. Our method is compared to AlphaPruning, each paired with Wanda and SparseGPT. Lower perplexity indicates improved model performance.

Sparsity Ratio	o Layer-wise LLaMA-V2-7B		LLaN	LLaMA-V2-13B		A-V3.2-3B	LLaMA-V3.1-8B		
	Method	Wanda	SparseGPT	Wanda	SparseGPT	Wanda	SparseGPT	Wanda	SparseGPT
0.7	AlphaPruning	34.71	20.92	15.37	13.69	123.19	64.33	105.64	38.43
	Ours	31.09	20.49	15.36	13.64	120.03	63.78	107.00	37.81
0.75	AlphaPruning	170.74	41.39	35.26	23.21	356.69	121.53	240.17	81.30
	Ours	161.58	39.98	33.67	22.98	311.73	117.19	229.63	81.93
0.8	AlphaPruning	868.66	89.73	150.01	46.32	1412.53	262.44	688.23	180.40
	Ours	831.37	88.25	127.69	44.74	1219.10	212.65	607.58	175.93
0.85	AlphaPruning	6425.25	217.79	874.97	107.21	5857.39	505.8	3498.94	400.69
	Ours	4437.46	204.44	748.91	98.99	4407.94	463.34	3766.13	411.35

## C.4. Zero-shot Tasks Performance

We demonstrate the task-wise performance in detail in Table 10 and Table 11.

### C.5. Detailed PL\_Alpha\_Hill Distribution

We provide additional experimental results on LLM pruning and SciML fine-tuning to support the PL\_Alpha\_Hill distribution analysis presented in Section 4.5. The results in Figure 10 and 11 show that FARMS consistently helps models achieve better layer quality across different experimental settings.



Figure 10. Comparing the distribution of PL Alpha Hill of DPOT-Tiny and DPOT-Small in different fine-tuning methods and data ratios.

Sparsity Ratio	Method	BoolQ	RTE	HellaSwag	WinoGrande	ARC-e	ARC-c	OBQA	Mean
	Magnitude	38.29	52.71	25.59	51.22	26.73	19.62	11.60	32.25
	AlphaPruning w. Magnitude	41.31	52.71	30.37	51.46	35.44	22.01	16.40	35.67
	Ours w. Magnitude	40.09	52.71	30.49	53.83	34.22	23.21	17.20	35.96
	Wanda	56.06	55.60	28.90	50.99	32.11	18.26	13.60	36.50
0.7	AlphaPruning $w$ . Wanda	64.34	57.40	35.57	61.09	45.58	24.49	17.20	43.67
	Ours w. Wanda	64.40	59.93	36.54	60.62	45.24	24.66	18.40	44.26
	SparseGPT	65.14	53.79	33.95	58.72	44.44	23.98	17.20	42.46
	AlphaPruning w. SparseGPT	65.74	53.07	37.72	64.01	47.35	26.88	18.80	44.79
	Ours w. SparseGPT	65.54	53.07	36.85	64.33	48.11	26.37	19.60	44.84
	Magnitude	42.26	52.35	25.88	48.54	26.68	21.42	14.00	33.02
	AlphaPruning w. Magnitude	50.67	52.71	26.80	49.96	27.19	21.42	13.40	34.59
	Ours w. Magnitude	55.54	53.07	27.80	49.25	29.21	21.08	15.20	35.88
	Wanda	37.83	53.79	27.01	49.96	27.74	19.37	12.60	32.61
0.75	AlphaPruning $w$ . Wanda	62.17	53.43	29.52	53.75	33.04	20.82	13.20	37.99
	Ours w. Wanda	62.17	58.12	30.99	55.49	35.69	21.76	13.40	39.66
	SparseGPT	62.14	53.43	29.76	51.22	33.80	19.11	13.40	37.55
	AlphaPruning w. SparseGPT	63.71	52.71	33.11	59.91	38.68	23.89	14.20	40.89
	Ours w. SparseGPT	64.07	53.07	32.74	59.83	38.43	23.55	14.80	40.93
-	Magnitude	48.81	49.10	25.59	48.78	24.92	22.01	14.20	33.34
	AlphaPruning w. Magnitude	44.40	53.07	26.17	50.75	25.67	22.27	14.20	33.79
	Ours w. Magnitude	46.76	53.79	26.08	52.33	24.58	22.95	13.80	34.33
	Wanda	61.69	51.42	25.85	50.24	26.23	20.62	14.00	35.72
0.8	AlphaPruning $w$ . Wanda	52.75	51.62	26.53	48.54	26.81	20.39	11.80	34.06
	Ours w. Wanda	62.14	51.62	26.81	50.20	27.57	20.56	11.40	35.76
	SparseGPT	43.55	52.71	27.87	48.86	29.34	18.34	13.40	33.44
	AlphaPruning w. SparseGPT	61.62	52.35	28.29	52.25	30.64	19.28	12.00	36.63
	Ours w. SparseGPT	62.26	53.43	29.29	54.22	30.18	19.71	13.40	37.50
	Magnitude	48.75	51.62	25.58	48.46	25.76	22.61	14.60	33.91
	AlphaPruning w. Magnitude	43.18	52.71	25.87	49.49	25.00	22.35	14.40	33.29
	Ours w. Magnitude	56.64	48.01	25.60	49.49	25.08	22.44	12.60	34.27
	Wanda	51.74	45.85	26.12	47.59	25.88	20.48	15.00	33.24
0.85	AlphaPruning $w$ . Wanda	38.23	47.29	25.87	50.12	25.76	22.18	12.40	31.69
	Ours w. Wanda	49.91	48.38	25.92	50.12	26.09	20.22	11.00	33.09
	SparseGPT	37.89	53.07	26.69	50.36	26.98	19.28	11.40	32.24
	AlphaPruning w. SparseGPT	57.61	52.35	27.08	48.70	26.64	19.45	12.20	34.86
	Ours w. SparseGPT	55.69	51.62	27.19	52.88	26.98	19.97	12.40	35.25

Table 10. Accuracies (%) of LLaMA-7B for 7 zero-shot tasks with unstructured sparsity from 70% to 85%. We compare FARMS with uniform pruning ratios and AlphaPruning using Magnitude-based pruning, Wanda and SparseGPT.

## C.6. Layer-wise Visualization over Training

We provide visualization to compare how the PL\_Alpha\_Hill and learning rates are distributed over layers during the training with different layer-wise optimization settings. In Figure 12, we report the learning rate and PL\_Alpha\_Hill every five epochs throughout the 200-epoch training duration. We can find that in Figure 12a, 12e and Figure 12b, 12f, if the previous TempBalance does not apply LS to exclude layers with aspect ratio bias (e.g., the first and last layers of the model), the layer-wise learning rate allocation will be inaccurate. This misallocation leads to a few layers having excessively large learning rates while most layers have very small ones. This ultimately results in poor model performance or an imbalanced training process. In contrast, FARMS (see results in Figures 12c, 12d, 12g and 12h) effectively eliminates the measurement inaccuracies caused by aspect ratio bias. As a result, it ensures that learning rates are properly allocated regardless of whether LS is applied.

### **C.7.** Computation Cost Analysis

In Table 12, we report the computational cost of eigenspectrum analysis across different models and methods. The most computation-intensive aspect of these layer analysis methods involves performing SVD on weight matrices, which can be optimized using parallel processing techniques. In the LLM pruning task, we use 8 L40 GPUs for weight analysis and record the PL\_Alpha\_Hill values for each layer. We select four different sparsity ratios, three pruning methods, and two evaluation approaches, yielding a total of 24 experimental configurations per model. However, since weight analysis is

Sparsity Ratio	Method	BoolQ	RTE	HellaSwag	WinoGrande	ARC-e	ARC-c	OBQA	Mean
	Magnitude	52.87	50.54	26.57	50.83	28.45	20.56	14.80	34.95
	AlphaPruning w. Magnitude	61.28	46.93	30.24	50.43	31.23	26.28	21.20	38.23
	Ours w. Magnitude	57.16	43.32	33.90	53.43	34.97	26.88	22.40	38.87
	Wanda	62.08	52.71	30.38	52.41	40.53	17.49	16.00	38.80
0.7	AlphaPruning $w$ . Wanda	63.43	53.43	42.16	65.59	57.53	28.67	21.40	47.46
	Ours w. Wanda	66.18	52.71	41.94	64.80	57.87	28.58	22.20	47.75
	SparseGPT	69.17	52.71	37.25	63.22	51.98	24.83	21.60	45.82
	AlphaPruning w. SparseGPT	72.05	54.15	42.44	68.35	55.60	28.50	22.40	49.07
	Ours w. SparseGPT	73.03	54.51	41.98	69.06	57.45	29.27	22.00	49.61
	Magnitude	45.05	50.90	25.93	50.43	25.93	20.82	13.60	33.24
	AlphaPruning w. Magnitude	60.03	50.90	30.10	52.88	28.20	25.60	19.40	38.16
	Ours w. Magnitude	60.06	44.04	28.29	52.57	32.53	23.89	22.40	37.68
	Wanda	42.51	52.71	27.88	49.64	29.17	17.83	12.00	33.11
0.75	AlphaPruning $w$ . Wanda	62.17	52.71	36.09	62.67	44.15	23.29	18.00	42.73
	Ours w. Wanda	62.42	52.71	35.41	61.33	46.93	23.46	16.40	42.66
	SparseGPT	62.69	52.71	31.66	57.06	40.74	20.56	15.00	40.06
	AlphaPruning w. SparseGPT	64.19	53.07	37.44	64.88	45.79	26.45	17.40	44.17
	Ours w. SparseGPT	65.69	52.71	36.89	64.72	47.43	26.02	17.80	44.47
	Magnitude	38.50	53.43	25.95	48.86	25.51	22.10	13.40	32.53
	AlphaPruning w. Magnitude	53.30	51.26	26.67	51.70	26.60	23.38	16.20	35.59
	Ours w. Magnitude	59.79	48.38	28.32	54.38	27.10	24.23	16.40	36.94
	Wanda	37.86	52.71	26.64	48.30	27.15	19.97	13.20	32.26
0.8	AlphaPruning $w$ . Wanda	62.17	52.71	29.59	55.09	34.93	20.22	14.20	38.42
	Ours w. Wanda	62.17	52.71	30.45	57.38	36.41	21.08	14.40	39.23
	SparseGPT	60.80	52.71	28.71	50.28	30.22	18.26	13.00	36.28
	AlphaPruning w. SparseGPT	62.20	52.71	31.46	57.62	35.48	19.62	14.40	39.07
	Ours w. SparseGPT	62.20	52.71	32.37	59.19	37.16	22.18	15.40	40.18
	Magnitude	38.29	54.51	25.95	49.96	25.21	23.12	14.00	33.01
	AlphaPruning w. Magnitude	39.11	51.26	25.47	50.83	26.39	22.10	16.60	33.11
	Ours w. Magnitude	41.22	51.62	25.73	50.43	26.01	22.01	16.00	33.29
	Wanda	37.83	52.71	26.09	47.91	25.88	21.16	12.80	32.05
0.85	AlphaPruning $w$ . Wanda	37.83	52.71	26.72	49.80	26.09	19.62	11.60	32.05
	Ours w. Wanda	38.65	53.79	26.31	48.46	26.77	20.31	12.60	32.41
	SparseGPT	38.41	52.71	27.31	50.04	26.30	17.66	12.60	32.15
	AlphaPruning w. SparseGPT	62.17	52.71	28.64	51.70	29.67	18.86	13.40	36.73
	Ours w. SparseGPT	62.17	52.71	28.59	54.14	30.09	18.77	12.80	37.04

Table 11. Accuracies (%) of LLaMA-13B for 7 zero-shot tasks with unstructured sparsity from 70% to 85%. We compare FARMS with uniform pruning ratios and AlphaPruning using Magnitude-based pruning, Wanda and SparseGPT.

performed only once per model, the additional computational cost per experiment remains minimal. When the number of submatrices becomes particularly large, the additional computational overhead introduced by FARMS increases accordingly. In CV and SciML experiments, we use a single L40 GPU and do weight analysis every epoch during the training and fine-tuning. Compared to previous methods, the additional computational cost introduced by FARMS is not particularly significant.

We acknowledge that computational cost can indeed be higher than previous methods. But there are certain ways to mitigate the issue, e.g., by using a larger window size (like 4096 for LLaMA 7B/5120 for LLaMA 13B) and a limited number of sampling steps. For example, in Table 6, when we use larger window size(such as 4096) and smaller sampling steps(such as 5), our method can still improve model performance and reduce the computational cost.

### C.8. LLM Pruning Stability Analysis

The experiment results from (Yin et al., 2023) show that layer-wise pruning LLMs at high sparsity is a challenging optimization problem. If sparsity ratios are not properly allocated, the performance of the model can become very unstable. Therefore, here we aim to demonstrate that the performance improvements observed in our experiments are due to more accurate layer-wise analysis, rather than accidental factors such as random seeds.

We follow the experiment settings from Table 6 and visualize the sparsity ratio assigned to each transformer block in





Figure 11. Comparing the distribution of PL Alpha Hill of LLaMA-7B and LLaMA-13B in different layer-wise strategies and sparsity ratios (shown as "SR"). The pruning method is SparseGPT.

Table 12. Computation cost for each experiment.

			Model	Method	Weight Analysis Time (sec/epoch)
			DecNet 19	TB	0.887
			Resident 10	Ours	1.054
Model	Experiment Settings	Weight Analysis Time (sec/experiment)	PacNat 24	TB	1.761
LLaMA-7B	AlphaPruning	2.65 6.08	Ours	1.946	
	Ours (4096, 5)		VCC 16	TB	1.048
	Ours (2000, $15\times15)$	134.5	VGG 10	Ours	1.200
LLaMA-13B	AlphaPruning	6.23	VCC 10	TB	1.421
	Ours (5120, 5)	16.65	VGG 19	Ours	1.578
	Ours (2000, $15 \times 15$ )	163.26	DBOT T	TB_Sig	0.235
			DPO1-1iny	Ours	0.278
			DBOT Small	TB_Sig	1.255
			Dr01-Small	Ours	1 340

LLaMA-7B under different sampling settings in Figure 13. This visualization indirectly reflects the degree of heavytailedness in the ESD of each transformer block's model layer as analyzed by our method. We observe that the differences in sparsity ratio assignments across these settings are not significant. However, the assignment from the best FARMS setting is still distinct from the worst setting, which explains the performance difference in Table 6.

# **D.** Training Quality Analysis

### D.1. Previous work on training quality

Previous work on HT-SR has established that the heavy-tailness of ESDs is strongly correlated with the test accuracy of models (Martin & Mahoney, 2021; Martin et al., 2021). While this does not imply that "training quality" is identical to "test accuracy," the correlation between heavy-tailedness and test accuracy has been used to justify HT-SR metrics. Therefore, improving test accuracy or similar performance metrics (e.g., perplexity) remains our primary goal.

Although previous work on HT-SR does not explicitly define "training quality," several related quantities have been mentioned: (1) strong correlation between weight elements (Martin & Mahoney, 2021; Martin et al., 2021) and (2) feature spikes and their alignment with the target teacher model (Ba et al., 2022; Wang et al., 2023). The feature spike, analyzed in the context of a single-index teacher and a two-layer student model (Ba et al., 2022; Wang et al., 2023), is approximately a rank-one update to the model (in the limit of infinite matrix size with fixed aspect ratio) and also persists after matrix subsampling. This is because the specific form of the rank-one update makes it cover the whole matrix with probability one.



Figure 12. Visualization of layer-wise Learning Rate and PL\_Alpha\_Hill (Alpha) over training. The top rows of each subfigure presents the evolution of layer-wise learning rates and the bottom rows presents PL\_Alpha\_Hill during training for different optimizing configurations of ResNet 34 and VGG 16 on CIFAR 100. The color gradient represents training epochs, transitioning from dark purple (early epochs) to yellow (later epochs).

#### D.2. A toy experiment to measure training quality

We designed a toy experiment to test the correlation between "training quality" and the new HT-SR metric measured using FARMS. Following (Ba et al., 2022; Wang et al., 2023), we use a single-index teacher to generate signals to train a two-layer student. The first layer of the student model is a weight matrix, while the second layer is a weight vector. We only update the weight of first layer. To measure "training quality", during training, we measure the alignment between the weight matrix and the ground truth target vector of the teacher model similar to (Ba et al., 2022; Wang et al., 2023), and we define this alignment to be the "training quality" of the student model.

Throughout the training process, we select the student network checkpoint with the highest alignment and report both the alignment value and the PL\_Alpha\_Hill value. We then vary the sizes of the student model with different weight matrix aspect ratios on a fixed input dimension 500 to conduct multiple experiments. Each experiment provides one



*Figure 13.* Block-wise Sparsity Ratio for LLaMA-7B assigned by FARMS. We set up FARMS with four different window sizes  $\{500, 1000, 2000, 4096\}$  and four sampling setps  $\{5, 10, 15, 20\}$ . The pruning method is SparseGPT and the base sparsity ratio is 0.8. The FARMS Best indicates using this sparsity ratio distribution makes minimal perplexity **79.42**±3.86 while the FARMS worst makes largest perplexity **99.23**±3.53. The distribution of block-wise sparsity ratios of FARMS Average represents the mean under the  $4 \times 4$  settings, with the standard deviation intervals also illustrated.

PL\_Alpha\_Hill value and one alignment value. The multiple experiments (conducted using varying sizes) produce one curve for PL\_Alpha\_Hill and one curve for the alignment value.

We then plot the two curves using both existing methods for estimating PL\_Alpha\_Hill and our method FARMS. As shown in Figure 14, FARMS reveals a clear negative correlation between the two curves: the better the training quality, the larger the alignment, and the smaller the PL\_Alpha\_Hill. However, for the existing method, due to the aspect ratio bias, the correlation is incorrect.

## **E. Broader Discussion**

In this section, we discuss more related literature as well as some explanations regarding aspect ratio bias and issues related to our method.

#### E.1. Relationship with Matrix Shape

The importance of recognizing that the numerical value of the same property measured in different network layers can naturally have different scales, and thus should not be directly compared without proper normalization. In some real-world application scenarios, such as in LLM Pruning (Lu et al., 2024; Liu et al., 2025a), Model Merging (Yang et al., 2024; Zhu et al., 2025), and LLM Reinforcement Learning (Wang et al., 2025; 2024; Chen et al., 2024), this factor need to be considered. Indeed, similar considerations regarding the different shapes of weight matrices in each layer have previously led to impactful theoretical and practical advancements. For example, works such as Tensor Programs IV (Yang & Hu, 2021) and Tensor Programs V (Yang et al., 2022) have explored how layer dimensions affect the optimal scaling of initializations and learning rates. Tensor Programs IV introduced the Maximal Update Parametrization ( $\mu$ P) to ensure feature learning by carefully choosing parameter scaling rules based on these dimensions, addressing how standard parametrizations might otherwise collapse to kernel regimes in the infinite-width limit. Building on this, Tensor Programs V demonstrated that  $\mu$ P can enable zero-shot hyperparameter transfer, where hyperparameters tuned on smaller models remain optimal for significantly larger ones. In this context, "shape" awareness pertains to designing parametrizations that maintain stable training dynamics.

Our work also concerned with the influence of matrix shape, addresses a different challenge: the bias introduced by the aspect ratio (number of rows versus columns) of an individual weight matrix on the measurement of its ESD. We demonstrate that

varying aspect ratios can artificially stretch or compress the ESD. This distortion confounds the interpretation of heavy-tailed (HT) metrics. To mitigate this measurement bias, we propose FARMS to technique analyze average ESD of submatrices sampled at a fixed aspect ratio. This approach provides a normalized HT metric, enabling more reliable comparisons of such spectral diagnostics across different layers within a given network. Thus, while and focus on shape-aware parametrization for training stability and transfer, our contribution lies in a shape-aware analysis technique (FARMS) aimed at correcting measurement bias in spectral diagnostics.

### E.2. Why does the FARMS preserve important spectral property about the original matrix?

The goal of measuring heavy-tailness in HT-SR is to evaluate the strength of correlations introduced by training, as established in previous work (Martin & Mahoney, 2021). However, when we subsample a single submatrix and measure correlations only within that submatrix, some correlations between elements in the subsampled matrix and those outside it are inevitably lost. This motivates our approach of using multiple submatrices to capture a broader range of correlations.

#### E.3. Does this approach introduce additional bias?

Our approach could be viewed as introducing a form of "bias"; however, we interpret this more specifically as achieving partial coverage of the entire matrix. Conceptually, this is similar to the principle behind bootstrap sampling in random forests, where multiple samples, each with potentially limited coverage, are used collectively to mitigate the effects of this partial view and improve overall model robustness.

Further justification for this perspective comes from recent work (Wang et al., 2023; Kothapalli et al., 2024) that aims to theoretically quantify heavy-tailedness. These studies interpret heavy-tailedness as the accumulation and evolution of feature spikes in the ESD that align with the teacher model's features. Critically, these feature spikes are characterized as being approximately rank-one updates to the original matrix Because a rank-one component inherently covers the whole matrix, sampling a submatrix will, with high probability, capture that rank-one component. Therefore, this subsampling process is unlikely to miss the feature spikes, which are identified by previous work as the cause of the heavy-tail structure. We believe this provides substantial evidence that FARMS can preserve important spectral information, specifically as measured by these feature spikes in the ESD.



Figure 14. Compare the PL\_Alpha\_Hill from FARMS and Baseline in measuring the training quality of a single layer. The Correlation Coefficient between FARMS and training quality is -0.89 and for baseline is -0.51. We can find that FARMS can measure the training quality more precisely.

### F. Hyperparameter Adjustment

In this section, we report the hyperparameters that we use in the experiments shown in the main paper (Section 4).

First, we report the common hyperparameters shared by Image Classification experiments (Section 4.3): the optimizer is SGD, batch size 128, number of total training epochs 200, weight decay 5e-4, and momentum 0.9. For each experiment setting, we repeat our experiments with three random seeds  $\{43, 37, 13\}$ . We also report the mean and standard deviation of the test accuracy across these seeds. In Table 13, we report the details of experiments for each model and method. We use the same learning rate range from (Zhou et al., 2024) and we expand the scaling ratio range into [(0.1, 1.9), (0.2, 1.8), (0.3, 1.9)]

1.7), (0.4, 1.6), (0.5, 1.5), (0.6, 1.4), (0.7, 1.3), (0.8, 1.2), (0.9, 1.1)] nine choices.

Second, we provide the hyperparameters used in experiments of LLM pruning and SciML. We follow the common hyperparameter settings as described in Lu et al. (2024); Liu et al. (2024). See more details for other hyperparameters like  $\tau$  in LLM pruning and scaling ratios in SciML in Table 14.

Finally, we report the detailed matrix subsampling settings used in every model in Table 15. We cannot use a very small window size or sampling steps because doing so may not cover the entire matrix. Conversely, selecting a very large size would result in too much overlap between sampled matrices. For ResNet, VGG and DPOT series models, we use the minimum dimension of the weight matrices to construct the sampled submatrices based on the parameter Q. We also select the  $\lfloor m/n \rfloor$  for the submatrices number, where m, n is the dimension of weight matrix,  $m \ge n$ . But for the final layer of ResNet and VGG models, we select nine submatrices based on experiments results. For LLaMA series models, we apply sliding window sampling using multiple moderately sized submatrices, resulting in a smoother PL\_Alpha\_Hill estimation.

Model	Method	Initial	Scaling Ratio	Test Accuracy	
		Learning Rate	$(s_1,s_2)$		
	CAL	0.05, <b>0.1</b> , 0.15	-	$78.23{\scriptstyle\pm0.087}$	
	TB(no LS)	0.1	(0.6, 1.4)	$78.76{\scriptstyle\pm0.111}$	
ResNet 18	LS+TB	0.1	(0.2, 1.8)	$79.31{\scriptstyle \pm 0.180}$	
	Ours(no LS)	0.1	(0.1, 1.9)	$79.49{\scriptstyle\pm0.080}$	
	LS+Ours	0.1	(0.1, 1.9)	$79.53{\scriptstyle \pm 0.177}$	
	CAL	<b>0.05</b> , 0.1, 0.15	-	$78.99{\scriptstyle \pm 0.137}$	
	TB(no LS)	0.1	(0.5, 1.5)	$79.64{\scriptstyle\pm0.029}$	
ResNet 34	LS+TB	0.1	(0.3, 1.7)	$80.00{\scriptstyle\pm0.090}$	
	Ours(no LS)	0.1	(0.2, 1.8)	$80.17{\scriptstyle\pm0.213}$	
	LS+Ours	0.1	(0.3, 1.7)	$80.20{\scriptstyle\pm0.221}$	
	CAL	0.025, <b>0.05</b> , 0.1	-	$74.30{\scriptstyle\pm0.078}$	
	TB(no LS)	0.05	(0.6, 1.4)	$74.43{\scriptstyle \pm 0.158}$	
VGG 16	LS+TB	0.05	(0.3, 1.7)	$75.19 \pm 0.131$	
	Ours(no LS)	0.05	(0.2, 1.8)	$75.36{\scriptstyle \pm 0.118}$	
	LS+Ours	0.05	(0.2, 1.8)	$75.15{\scriptstyle \pm 0.247}$	
	CAL	0.025, <b>0.05</b> , 0.1	-	73.11±0.113	
	TB(no LS)	0.05	(0.6, 1.4)	$73.22 \pm 0.277$	
VGG 19	LS+TB	0.05	(0.2, 1.8)	$74.19{\scriptstyle \pm 0.159}$	
	Ours(no LS)	0.05	(0.2, 1.8)	$74.28{\scriptstyle\pm0.392}$	
	LS+Ours	0.05	(0.4, 1.6)	$73.99{\scriptstyle\pm0.300}$	

Table 13. Parameter settings of the experiment reported in Section 4.3.

Table 14. Hyperparameters for LLaMA and DPOT models. (Left) The range of  $\tau$  used for LLM pruning. (Right) Learning rate and scaling ratio settings for DPOT series models at different subsampling ratios.

		Model	DPOT-Tiny		DPOT-Small	
Sparsity Ratio	LLaMA-7B/13B	Hyperparameters	Learning Rate	Scaling Ratio	Learning Rate	Scaling Ratio
0.7	0.1, 0.2, 0.3, 0.4, 0.5, 0.6	5%	2.5e-4	(1.0, 1.0)	1e-4	(1.0, 1.0)
0.75	0.1, 0.2, 0.3, 0.4, 0.5, 0.6	10%	2.5e-4	(1.0, 1.0)	1e-4	(1.0, 1.0)
0.8	0.1, 0.2, 0.25, 0.3, 0.4, 0.5	25%	2.5e-4	(1.0, 1.0)	2.5e-4	(1.0, 1.0)
0.85	0.1, 0.15, 0.2, 0.25, 0.3	50%	5e-4	(1.0, 1.0)	2.5e-4	(1.0, 1.0)
		100%	5e-4	(1.0, 1.0)	2.5e-4	(1.0, 1.0)

Table 15. Subsampling Hyperparameters for different models.

Model	Aspect Ratio(Q)	Window Size	Submatrices Number
ResNet 18/34, VGG 16/19	1.0	Minimum Dimension	$\lfloor m/n  floor$ , 9
DPOT-Tiny/Small	1.0	Minimum Dimension	$\lfloor m/n  floor$
	1.0	2000	$15 \times 15$
LLaMA-7B	1.0	2000	$10 \times 10$
	1.0	1000	$10 \times 10$
LLaMA-13B	1.0	2000	$15 \times 15$