A Novel Ant Colony Optimization Approach for Core–Periphery Detection in Networks

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Introduction

In network science, mesoscale structures have gained significant attention in recent years. Algorithmic detection of these mesoscale structures enables discovery of network characteristics that are not easily observable at the local level of nodes and edges or through global summary statistics [1]. Among the various types of mesoscale structures that emerge in networks, core-periphery structures have received particular focus due to their distinct organization and relevance across diverse domains. In a core-periphery structure, the core nodes are adjacent to each other and adjacent to some periphery nodes, while the periphery nodes are not adjacent to each other [1].

Optimization problems are of great importance in both industrial and scientific domains. Ant Colony Optimization(ACO), introduced in the early 1990s, is a metaheuristic designed to solve complex combinatorial optimization problems [3]. This paper introduces an approach to detecting the core-periphery structure using artificial ants. The proposed approach, inspired by the foraging behavior of ants, employs artificial pheromone trails to construct and refine solutions iteratively, thus eliminating the need for arbitrary partitions that often constrain traditional methods. Our method is applied to various real-world networks, highlighting its adaptability and robustness. We systematically compare the performance of our approach with established core-periphery detection techniques, highlighting differences in node classification between the core and the periphery. Benchmarking against existing methods by Rossa [5], Rombach [4], and Boyd et al. [2] highlights the superior performance of our proposed technique, showcasing its enhanced flexibility and precision. This study advances the field of network analysis and sets a precedent for the integration of bioinspired algorithms in the study of complex systems.

Proposed Method

Let $G = (V, E)$ be a weighted, simple, and undirected graph, where V denotes the set of vertices, and E represents the set of edges. Each edge $(i, j) \in E$ is assigned a weight w_{ij}, which represents the strength of the connection between vertices i and j. We employ the original Ant System (AS) for our analysis, which iteratively updates pheromone values on edges, allowing us to identify high-weight core edges in the network. The pheromone update mechanism, a critical part of AS, reinforces the edges that form a part of high-quality solutions (i.e., stronger core-periphery structures). The update formula for edge pheromone levels (i, j) is as follows:

$$\tau_{ij} \leftarrow (1-\rho) \cdot \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k} \tag{1}$$

where $\rho \in (0, 1]$ is the rate of evaporation of the pheromone, which prevents the system from overcommitting to early solutions, and m is the number of ants.

$\Delta\tau_{ij}^{k}$ is the pheromone deposited on the edge (i, j) by the ant k, defined as:

$$\Delta\tau_{ij}^{k} = \begin{cases} \frac{1}{L_k}, & \text{if ant } k \text{ uses edge } (i, j), \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

where L_k represents the total accumulated weight of the path constructed by ant k.

The transition probability p_{ij}^{k} of ant k, currently located at the vertex i, moving from the vertex i to j is given by the following:

$$p_{ij}^{k} = \begin{cases} \frac{(\tau_{ij})^{\alpha} \cdot (\eta_{ij})^{\beta}}{\sum\limits_{l \in N_i^k} (\tau_{il})^{\alpha} \cdot (\eta_{il})^{\beta}} & \text{if } j \in N_i^k, \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where N_i^k denotes the set of feasible neighbors of the ant k when at the vertex i. The parameters α and β control the relative importance of the pheromone and heuristic information η_{ij}, where $\eta_{ij} = w_{ij}$, representing the weight between the vertices i and j.

We define the cohesiveness measure Ψ_S for a subgraph $S \subseteq G$ as

$$\Psi_S = \frac{\sum_{i \in S, j \in S} (\tau_{ij})^{\alpha} (\eta_{ij})^{\beta}}{\sum_{l \in S, i \in V(G)} (\tau_{il})^{\alpha} (\eta_{il})^{\beta}} \tag{4}$$

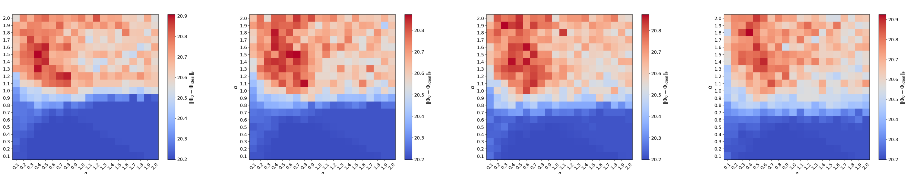
This measure Ψ_S reflects the likelihood that an ant, currently positioned at any vertex in S, remains within S in the next step, thus serving as an indicator of the cohesiveness of the subset within the network.

The determination of the *coreness* for each vertex in the network is carried out by an iterative procedure. The process begins by selecting the vertex with the smallest weighted degree and defining the set containing this vertex as S_1. Without loss of generality, let $S_1 = \{1\}$, and set the cohesiveness measure $\Psi_1 := \Psi_{S_1} = 0$. In the subsequent step, we consider the subsets $S_2^{(j)} := S_1 \cup \{j\}$ for all $2 \leq j \leq N$ and compute the cohesiveness measure $\Psi_{S_2^{(j)}}$ for each subset. Let $\Psi_{S_2^{(k)}}$ represent the minimum cohesiveness measure among all the subsets. We then update the set to $S_2 := S_2^{(k)}$ and assign the coreness value $\Psi_2 := \Psi_{S_2^{(k)}} = \Psi_{S_2}$, which corresponds to the *coreness* of vertex k. In particular, S_2 now consists of two vertices with the lowest transition probability, ensuring the condition $\Psi_1 \leq \Psi_2$. This procedure is iterated to construct S_3 from S_2, compute Ψ_3, the coreness of the third vertex, and maintain the inequality $\Psi_1 \leq \Psi_2 \leq \Psi_3$. The process continues in this manner until all vertices are included in the set. Ultimately, we obtain the sequence $\Psi_1 \leq \Psi_2 \leq \cdots \leq \Psi_N$.

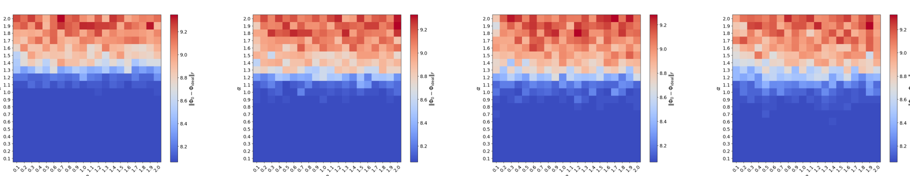
Results & Discussion

Data Description: The data sets used in this study cover a wide array of network types. These include Zachary's Karate Club, Florentine Families, Davis Southern Women, Krackhardt Kite, Les Misérables, Word Adjacencies, American College Football, Dolphins, and Books About US Politics (compiled by V. Krebs, available at http://www.orgnet.com).

Sensitivity Analysis: For sensitivity analysis, we computed the difference in the Frobenius norm between the normalized permuted matrix Φ_0 and the idealized core-periphery matrix Φ_{ideal}. From the figure 1, it is clear that as the parameters α and β increase from their initial values, the difference in the Frobenius norm between the permuted adjacency matrix and the ideal core-periphery matrix remains relatively stable. However, once α surpasses a threshold of 1; this difference increases for any value of β. This result is consistent with various values of the evaporation rate, including $\rho = 0.1$, 0.3, 0.5, and 0.7. Therefore, for the rest of the paper, we use the optimum values of $\alpha = 0.5$, $\beta = 1$ and $\beta = 0.5$.



$\rho = 0.1$ $\rho = 0.3$ $\rho = 0.5$ $\rho = 0.7$



$\rho = 0.1$ $\rho = 0.3$ $\rho = 0.5$ $\rho = 0.7$

Fig. 1: Frobenius norm difference $\|\Phi_{\text{ideal}} - \Phi_0\|_F$ as a function of parameters α and β for varying values of ρ, on (a) Zachary's Karate Club and (b) Florentine Families networks.

Comparative Study: Figure 2 shows the ground-truth adjacency matrices for Word Adjacencies and American College Football. The matrices are ordered by decreasing core scores obtained from our proposed method and the Rossa, Rombach, and Boyd methods. The results indicate that our method produces clearer core-core, core-periphery, and periphery-periphery blocks compared to the Rombach and Boyd methods, while performing competitively with the Rossa method. Similar results were obtained for other networks.

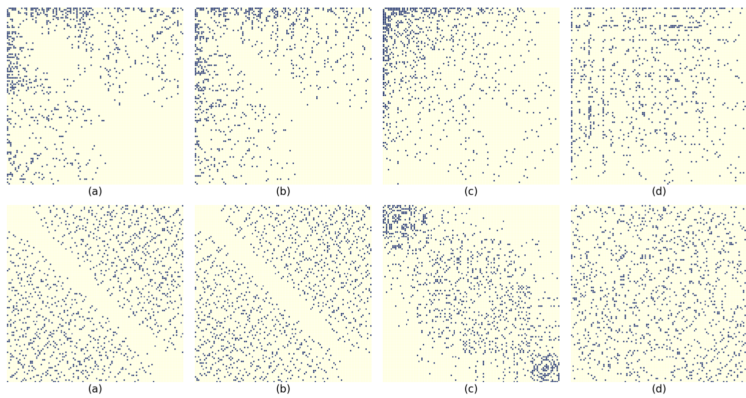


Fig. 2: Ground-truth adjacency matrices for Word Adjacencies and American College Football. The matrices are ordered in decreasing order of core scores obtained using (a) the proposed method, (b) the Rossa method, (c) the Rombach method, and (d) the Boyd method.

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