FREEDYG: FREQUENCY ENHANCED CONTINUOUS-TIME DYNAMIC GRAPH MODEL FOR LINK PREDIC-TION

Yuxing Tian^{1*†} Yiyan Qi^{1*‡} FanGuo² ¹IDEA Research, International Digital Economy Academy ²Jiangxi Normal University

Abstract

Link prediction is a crucial task in dynamic graph learning. Recent advancements in continuous-time dynamic graph models, primarily by leveraging richer temporal details, have significantly improved link prediction performance. However, due to their complex modules, they still face several challenges, such as overfitting and optimization difficulties. More importantly, it is challenging for these methods to capture the 'shift' phenomenon, where node interaction patterns change over time. To address these issues, we propose a simple yet novel method called Frequency Enhanced Continuous-Time Dynamic Graph (FreeDyG) model for link prediction. Specifically, we propose a node interaction frequency encoding module that both explicitly captures the proportion of common neighbors and the frequency of the interaction of the node pair. Unlike previous works that primarily focus on the time domain, we delve into the frequency domain, allowing a deeper and more nuanced extraction of interaction patterns, revealing periodic and "shift" behaviors. Extensive experiments conducted on seven real-world continuous-time dynamic graph datasets validate the effectiveness of FreeDyG. The results consistently demonstrate that FreeDyG outperforms existing methods in both transductive and inductive settings. Our code is available at this repository: https://github.com/Tianxzz/FreeDyG

1 INTRODUCTION

Link prediction on dynamic graphs is a fundamental problem in various real-world applications, such as social media analysis Huo et al. (2018); Alvarez-Rodriguez et al. (2021), recommendation systems Song et al. (2019); Yuxiao et al. (2012); Wang et al. (2021b), and drug discovery Abbas et al. (2021). Recently, dynamic graph representation learning methods have become widespread in both industry and academia due to their remarkable performance in solving this problem.

Existing dynamic graph learning methods can be divided into two main categories: discrete-time dynamic graph (DTDG) models Pareja et al. (2020); Zhao et al. (2019); Yang et al. and continuoustime dynamic graph (CTDG) models Kumar et al. (2019); Wang et al. (2021c); Xu et al. (2020); Trivedi et al. (2019); Yu et al. (2023); Wang et al. (2021a;d); Jin et al. (2022a); Luo et al. (2023); Chang et al. (2020); Huang et al. (2020). Notably, there has been a growing interest in CTDG algorithms, primarily because of their ability to preserve time information effectively.

Although the above CTDG methods have achieved impressive results, they still have some limitations. Firstly, some methods rely on random walks (RW) Wang et al. (2021d); Jin et al. (2022b), temporal point processes (TPP) Chang et al. (2020); Huang et al. (2020), or ordinary differential equations (ODE) Luo et al. (2023); Liang et al. (2022) to enhance their learning ability. However, these methods are computationally expensive and may lead to overfitting of the historical interactions. Secondly, most existing methods encode interaction sequences independently, disregarding potential relationships between them. Although Yu et al. (2023) proposes the neighbor

^{*}Equal contribution.

[†]Work done during internship at IDEA Research.

[‡]Corresponding author.

co-occurrence mechanism, it relies solely on the number of historical common neighbors between node pairs to predict the likelihood of node pairs interacting in the future. As a result, it is difficult for them to capture the "shift" phenomenon which is commonly hidden behind time series data. For instance, in a co-author network, researchers who are acquainted with each other tend to collaborate during the same period each year, as conferences are typically held around the same time annually. However, when a new and popular research topic emerges suddenly, such as large language models (LLMs), there is a notable shift in these collaboration patterns. Researchers then show a preference for establishing collaborations with unfamiliar researchers who specialize in that area.

To address the above challenges, we delve into the frequency domain and propose a simple yet novel method called **Fre**quency Enhanced Continuous-Time **Dy**namic **G**raph (**FreeDyG**) model for link prediction. FreeDyG comprises two essential components: the encoding layer and the frequency-enhanced MLP-Mixer layer. The encoding layer is designed to transform each interaction (u, v, t) into an embedding sequence, considering the historical neighbors of both node u and v. In contrast to existing methods, we also introduce a new node interaction frequency encoding approach to explicitly capture the interaction frequency between node pairs. Then, we propose a novel frequency-enhanced MLP-Mixer layer to efficiently capture the periodic temporal patterns and the "shift" phenomenon hidden in the frequency domain. Specifically, we apply the Fourier Transform to the encoded embedding sequence, followed by multiplication with a learnable complex number tensor. This can adaptively enhance desired frequencies, thereby improving the model's ability to capture relevant information.

Our contributions can be summarized as follows: Firstly, we introduce a novel frequency-enhanced dynamic graph model called FreeDyG for the task of link prediction. Instead of the temporal domain, FreeDyG tries to address this problem by delving into the frequency domain. Secondly, in addition to the traditional encoding of node/link features and time information, we propose a node interaction frequency encoding approach that explicitly captures the frequency of neighbor nodes from both the source and target nodes. Furthermore, we design a novel frequency-enhanced MLP-Mixer layer to further capture periodic temporal patterns and the "shift" phenomenon present in the frequency domain. Finally, we conduct extensive experiments on seven widely used real-world continuous-time dynamic graph datasets, evaluating both transductive and inductive settings. The experimental results demonstrate the superior performance of FreeDyG compared to state-of-the-art methods.

2 PRELIMINARIES

Task Definition. Following prevailing methods Poursafaei et al. (2022); Yu et al. (2023); Rossi et al. (2020), our study primarily focuses on CTDGs with edge addition events. A CTDG G can be represented as a chronological sequence of interactions between specific node pairs: $G = \{(u_0, v_0, t_0), (u_1, v_1, t_1), \ldots, (u_n, v_n, t_n)\}$, where t_i denotes the timestamp and the timestamps are ordered as $(0 \le t_0 \le t_1 \le \ldots \le t_n)$. $u_i, v_i \in V$ denote the node IDs of the i - th interaction at timestamp t_i , V is the entire node set. Each node $u \in V$ is associated with node feature $x_u \in \mathbb{R}^{d_N}$, and each interaction (u, v, t) has edge feature $e_{u,v}^t \in \mathbb{R}^{d_E}$, where d_N and d_E denote the dimensions of the node and link feature respectively. Based on the above definitions, link prediction in CTDGs can be formulated as: given a pair of nodes with a specific timestamp t, we aim to predict whether the two nodes are connected at t based on all the available historical data.

Discrete Fourier Transform (DFT). Given a sequence of data $\{x_n\}_{n=1}^N$, the Discrete Fourier Transform (DFT) is utilized to convert the sequence into the frequency domain. The DFT operation can be defined as:

$$X_{k} = \sum_{k=1}^{N} x_{n} e^{-\frac{2\pi i}{N}nk}, \quad 1 \le k \le N$$
(1)

where *i* denotes the imaginary unit and $\{X_k\}_{k=1}^N$ is a sequence of complex numbers in the frequency domain. Thus, X_k captures the spectrum of the sequence $\{x_n\}_{n=1}^N$ at the frequency $\omega_k = 2\pi k/M$. We note that the DFT is a one-to-one transformation, enabling the recovery of the original sequence through the inverse DFT (IDFT):

$$x_n = \frac{1}{N} \sum_{k=1}^{N} X_k e^{\frac{2\pi i}{N}nk}$$
(2)



Figure 1: The overview of FreeDyG.

The computation complexity of DFT is $O(N^2)$. In practice, the Fast Fourier Transform (FFT) is commonly used to compute the DFT efficiently, which could recursively express the DFT of a sequence of length N and reduce the computation complexity from $O(N^2)$ to $O(N \log N)$. Similarly, the IDFT in Equation 2 can be efficiently computed using the inverse Fast Fourier Transform (IFFT). Due to the page limit, we omit the details of DFT and FFT, which can be found in Bruun (1978).

3 Metholodgy

In this section, we provide a detailed description of our FreeDyG. As shown in Figure 1, the model operates on a node pair (u, v) and a specific timestamp (t). Initially, we sample L first-hop historical neighbors for both nodes, ordered based on temporal proximity, to construct two interaction sequences, S_u^t and S_v^t . If a node has fewer than L historical neighbors, zero-padding is used to fill the gap. Subsequently, we employ an encoding layer to encode the features of each node, link, and timestamp within the sequences. Additionally, we incorporate the frequencies of neighbor appearances in both S_u^t and S_v^t to exploit correlations between nodes. The encoded representations are then aligned and fed into a frequency-enhanced MLP-Mixer layer, enabling the capture of information at different frequencies. The outputs are aggregated to generate time-aware representations of u and v at timestamp t (i.e., h_u^t and h_v^t). The final prediction is generated by the link prediction layer.

3.1 ENCODING LAYER

Node/Edge Encoding. In dynamic graphs, both nodes and edges (links) frequently possess associated features. To derive embeddings for interactions, it is sufficient to extract the intrinsic features of the neighboring nodes and edges based on the sequence S_*^t , where * can be either u or v. Similar to existing works, we encode the nodes and links as $Z_{*,N}^t \in \mathbb{R}^{L \times d_N}$ and $Z_{*,E}^t \in \mathbb{R}^{L \times d_E}$ respectively, where d_N , d_E are the dimensions of node and edge embeddings respectively.

Time Encoding. Time encoding is employed to map constant timestamps to vectors $Z_{*,T}^t \in \mathbb{R}^{L \times d_T}$, where d_T is the dimension of time embeddings. Specifically, we utilize the widely adopted time encoding function $cos(t_n\omega)$, where $\omega = \left\{\alpha^{-(i-1)/\beta}\right\}_{i=1}^{d_T}$ is employed to encode timestamps. α and β are hyperparameters to make $t_{max} \times \alpha^{-(i-1)/\beta}$ close to 0 when i close to d_T . A cosine function is then applied to project these values into the range of [-1, +1]. Notably, we use relative timestamps instead of absolute timestamps for encoding. In other words, if the timestamp of the sampled interaction is t_0 , and the specific timestamp for link prediction is t, we utilize $cos((t_n-t)\omega)$ as the effective relative time encoding function. It is worth mentioning that ω remains constant and is not updated during the training phase. This technique makes the model easy to optimize and leads to performance improvement, as demonstrated in Cong et al. (2023).

Node Interaction Frequency (NIF) Encoding. Most existing methods encode interaction sequences, i.e., S_u^t and S_v^t , independently, disregarding potential relationships between them. Yu et al. (2023) introduces a neighbor co-occurrence scheme, suggesting that nodes with a higher number of shared historical neighbors are more likely to interact in the future. However, while a significant portion of edges in networks exhibit recurring patterns over time, these recurrence patterns vary considerably across different networks and domains Poursafaei et al. (2022).

To address this issue, we introduce a node interaction frequency encoding approach, which takes into account not only the appearance frequency of common neighbors but also the interaction frequency between the node pairs. It recognizes that the potential for future interactions between two nodes is influenced by both their common neighbors and their past interactions. Specifically, given the interaction sequences S_u^t and S_v^t , we count the frequency of each neighbor in both S_u^t and S_v^t . In addition, We specifically encode for the frequency of the interaction of the node pair and get the node interaction frequency features for u and v, which are represented by $Z_{*,C}^t$. For example, suppose the historical interacted nodes of u and v are $\{c, v, c, d, v\}$ and $\{d, u, d, c, u\}$. The appearing frequencies of c, d in u/v's historical interactions are 2/1, 1/2 respectively. And the node pair interaction frequency is [2, 2]. Therefore, the node interaction frequency features of u and v are $v_v = [[1, 2], [2, 2], [1, 2], [2, 2]]^T$. Then, we encode the node interaction frequency features by:

$$Z_{*,F}^{t} = f\left(F_{*}^{t}[:,0]\right) + f\left(F_{*}^{t}[:,1]\right) \in \mathbb{R}^{L \times d_{F}},\tag{3}$$

where * could be u or v. f() represents a two-layer perceptron with ReLU activation. And the dimension of input and output of f() are 1 and d_F .

Finally, we concatenate all encodings mentioned above to an embedding of dimension d with trainable weight $W_* \in R^{d_* \times d}$ and $b_* \in R^d$, resulting in $Z_{u,*}^t \in R^{l_u^t \times d}$ and $Z_{v,*}^t \in R^{l_v^t \times d}$. Here, * can be N, E, T or F. Subsequently, the concatenated encodings for nodes are summed as $Z_u^t \in \mathbb{R}^{L \times d} = Z_{u,N}^t + Z_{u,E}^t + Z_{u,T}^t + Z_{u,F}^t$ and $Z_v^t \in \mathbb{R}^{L \times d} = Z_{v,N}^t + Z_{v,F}^t + Z_{v,F}^t$, respectively.

3.2 FREQUENCY-ENHANCED MLP-MIXER LAYER

We derive the historical interaction embedding sequences for the node pair by utilizing the encoding layer. Since each interaction embedding is arranged chronologically, it can be viewed as discrete time series data. Consequently, we naturally leverage the Fourier transform to decompose of time series data into their constituent frequencies, which can effectively capture the interaction patterns of nodes across various frequencies. To this end, we introduce the frequency-enhanced MLP-Mixer layer, which contains two sublayers: the frequency-enhancing layer and the MLP-Mixer layer.

Frequency Enhancing (FE) layer. Given the input Z_*^l of the *l*-th layer and the first layer input is Z_*^t . For simplicity, Z_*^l is short for $Z_*^{l,t}$. To better identify important frequencies within the historical interaction sequence data, we perform the Fast Fourier Transform (FFT) along the first dimension of $Z_*^l \in \mathbb{R}^{L \times d}$ to convert it into the frequency domain:

$$\mathcal{Z}_*^l = \mathcal{F}(Z_*^l),\tag{4}$$

where \mathcal{F} denotes the 1D FFT. $\mathcal{Z}_*^l \in \mathbb{C}^{\{\frac{L}{2}+1\}\times d}$ denotes the frequency components of Z_*^l and \mathbb{C} denotes the complex number domain. Then we can adaptively enhance the frequency components by multiplying it with a learnable complex number tensor $\mathcal{W} \in \mathbb{C}^{\{\frac{L}{2}+1\}\times d}$.

$$\widehat{\mathcal{Z}}_*^l = \mathcal{W} \cdot \mathcal{Z}_*^l \tag{5}$$

where \cdot denotes the element-wise multiplication, $\hat{\mathcal{Z}}_*^l \in \mathbb{C}^{\{\frac{L}{2}+1\}\times d}$ represents the enhanced frequency components. Finally, we transform $\hat{\mathcal{Z}}_*^l$ back to the time domain.

$$\widetilde{Z}_*^l \leftarrow \mathcal{F}^{-1}\left(\widehat{\mathcal{Z}}_*^l\right). \tag{6}$$

where $\mathcal{F}^{-1}()$ denotes the inverse 1D FFT, which converts the complex number tensor into a real number tensor. Then we use the residual connection and dropout layer as:

$$Z_*^l = Z_*^l + \text{Dropout}\left(\widetilde{Z}_*^l\right) \tag{7}$$

MLP-Mixer layer. After that, we employ MLP-Mixer Tolstikhin et al. (2021) to further capture the non-linearity characteristics, which contains two types of layers: token mixing MLP layer and channel mixing MLP layer. The computation is listed as:

$$Z_{*,\text{token}}^{l} = Z_{*}^{l} + \mathbf{W}_{\text{token}}^{2} \sigma(\mathbf{W}_{\text{token}}^{1} \text{LayerNorm}(Z_{*}^{l})),$$

$$Z_{*,\text{channel}}^{l} = Z_{*,\text{token}}^{l} + \sigma(\text{LayerNorm}(Z_{*,\text{token}}^{l})\mathbf{W}_{\text{channel}}^{1})\mathbf{W}_{\text{channel}}^{2}$$
(8)

where σ is an element-wise nonlinearity, $\mathbf{W}^*_{\text{token}} \in \mathbb{R}^{d \times d}$ and $\mathbf{W}^*_{\text{channel}} \in \mathbb{R}^{d \times d}$ are trainable parameters in the token-mixing and channel-mixing MLPs, respectively. The same channel-mixing MLP layer (token-mixing MLP layer) is applied to every row (column) of input.

Theorem 1. Given an input $Z \in \mathbb{R}^{N \times D}$ and a learnable matrix $W \in \mathbb{R}^{N \times D}$ and their corresponding frequency domain conversion, the multiplication in the frequency domain is equivalent to the global convolution in the time domain.

Proof. We regard $Z \in \mathbb{R}^{N \times D}$ as $\{z_n \in \mathbb{R}^D\}_{n=1}^N$ and $W \in \mathbb{R}^{N \times D}$ as $\{w_n \in \mathbb{R}^D\}_{n=1}^N$. Denote the *d*-th dimension features of *Z* and *W* as $\{z_n^{(d)}\}_{n=1}^N$ and $\{w_n^{(d)}\}_{n=1}^N$. Then, the global convolution can be defined as:

$$\{z_n^{(d)}\}_{n=1}^N \circledast \{w_n^{(d)}\}_{n=1}^N = \sum_{m=1}^N w_m^{(d)} \cdot z_{(n-m) \bmod N}^{(d)}$$
(9)

where \circledast and mod denote the convolution and integer modulo operation, respectively. Then, according to Equation 1, the multiplication in the frequency domain can be written as:

$$\mathcal{DFT}(\{w_n^{(d)}\}_{n=1}^N) \cdot \mathcal{DFT}(\{z_n^{(d)}\}_{n=1}^N) = w_k^{(d)} \sum_{n=1}^N z_n^{(d)} e^{-\frac{2\pi i}{N}kn}$$

$$= w_k^{(d)} \left(\sum_{n=1}^{N-m} z_n^{(d)} e^{-\frac{2\pi i}{N}kn} + \sum_{n=N-m}^N z_n^{(d)} e^{-\frac{2\pi i}{N}kn} \right)$$

$$= w_k^{(d)} \left(\sum_{n=m}^N z_{n-m}^{(d)} e^{-\frac{2\pi i}{N}k(n-m)} + \sum_{n=1}^m z_{n-m+N}^{(d)} e^{-\frac{2\pi i}{N}k(n-m)} \right)$$

$$= \sum_{m=1}^N w_m^{(d)} e^{-\frac{2\pi i}{N}km} \sum_{n=1}^N z_{(n-m)\%N}^{(d)} e^{-\frac{2\pi i}{N}k(n-m)}$$

$$= \mathcal{DFT}(\{z_n^{(d)}\}_{n=1}^N \circledast \{w_n^{(d)}\}_{n=1}^N)$$
(10)

3.3 AGGREGATOR

Different from most previous methods that simply aggregate the sequence of each appearing element using average pooling, we get the time-aware representations $h_u^t \in \mathbb{R}^d$ and $h_v^t \in \mathbb{R}^d$ of node u and v at timestamp t by the following weighted aggregation equation:

$$h_*^t = \left(\left(W^{agg} \cdot Z^l_{*,\text{channel}} \right)^T \cdot Z^l_{*,\text{channel}} \right)^T \tag{11}$$

where $W^{agg} \in \mathbb{R}^{1 \times L}$ is a trainable vector designed to adaptively learn the importance of various interactions.

3.4 LINK PREDICTION LAYER

The prediction \hat{y} is computed by adopting a 2-layer multilayer perceptron (MLP) and using softmax to convert it into link probability on the concatenation of the above two nodes' representation:

$$\hat{y} = Softmax(MLP(RELU(MLP(h_u^t || h_v^t)))).$$
(12)

3.5 Loss Function

For link prediction loss, we adopt binary cross-entropy loss function, which is defined as:

$$\mathcal{L}_p = -\frac{1}{K} \sum_{i=1}^{S} (y_i * \log \hat{y}_i + (1 - y_i) * \log(1 - \hat{y}_i)),$$
(13)

where K is the number of positive/negative samples, y_i represents the ground-truth label of *i*-th sample and the \hat{y}_i represents the prediction value.



Figure 2: Comparison of model performance, parameter size and training time per epoch on WIKI and Reddit.

4 EXPERIMENTS

4.1 DATASETS

We utilize seven publicly available real-world datasets: Wiki, REDDIT, MOOC, LastFM, Enron, Social Evo, and UCI, in our study. Appendix A provides a detailed description and the statistics of the datasets are shown in Table 4. The sparsity of the graphs is quantified using the density score, calculated as $\frac{2|E|}{|V|(|V|-1)}$, where |E| and |V| represent the number of links and nodes in the training set, respectively. To facilitate training, validation, and testing, we split these datasets into three chronological segments with ratios of 70%-15%-15%.

4.2 **BASELINES**

To evaluate the performance of our method, we conduct experiments comparing it with previous state-of-the-arts, including JODIE (Kumar et al., 2019), DyRep (Trivedi et al., 2019), TGAT (Xu et al., 2020), TGN (Rossi et al., 2020), CAWN (Wang et al., 2021d), EdgeBank (Poursafaei et al., 2022), TCL (Wang et al., 2021a), GraphMixer (Cong et al., 2023), DyGFormer (Yu et al., 2023). Detailed descriptions of these methods can be found in Appendix B.

4.3 EVALUATION METRICS AND EXPERIMENTAL SETTINGS

For evaluating our method, we employ Average Precision (AP) and Area Under the Receiver Operating Characteristic Curve (AUC-ROC) as the evaluation metrics. The link prediction task encompasses two settings: 1) **transductive setting**, which focuses on predicting future links between nodes observed during training, and 2) **inductive setting**, which involves predicting future links between unseen nodes. To ensure comprehensive comparisons, we also evaluate each method with three negative sampling strategies as Poursafaei et al. (2022), i.e., random (rnd), historical (hist), and inductive (ind) negative sampling, which the latter two are more challenging. The evaluation details of all three strategies can be found in Appendix C.

All models are trained for a maximum of 200 epochs using the early stopping strategy with patience of 20. The model that achieves the highest performance on the validation set is selected for testing. For all models, we employ the Adam optimizer and set the learning rate and batch size to 0.0001 and 200, respectively. The hyperparameter configurations of the baselines align with those specified in their respective papers. For our FreeDyG, we set the d_T to 100,and both α and β to 10. The number of frequency-enhanced MLP-Mixer layers are 2. We conduct ten runs of each method with different seeds and report the average performance to eliminate deviations. All experiments are performed on an NVIDIA A100-SXM4 40GB GPU.

4.4 COMPARISON WITH SOTA

In this section, we compare our FreeDyG with the previous SOTA in both transductive and inductive settings. Table 10 and Table 2 show the AP of all datasets in these two settings respectively. To provide a more comprehensive study of our FreeDyG, we show results among all three negative sampling strategies. Due to the limitation of space, we put similar results on AUC-ROC in Table 5 and Table 6 in Appendix. We note that EdgeBank Poursafaei et al. (2022) is only designed for the Published as a conference paper at ICLR 2024

NSS D	Datasets	IODIE									
-	Succes	JODIE	DyRep	TGAT	TGN	CAWN	EdgeBank	TCL	GraphMixer	DyGFormer	FreeDyG
	Wiki	96.69 ± 0.25	95.23 ± 0.50	96.95 ± 0.17	98.42 ± 0.05	98.65 ± 0.04	90.37 ± 0.00	96.47 ± 0.16	97.17 ± 0.05	$\underline{98.82\pm0.02}$	99.26 ± 0.01
I	Reddit	$97.83 {\pm} 0.21$	$98.17 {\pm} 0.02$	$98.47 \!\pm\! 0.03$	$98.67 {\pm} 0.04$	$99.11 {\pm} 0.01$	$94.86{\pm}0.00$	$97.61 \!\pm\! 0.03$	$97.37{\pm}0.01$	$\underline{99.11{\pm}0.02}$	$99.48{\pm}0.01$
N	моос	77.20 ± 1.32	$79.97 {\pm} 0.82$	$85.44 {\pm} 0.76$	<u>89.43±2.95</u>	$78.66 {\pm} 0.31$	$57.97{\pm}0.00$	$81.12 {\pm} 0.43$	82.73 ± 0.16	$87.23 {\pm} 0.45$	$89.61{\pm}0.19$
rnd L	LastFM	68.54±2.95	$70.79 {\pm} 1.87$	$73.76 {\pm} 0.45$	$78.69 {\pm} 2.71$	$86.58 {\pm} 0.10$	$79.29{\pm}0.00$	$65.64{\pm}2.52$	$75.64 {\pm} 0.23$	$\underline{92.07{\pm}0.28}$	$92.15{\pm}0.16$
1	Enron	79.10±0.85	82.02 ± 3.07	$72.58 {\pm} 0.79$	85.33±1.05	89.56±0.09	$83.53 {\pm} 0.00$	$79.70 {\pm} 0.71$	$81.08 {\pm} 0.73$	92.47±0.12	$92.51{\pm}0.05$
Soc	cial Evo.	88.12±0.74	88.87±0.30	93.16±0.17	93.57±0.17	84.96±0.09	$74.95 {\pm} 0.00$	93.13±0.16	93.37±0.07	<u>94.73±0.01</u>	94.91±0.01
	UCI	87.65±1.85	$70.24 {\pm} 0.32$	$79.55 {\pm} 0.83$	90.69±0.45	94.35±0.11	$76.20{\pm}0.00$	$88.12 {\pm} 2.73$	93.50±0.49	<u>95.76±0.15</u>	96.28±0.11
Avg.R	Rank	7.14	6.08	5.85	3.31	2.94	8.79	4.56	5.23	2.14	1
	Wiki	81.19±0.48	78.32±0.71	87.01±0.19	86.96±0.36	72.38±1.85	73.35 ± 0.00	88.75±0.27	90.87±0.08	82.23 ± 2.54	91.59±0.57
I	Reddit	80.03 ± 0.36	79.83 ± 0.31	79.55 ± 0.20	$\underline{81.75\pm0.36}$	80.82 ± 0.45	73.59 ± 0.00	77.14 ± 0.16	78.44 ± 0.18	81.02 ± 0.59	85.67 ± 1.01
N	моос	78.94 ± 1.25	75.60 ± 1.12	82.19 ± 0.62	87.06 ± 1.93	74.05 ± 0.95	60.71 ± 0.00	77.06 ± 0.41	77.77 ± 0.92	85.85 ± 0.66	86.71 ± 0.81
hist L	LastFM	74.35 ± 3.81	74.92 ± 2.46	71.59 ± 0.24	76.87 ± 4.64	69.86 ± 0.43	73.03 ± 0.00	59.30 ± 2.31	72.47 ± 0.49	81.57 ± 0.48	$\underline{79.71 \pm 0.51}$
1	Enron	69.85 ± 2.70	71.19 ± 2.76	64.07 ± 1.05	73.91 ± 1.76	64.73 ± 0.36	76.53 ± 0.00	70.66 ± 0.39	$\underline{77.98 \pm 0.92}$	75.63 ± 0.73	$\textbf{78.87} \pm \textbf{0.82}$
So	ocial Evo.	87.44 ± 6.78	93.29 ± 0.43	95.01 ± 0.44	94.45 ± 0.56	85.53 ± 0.38	80.57 ± 0.00	94.74 ± 0.31	94.93 ± 0.31	$\underline{97.38 \pm 0.14}$	97.79 ± 0.23
	UCI	75.24 ± 5.80	55.10 ± 3.14	68.27 ± 1.37	80.43 ± 2.12	65.30 ± 0.43	65.50 ± 0.00	80.25 ± 2.74	<u>84.11 ± 1.35</u>	82.17 ± 0.82	86.10 ± 1.19
Avg.R	Rank	5.46	5.08	5.08	3.85	7.54	5.92	5.46	4.00	2.85	1.28
	Wiki	75.65 ± 0.79	70.21 ± 1.58	87.00 ± 0.16	85.62 ± 0.44	74.06 ± 2.62	80.63 ± 0.00	86.76 ± 0.72	$\underline{88.59 \pm 0.17}$	78.29 ± 5.38	90.05 ± 0.79
I	Reddit	86.98 ± 0.16	86.30 ± 0.26	89.59 ± 0.24	88.10 ± 0.24	91.67 ± 0.24	85.48 ± 0.00	87.45 ± 0.29	85.26 ± 0.11	91.11 ± 0.40	90.74 ± 0.17
N	моос	65.23 ± 2.19	61.66 ± 0.95	75.95 ± 0.64	77.50 ± 2.91	73.51 ± 0.94	49.43 ± 0.00	74.65 ± 0.54	74.27 ± 0.92	$\underline{81.24 \pm 0.69}$	83.01 ± 0.87
ind L	LastFM	62.67 ± 4.49	64.41 ± 2.70	71.13 ± 0.17	65.95 ± 5.98	67.48 ± 0.77	75.49 ± 0.00	58.21 ± 0.89	68.12 ± 0.33	$\underline{73.97 \pm 0.50}$	72.19 ± 0.24
1	Enron	68.96 ± 0.98	67.79 ± 1.53	63.94 ± 1.36	70.89 ± 2.72	75.15 ± 0.58	73.89 ± 0.00	71.29 ± 0.32	75.01 ± 0.79	$\underline{77.41 \pm 0.89}$	77.81 ± 0.65
So	ocial Evo.	89.82 ± 4.11	93.28 ± 0.48	94.84 ± 0.44	95.13 ± 0.56	88.32 ± 0.27	83.69 ± 0.00	94.90 ± 0.36	94.72 ± 0.33	97.68 ± 0.10	$\underline{97.57 \pm 0.15}$
	UCI	65.99 ± 1.40	54.79 ± 1.76	68.67 ± 0.84	70.94 ± 0.71	64.61 ± 0.48	57.43 ± 0.00	76.01 ± 1.11	$\underline{80.10\pm0.51}$	72.25 ± 1.71	82.35 ± 0.73
Avg.R	Rank	6.62	6.38	4.15	4.38	5.46	5.62	4.69	4.46	2.43	1.71

Table 1: AP for transductive link prediction with three different negative sampling strategies.

NSS	Datasets	JODIE	DyRep	TGAT	TGN	CAWN	TCL	GraphMixer	DyGFormer	FreeDyG
	Wikipedia	94.82 ± 0.20	92.43 ± 0.37	96.22 ± 0.07	97.83 ± 0.04	98.24 ± 0.03	96.22 ± 0.17	96.65 ± 0.02	$\underline{98.59 \pm 0.03}$	98.97±0.01
	Reddit	96.50 ± 0.13	96.09 ± 0.11	97.09 ± 0.04	97.50 ± 0.07	98.62 ± 0.01	94.09 ± 0.07	95.26 ± 0.02	$\underline{98.84 \pm 0.02}$	98.91 ± 0.01
	MOOC	79.63 ± 1.92	81.07 ± 0.44	85.50 ± 0.19	89.04 ± 1.17	81.42 ± 0.24	80.60 ± 0.22	81.41 ± 0.21	86.96 ± 0.43	87.75±0.62
rnd	LastFM	81.61 ± 3.82	83.02 ± 1.48	78.63 ± 0.31	81.45 ± 4.29	89.42 ± 0.07	73.53 ± 1.66	82.11 ± 0.42	$\underline{94.23 \pm 0.09}$	94.89 ± 0.01
	Enron	80.72 ± 1.39	74.55 ± 3.95	67.05 ± 1.51	77.94 ± 1.02	86.35 ± 0.51	76.14 ± 0.79	75.88 ± 0.48	89.76 ± 0.34	$\underline{89.69 \pm 0.17}$
	Social Evo.	91.96 ± 0.48	90.04 ± 0.47	91.41 ± 0.16	90.77 ± 0.86	79.94 ± 0.18	91.55 ± 0.09	91.86 ± 0.06	$\underline{93.14\pm0.04}$	94.76 ± 0.05
	UCI	79.86 ± 1.48	57.48 ± 1.87	79.54 ± 0.48	88.12 ± 2.05	92.73 ± 0.06	87.36 ± 2.03	91.19 ± 0.42	$\underline{94.54\pm0.12}$	94.85 ± 0.10
A	vg.Rank	6.14	6.14	5.45	3.71	3.14	4.56	4.45	<u>1.71</u>	1.14
	Wikipedia	68.69 ± 0.39	62.18 ± 1.27	84.17 ± 0.22	81.76 ± 0.32	67.27 ± 1.63	82.20 ± 2.18	87.60 ± 0.30	71.42 ± 4.43	82.78 ±0.30
	Reddit	62.34 ± 0.54	61.60 ± 0.72	63.47 ± 0.36	64.85 ± 0.85	63.67 ± 0.41	60.83 ± 0.25	64.50 ± 0.26	$\underline{65.37 \pm 0.60}$	66.02 ± 0.41
	MOOC	63.22 ± 1.55	62.93 ± 1.24	76.73 ± 0.29	77.07 ± 3.41	74.68 ± 0.68	74.27 ± 0.53	74.00 ± 0.97	$\underline{80.82\pm0.30}$	81.63 ± 0.33
hist	LastFM	70.39 ± 4.31	71.45 ± 1.76	76.27 ± 0.25	66.65 ± 6.11	71.33 ± 0.47	65.78 ± 0.65	$\underline{76.42 \pm 0.22}$	76.35 ± 0.52	$\textbf{77.28} \pm \textbf{0.21}$
	Enron	65.86 ± 3.71	62.08 ± 2.27	61.40 ± 1.31	62.91 ± 1.16	60.70 ± 0.36	67.11 ± 0.62	$\underline{72.37 \pm 1.37}$	67.07 ± 0.62	$\textbf{73.01} \pm \textbf{0.88}$
	Social Evo.	88.51 ± 0.87	88.72 ± 1.10	93.97 ± 0.54	90.66 ± 1.62	79.83 ± 0.38	94.10 ± 0.31	94.01 ± 0.47	96.82 ± 0.16	$\underline{96.69 \pm 0.14}$
	UCI	63.11 ± 2.27	52.47 ± 2.06	70.52 ± 0.93	70.78 ± 0.78	64.54 ± 0.47	76.71 ± 1.00	82.66 ± 0.49	72.13 ± 1.87	$\underline{82.35\pm0.39}$
A	vg.Rank	5.71	6.14	5.46	3.71	4.14	3.85	2.85	<u>2.57</u>	1.28
	Wikipedia	68.70 ± 0.39	62.19 ± 1.28	84.17 ± 0.22	81.77 ± 0.32	67.24 ± 1.63	82.20 ± 2.18	87.60 ± 0.29	71.42 ± 4.43	$\underline{87.54\pm0.26}$
	Reddit	62.32 ± 0.54	61.58 ± 0.72	63.40 ± 0.36	64.84 ± 0.84	63.65 ± 0.41	60.81 ± 0.26	64.49 ± 0.25	65.35 ± 0.60	$\underline{64.98\pm0.20}$
	MOOC	63.22 ± 1.55	62.92 ± 1.24	76.72 ± 0.30	77.07 ± 3.40	74.69 ± 0.68	74.28 ± 0.53	73.99 ± 0.97	$\underline{80.82\pm0.30}$	81.41 ± 0.31
ind	LastFM	70.39 ± 4.31	71.45 ± 1.75	76.28 ± 0.25	69.46 ± 4.65	71.33 ± 0.47	65.78 ± 0.65	$\underline{76.42 \pm 0.22}$	76.35 ± 0.52	$\textbf{77.01} \pm \textbf{0.43}$
	Enron	65.86 ± 3.71	62.08 ± 2.27	61.40 ± 1.30	62.90 ± 1.16	60.72 ± 0.36	67.11 ± 0.62	$\underline{72.37 \pm 1.38}$	67.07 ± 0.62	$\textbf{72.85} \pm \textbf{0.81}$
	Social Evo.	88.51 ± 0.87	88.72 ± 1.10	93.97 ± 0.54	90.65 ± 1.62	79.83 ± 0.39	94.10 ± 0.32	94.01 ± 0.47	$\underline{96.82\pm0.17}$	96.91 ± 0.12
	UCI	63.16 ± 2.27	52.47 ± 2.09	70.49 ± 0.93	70.73 ± 0.79	64.54 ± 0.47	76.65 ± 0.99	$\underline{81.64\pm0.49}$	72.13 ± 1.86	82.06 ± 0.58
A	vg.Rank	6.62	7.14	4.15	4.14	5.71	4.57	3.57	2.43	1.28

Table 2: AP for inductive link prediction with three different negative sampling strategies.

transductive setting and thus in Table 2 and 6 we omit this method. We find that our FreeDyG outperforms other baselines in most scenarios and the average ranking of FreeDyG is close to 1, which is far superior to the second one, i.e., DyGFormer. In addition, it's notable that DyGFormer, when adopting the "hist" negative sampling strategy, experiences a much more significant performance decline compared to the "rnd" strategy. This underscores a limitation in DyGFormer's neighbor encoding approach, which simply relies on the historical co-occurrence counts of neighbors to determine the likelihood of node interactions. In contrast, our method takes into account the historical frequency patterns of node interactions, leading to improved performance.



Figure 3: Ablation study of FreeDyG under historical negative sampling setting, where w/o NS and w/o FE represent FreeDyG without node interaction frequency encoding module and FE layer, respectively.

We also conduct a comparative analysis of performance, training time per epoch (measured in seconds), and the size of trainable parameters (measured in Megabyte, i.e., MB) between FreeDyG and baseline methods on the WIKI and Reddit datasets. The results are shown in Figure 2. It is evident that the RW-based method, i.e., CAWN, not only requires a longer training time but also has a substantial number of parameters. On the other hand, simpler methods such as JODIE may have fewer parameters, but there is a significant performance gap compared to the best-performing method. In contrast, FreeDyG achieves the best performance with a small size of trainable parameters and a moderate training time required per epoch.

4.5 Ablation Study

Next, we conduct an ablation study on our FreeDyG model to assess the effectiveness of the proposed node interaction frequency encoding approach and frequency-enhancing layer. We refer to FreeDyG without these two modules as **w/o NIF** and **w/o FE** respectively. Figure 3 illustrates the performance comparison under the historical negative sampling setting. Due to the page limit, the performance comparison under the random negative sampling setting is shown in Figure 5 in the Appendix. Both variants exhibit inferior performance across all datasets, highlighting the efficacy of the modules we proposed. It is noteworthy that under different negative sampling settings, distinct modules exhibit varying degrees of importance. In the historical negative sampling setting, the performance decreases more significantly without FE layer. Whereas, under the random negative sampling setting, the NIF encoding module has a more pronounced impact on performance.

The reason behind this phenomenon is as follows: under the random negative sampling setting, where the target node of a negative edge is sampled from the entire graph, the negative samples are significantly easier to distinguish, and the model tends to predict future interactions between node pairs that have previously interacted. Consequently, the NIF encoding, which captures neighbor interactions, assumes a more critical role. However, in the historical negative sampling setting, where the target node of a negative edge is sampled from the source node's historical neighbors, the NIF encoding may introduce more false positives on negative samples. This necessitates the importance of FE layer in capturing temporal or shifting signals. It is evident that the FE component holds greater significance in this setting.

4.6 HYPERPARAMETER STUDY

In this section, we analyze the impact of the number of sampled neighbors (L) on the performance of FreeDyG. The results are presented in Table 3. We observe that the performance tends to be suboptimal when only a few neighbors are sampled (e.g., L = 10), primarily due to the lack of sufficient information. Furthermore, the optimal number of sampled neighbors varies across different datasets.

Published as a conference paper at ICLR 2024

	L=	:10	L=20		L=	=32	L=	:64	L=100	
	AP	AUC-ROC								
Wikipedia	99.15 ± 0.01	99.30 ± 0.01	99.26 ± 0.01	99.41 ± 0.01	99.23 ± 0.01	99.40 ± 0.01	99.14 ± 0.03	99.19 ± 0.02	99.01 ± 0.04	99.10 ± 0.09
Reddit	99.00 ± 0.02	99.06 ± 0.02	99.21 ± 0.01	99.22 ± 0.01	99.37 ± 0.01	99.40 ± 0.01	99.48 ± 0.01	99.50 ± 0.01	99.45 ± 0.01	99.46 ± 0.01
MOOC	87.10 ± 0.83	87.44 ± 1.01	88.01 ± 0.30	88.17 ± 0.39	89.26 ± 0.20	89.37 ± 0.61	89.61 ± 0.19	89.93 ± 0.35	88.57 ± 0.52	88.71 ± 0.74
LastFM	85.71 ± 0.22	86.64 ± 0.20	87.14 ± 0.20	87.82 ± 0.26	91.13 ± 0.17	90.71 ± 0.20	90.71 ± 0.22	91.53 ± 0.19	92.15 ± 0.16	93.42 ± 0.15
Enron	91.46 ± 0.19	92.09 ± 0.33	92.51 ± 0.05	94.01 ± 0.11	92.24 ± 0.08	93.71 ± 0.09	92.43 ± 0.12	94.00 ± 0.20	91.93 ± 0.31	92.21 ± 0.40
Social Evo.	94.01 ± 0.04	95.74 ± 0.05	94.58 ± 0.02	96.46 ± 0.4	94.91 ± 0.01	96.59 ± 0.04	94.03 ± 0.07	95.70 ± 0.13	94.06 ± 0.20	94.91 ± 0.18
UCI	95.58 ± 0.19	94.21 ± 0.33	96.23 ± 0.11	94.97 ± 0.26	96.28 ± 0.11	95.00 ± 0.21	95.93 ± 0.17	94.46 ± 0.28	95.99 ± 0.27	94.52 ± 0.43

Table 3: Performance comparison of different sample numbers of historical neighbors.

Specifically, datasets characterized by a higher frequency of the "shift" phenomenon (e.g., MOOC, LastFM) Poursafaei et al. (2022) require a larger number of samples to effectively capture hidden patterns and achieve optimal performance. This arises from the essential requirement of FreeDyG to learn from more nuanced frequency domain features of interaction behaviors over time (i.e., DFT of a longer time sequence comprises a greater number of refined frequency domain components).

5 RELATED WORK

Dynamic graph representation learning. Existing methods can be roughly grouped into two categories: discrete-time dynamic graph (DTDG) models and continuous-time dynamic graph (CTDG) models. DTDG models typically generate discrete snapshots of graphs and fuse information extracted from different snapshots Pareja et al. (2020); Zhao et al. (2019); Yang et al.. However, they often suffer from information loss, as time discretization can miss out on capturing important interactions. To solve it, there is growing interest in designing CTDG models that treat dynamic graph data as link streams and directly learn node representations from interactions that occur continuously. Specifically, existing CTDG models Xu et al. (2020); Wang et al. (2021c;a) commonly employ Recurrent Neural Networks (RNNs) or Self-Attention mechanisms as their basic mechanism. Building upon this, some methods incorporate additional techniques like memory networks Rossi et al. (2020); Kumar et al. (2019); Trivedi et al. (2019), ordinary differential equations (ODE) Luo et al. (2023); Liang et al. (2022), random walk (RW) Wang et al. (2021d); Jin et al. (2022b), temporal point processes (TPP) Chang et al. (2020); Huang et al. (2020) to better learn the continuous temporal information.

Fourier Transform. Fourier Transform is a basic technique in the digital signal processing domain Reddy (2018); Pitas (2000) over the decades. Numerous studies have integrated it into areas like computer vision Huang et al. (2023); Wang et al. (2020) and natural language processing Tamkin et al. (2020); Lee-Thorp et al. (2021). More recent research efforts have sought to harness the power of Fourier transform-enhanced models for tasks such as long-term time series forecasting Wu et al. (2023); Zhou et al. (2022; 2021).

6 CONCLUSION

In this paper, we present FreeDyG, a frequency-enhanced continuous-time dynamic graph model designed specifically for link prediction. Our approach includes a novel frequency-enhanced MLP-Mixer layer, which effectively captures periodic temporal patterns and the "shift" phenomenon observed in the frequency domain. We also introduce a node interaction frequency encoder that simultaneously extracts the information of interaction frequency and the proportion of common neighbors between node pairs. Extensive experiments conducted on various real-world datasets demonstrate the effectiveness and efficiency of our proposed FreeDyG model. In future work, we plan to extend our method to handle continuous-time dynamic graphs with both edge insertion and deletion events.

7 ACKNOWLEDGEMENT

The research presented in this paper is supported in part by the National Natural Science Foundation of China (62372362).

REFERENCES

- Khushnood Abbas, Alireza Abbasi, Shi Dong, Ling Niu, Laihang Yu, Bolun Chen, Shi-Min Cai, and Qambar Hasan. Application of network link prediction in drug discovery. *BMC Bioinformatics*, 22, 04 2021. doi: 10.1186/s12859-021-04082-y.
- Unai Alvarez-Rodriguez, Federico Battiston, Guilherme Ferraz de Arruda, Yamir Moreno, Matjaž Perc, and Vito Latora. Evolutionary dynamics of higher-order interactions in social networks. *Nature Human Behaviour*, 5:1–10, 05 2021. doi: 10.1038/s41562-020-01024-1.
- G. Bruun. z-transform dft filters and fft's. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 26(1):56–63, 1978.
- Xiaofu Chang, Xuqin Liu, Jianfeng Wen, Shuang Li, Yanming Fang, Le Song, and Yuan Qi. Continuous-time dynamic graph learning via neural interaction processes. In *CIKM*, 2020.
- Weilin Cong, Si Zhang, Jian Kang, Baichuan Yuan, Hao Wu, Xin Zhou, Hanghang Tong, and Mehrdad Mahdavi. Do we really need complicated model architectures for temporal networks? In *ICLR*, 2023.
- Zhipeng Huang, Zhizheng Zhang, Cuiling Lan, Zheng-Jun Zha, Yan Lu, and Baining Guo. Adaptive frequency filters as efficient global token mixers. In *ICCV*, 2023.
- Zijie Huang, Yizhou Sun, and Wei Wang. Learning continuous system dynamics from irregularlysampled partial observations. In *NeurIPS*, 2020.
- Zepeng Huo, Xiao Huang, and Xia Hu. Link prediction with personalized social influence. *AAAI*, 32, 04 2018.
- Ming Jin, Yuan-Fang Li, and Shirui Pan. Neural temporal walks: Motif-aware representation learning on continuous-time dynamic graphs. In *NeurIPS*, 2022a.
- Ming Jin, Yuan-Fang Li, and Shirui Pan. Neural temporal walks: Motif-aware representation learning on continuous-time dynamic graphs. In *NeurIPS*, 2022b.
- Srijan Kumar, Xikun Zhang, and Jure Leskovec. Predicting dynamic embedding trajectory in temporal interaction networks. In *KDD*, 2019.
- James Lee-Thorp, Joshua Ainslie, Ilya Eckstein, and Santiago Ontanon. Fnet: Mixing tokens with fourier transforms. *arXiv:2105.03824*, 2021.
- Bo Liang, Lin Wang, and Xiaofan Wang. Autoregressive gnn-ode gru model for network dynamics. arXiv:2211.10594, 2022.
- Linhao Luo, Gholamreza Haffari, and Shirui Pan. Graph sequential neural ODE process for link prediction on dynamic and sparse graphs. In Tat-Seng Chua, Hady W. Lauw, Luo Si, Evimaria Terzi, and Panayiotis Tsaparas (eds.), *WSDM*, 2023.
- Aldo Pareja, Giacomo Domeniconi, Jie Chen, Tengfei Ma, Toyotaro Suzumura, Hiroki Kanezashi, Tim Kaler, Tao B. Schardl, and Charles E. Leiserson. Evolvegcn: Evolving graph convolutional networks for dynamic graphs. In AAAI, 2020.
- Ioannis Pitas. Digital Image Processing Algorithms and Applications. John Wiley & Sons, Inc., USA, 1st edition, 2000. ISBN 0471377392.
- Farimah Poursafaei, Shenyang Huang, Kellin Pelrine, and Reihaneh Rabbany. Towards better evaluation for dynamic link prediction. In *NeurIPS*, 2022.
- G. P. Obi Reddy. *Digital Image Processing: Principles and Applications*. Springer International Publishing, 2018.
- Emanuele Rossi, Ben Chamberlain, Fabrizio Frasca, Davide Eynard, Federico Monti, and Michael M. Bronstein. Temporal graph networks for deep learning on dynamic graphs. *CoRR*, abs/2006.10637, 2020.

- Weiping Song, Zhiping Xiao, Yifan Wang, Laurent Charlin, Ming Zhang, and Jian Tang. Sessionbased social recommendation via dynamic graph attention networks. In WSDM, 2019.
- Alex Tamkin, Dan Jurafsky, and Noah Goodman. Language through a prism: A spectral approach for multiscale language representations. In *NeurIPS*, 2020.
- Ilya Tolstikhin, Neil Houlsby, Alexander Kolesnikov, Lucas Beyer, Xiaohua Zhai, Thomas Unterthiner, Jessica Yung, Daniel Keysers, Jakob Uszkoreit, Mario Lucic, and Alexey Dosovitskiy. Mlp-mixer: An all-mlp architecture for vision, 2021.
- Rakshit S. Trivedi, Mehrdad Farajtabar, Prasenjeet Biswal, and Hongyuan Zha. Dyrep: Learning representations over dynamic graphs. In *ICLR*, 2019.
- Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N. Gomez, Łukasz Kaiser, and Illia Polosukhin. Attention is all you need. In *NeurIPS*, 2017.
- Haohan Wang, Xindi Wu, Zeyi Huang, and Eric P. Xing. High-frequency component helps explain the generalization of convolutional neural networks. In *CVPR*, pp. 8681–8691, 2020.
- Lu Wang, Xiaofu Chang, Shuang Li, Yunfei Chu, Hui Li, Wei Zhang, Xiaofeng He, Le Song, Jingren Zhou, and Hongxia Yang. TCL: transformer-based dynamic graph modelling via contrastive learning. *CoRR*, abs/2105.07944, 2021a.
- Wen Wang, Wei Zhang, Shukai Liu, Qi Liu, Bo Zhang, Leyu Lin, and Hongyuan Zha. Incorporating link prediction into multi-relational item graph modeling for session-based recommendation. *IEEE Transactions on Knowledge and Data Engineering*, PP:1–1, 09 2021b.
- Xuhong Wang, Ding Lyu, Mengjian Li, Yang Xia, Qi Yang, Xinwen Wang, Xinguang Wang, Ping Cui, Yupu Yang, Bowen Sun, and Zhenyu Guo. APAN: asynchronous propagation attention network for real-time temporal graph embedding. In SIGMOD, 2021c.
- Yanbang Wang, Yen-Yu Chang, Yunyu Liu, Jure Leskovec, and Pan Li. Inductive representation learning in temporal networks via causal anonymous walks. In *ICLR*, 2021d.
- Haixu Wu, Tengge Hu, Yong Liu, Hang Zhou, Jianmin Wang, and Mingsheng Long. Timesnet: Temporal 2d-variation modeling for general time series analysis. In *ICLR*, 2023.
- Da Xu, chuanwei ruan, evren korpeoglu, sushant kumar, and kannan achan. Inductive representation learning on temporal graphs. In *ICLR*, 2020.
- Menglin Yang, Min Zhou, Marcus Kalander, Zengfeng Huang, and Irwin King. Discrete-time temporal network embedding via implicit hierarchical learning in hyperbolic space. In *KDD*.
- Le Yu, Leilei Sun, Bowen Du, and Weifeng Lv. Towards better dynamic graph learning: New architecture and unified library. *CoRR*, abs/2303.13047, 2023.
- Dong Yuxiao, Jie Tang, Sen Wu, Jilei Tian, Nitesh Chawla, Jinghai Rao, and Huanhuan Cao. Link prediction and recommendation across heterogeneous social networks. In *ICDM*, 2012.
- Ling Zhao, Yujiao Song, Chao Zhang, Yu Liu, Pu Wang, Tao Lin, Min Deng, and Haifeng Li. T-gcn: A temporal graph convolutional network for traffic prediction. *IEEE transactions on intelligent* transportation systems, 21(9):3848–3858, 2019.
- Haoyi Zhou, Shanghang Zhang, Jieqi Peng, Shuai Zhang, Jianxin Li, Hui Xiong, and Wancai Zhang. Informer: Beyond efficient transformer for long sequence time-series forecasting. In AAAI, 2021.
- Tian Zhou, Ziqing Ma, Qingsong Wen, Xue Wang, Liang Sun, and Rong Jin. FEDformer: Frequency enhanced decomposed transformer for long-term series forecasting. In *ICML*, 2022.

Dataset	Nodes	Edges	Unique Edges	Node/Link Feature	Time Granularity	Duration	density
WIKI	9227	157474	18257	0/172	Unix timestamp	1 month	4.30E-03
REDDIT	10984	672447	78516	0/172	Unix timestamp	1 month	8.51E-03
MOOC	7144	411749	178443	0/4	Unix timestamp	17 month	1.26E-02
LastFm	1980	1293103	154993	0/0	Unix timestamp	1 month	5.57E-01
Enron	184	125235	3125	0/0	Unix timestamp	3 years	5.53E+00
Social Evo.	74	2099519	4486	0/2	Unix timestamp	8 months	5.36E+02
UCI	1899	59835	20296	0/0	Unix timestamp	196 days	3.66E-02

DETAILS OF DATASETS. А

Table 4: Dataset statistics

Wiki¹: A dataset tracking user edits on Wikipedia pages, is represented as a bipartite interaction graph that contains interactions(edits) between users and Wikipedia pages. Nodes represent users and pages, and links denote the editing behaviors with timestamps. Each interaction is associated with a 172-dimensional Linguistic Inquiry and Word Count (LIWC) feature. This dataset additionally contains dynamic labels that indicate whether users are temporarily banned from editing.

Reddit²: A dataset tracking users posting in Reddit, is bipartite and records the posts of users under subreddits for one month. Users and subreddits are the nodes, and links are the timestamped posting requests. Each link has a 172-dimensional LIWC feature. This dataset also includes dynamic labels representing whether users are banned from posting.

MOOC³: is a bipartite interaction network of online sources, where nodes are students and course content units (e.g., videos and problem sets). Each link denotes a student's access behavior to a specific content unit and is assigned a 4-dimensional feature.

LastFM⁴: is bipartite and consists of the information about which songs were listened to by which users over one month. Users and songs are nodes, and links denote the listening behaviors of users.

Enron⁵: is an email correspondence dataset containing around 50K emails exchanged among employees of the ENRON energy company over a three-year period. This dataset has no attributes.

Social Evo.⁶: is a mobile phone proximity network that monitors the daily activities of an entire undergraduate dormitory for a period of eight months, where each link has a 2-dimensional feature.

UCI⁷: is a Facebook-like, unattributed online communication network among students of the University of California at Irvine, along with timestamps with the temporal granularity of seconds.

В DETAIL DESCRIPTIONS OF BASELINES

JODIE is an RNN-based method. Denote $x_i(t)$ as the embedding of node v_i at timestamp $t, x_{i,i}^{link}(t)$ as the link feature between v_i , v_j at timestamp t, and m_i as the timestamp that v_i latest interact with other node. When an interaction between v_i , v_j happens at timestamp t, JODIE updates the temporal embedding using RNN by $x_i(t) = RNN(x_i(m_i), x_j(m_j), x_{ij}^{link}(t), t - m_i)$, Then, the embedding of node v_i at timestamp t_0 is computed by $h_i(t_0) = (1 + (t_0 - m_i)w) \cdot x_i(m_i)$.

TGAT is a self-attention-based method that could capture spatial and temporal information simultaneously. TGAT first concatenates the raw feature x_i with a trainable time encoding z(t), i.e., $x_i(t) = [x_i||z(t)]$ and z(t) = cos(tw + b). Then, self-attention is applied to produce node representation $hi(t_0) = SAM(x_i(t_0), x_u(h_u) | u \in N_{t_0}(i))$, where $N_{t_0}(i)$ denotes the neighbors of node v_i

¹Download from http://snap.stanford.edu/jodie/wikipedia.csv

²Download from http://snap.stanford.edu/jodie/reddit.csv

³Download from http://snap.stanford.edu/jodie/mooc.csv

⁴Download from http://snap.stanford.edu/jodie/lastfm.csv

⁵Download from https://zenodo.org/record/7213796#.Y1cO6y8r30o

⁶Download from https://zenodo.org/record/7213796#.Y1cO6y8r30o

⁷Download from https://zenodo.org/record/7213796#.Y1cO6y8r30o

at time t_0 and h_u denotes the timestamp of the latest interaction of node u. Finally, the prediction on any node pair at time t_0 is computed by $MLP([h_i(t_0)||h_j(t_0)])$.

TGN is a mixture of RNN- and self-attention-based method. TGN utilizes a memory module to store and update the (memory) state $s_i(t)$ of node *i*. The state of node *i* is expected to represent *i*'s history in a compressed format. Given the memory updater as mem, when an link $e_{ij}(t)$ connecting node *i* is observed, node *i*'s state is updated as $s_i(t) = mem(s_i(t^-), s_j(t^-)||e_{ij}(t))$. where $s_i(t^-)$ is the memory state of node *i* just before time *t*. || is the concatenation operator, and node *j* is *i*'s neighbor connected by $e_{ij}(t)$. The implementation of mem is a recurrent neural network (RNN), and node *i*'s embedding is computed by aggregating information from its K-hop temporal neighborhood using self-attention.

DyRep is an RNN-based method that updates node states upon each interaction. It also includes a temporal-attentive aggregation module to consider the temporally evolving structural information in dynamic graphs.

DyGFormer is a self-attention based method. Specifically, for node n_i , DyGFormer just retrieves the features of involved neighbors and links based on the given features to represent their encodings. Instead of learning at the interaction level, DyGFormer splits each source/destination node's sequence into multiple patches and then feeds them to transformer (Vaswani et al., 2017).

GraphMixer is a simple MLP-based architecture, that uses a fixed time encoding function rather than the trainable version and incorporates it into a link encoder based on MLP-Mixer to learn from temporal links. A node encoder with neighbor mean-pooing is employed to summarize node features.

TCL is a self-attention based method. It first generates each node's interaction sequence by performing a breadth-first search algorithm on the temporal dependency interaction sub-graph. Then, it presents a graph transformer that considers both graph topology and temporal information to learn node representations. It also incorporates a cross-attention operation for modeling the interdependencies of two interaction nodes.

CAWN is a mixer of RNN- and self-attention-based method that proposes to represent network dynamics by extracting temporal network motifs using temporal random walks (CAWs). CAWs replace node identities with the hitting counts of the nodes based on a set of sampled walks to establish the correlation between motifs. Then, the extracted motifs are fed into RNNs to encode each walk as a representation and use self-attention to aggregate the representations of multi-walks into a single vector for downstream tasks.

EdgeBank is a pure memory-based approach without trainable parameters for transductive dynamic link prediction. It stores the observed interactions in the memory unit and updates the memory through various strategies. An interaction will be predicted as positive if it is retained in the memory, and negative otherwise.

C EVALUATION DETAILS OF SAMPLING STRATEGIES

In the evaluation stage, we combine the original test set, treated as positive samples, with additional negative samples. Specifically, the ratio of positive samples to negative samples is set to 1:1. We employ three different negative sampling strategies, as described in Poursafaei et al. (2022): 1) **Random Negative Sampling Strategy**: This strategy randomly samples negative edges from nearly all possible node pairs within the graphs. 2) **Historical Negative Sampling Strategy**: Here, negative edges are sampled from the set of edges observed in previous timestamps but are absent in the current step. 3) **Inductive Negative Sampling Strategy**: Negative edges are sampled from unseen edges that were not encountered during the training phase.

NSS	Datasets	JODIE	DyRep	TGAT	TGN	CAWN	EdgeBank	TCL	GraphMixer	DyGFormer	FreeDyG
	Wikipedia	96.33 ± 0.07	94.37 ± 0.09	96.67 ± 0.07	98.37 ± 0.07	98.54 ± 0.04	90.78 ± 0.00	95.84 ± 0.18	96.92 ± 0.03	$\underline{98.91 \pm 0.02}$	99.41 ± 0.01
	Reddit	98.31 ± 0.05	98.17 ± 0.05	98.47 ± 0.02	98.60 ± 0.06	99.01 ± 0.01	95.37 ± 0.00	97.42 ± 0.02	97.17 ± 0.02	<u>99.15 ± 0.01</u>	99.50 ± 0.01
rnd A hist ind	MOOC	83.81 ± 2.09	85.03 ± 0.58	87.11 ± 0.19	91.21 ± 1.15	80.38 ± 0.26	60.86 ± 0.00	83.12 ± 0.18	84.01 ± 0.17	87.91 ± 0.58	89.93 ± 0.35
ma	LastFM	70.49 ± 1.66	71.16 ± 1.89	71.59 ± 0.18	78.47 ± 2.94	85.92 ± 0.10	83.77 ± 0.00	64.06 ± 1.16	73.53 ± 0.12	$\underline{93.05\pm0.10}$	93.42 ± 0.15
	Enron	87.96 ± 0.52	84.89 ± 3.00	68.89 ± 1.10	88.32 ± 0.99	90.45 ± 0.14	87.05 ± 0.00	75.74 ± 0.72	84.38 ± 0.21	$\underline{93.33\pm0.13}$	94.01 ± 0.11
	Social Evo.	92.05 ± 0.46	90.76 ± 0.21	94.76 ± 0.16	95.39 ± 0.17	87.34 ± 0.08	81.60 ± 0.00	94.84 ± 0.17	95.23 ± 0.07	$\underline{96.30\pm0.01}$	96.59 ± 0.04
	UCI	90.44 ± 0.49	68.77 ± 2.34	78.53 ± 0.74	92.03 ± 1.13	93.87 ± 0.08	77.30 ± 0.00	87.82 ± 1.36	91.81 ± 0.67	$\underline{94.49\pm0.26}$	95.00 ± 0.21
A	vg.Rank	7.14	8.57	6.14	3.31	3.78	4.86	6.14	4.78	<u>2.14</u>	1.14
	Wikipedia	80.77 ± 0.73	77.74 ± 0.33	82.87 ± 0.22	82.74 ± 0.32	67.84 ± 0.64	77.27 ± 0.00	85.76 ± 0.46	$\textbf{87.68} \pm \textbf{0.17}$	78.80 ± 1.95	82.78±0.30
	Reddit	80.52 ± 0.32	80.15 ± 0.18	79.33 ± 0.16	81.11 ± 0.19	80.27 ± 0.30	78.58 ± 0.00	76.49 ± 0.16	77.80 ± 0.12	80.54 ± 0.29	85.92 ± 0.10
hiet	MOOC	82.75 ± 0.83	81.06 ± 0.94	80.81 ± 0.67	$\underline{88.00 \pm 1.80}$	71.57 ± 1.07	61.90 ± 0.00	72.09 ± 0.56	76.68 ± 1.40	87.04 ± 0.35	88.32 ± 0.99
mst	LastFM	75.22 ± 2.36	74.65 ± 1.98	64.27 ± 0.26	77.97 ± 3.04	67.88 ± 0.24	$\underline{78.09 \pm 0.00}$	47.24 ± 3.13	64.21 ± 0.73	$\textbf{78.78} \pm \textbf{0.35}$	73.53 ± 0.12
	Enron	75.39 ± 2.37	74.69 ± 3.55	61.85 ± 1.43	$\underline{77.09 \pm 2.22}$	65.10 ± 0.34	79.59 ± 0.00	67.95 ± 0.88	75.27 ± 1.14	76.55 ± 0.52	75.74 ± 0.72
	Social Evo.	90.06 ± 3.15	93.12 ± 0.34	93.08 ± 0.59	94.71 ± 0.53	87.43 ± 0.15	85.81 ± 0.00	93.44 ± 0.68	94.39 ± 0.31	$\underline{97.28\pm0.07}$	97.42 ± 0.02
	UCI	$\underline{78.64 \pm 3.50}$	57.91 ± 3.12	58.89 ± 1.57	77.25 ± 2.68	57.86 ± 0.15	69.56 ± 0.00	72.25 ± 3.46	77.54 ± 2.02	76.97 ± 0.24	80.38 ± 0.26
A	vg.Rank	4.78	5.85	6.01	3.85	6.54	7.14	4.85	4.14	2.85	2.14
	Wikipedia	70.96 ± 0.78	67.36 ± 0.96	81.93 ± 0.22	80.97 ± 0.31	70.95 ± 0.95	81.73 ± 0.00	82.19 ± 0.48	84.28 ± 0.30	75.09 ± 3.70	82.74 ± 0.32
	Reddit	83.51 ± 0.15	82.90 ± 0.31	87.13 ± 0.20	84.56 ± 0.24	$\textbf{88.04} \pm \textbf{0.29}$	85.93 ± 0.00	84.67 ± 0.29	82.21 ± 0.13	$\underline{86.23\pm0.51}$	84.38 ± 0.21
ind	MOOC	66.63 ± 2.30	63.26 ± 1.01	73.18 ± 0.33	77.44 ± 2.86	70.32 ± 1.43	48.18 ± 0.00	70.36 ± 0.37	72.45 ± 0.72	80.76 ± 0.76	78.47 ± 0.94
ma	LastFM	61.32 ± 3.49	62.15 ± 2.12	63.99 ± 0.21	65.46 ± 4.27	67.92 ± 0.44	77.37 ± 0.00	46.93 ± 2.59	60.22 ± 0.32	69.25 ± 0.36	$\underline{72.30 \pm 0.59}$
	Enron	70.92 ± 1.05	68.73 ± 1.34	60.45 ± 2.12	71.34 ± 2.46	$\underline{75.17\pm0.50}$	75.00 ± 0.00	67.64 ± 0.86	71.53 ± 0.85	74.07 ± 0.64	77.27 ± 0.61
	Social Evo.	90.01 ± 3.19	93.07 ± 0.38	92.94 ± 0.61	95.24 ± 0.56	89.93 ± 0.15	87.88 ± 0.00	93.44 ± 0.72	94.22 ± 0.32	$\underline{97.51 \pm 0.06}$	98.47 ± 0.02
	UCI	64.14 ± 1.26	54.25 ± 2.01	60.80 ± 1.01	64.11 ± 1.04	58.06 ± 0.26	58.03 ± 0.00	70.05 ± 1.86	$\underline{74.59 \pm 0.74}$	65.96 ± 1.18	75.39 ± 0.57
A	vg.Rank	6.0	5.77	4.77	3.46	4.0	5.35	5.46	4.57	2.85	2.14

Table 5: AUC-ROC for transductive dynamic link prediction with different sampling strategies.

NSS	Datasets	JODIE	DyRep	TGAT	TGN	CAWN	TCL	GraphMixer	DyGFormer	FreeDyG
	Wikipedia	94.33 ± 0.27	91.49 ± 0.45	95.90 ± 0.09	97.72 ± 0.03	98.03 ± 0.04	95.57 ± 0.20	96.30 ± 0.04	$\underline{98.48 \pm 0.03}$	99.01 ± 0.02
	Reddit	96.52 ± 0.13	96.05 ± 0.12	96.98 ± 0.04	97.39 ± 0.07	98.42 ± 0.02	93.80 ± 0.07	94.97 ± 0.05	$\underline{98.71 \pm 0.01}$	$\textbf{98.84} \pm \textbf{0.01}$
rnd	MOOC	83.16 ± 1.30	84.03 ± 0.49	86.84 ± 0.17	91.24 ± 0.99	81.86 ± 0.25	81.43 ± 0.19	82.77 ± 0.24	$\underline{87.62\pm0.51}$	87.01 ± 0.74
mu	LastFM	81.13 ± 3.39	82.24 ± 1.51	76.99 ± 0.29	82.61 ± 3.15	87.82 ± 0.12	70.84 ± 0.85	80.37 ± 0.18	$\underline{94.08\pm0.08}$	94.32 ± 0.03
	Enron	81.96 ± 1.34	76.34 ± 4.20	64.63 ± 1.74	78.83 ± 1.11	87.02 ± 0.50	72.33 ± 0.99	76.51 ± 0.71	90.69 ± 0.26	$\underline{89.51\pm0.20}$
	Social Evo.	93.70 ± 0.29	91.18 ± 0.49	93.41 ± 0.19	93.43 ± 0.59	84.73 ± 0.27	93.71 ± 0.18	94.09 ± 0.07	$\underline{95.29\pm0.03}$	$\textbf{96.41} \pm \textbf{0.07}$
	UCI	78.80 ± 0.94	58.08 ± 1.81	77.64 ± 0.38	86.68 ± 2.29	90.40 ± 0.11	84.49 ± 1.82	89.30 ± 0.57	$\underline{92.63\pm0.13}$	93.01 ± 0.08
А	vg.Rank	4.69	5.85	5.31	2.85	3.38	5.31	6.0	1.86	1.43
	Wikipedia	61.86 ± 0.53	57.54 ± 1.09	78.38 ± 0.20	75.75 ± 0.29	62.04 ± 0.65	79.79 ± 0.96	82.87 ± 0.21	70.33 ± 0.25	$\underline{82.08\pm0.32}$
	Reddit	61.69 ± 0.39	60.45 ± 0.37	64.43 ± 0.27	64.55 ± 0.50	64.94 ± 0.21	61.43 ± 0.26	64.27 ± 0.13	$\underline{66.08 \pm 0.34}$	66.79 ± 0.31
hist	MOOC	64.48 ± 1.64	64.23 ± 1.29	74.08 ± 0.27	77.69 ± 3.55	71.68 ± 0.94	69.82 ± 0.32	72.53 ± 0.84	80.77 ± 0.63	81.52 ± 0.37
mst	LastFM	68.44 ± 3.26	68.79 ± 1.08	69.89 ± 0.28	66.99 ± 5.62	67.69 ± 0.24	55.88 ± 1.85	70.07 ± 0.20	$\underline{70.73\pm0.37}$	72.63 ± 0.16
	Enron	65.32 ± 3.57	61.50 ± 2.50	57.84 ± 2.18	62.68 ± 1.09	62.25 ± 0.40	64.06 ± 1.02	68.20 ± 1.62	65.78 ± 0.42	$\textbf{70.09} \pm \textbf{0.65}$
	Social Evo.	88.53 ± 0.55	87.93 ± 1.05	91.87 ± 0.72	92.10 ± 1.22	83.54 ± 0.24	93.28 ± 0.60	93.62 ± 0.35	$\underline{96.91\pm0.09}$	$\textbf{96.94} \pm \textbf{0.17}$
	UCI	60.24 ± 1.94	51.25 ± 2.37	62.32 ± 1.18	62.69 ± 0.90	56.39 ± 0.10	70.46 ± 1.94	$\underline{75.98 \pm 0.84}$	65.55 ± 1.01	76.01 ± 0.75
Α	vg.Rank	5.08	6.00	4.23	4.38	5.38	3.69	3.08	<u>2.85</u>	1.14
	Wikipedia	61.87 ± 0.53	57.54 ± 1.09	78.38 ± 0.20	$0.75.76 \pm 0.29$	62.02 ± 0.65	79.79 ± 0.96	$\underline{82.88 \pm 0.21}$	68.33 ± 2.82	83.17 ± 0.31
	Reddit	61.69 ± 0.39	60.44 ± 0.37	64.39 ± 0.27	64.55 ± 0.50	64.91 ± 0.21	61.36 ± 0.26	64.27 ± 0.13	$\underline{64.80\pm0.25}$	64.51 ± 0.19
ind	MOOC	64.48 ± 1.64	64.22 ± 1.29	74.07 ± 0.27	77.68 ± 3.55	71.69 ± 0.94	69.83 ± 0.32	72.52 ± 0.84	80.77 ± 0.63	75.81 ± 0.69
mu	LastFM	68.44 ± 3.26	68.79 ± 1.08	69.89 ± 0.28	66.99 ± 5.61	67.68 ± 0.24	55.88 ± 1.85	70.07 ± 0.20	$\underline{70.73 \pm 0.37}$	71.42 ± 0.33
	Enron	65.32 ± 3.57	61.50 ± 2.50	57.83 ± 2.18	62.68 ± 1.09	62.27 ± 0.40	64.05 ± 1.02	$\underline{68.19 \pm 1.63}$	65.79 ± 0.42	68.79 ± 0.91
	Social Evo.	88.53 ± 0.55	87.93 ± 1.05	91.88 ± 0.72	92.10 ± 1.22	83.54 ± 0.24	93.28 ± 0.60	93.62 ± 0.35	96.91 ± 0.09	$\underline{96.79\pm0.17}$
	UCI	60.27 ± 1.94	51.26 ± 2.40	62.29 ± 1.17	62.66 ± 0.91	56.39 ± 0.11	70.42 ± 1.93	75.97 ± 0.85	65.58 ± 1.00	$\underline{73.41 \pm 0.88}$
А	vg.Rank	4.69	5.85	5.31	2.85	3.38	5.31	6.0	1.86	1.43

Table 6: AUC-ROC for inductive dynamic link prediction with different sampling strategies.

D SUPPLEMENTARY MATERIALS

THE PSEUDO-CODE OF FREEDYG D.1

In Algorithm 1, we show the pseudo-code of the training process of FreeDyG. In addition, following the suggestion of the reviewers, we briefly describe the procedure of FFT in Algorithm 2.

Algorithm 1 Training pipeline for FreeDyG

Input: CTDG G, a node pair (u, v) with a specific timestamp (t), the neighbor sample number L, maximum training epoch of 200, early stopping strategy with patience = 20.

Output: The probability of the node pair interacting at timestamp t

```
1: initial patience = 0
```

- 2: for training epoch = 1, 2, 3... do
- 3: Acquire the L most recent first-hop interaction neighbors of nodes u and v from G prior to timestamp t as S_u^t and S_v^t ;
- for S_u^t and S_v^t in parallel do 4:
- Obtain node encoding $Z_{*,V}^t$ and edge encoding $Z_{*,E}^t$; 5:
- Obtain time encoding $Z_{*,T}^{t}$ as Cong et al. (2023); 6:
- Obtain NIF encoding Z_{*F}^{t} from Equation 3; 7:
- 8:
- $Z_*^t \leftarrow Z_{*,V}^t + Z_{*,E}^t + Z_{*,T}^t + Z_{*,F}^t;$ for Frequency-ehanced MLP-Mixer Layer do $Z_*^l \leftarrow \mathcal{F}(Z_*^t)$ with Equation 1; 9:
- 10:
- $\widehat{\mathcal{Z}}^l_{\star} \leftarrow W \cdot \mathcal{Z}^l_{\star};$ 11:

12:
$$\widetilde{Z}_*^l \leftarrow \mathcal{F}^{-1}\left(\widehat{\mathcal{Z}}_*^l\right)$$
 with Equation 2;

13:
$$Z_*^l \leftarrow \text{LayerNorm}\left(Z_*^l + \text{Dropout}\left(\widetilde{Z}_*^l\right)\right)$$

Feed Z_*^l into MLP-Mixer Layer with Equation 10; 14:

- 15: end for
- Weighted aggregation with Equation 11; 16:
- 17: end for

18: Conduct link prediction with
$$\hat{y} \leftarrow Softmax(MLP(RELU(MLP(h_u^t || h_v^t))))$$

Loss $\mathcal{L}_p \leftarrow -\frac{1}{K} \sum_{i=1}^{S} (y_i * \log \hat{y}_i + (1 - y_i) * \log(1 - \hat{y}_i))$ if current epoch's metrics worse than the previous epoch's **then** 19:

- 20:
- 21: patience = patience + 1
- else 22:
- Save the model parameters from the current epoch 23:
- 24: end if
- if patience = 20 then 25:
- 26: Exit training process
- 27: end if
- 28: end for

Algorithm 2 The Pseudo-Code of Fast Fourier Transform

1: **Input:** Sequence X of length L2: **Output:** Y {FFT of X} 3: 4: Function NextPowerOf2(L) {Return the next power of 2 greater than or equal to L} 5: $N \leftarrow 1$ {Initialize n to the smallest power of 2} 6: while N < L do $N \leftarrow N \times 2$ {Double *n* until it is greater than or equal to *L*} 7: 8: end while 9: return N 10: 11: **Function** FFT(X, N)12: **if** N = 1 **then** 13: return X 14: end if 15: $X_{\text{even}} \leftarrow \text{even-index elements of } X[0, \dots, N-1]$ 16: $X_{odd} \leftarrow odd$ -index elements of $X[0, \ldots, N-1]$ {Divide the sequence into two parts: evenindexed and odd-indexed} 17: $Y_{\text{even}} \leftarrow \text{FFT}(X_{\text{even}}, N/2)$ {Recursively apply FFT to even part} 18: $Y_{\text{odd}} \leftarrow \text{FFT}(X_{\text{odd}}, N/2)$ {Recursively apply FFT to odd part} 19: $Y \leftarrow$ new array of size N {Initialize an array Y of size N} 20: for k = 0 to N/2 - 1 do $t \leftarrow e^{-2\pi i k / N}$ {Compute the twiddle factor} 21: $Y[k] \leftarrow Y_{\text{even}}[k] + t \cdot Y_{\text{odd}}[k]$ {Compute the FFT value for index k} 22: $Y[k + N/2] \leftarrow Y_{\text{even}}[k] - t \cdot Y_{\text{odd}}[k]$ {Compute the FFT value for index k + N/2} 23: 24: end for 25: return Y 26: 27: Procedure: 28: $N \leftarrow \text{NextPowerOf2}(L)$ {Pad the sequence length to the next power of 2} 29: $X_{padded} \leftarrow X$ 30: while length of $X_{\text{padded}} < N \text{ do}$ append 0 to X_{padded} {Pad the sequence with zeros} 31: 32: end while 33: $Y \leftarrow \text{FFT}(X_{\text{padded}}, N)$ {Apply FFT to the padded sequence}

D.2 THE EXPERIMENTAL RESULTS ON MICRO-F1 AND MACRO-F1 METRIC

In Table 7-10, we conduct experiments on all datasets under three negative sampling strategies with both transductive and inductive link prediction. We use Micro-F1 and Macro-F1 scores as the metric. We note that the results are similar to those on AP and AUC-ROC. Specifically, our FreeDyG outperforms other baselines in most scenarios and the average ranking of FreeDyG is close to 1.

NSS	Datasets	JODIE	DyRep	TGAT	TGN	CAWN	TCL	GraphMixer	DyGFormer	FreeDyG
	Wikipedia	87.47 ± 0.99	87.12 ± 0.45	89.69 ± 0.15	93.09 ± 0.05	<u>94.63 ± 0.08</u>	88.06 ± 0.50	90.57 ± 0.24	94.60 ± 0.53	95.92 ± 0.09
	reddit	91.99 ± 0.49	93.03 ± 0.21	93.90 ± 0.17	94.15 ± 0.07	95.55 ± 0.03	91.68 ± 0.07	91.24 ± 0.04	$\underline{95.86 \pm 0.20}$	96.03 ± 0.07
NSS rnd hist ind	MOOC	73.02 ± 1.20	78.05 ± 1.58	$\underline{78.26\pm0.38}$	84.34 ± 1.72	70.05 ± 0.56	74.02 ± 0.32	74.76 ± 0.18	75.44 ± 1.04	76.74 ± 0.97
mu	LastFM	59.74 ± 3.96	61.01 ± 5.68	65.01 ± 0.26	54.93 ± 3.44	80.94 ± 0.47	59.50 ± 0.33	66.96 ± 0.44	$\underline{88.13 \pm 0.64}$	88.89 ± 0.38
	Enron	71.74 ± 0.84	77.01 ± 2.61	63.09 ± 2.39	78.36 ± 3.22	84.53 ± 0.16	70.88 ± 2.79	76.47 ± 0.16	$\underline{88.37\pm0.33}$	88.91 ± 0.20
	Social Evo.	77.46 ± 4.26	83.66 ± 0.62	90.42 ± 0.12	91.18 ± 0.23	$80.67{\pm}~0.31$	90.56 ± 0.18	90.67 ± 0.14	$\underline{93.26\pm0.10}$	93.87 ± 0.07
	UCI	67.61 ± 2.97	63.49 ± 9.17	70.29 ± 1.18	79.39 ± 0.67	87.84 ± 0.15	76.15 ± 0.58	83.30 ± 0.99	$\underline{89.34\pm0.07}$	$90.25 {\pm}~0.08$
A	vg.Rank	7.71	6.43	5.71	4.28	4.43	7.00	5.43	2.57	1.43
	Wikipedia	65.17 ± 1.27	63.35 ± 0.61	62.53 ± 0.43	59.67 ± 1.26	50.18 ± 0.23	$\underline{67.61 \pm 0.88}$	69.94 ± 1.44	52.87 ± 1.88	66.73 ± 1.32
	Reddit	59.76 ± 0.26	61.40 ± 0.76	59.27 ± 0.54	60.55 ± 0.33	54.04 ± 0.03	59.22 ± 0.14	$\underline{61.74\pm0.10}$	49.87 ± 0.14	62.74 ± 0.61
	MOOC	63.74 ± 9.08	75.35 ± 0.54	72.90 ± 0.37	77.77 ± 0.87	64.20 ± 1.42	59.69 ± 2.06	65.58 ± 0.12	75.10 ± 5.73	$\underline{76.18 \pm 0.87}$
hist	LastFM	49.71 ± 1.53	59.42 ± 2.52	55.26 ± 0.49	50.86 ± 1.06	59.85 ± 0.68	38.98 ± 2.52	56.39 ± 0.62	68.59 ± 0.55	$\underline{67.81 \pm 0.40}$
	Enron	65.91 ± 0.31	68.11 ± 2.64	56.38 ± 0.64	67.63 ± 1.81	55.72 ± 0.94	66.74 ± 0.34	61.48 ± 1.79	66.52 ± 1.64	66.97 ± 1.28
	Social Evo.	73.30 ± 0.25	86.41 ± 0.38	83.47 ± 0.49	84.81 ± 1.28	86.81 ± 0.17	87.70 ± 5.17	84.96 ± 0.96	$\underline{89.42 \pm 0.69}$	90.02 ± 0.83
	UCI	62.13 ± 3.38	51.82 ± 9.65	49.44 ± 0.48	56.82 ± 0.70	44.78 ± 0.02	48.93 ± 0.43	$\textbf{78.66} \pm \textbf{0.02}$	59.96 ± 2.00	$\underline{64.39 \pm 1.29}$
Α	vg.Rank	6.14	3.86	6.57	4.71	7.00	6.00	4.00	4.71	2.00
	Wikipedia	53.90 ± 0.70	53.91 ± 0.01	59.76 ± 0.28	57.25 ± 0.23	52.73 ± 0.22	$\underline{63.64 \pm 0.17}$	63.78 ± 1.34	49.49 ± 0.02	58.10 ± 0.74
	Reddit	58.07 ± 0.12	59.52 ± 0.18	$\underline{65.52\pm0.65}$	58.57 ± 0.41	58.76 ± 0.41	66.09 ± 0.03	63.94 ± 0.19	48.65 ± 0.38	63.09 ± 0.73
	MOOC	51.69 ± 3.58	57.96 ± 0.32	64.54 ± 0.64	61.75 ± 2.58	62.90 ± 0.91	60.11 ± 1.55	62.50 ± 0.38	<u>67.13 ± 2.26</u>	70.06 ± 1.45
ind	LastFM	47.55 ± 1.53	51.57 ± 2.24	55.14 ± 0.66	50.34 ± 0.45	63.87 ± 0.22	39.30 ± 2.32	53.42 ± 0.27	$\underline{66.31 \pm 0.30}$	66.57 ± 0.71
	Enron	58.57 ± 0.40	61.26 ± 2.27	54.75 ± 1.25	60.51 ± 0.68	<u>64.57 ± 1.18</u>	65.66 ± 0.50	58.24 ± 0.73	56.80 ± 0.74	60.09 ± 1.43
	Social Evo.	73.42 ± 0.40	86.49 ± 0.40	83.68 ± 0.46	86.13 ± 1.02	87.72 ± 0.35	87.62 ± 5.33	84.94 ± 1.05	$\underline{89.40 \pm 0.65}$	89.87 ± 0.52
	UCI	58.34 ± 1.64	53.07 ± 2.71	51.69 ± 0.25	49.75 ± 0.46	46.14 ± 0.02	50.76 ± 0.84	69.43 ± 0.21	46.53 ± 0.84	$\underline{60.02 \pm 2.61}$
А	vg.Rank	7.14	5.28	4.86	6.00	5.00	4.28	4.14	5.71	2.57

Table 7: micro-F1 for transductive link prediction with three different negative sampling strategies.

NSS	Datasets	JODIE	DyRep	TGAT	TGN	CAWN	TCL	GraphMixer	DyGFormer	FreeDyG
	Wikipedia	87.45 ± 0.99	87.10 ± 0.45	89.69 ± 0.15	93.09 ± 0.05	$\underline{94.63 \pm 0.08}$	88.06 ± 0.50	90.57 ± 0.24	94.60 ± 0.53	95.92 ± 0.09
	reddit	91.99 ± 0.49	93.03 ± 0.21	93.90 ± 0.17	94.14 ± 0.07	95.55 ± 0.03	91.68 ± 0.07	91.24 ± 0.04	$\underline{95.86 \pm 0.20}$	96.03 ± 0.07
NSS rnd hist ind	MOOC	72.13 ± 1.45	77.89 ± 1.69	$\underline{78.23 \pm 0.38}$	84.29 ± 1.75	69.63 ± 0.67	73.91 ± 0.35	74.65 ± 0.18	75.37 ± 1.09	76.63 ± 1.01
mu	LastFM	55.61 ± 4.77	55.77 ±5.49	64.90 ± 0.27	42.42 ± 5.57	80.86 ± 0.44	58.93 ± 0.60	66.70 ± 0.44	$\underline{87.95 \pm 0.60}$	88.02 ± 0.47
	Enron	69.51 ± 1.25	76.55 ± 2.95	62.28 ± 2.65	77.75 ± 3.63	84.23 ± 0.20	69.66 ± 3.13	76.04 ± 0.17	$\underline{88.01 \pm 0.32}$	$\textbf{88.83} \pm \textbf{0.21}$
	Social Evo.	78.47 ± 3.55	83.58 ± 0.66	90.41 ± 0.12	91.17 ± 0.23	$80.58 {\pm}~0.29$	90.55 ± 0.18	90.67 ± 0.14	$\underline{93.26\pm0.10}$	93.87 ± 0.07
	UCI	63.43 ± 4.37	61.57 ± 11.71	70.19 ± 1.18	78.88 ± 0.76	87.76 ± 0.13	76.01 ± 0.62	82.87 ± 1.07	$\underline{89.24\pm0.09}$	90.22 ± 0.08
A	vg.Rank	8.00	6.57	5.71	4.28	4.43	6.57	5.43	<u>2.57</u>	1.43
	Wikipedia	62.92 ± 1.80	61.21 ± 0.79	59.41 ± 0.55	55.02 ± 1.99	38.90 ± 0.58	$\underline{66.34 \pm 1.02}$	68.70 ± 1.74	45.72 ± 1.45	61.52 ± 1.25
	Reddit	54.11 ± 0.32	56.89 ± 1.19	53.73 ± 0.92	55.45 ± 0.59	$44.95{\pm}~0.37$	54.43 ± 0.27	$\underline{58.19\pm0.19}$	36.58 ± 0.18	58.71 ± 0.59
	MOOC	59.01 ± 14.15	$\underline{77.14 \pm 0.50}$	72.73 ± 0.46	$\textbf{77.44} \pm \textbf{0.80}$	63.98 ± 1.37	58.61 ± 2.32	65.07 ± 0.15	74.92 ± 5.87	76.03 ± 0.87
hist	LastFM	48.17 ± 1.92	53.69 ± 2.28	54.23 ± 0.78	35.63 ± 2.79	58.75 ± 0.94	35.34 ± 6.15	55.68 ± 0.71	$\underline{67.28\pm0.37}$	67.48 ± 0.42
	Enron	61.92 ± 0.06	66.87 ± 2.99	55.35 ± 0.73	65.93 ± 2.12	51.65 ± 2.12	$\underline{65.93 \pm 0.57}$	59.69 ± 1.83	64.98 ± 1.91	65.33 ± 1.70
	Social Evo.	71.95 ± 0.07	86.37 ± 0.38	83.37 ± 0.50	84.69 ± 1.35	87.75 ± 0.34	87.66 ± 5.21	84.89 ± 0.98	$\underline{89.23 \pm 0.71}$	90.17 ± 0.64
	UCI	57.97 ± 5.87	48.76 ± 12.80	46.15 ± 1.10	49.27 ± 0.75	35.98 ± 0.05	42.58 ± 1.60	$\textbf{78.43} \pm \textbf{0.01}$	57.90 ± 2.73	$\underline{65.07 \pm 1.15}$
А	vg.Rank	6.00	4.00	6.57	4.85	6.85	5.71	3.85	4.85	2.28
	Wikipedia	46.88 ± 1.59	47.95 ± 0.05	55.48 ± 0.49	51.12 ± 0.58	42.85 ± 0.57	61.48 ± 0.18	$\underline{61.07 \pm 1.82}$	36.76 ± 1.03	52.71 ± 1.36
	Reddit	51.34 ± 0.27	54.00 ± 0.41	62.43 ± 0.94	52.29 ± 0.75	52.47 ± 0.51	63.64 ± 0.07	61.04 ± 0.29	33.66 ± 0.30	$\underline{61.25\pm0.81}$
	MOOC	40.70 ± 9.95	54.12 ± 0.92	63.63 ± 0.98	58.48 ± 2.91	62.73 ± 0.88	59.11 ± 1.72	61.73 ± 0.45	$\underline{66.74 \pm 2.20}$	68.31 ± 1.54
ind	LastFM	42.08 ± 2.31	40.70 ± 4.93	54.07 ± 0.97	34.61 ± 1.62	$63.29 {\pm}~0.89$	35.78 ± 5.79	52.41 ± 0.25	51.37 ± 1.04	$\underline{60.42 \pm 1.59}$
	Enron	50.58 ± 1.45	58.24 ± 2.84	53.31 ± 1.42	56.00 ± 0.59	$\underline{62.66 \pm 1.94}$	64.93 ± 0.70	55.67 ± 0.51	51.37 ± 1.04	55.79 ± 1.06
	Social Evo.	72.07 ± 0.25	86.45 ± 0.40	83.60 ± 0.48	86.05 ± 1.07	88.31 ± 0.16	87.57 ± 5.38	84.88 ± 1.08	89.25 ± 0.70	89.94 ± 0.47
	UCI	54.41 ± 4.13	48.49 ± 6.79	49.11 ± 0.07	35.97 ± 0.32	37.56 ± 0.05	45.18 ± 2.13	69.24 ± 0.21	38.10 ± 2.10	<u>58.47 ± 2.41</u>
А	vg.Rank	7.28	5.57	4.28	6.71	4.57	3.85	4.14	6.00	2.57

Table 8: macro-F1 for transductive link prediction with three different negative sampling strategies.

NSS	Datasets	JODIE	DyRep	TGAT	TGN	CAWN	TCL	GraphMixer	DyGFormer	FreeDyG
	Wikipedia	84.05 ± 0.89	82.60 ± 0.26	88.20 ± 0.14	91.22 ± 0.27	92.86 ± 0.11	87.09 ± 0.27	89.23 ± 0.09	$\underline{93.13 \pm 0.50}$	94.42 ± 0.05
	reddit	88.84 ± 0.73	88.99 ± 0.48	91.06 ± 0.42	91.60 ± 0.19	94.24 ± 0.04	86.39 ± 0.21	87.88 ± 0.15	<u>94.62 ± 0.22</u>	95.33 ± 0.09
rnd	MOOC	71.19 ± 1.66	76.89 ± 0.66	$\underline{78.00\pm0.32}$	83.91 ± 0.76	72.01 ± 0.27	72.55 ± 0.26	73.53 ± 0.17	75.24 ± 1.42	77.81 ± 1.12
mu	LastFM	70.71 ± 4.65	70.68 ± 6.38	70.10 ± 0.36	58.14 ± 6.06	83.55 ± 0.08	65.86 ± 0.48	72.93 ± 0.85	$\underline{90.03 \pm 0.25}$	90.49 ± 0.23
	Enron	65.50 ± 1.05	66.01 ± 3.88	59.29 ± 2.13	71.54 ± 2.98	80.29 ± 0.24	68.47 ± 3.03	69.50 ± 0.18	$\underline{85.15\pm0.24}$	86.34 ± 0.26
	Social Evo.	84.78 ± 1.22	83.54 ± 1.94	88.80 ± 0.12	88.49 ± 0.53	79.42 ± 0.41	89.74 ± 0.13	89.67 ± 0.08	$\underline{92.16\pm0.08}$	92.59 ± 0.07
	UCI	61.25 ± 2.27	56.51 ± 5.09	68.41 ± 0.89	69.41 ± 2.12	85.26 ± 0.55	71.13 ± 0.78	82.44 ± 0.68	$\underline{88.40\pm0.07}$	89.11 ± 0.05
Α	vg.Rank	7.43	7.00	5.85	4.85	4.57	6.43	5.14	2.43	1.28
	Wikipedia	48.81 ± 0.11	48.41 ± 0.06	56.76 ± 0.38	52.89 ± 0.24	49.37 ± 0.17	$\underline{61.10\pm0.25}$	61.11 ± 1.25	48.48 ± 0.27	57.43 ± 0.75
	Reddit	51.50 ± 0.19	51.64 ± 0.03	54.12 ± 0.11	51.67 ± 0.23	50.87 ± 0.33	$\underline{54.14\pm0.27}$	54.48 ± 0.16	48.10 ± 0.38	51.07 ± 0.36
	MOOC	52.98 ± 4.84	59.02 ± 0.65	65.59 ± 0.25	63.13 ± 1.90	64.37 ± 1.07	59.76 ± 1.56	62.65 ± 0.24	66.80 ± 1.86	$\underline{66.02\pm0.75}$
hist	LastFM	52.09 ± 3.15	53.35 ± 4.66	57.99 ± 0.61	50.37 ± 0.55	54.64 ± 0.31	42.84 ± 3.28	57.88 ± 0.24	69.75 ± 0.53	$\underline{68.54 \pm 0.48}$
	Enron	60.29 ± 0.83	55.27 ± 1.11	52.03 ± 0.60	56.84 ± 2.40	58.71 ± 0.75	63.38 ± 0.57	56.05 ± 0.34	57.31 ± 0.47	58.23 ± 1.07
	Social Evo.	69.77 ± 0.74	71.25 ± 3.58	81.07 ± 0.94	81.29 ± 3.48	88.14 ± 0.73	87.69 ± 4.42	83.82 ± 0.79	$\underline{89.01 \pm 0.53}$	90.13 ± 0.58
	UCI	56.71 ± 1.55	52.27 ± 0.24	52.72 ± 0.09	49.16 ± 0.49	44.71 ± 0.05	50.54 ± 0.89	$\textbf{71.02} \pm \textbf{0.11}$	46.79 ± 0.98	$\underline{62.19 \pm 2.09}$
Α	vg.Rank	6.14	7.00	4.71	5.85	5.43	4.43	3.57	4.86	3.00
	Wikipedia	48.81 ± 0.11	48.41 ± 0.06	56.76 ± 0.38	52.89 ± 0.24	49.37 ± 0.17	$\underline{61.09\pm0.25}$	61.10 ± 1.26	48.48 ± 0.27	57.42 ± 0.75
	Reddit	51.50 ± 0.19	51.64 ± 0.03	$\underline{54.07\pm0.03}$	51.67 ± 0.23	50.86 ± 0.30	54.16 ± 0.24	54.48 ± 0.16	48.10 ± 0.38	51.07 ± 0.36
	MOOC	52.98 ± 4.85	59.02 ± 0.65	65.60 ± 0.25	63.13 ± 1.89	64.37 ± 1.07	59.76 ± 1.56	62.66 ± 0.24	66.80 ± 1.87	66.02 ± 0.75
ind	LastFM	52.09 ± 3.1	53.35 ± 4.66	57.99 ± 0.61	50.37 ± 0.55	54.64 ± 0.31	42.83 ± 3.28	57.89 ± 0.24	69.76 ± 0.53	$\underline{68.54 \pm 0.48}$
	Enron	$\underline{60.29 \pm 0.83}$	55.27 ± 1.11	52.04 ± 0.60	56.85 ± 2.40	58.72 ± 0.76	63.39 ± 0.58	56.06 ± 0.35	57.31 ± 0.47	58.23 ± 1.07
	Social Evo.	69.77 ± 0.74	71.25 ± 3.58	81.73 ± 0.92	81.30 ± 3.47	88.20 ± 0.75	87.70 ± 4.42	83.82 ± 0.79	$\underline{89.01 \pm 0.53}$	90.13 ± 0.58
	UCI	56.71 ± 1.53	52.29 ± 0.25	52.62 ± 0.07	49.16 ± 0.51	44.71 ± 0.05	50.58 ± 0.90	$\textbf{71.01} \pm \textbf{0.11}$	46.79 ± 0.99	$\underline{62.19 \pm 2.09}$
A	vg.Rank	6.14	7.00	4.57	6.00	5.43	4.43	3.57	4.85	3.00

Table 9: micro-F1 for inductive link prediction with three different negative sampling strategies.

NSS	Datasets	JODIE	DyRep	TGAT	TGN	CAWN	TCL	GraphMixer	DyGFormer	FreeDyG
	Wikipedia	84.02 ± 0.89	82.55 ± 0.27	88.19 ± 0.14	91.22 ± 0.27	92.86 ± 0.11	87.09 ± 0.27	89.23 ± 0.09	$\underline{93.13 \pm 0.50}$	94.42 ± 0.05
	reddit	88.85 ± 0.72	88.99 ± 0.48	91.06 ± 0.42	91.60 ± 0.19	94.24 ± 0.04	86.39 ± 0.21	87.88 ± 0.15	$\underline{94.62 \pm 0.22}$	95.33 ± 0.09
NSS rnd A hist A ind	MOOC	70.55 ± 1.47	76.80 ± 0.70	77.97 ± 0.33	83.87 ± 0.76	71.78 ± 0.31	72.44 ± 0.28	73.43 ± 0.17	75.18 ± 1.47	77.71 ± 1.14
ma	LastFM	66.82 ± 5.98	67.76 ± 10.04	70.01 ± 0.36	47.46 ± 9.02	83.42 ± 0.08	65.40 ± 0.52	72.80 ± 0.87	90.14 ± 0.24	90.65 ± 0.25
	Enron	68.11 ± 0.67	64.21 ± 4.73	58.61 ± 2.40	70.39 ± 3.71	79.97 ± 0.30	67.34 ± 3.43	68.75 ± 0.18	$\underline{84.84 \pm 0.24}$	86.10 ± 0.16
	Social Evo.	85.03 ± 1.14	83.29 ± 2.06	88.78 ± 0.12	88.43 ± 0.56	79.16 ± 0.40	89.74 ± 0.13	89.67 ± 0.08	$\underline{92.16\pm0.08}$	92.59 ± 0.07
	UCI	55.66 ± 3.90	54.23 ± 7.99	68.23 ± 0.88	67.33 ± 2.91	85.21 ± 0.53	70.84 ± 0.87	82.10 ± 0.75	$\underline{88.31 \pm 0.07}$	89.05 ± 0.04
A	vg.Rank	7.43	7.14	5.43	5.00	4.57	6.57	5.14	2.43	1.28
	Wikipedia	40.56 ± 0.65	41.01 ± 0.21	51.08 ± 0.68	44.34 ± 0.78	49.37 ± 0.17	57.98 ± 0.33	<u>57.33 ± 1.96</u>	35.52 ± 0.87	50.49 ± 0.59
	Reddit	42.53 ± 0.57	43.77 ± 0.15	46.74 ± 0.37	41.72 ± 0.54	44.95 ± 0.25	$\underline{49.15\pm0.08}$	48.60 ± 0.35	33.16 ± 0.21	49.81 ± 0.76
	MOOC	44.48 ± 12.08	56.69 ± 1.20	64.92 ± 0.50	60.30 ± 2.21	64.31 ± 1.06	58.89 ± 1.73	62.02 ± 0.30	66.39 ± 1.80	$\underline{65.63 \pm 1.53}$
hist	LastFM	49.01 ± 3.41	47.68 ± 5.06	56.77 ± 0.93	34.73 ± 1.83	52.05 ± 0.85	38.60 ± 6.90	56.41 ± 0.22	68.61 ± 0.37	$\underline{68.11 \pm 0.40}$
	Enron	<u>55.43 ± 1.92</u>	49.32 ± 1.57	50.45 ± 0.63	52.43 ± 3.86	55.24 ± 1.54	62.66 ± 0.68	53.53 ± 0.11	52.70 ± 0.87	54.70 ± 0.82
	Social Evo.	67.53 ± 1.25	69.55 ± 4.61	80.90 ± 0.97	80.88 ± 3.84	88.09 ± 0.57	87.65 ± 4.46	83.73 ± 0.81	$\underline{88.92 \pm 0.46}$	89.28 ± 0.51
	UCI	50.77 ± 4.58	47.47 ± 5.98	50.45 ± 0.38	35.17 ± 0.01	35.48 ± 0.07	45.52 ± 2.05	$\textbf{70.83} \pm \textbf{0.10}$	38.54 ± 2.33	50.05 ± 3.15
А	vg.Rank	6.14	7.14	4.29	7.43	4.71	4.14	3.57	5.00	2.57
	Wikipedia	40.55 ± 0.64	41.01 ± 0.21	51.08 ± 0.69	44.34 ± 0.77	37.36 ± 0.41	57.97 ± 0.34	$\underline{57.32 \pm 1.97}$	35.52 ± 0.87	50.49 ± 0.59
	Reddit	42.53 ± 0.57	43.77 ± 0.15	46.65 ± 0.27	41.72 ± 0.54	39.42 ± 0.38	$\underline{49.14\pm0.04}$	48.58 ± 0.35	33.16 ± 0.21	49.81 ± 0.76
	MOOC	44.48 ± 12.08	56.69 ± 1.20	64.92 ± 0.50	60.30 ± 2.20	64.31 ± 1.06	58.90 ± 1.73	62.03 ± 0.30	66.39 ± 1.80	$\underline{65.63 \pm 1.53}$
ind	LastFM	49.01 ± 3.41	47.68 ± 5.06	56.78 ± 0.93	34.73 ± 1.83	52.05 ± 0.85	38.60 ± 6.90	56.41 ± 0.22	68.63 ± 0.37	$\underline{68.11 \pm 0.40}$
	Enron	<u>55.43 ± 1.92</u>	49.32 ± 1.57	50.45 ± 0.62	52.44 ± 3.86	55.25 ± 1.55	62.67 ± 0.69	53.54 ± 0.12	52.70 ± 0.87	54.70 ± 0.82
	Social Evo.	67.53 ± 1.25	69.55 ± 4.61	81.58 ± 0.92	80.88 ± 3.83	88.12 ± 0.51	87.66 ± 4.46	83.72 ± 0.81	$\underline{88.92\pm0.46}$	89.28 ± 0.51
	UCI	$\underline{50.76 \pm 4.56}$	47.50 ± 6.01	50.32 ± 0.37	35.17 ± 0.02	35.49 ± 0.06	45.57 ± 2.05	$\textbf{70.82} \pm \textbf{0.11}$	38.56 ± 2.35	50.05 ± 3.15
А	vg.Rank	5.86	6.86	4.29	7.14	5.57	4.14	3.57	5.00	2.57

Table 10: macro-F1 for inductive link prediction with three different negative sampling strategies.

D.3 HYPERPARAMETER STUDY

To enhance the presentation of the results in Table 3, we employ Figure 4 to visually depict the trends observed across varying numbers of sampled historical neighbors.



Figure 4: Performance comparison on AP of different number of sampled historical neighbors



D.4 Ablation study of FreeDyG under random negative sampling setting

Figure 5: Ablation study of FreeDyG under random negative sampling setting, where w/o NS and w/o FE represent our FreeDyG without node interaction frequency encoding module and FE layer respectively.