Distributed Dynamic Average Consensus for Nonholonomic Mobile Robots via Game-Theoretic Nash Equilibrium Approach

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***Abstract*—This paper proposes a distributed algorithm to achieve dynamic average consensus for multiple nonholonomic mobile robot systems, which is critical for practical applications such as enemy capture, island defense, and escorting, among others. Unlike most existing studies on dynamic average consensus, this approach requires both the formation of the desired shape and the optimization of a global distance function for multiple nonholonomic mobile robot systems. By formulating the problem using a game-based approach, the dynamic average consensus control issue is converted into a least-norm Nash equilibrium seeking problem. A distributed Nash equilibrium seeking strategy is proposed by integrating adaptive control laws with regularization-based optimization algorithms. The adaptive control laws are designed to handle unknown parameters within the multiple nonholonomic mobile robot systems, while the regularization-based optimization algorithms are utilized to achieve both formation and distance optimization simultaneously. The proposed algorithm is proven to globally asymptotically direct the states of multiple nonholonomic mobile robots to the least-norm Nash equilibrium, thereby achieving dynamic average consensus control. Finally, the effectiveness of the proposed algorithm is demonstrated through simulation studies.**

***Index Terms*—Dynamic average consensus, Nonholonomic mobile robots, Nash equilibrium seeking, Adaptive control laws.**

I. Introduction

The control of multiple nonholonomic mobile robot systems has garnered significant attention due to its wide range of practical applications, including scenarios such as enemy capture, island defense, and escort missions. A critical problem in this context is achieving dynamic average consensus among the robots, where each robot's state should converge to the average of the dynamically changing states of the others. This problem is particularly challenging when it must be accomplished alongside other objectives, such as maintaining a desired formation and optimizing a global distance function among the robots.

Traditional approaches to dynamic average consensus typically focus on consensus alone, without considering the simultaneous need for formation control and distance optimization. However, in practical applications involving nonholonomic mobile robots, these additional constraints are often essential for mission success. To address these challenges, this paper proposes a novel distributed algorithm that integrates formation control and distance optimization within the dynamic average consensus framework for multiple nonholonomic mobile robot systems.

The problem is formulated using a game-based approach, where the dynamic average consensus control issue is transformed into a least-norm Nash equilibrium seeking problem. This formulation allows for the simultaneous achievement of consensus, formation, and distance optimization, making the proposed approach particularly suitable for complex multi-robot systems. By employing adaptive control laws to handle unknown parameters and regularization-based optimization algorithms for formation and distance control, the proposed strategy ensures robust and efficient performance in achieving the desired objectives.

II. Contributions

The primary contributions of this paper are as follows:

1) **Novel Problem Formulation:** The dynamic average consensus problem for multiple nonholonomic mobile robot systems is reformulated as a least-norm Nash equilibrium seeking problem using a game-based approach. This formulation allows for the integration of formation control and global distance optimization within the consensus framework.
2) **Distributed Nash Equilibrium Seeking Strategy:** We propose a distributed strategy that combines adaptive control laws with regularization-based optimization algorithms. The adaptive control laws address the challenges posed by unknown system parameters, while the regularization-based optimization algorithms ensure that both the formation and distance optimization objectives are met simultaneously.
3) **Global Asymptotic Convergence:** The proposed algorithm is theoretically proven to globally asymptotically direct the states of the multiple nonholonomic mobile robots to the least-norm Nash equilibrium. This result

guarantees the achievement of dynamic average consensus control with the desired formation and distance optimization.

4) **Simulation Validation:** Extensive simulation studies are conducted to demonstrate the effectiveness of the proposed algorithm. The results confirm that the algorithm successfully achieves dynamic average consensus, formation control, and distance optimization in various scenarios, highlighting its practical applicability and robustness.

III. Conclusion

In conclusion, this paper presents a novel distributed algorithm designed to achieve dynamic average consensus for multiple nonholonomic mobile robot systems, addressing the simultaneous challenges of formation control and global distance optimization. By reformulating the problem as a least-norm Nash equilibrium seeking issue and integrating adaptive control laws with regularization-based optimization algorithms, the proposed approach ensures robust and efficient performance in achieving the desired objectives. Theoretical analysis proves the global asymptotic convergence of the algorithm to the least-norm Nash equilibrium, and extensive simulation studies validate its effectiveness and practical relevance. Future work may explore the application of this algorithm to more complex multi-robot systems and investigate its performance under different environmental conditions and mission requirements.

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