# Normalization of Language Embeddings for Cross-Lingual Alignment

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## Abstract

Learning a good transfer function to map the word vectors from two languages 1 2 into a shared cross-lingual word vector space plays a crucial role in cross-lingual 3 NLP. It is useful in translation tasks and important in allowing complex models built on a high-resource language like English to be directly applied on an aligned 4 low resource language. While Procrustes and other techniques can align language 5 models with some success, it has recently been identified that structural differences 6 (for instance, due to differing word frequency) create different profiles for various 7 8 monolingual embedding. When these profiles differ across languages, it corre-9 lates with how well languages can align and their performance on cross-lingual downstream tasks. In this work, we develop a very general language embedding 10 normalization procedure, building and subsuming various previous approaches, 11 which removes these structural profiles across languages without destroying their 12 intrinsic meaning. We demonstrate that meaning is retained and alignment is 13 improved on similarity, translation, and cross-language classification tasks. Our 14 proposed normalization clearly outperforms all prior approaches like centering and 15 vector normalization on each task and with each alignment approach. 16

# 17 **1 Introduction**

The best multilingual NLP approaches typically do not jointly learn a single embedding, since words 18 of the same language tend to cluster, and thus are not useful for translation and cross-lingual learning 19 tasks. Rather, after learning individual embeddings, the standard approach is to map word vectors 20 from multiple languages into a shared cross-lingual word vector space [15]. This shared space creates 21 a cross-lingual word embedding (CLWE) [22, 43]. These serve as a valuable tool for transferring 22 23 data across different languages, understanding cross-linguistic differences, and cross-lingual transfer 24 for downstream tasks, such as direct translation [16, 20, 24], cross-lingual information retrieval [42], cross-lingual document classification [23], and cross-lingual dependency parsing [17, 39]. 25

A common element of almost all CLWE methods is the use of a rigid, orthogonal transformation 26 mapping one embedding onto another so they inhabit a shared linguistic space. An orthogonal 27 transformation is a special class of transformations that can be interpreted as the space of (in our 28 case, high-dimensional) rotations around the origin, and also allowing a mirror flip. This family of 29 transformations preserves (a) linear and (b) angular properties. By linear properties, we mean that 30 the straight-line Euclidean distance between elements is preserved, as are more powerful properties 31 like analogies (e.g., Paris - France + Italy  $\approx$  Rome). Angular properties refer to measuring angles 32 between pairs of points (from the origin), and as a result, cosine distance is preserved. Given a 33 correspondence between pairs of objects across two embeddings, the classic Procrustes method, 34 provides a closed-form solution which minimizes the sum of Euclidean distances. Moreover, if the 35 vectors are all first made as unit vectors, then this also maximizes the sum of cosine similarities [10]. 36

Under this framework, there has been a flurry of work significantly improving CLWE model per-37 formance along two directions. Semi-supervised and unsupervised models make these approaches 38 require less input, and more amenable to lower-resource languages. For example Bootstrap Procrustes 39 (PROC-B) [15, 41] is semi-supervised in that it starts with a small pairwise correspondence (of 40 500-1000 words), aligns those to infer a larger correspondence, and repeats applying Procrustes 41 alignment. Methods like MUSE [8] are unsupervised, and use a GAN to estimate a correspondence 42 43 before applying a Procrustes procedure. The second direction is preprocessing the embeddings before applying the Procrustes alignment. 44

These involve methods like removing the mean, removing principal components, and normalization which we will discuss in depth later. In principle, these methods aim to remove the geometry of data intrinsic to particular languages (but not shared across languages) while preserving similarity properties as assured by orthogonal alignment. The space of transformations allowed under orthogonal alignments is quite large, and we make the point that unless this data geometry is "normalized" it

<sup>50</sup> inhibits the alignment from optimizing over the entirety of this large space.

Finally, we note that methods like Canonical Correlation Analysis (CCA) [13], Discriminative Latent
Variable (DLV) [36], and Ranking-based optimization (RCSLS) [21] have also been applied towards
finding an orthogonal alignment (or pair of alignments) which minimizes a different optimization
function – since the objective function may not align with sums of squared Euclidean of cosine
distance [8, 38]. Unlike the others, the RCSLS method notably does not require a rigid transformation.

The focus of this paper is on embedding *preprocessing*, and is agnostic to the method of alignment used afterward, whether it is Procrustes-based, or optimizing something else.

58 **Our contribution.** This work proposes a new and general approach to preprocessing word embeddings, subsuming many previous approaches. The key is Spectral Normalization which regularizes 59 the spectral properties of monolingual embeddings by setting all of the top singular vectors to have 60 the same singular value. However, it leaves alone the smaller singular value; these capture important 61 information and cannot be zeroed out, but making them the same value as the top singular vectors 62 introduces too much noise. Spectral normalization already performs as well as the best previous 63 approaches on alignment and translation tasks, and since it applies a fairly uniform stretching to the 64 embeddings it does not distort monolingual similarity performance. Moreover, we show layering 65 Spectral Normalization within an iterative sequence with also centering and vector length normaliza-66 67 tion improves results further. We first demonstrate this improvement on the standard translation task 68 (BLI). We also show that this normalization preserves the core ontological structure of embeddings 69 across languages, and that applying our normalization before aligning a low resource language to English improves performance on topic classification and on a natural language inference task. 70

# 71 2 Existing Methods for Orthogonal Vector Spaces Alignment

Given a language L, our starting point is an embedded representation of a set of n words. Indexing words from i = 1 ... n, each word is associated with a vector  $x_L^i \in \mathbb{R}^d$ . And let  $X_L = \{x_L^1, ..., x_L^n\}$ be the set of n words as their vector representation. These vector representations (derived by methods like word2vec [28], GloVe [33], or FastText [4]) are chosen so words with similar pairwise cosine similarity are found in the similar local context in large text corpora on which they are trained. Higher-level linear structure is shown to emerge, such as concept subspaces and analogies [29].

The focus of this paper is on aligning embeddings of two languages  $L_1$  and  $L_2$ . Each embedding  $X_{L_1}$  and  $X_{L_2}$ , only is designed to ensure pairwise relationships between its word vectors, but the actual coordinates of those vectors do not have any explicit meaning. Yet, previous work has clearly demonstrated that there exists significant overall structural similarity, and alignment seeks to make correspondences between those structures for translation and joint understanding.

Most methods start with a known correspondence (or build one) between a set of K words in two languages, wlog let these be the same first K indexed words in those languages, denoted  $X_{L_1}^K = \{x_{L_1}^1 \dots x_{L_1}^K\}$  and  $X_{L_2}^K = \{x_{L_2}^1 \dots x_{L_2}^K\}$ . Then the *Procrustes Problem* solve for an orthogonal matrix  $W^* = \arg\min_W \|X_{L_1}^K W - X_{L_2}^K\|_2^2$ . There exists a simple solution [2, 38, 45, 10] as  $W^* = UV^{\top}$  where  $U\Sigma V^{\top} = \operatorname{svd}(X_{L_1}^K (X_{L_2}^K)^{\top})$ . Dev *et.al.* [10] also point out that if all vectors are normalized first, then this procedure also maximizes the sum of cosine similarities.

#### 89 2.1 Pre-processing Embeddings before Orthogonal Alignment

It turns out directly aligning embeddings from two languages (even using the "optimal" Procrustes 90 solution) does not provide the best possible joint embedding for translation tasks. While word 91 meaning appears to hold a similar structure, languages have other properties such as differing word 92 frequency, and this for instance leads to more frequent words having longer vectors in embeddings. 93 This extra language-specific structure tends to interfere with alignment. As a result, a number 94 of techniques have been developed to "normalize" the embeddings before Procrustes (or other) 95 alignment. This in some sense allows the word meaning to dominate the optimization tasks without 96 other confounding factors. We review the most common normalization approaches. 97

Mean Centering (C) subtracts the mean of all vectors in an embedding from each vector in that embedding. The result is that the mean of all vectors is 0. This is a rigid transformation, and so does not change the Euclidean distance between any pair of points in an embedding, and also preserves any linear property like analogies (e.g., Paris - France + Italy  $\approx$  Rome). Dev *et. al.* [10] points out that this is the first step (followed by the Procrustes orthogonal transformation) to minimize the sum of squared Euclidean distances among paired words, under any rigid transformation. However, this *does change* the cosine distance between pairs of points.

Length Normalization (L) makes each vector have a 2-norm equal to 1, but retains its direction from the origin [2, 45]. This preprocessing step does not change the cosine distance between any pair of points in an embedding. But, it *does change* the Euclidean distance between pairs of points.

Despite these contrasting goals, these two normalizations each turn out to be individually effective in regularizing the geometry of the embeddings, and allow for better CLWE. Wang *et. al.* [47], realized doing both was even more effective, and showed that iterating these two steps achieves the stateof-the-art way to preprocess, we denote as **I-C+L**. Iterative Normalization transforms monolingual word embeddings to have unit-length and zero-mean simultaneously (in practice they terminate this iterative process after a few steps before it achieves these two goals exactly).

**PCA Removal (PR)** computes the principal component analysis (PCA) of an embedding, and then 114 projects away from the direction of the top principal component, removing it [31]. Mu et. al. [31] 115 observed that the top singular values typically do not encode essential semantic relationships between 116 words but rather align strongly with word frequency. Also, they notice that the top principal values 117 are much larger than the other values. After eliminating d/100 principal components, where d is the 118 dimension of a word representation, they achieve better performance on both intrinsic and extrinsic 119 tasks. Sachidananda et. al. [37] applied this simple pre-processing approach to their Filtered Inner 120 Product Projection (FIPP) alignment method, and they significantly improve on the BLI task. 121

## 122 2.2 Spectral Statistics of Embeddings

Dubossarsky et. al. [11] recently documented how cross-lingual alignment is strongly affected by the 123 spectral statistics of monolingual embeddings. We stack the embedded vectors  $x_L^i \in \mathbb{R}^d$  as rows in 124 a  $n \times d$  matrix  $A \in \mathbb{R}^{n \times d}$ . The SVD decomposes A into  $U \Sigma V^{\top}$  where U and V contain the left 125 and right singular vectors, and the singular values  $\sigma_1 \ge \sigma_2 \ge \ldots \ge \sigma_d \ge 0$  are on the diagonal of  $\Sigma$ . The *effective rank* of A is a smoother analog to rank (when there is noise in low rank components), 126 127 defined  $\operatorname{er}(A) = e^{H}(\Sigma)$  where  $H(\Sigma) = -\sum_{i=1}^{d} \bar{\sigma}_{i} \log \bar{\sigma}_{i}$  with  $\bar{\sigma}_{i} = \sigma_{i} / \sum_{i=1}^{d} \sigma_{i}$ . The effective condition number  $\kappa_{\operatorname{eff}}(A) = \sigma_{1} / \sigma_{\operatorname{er}(A)}$ , which replaces the numerator (of condition number,  $\sigma_{d}$ ) 128 129 with the more robust singular value at the effective rank. This is desired to be small in stable data 130 sets. The joint effective condition number measures the harmonic mean of the effective condition 131 number across two matrices A, A' as ECOND-HM $(A, A') = \frac{2\kappa_{\text{eff}}(A)\kappa_{\text{eff}}(A')}{\kappa_{\text{eff}}(A) + \kappa_{\text{eff}}(A')}$ . The singular value gap 132 measures how similar the singular value sequences are between two matrices as SVG(A, A') =133  $\sum_{i=1}^{d} (\log \sigma_i - \log \sigma'_i)^2$ . These should both be smaller, for more comparable data sets. 134

Dubossarsky *et. al.* [11] applied these to monolingual embeddings and demonstrated that the performance of several CLWE methods were closely tied to these spectral properties. Basically embeddings align better if they are better jointly conditioned, especially measured via joint effective condition number and the singular value gap. Motivated by this idea, we propose methods that spectrally normalize embeddings improving these statistics while retaining intra-embedding meaning.

# **140 3 New Normalization Methods**

We introduce more direct and more general techniques to normalize monolingual word embeddings
 to more effectively prepare them for alignment. The goal is to remove language-specific geometry
 while maintaining the intrinsic similarity and structure captured within them.

#### 144 3.1 Geometric Median Normalization

Iterative Normalization enforces individual word embeddings to have a unit length and each monolingual embedding to have a zero mean through an iterative technique. Wang *et. al.* [47] showed that iterating solutions for these distinct goals will eventually converge to a solution which satisfies both.

In this paper, we observe that both goals can be done in one shot without iterating – by solving the Fermat-Weber problem [27, 25]. This dates to the 17th century, and corresponds with identifying the geometric median of a point set. Formally, the goal is a point  $x^* \in \mathbb{R}^d$  that minimizes the sum of distances from *n* anchor points  $\{a_1, \ldots, a_n\} \subset \mathbb{R}^d$  which are not collinear:  $x^* = \min_x \sum_{i=1}^n ||x - a_i||$ . Several methods [6, 12, 32] have been proposed; the most popular is the Weiszfeld's algorithm (Weiszfeld, see Appendix A). It is folklore that the solution  $x^*$  satisfies that  $0 = \sum_{i=1}^n \frac{a_i - x^*}{||a_i - x^*||}$ ; we do not know of a written proof, so prove this in Appendix A.1 for completeness.

Using this characteristic of the geometric median, we can simultaneously enforce monolingual word embeddings to have unit-length and zero-mean in just one step. This can be done using the Geometric Median normalization (GeoMediaN) algorithm (as Algorithm 1). Given a monolingual

word embedding A, we compute the geometric median  $x^*$ , and "center" the data on this point, and

<sup>159</sup> unit length normalizes the centered embedding.

Algorithm 1 Geometric Median Normalization: GeoMediaN(A)	
1: $x^* \leftarrow Weiszfeld(A)$	
2: for all $a_i \in A$ do $a_i \leftarrow \frac{a_i - x^*}{\ a_i - x^*\ }$	
3: return A	
	-

After these steps, all vectors are unit length, and because of the folklore property (Theorem A.1), the mean of those points is also 0. As a result, we can state the following property.

## **Theorem 3.1.** The output of GeoMediaN(A) is centered and length normalized.

Despite the Geometric Median Normalization algorithm's ability to enforce unit-length and zeromean in just one step, we will observe that it does not perform especially well on the BLI task.
Both GeoMediaN and I-C+L achieve one of many solutions which achieve these joint goals. We
next investigate another one that works better: it preserves meaning and structure, and removes
language-specific geometry allowing improved alignment.

#### 168 3.2 Spectral Normalization

The predominant effect of unit-length and zero-mean normalization on monolingual word embeddings 169 is that it makes embedding vectors from a language lie on a hypersphere with the center of the 170 171 hypersphere centered at the origin. However, this does not take into account how the word embeddings vectors are spread out or clustered on the hypersphere. Approaches like PCA removal and mean 172 centering have the effect of reducing the top principal component or top singular vector. As a result, 173 if the spectral properties are extreme, it can help regularize them. However, this approach can be a 174 bit blunt. PCA removal makes the top singular value exactly 0, so the condition number becomes 175 infinite. Other quantities like the effective condition number,  $\kappa_{\text{eff}}(A)$ , do however, tend to decrease. 176 To this effect, we propose a new algorithm Spectral Normalization that more gently regularizes 177

the spectral properties of word embeddings; see Algorithm 2. We will then combine it with other approaches to again ensure the embedding vectors lie on the unit sphere.

Algorithm 2 Spectral Normalization (SpecNorm $(A, \beta)$ )

Compute svd(A) = UΣV<sup>T</sup>; Let D ∈ ℝ<sup>d</sup> be a diagonal matrix.
 Compute η = √||A||<sub>F</sub><sup>2</sup>/d, where d is the dimension of the word embedding
 for i = 1,..., d do
 if (Σ<sub>ii</sub> > βη) then D<sub>ii</sub> ← Σ<sub>ii</sub>/(βη)
 else D<sub>ii</sub> = 1.
 return AVD<sup>-1</sup>

Given a monolingual word embedding A it updates part of the spectral properties of A as a whole, 180 using on a parameter  $\beta \ge 0$ . Based on an average of singular values  $\eta = \sqrt{\|A\|_F^2/d}$ , if a value is 181 above  $\beta$  times that average, it adjusts it to  $\beta\eta$ . Hence, all of the top directions are given the same 182 singular value. Otherwise, if it is below  $\beta\eta$ , it is considered a minor effect (some are quite small, 183 and fairly noisy), and it is left alone. If these small ones are completely zeroed out, the critical 184 information within is destroyed. However, if these small ones are also given the same value (i.e.,  $\beta\eta$ ) 185 then components which do not contribute to the most prevalent aspects of a vectors similarity is given 186 more importance, and we observed (see Section 4.1) that the usefulness of the embedding decreased. 187

**Iterative Spectral Normalization.** Spectral normalization makes the most sense (see Appendix G) in a setting where the vectors are already centered, and also unit length. While SpecNorm does not change the center of the data, it does not maintain the length of individual vectors. As such, we advocate combining these methods into a single iterative algorithm: I-C+SN+L as in Algorithm 3.

Algorithm 3 Iterative Spectral Normalization with C+L normalization (I-C+SN+L(A, #Iter))

1: for #Iter steps do 2:  $A \leftarrow \text{Center } A$ 3:  $A \leftarrow \text{SpecNorm}(A)$ 4:  $A \leftarrow \text{Unit length normalization of } A$ 5: return A

We observe in Figure 1 that this process significantly improves the spectral properties, compare to any other approach. Without preprocessing (None), the languages (EN: English, DE: German, HI: Hindi, JA: Japanese shown) have large effective condition numbers – indicating that there is a large disparity between meaningful singular values. Note the y-axis is in log scale. Hence, aligning these languages without normalization would likely restrict alignment among top singular vectors, not allowing enough degree of freedom to align corresponding words.

In contrast, after preprocessing when these values are more uniform, rotations among the dimensions containing the top principal components will not have an influence on the data distribution, and can fully optimize the alignment between words. Moreover, Figure 1 shows that I-C+SN+L most decreases the effective condition number, joint effective condition number, and singular value gap. Further, these values are fairly uniform across languages, despite great variation beforehand (as shown with None). In fact, I-C+SN+L is much more effective than other methods.

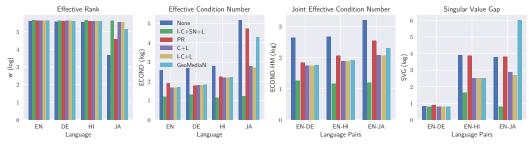


Figure 1: Spectral Measures of four (4) monolingual word embeddings, before (None) and after applying various normalization methods.

# **204 4 Experimental Analysis**

We provide an evaluation of our proposed preprocessing methods using 8 language embeddings pretrained on Wikipedia [4] of each language: Croatian (HR), English (EN), Finnish (FI), French (FR), German (DE), Italian (IT), Russian (RU), and Turkish (TR). We use the 300-dimensional fastText [4]<sup>1</sup> embeddings, and all vocabularies are trimmed to the 200K most frequent words.

Alignment evaluation tasks: BLI We evaluate and compare our proposed preprocessing methods 209 mostly on the Bilingual Lexicon Induction (BLI) task, a word translation task. We discuss two more 210 global applications Cross-lingual document classification (CLDC), and Cross-lingual transfer for 211 natural language inference (XNLI) later. BLI is more direct, and has become the de facto evaluation 212 task for CLWE models. For words in the source language, this task retrieves the nearest neighbors in 213 the target language after alignment to check if it contains the translation. It reports the mean average 214 215 precision (MAP) [15], which is equivalent to the mean reciprocal rank (MRR), of the translation. 216 Unless stated otherwise, reported values on baseline methods are taken from [15], and use the Google Translate (GTrans) dictionary from  $[15]^2$ . We trained (aligned) using 1k, 3k and 5k source words and 217 evaluated (tested) on separate 2k source test queries, unless noted otherwise. 218

Alignment Algorithms. We evaluated and compared the result of several supervised rigid-219 transformation CLWE models on the evaluation benchmarks using our proposed methods. All 220 have publically available code, links are found in the reference citation. These include Canonical 221 Correlation Analysis (CCA) [13], Procrustes (PROC) [2, 38, 45, 10], Bootstrapping Procrustes 222 (PROC-B) [15], and Discriminative Latent-Variable (DLV) [36], as discussed in Section 1. We also 223 consider Ranking-Based Optimization (RCSLS) [21] which is not a rigid alignment. In a few places, 224 we also compare with VECMAP [3] as an example of an unsupervised alignment process. This should 225 only use the geometry of the global embedding structure, e.g., derived from the natural ontology, and 226 our normalization method still helps when using this approach. 227

## 228 4.1 Hyperparameter Tuning

Our main proposed algorithm I-C+SN+L has a few simple parameters. To avoid overfitting, we choose these through cross-validation on English (EN) and a held-out set of 5 languages Hindi (HI), Russian (RU), Chinese (ZH), Japanese (JA), Turkish (TR). Ten (10) Language pairs of the form EN-X and X-EN were considered. The hyperparameters  $\beta \in \{1, 2, 3, 4, 5\}$  and #Iter (number of iterations)  $\in \{1, 2, 3, 4, 5\}$  were fine-tuned for I-C+SN+L.

We used the publicly available MUSE<sup>3</sup> translation dictionary [8] for hyperparameter tuning. The Procrustes alignment algorithm was trained on 5k source words and evaluated on 1.5k source test queries. We reported the mean average precision (MAP) in Table 1 for  $\beta \in \{1, 2, 3, 4, 5\}$  and with #Iter  $\in \{1, 2, 3, 4, 5\}$ . We observe the value of  $\beta = 2$  was consistently the best threshold (although any  $\beta \ge 2$  performed similarly). These singular values were normalized, and those below  $\beta$  times the average we judged as noise, and left as is. However, the result did not change much with respect to the number of iterations.

β	#Iter=1	#Iter=2	#Iter=3	#Iter=4	#Iter=5
1	0.363	0.340	0.328	0.322	0.317
2	0.385	0.386	0.386	0.386	0.386
3	0.381	0.384	0.384	0.384	0.384
4	0.381	0.382	0.382	0.382	0.382
5	0.380	0.381	0.381	0.381	0.381

Table 1: Cross-Validation for Hyperparameter Tuning: MAP after Procrustes for 10 language pairs.

The tie between the #Iter hyperparameter was broken using their performance on thirteen English word similarity benchmarks; see Appendix C for more details. In Table 2 below, we report the

<sup>&</sup>lt;sup>1</sup>https://github.com/facebookresearch/fastText

<sup>&</sup>lt;sup>2</sup>https://github.com/codogogo/xling-eval?utm\_source=catalyzex.com

<sup>&</sup>lt;sup>3</sup>https://github.com/facebookresearch/MUSE

average Spearman rank coefficient score on the word similarity task (None means no normalization).  $(\beta, \#Iter) = (2, 5)$  achieved the highest score. So hereafter, we applying I-C+SN+L with the

hyperparameter ( $\beta$ , #Iter) = (2, 5).

Table 2: Monolingual word similarity Task; Average Spearman rank coefficient

None	$(\beta, \# \mathrm{Iter}) = (2,2)$	$(\beta, \# \mathrm{Iter}) = (2,3)$	$(\beta, \# \mathrm{Iter}) = (2,4)$	$(\beta, \# \mathrm{Iter}) = (2,5)$
0.651	0.67077	0.67101	0.67108	0.67111

Note that the proposed approach (I-C+SN+L) only increased this score for these similarity tasks, so showed no signs of distorting inherent information. Although Spectral Normalization does not exactly preserve the linear properties or angular properties (as centering and length normalization do, one each, respectively), it does not suffer ill effects. We hypothesize this is because it is somewhat uniformly stretching words along the major modes of variation, and is effectively removing information not relevant to meaning, like frequency of occurrence. This benign effect is on contrast to other spectral adjustments (removing small singular values, or setting all to the same value) shown in Appendix C.

### 253 4.2 BLI Performance across Normalization and Alignment Algorithms

We compare and evaluate the BLI performance (MAP) of various normalization algorithms from 254 previous works to our proposed algorithms. Using the MUSE translation dictionary, we trained CCA, 255 PROC, PROC-B and RCSLS on 5k source words and evaluated on 1.5k source test queries. The 256 following normalization algorithms were used in the comparison analysis: PR (PCA Removal) [31], 257 GeoMediaN (Geometric Median Normalization), C+L (Mean centering and Length normalization, 258 1 round), I-C+L (Iterative Mean centering and Length normalization, 5 rounds) [47], SN (Spectral 259 Normalization, 1 round), C+SN+L (Mean centering, Spectral Normalization and Length normal-260 ization, 1 round), and I-C+SN+L (Iterative Mean centering, Spectral Normalization and Length 261 normalization, 5 rounds). Specifically, we evaluated 18 language pairs, i.e., English (EN) from/to 262 Bulgarian (BG), Catalan (CA), Czech (CS), German (DE), Spanish (ES), Korean (KO), Thai (TH) and 263 Chinese (ZH), separate from hyperparameter tuning. The average is reported in Table 3, all results 264 are in Appendix G. For almost all algorithms I-C+SN+L achieves the best scores (and especially on 265  $X_{L_2}$ -EN, often considerably better). The only exceptions are on non-rigid RCSLS when C+SN+L 266 (with no iteration) or just SN (with C+L) performs slightly better. So, Spectral Normalization, and in 267 268 particular I-C+SN+L, is shown as the best way to normalize languages before alignment.

		Method	$s: EN-X_L$	2	Methods : $\mathbf{X}_{L_1} - \mathbf{E}\mathbf{N}$				
Normalization	CCA	PROC	PROC-B	RCSLS	CCA	PROC	PROC-B	RCSLS	
None	0.358	0.365	0.377	0.394	0.398	0.399	0.405	0.428	
PR	0.394	0.391	0.404	0.373	0.434	0.430	0.442	0.425	
GeoMediaN	0.393	0.391	0.400	0.379	0.433	0.432	0.440	0.429	
C+L	0.393	0.394	0.408	0.404	0.439	0.437	0.445	0.464	
I-C+L	0.394	0.395	0.410	0.406	0.439	0.438	0.448	0.460	
SN	0.391	0.394	0.408	0.405	0.440	0.438	0.451	0.468	
C+SN+L	0.395	0.396	0.413	0.407	0.444	0.444	0.458	0.466	
I-C+SN+L	0.396	0.398	0.414	0.406	0.445	0.446	0.461	0.466	

Table 3: BLI performance (MAP) on aligning  $EN-X_{L_2}$  and  $X_{L_2}-EN$ 

We also compute the average BLI MAP score across all 28 language pair for more direct comparison to prior work [15], summarized in Table 4 and Appendix D. All results are in Appendix H. We compare I-C+SN+L (denoted with SN) against no normalization on various dictionary sizes: 1k, 3k and 5k source words and evaluated on 2k source test queries. In all cases, I-C+SN+L significantly improves over the baseline. This includes improvement over RCSLS which is non-rigid, so in

principle could "learn" adjustments similar to our normalization in the process of alignment. We also

<sup>275</sup> tested on VECMAP, an unsupervised approach; I-C+SN+L preprocessing also improves this result

276 from 0.375 to 0.410.

Table 4: Summary of BLI performance (MAP), average scores for all 28 language pairs. No normalization results from [15], against I-C+SN+L (denoted SN).

Dict	CCA	CCA <sup>SN</sup>	PROC	<b>PROC<sup>SN</sup></b>	PROC-B	PROC-B <sup>SN</sup>	DLV	DLV <sup>SN</sup>	RCSLS	RCSLS <sup>SN</sup>
1K	.289	.314	.299	.326	.379	.407	.289	.332	<b>.331</b>	.331
3K	.378	.401	.384	.408	.398	.415	.381	.429	.415	.427
5K	.400	.423	.405	.429	–	_	.403	.452	.437	.460

## 277 4.3 Downstream Tasks

We conclude by demonstrating that Spectral Normalization not only improves in direct translation tasks, but also captures an important global structure that generalizes from a high resource language (i.e., English, EN) to lower resource languages. In both examples, a powerful classifier is trained on the EN embedding (after normalization), and then we demonstrate that after a lower resource language (e.g., German, DE) has been normalized and align the analysis task can be directly applied to that language. And in particular, adding the simple process of our normalization (I-C+SN+L) dramatically improves the results over not doing that step.

**Cross-lingual Document Classification (CLDC).** The CLDC task builds a topic classification 285 using a language model on a high resource language (in our case English EN) across 15 topics. 286 The TED CLDC corpus assembled by [19] was used for training and evaluation. Following [15] 287 a simple CNN was used to train. Table 5 summarizes the average F1-score for all topic classifiers 288 on 5 language pairs. The CLWEs induced by PROC<sup>SN</sup>, PROC-B<sup>SN</sup>, DLV<sup>SN</sup>, and RCSLS<sup>SN</sup> (using 289 I-C+SN+L) outperformed the baseline result (with no normalization) on the CLDC task, significantly 290 improving the best average score from 0.421 to 0.461. Glavas et.al. [15] use only 12 of 15 topics, 291 but could not confirm which, so we re-ran all baselines using all 15 topics. 292

Model	Dict	EN-DE	EN-FR	EN-IT	EN-RU	EN-TR	Avg
PROC	5K	.366	.258	.338	.288	.278	.306
PROC <sup>SN</sup>	5K	.436	.366	.427	.517	.511	.451
PROC-B	3K	.364	.304	.299	.336	.317	.324
PROC-B <sup>SN</sup>	3K	.448	.396	.423	.522	.517	.461
DLV	5K	.419	.336	.397	.493	.458	.421
DLV <sup>SN</sup>	5K	.433	.323	.406	.499	.472	.427
RCSLS	5K	.466	.397	.403	.403	.406	.415
RCSLS <sup>SN</sup>	5K	.468	.500	.443	.488	.394	.459

Table 5: CLDC performance (micro-averaged  $F_1$  scores). Cross-lingual transfer EN-X

Cross-lingual Natural Language Inference (XNLI). We evaluated the CLWE on a cross-lingual 293 natural language inference (XNLI) task. We used a multi-lingual XNLI corpus created by [9], which 294 is a collection of sentence pairs from the English MultiNLI corpus [44] translated into 14 languages. 295 The MultiNLI corpus contains 433k sentence pairs with the labels entailment, contradiction, and 296 neutral. The intersection between XNLI languages and BLI languages result in four XNLI evaluation 297 pairs: EN-DE, EN-FR. EN-TR and EN-RU. We use the training setup in [15] with the Enhanced 298 Sequential Inference Model [7] on English after normalization. First, we aligned normalized versions 299 of each language onto the normalized EN embedding to obtain the shared cross-lingual embedding. 300 Then we used the 5k test pairs from the XNLI corpus to evaluate each language alignment. Table 6 301 shows the result for PROC, PROC-B, and RCSLS alignments (DLV and VECMAP transform the 302 EN embedding in the process, so were omitted). We compare against the same procedure without 303

normalization from Glavas *et.al.* [15] (I-C+SN+L normalization denoted <sup>SN</sup>). As in other experiments, our normalization improves the average test accuracy with each alignment approach.

Model	Dict	EN-DE	EN-FR	EN-TR	EN-RU	Avg
PROC	5K	.607	.534	.568	.585	.574
PROC <sup>SN</sup>	5K	.611	.638	.542	.596	<b>.597</b>
PROC-B	3K	.615	.532	.573	.599	.580
PROC-B <sup>SN</sup>	3K	.624	.638	.548	.601	<b>.603</b>
RCSLS	5K	.390	.363	.387	.399	.385
RCSLS <sup>SN</sup>	5K	.499	.482	.504	.556	.510

Table 6: XNLI performance (test set accuracy)

# **306 5 Conclusion & Discussion**

We introduce a new way to normalize embeddings, based on spectral normalization, for use in creating cross-lingual word embeddings. Our approach generalizes previous approaches, and effectively removes much of the clustering of words based on properties other than the similarity which encodes meaning. When used to individually preprocess monolingual embeddings, our approach allows any alignment procedure to find better alignments: resulting in improved performance on direct translation tasks as well as cross-lingual topic classification and natural language inference tasks.

Social impacts. The vast majority of NLP research and cutting-edge advancements are in English. This disadvantages those who primarily operate in other languages, with less developed models, or less data to train models. As large language models are the cornerstone of most NLP research and development in English, one of this work's main goals is to port these advances to other languages, and those who use them. This will help unlock this technology to many others around the world. As with most models, this accuracy and improvement may vary across tasks and languages.

While language models have many positive use cases including improving accessibility, better 319 recommendations, and increased automation, they have some negative effects as well. These include 320 requiring potentially large computational and hence environmental cost, encoding and exacerbating 321 bias, and aiding in automatically generating fake or deceitful content. While this paper is unlikely to 322 change the *desire* to use embeddings, it aims to reduce the burden of use and increase the effectiveness 323 in lower-resource settings. And in particular to port models trained in English to other languages. 324 This would reduce the cost of retraining in other languages if the English model can be reused, easing 325 environmental costs. We support the maturing efforts in attenuating bias in all such embeddings. 326 And while we acknowledge the possibility of this work aiding in the automated creation of deceitful 327 content and the harm it can cause, we believe the many benefits outweigh the harms. 328

Limitations. The overarching goal in this line of work is to port the many advances built on embeddings from high-resource settings (like built on the English language), to lower-resource settings (like Turkish). This work can apply a powerful model built on an English language model (e.g., for natural language inference) and automatically invoke it in Turkish after the embeddings have been aligned. However, the alignment will not be any better than the low-resource embedding. If the embedding is too noisy or limited, then the analysis will likely not be effective.

Also, this work focuses on non-contextual embeddings like FastText, but not contextual ones like RoBERTa which have proven almost universally more effective in NLP on English. While in principle a normalization function and alignment could be *applied* to contextual settings, we are unaware of any technique for *learning* these mappings. We believe it could be important future work.

**Data, Code, and Experiments.** All existing methods are compared with publicly available code with publicly available data, with links above or in references. The exception is code for CLDC and XNLI is shared by Glavás *et.al.* [15]. Everything is run with default parameters; the exception is RCSLS were we follow the suggested hyperparameter selection strategy [21] (with learning rate in {1, 10, 25, 50} and epoch number in {10, 20}). Our new code for SpecNorm is in Appendix E.

## 344 **References**

[1] Eneko Agirre, Enrique Alfonseca, Keith Hall, Jana Kravalova, Marius Paşca, and Aitor Soroa.
 A study on similarity and relatedness using distributional and WordNet-based approaches. In
 *Proceedings of Human Language Technologies: The 2009 Annual Conference of the North American Chapter of the Association for Computational Linguistics*, pages 19–27, Boulder,
 Colorado, June 2009. Association for Computational Linguistics.

- [2] Mikel Artetxe, Gorka Labaka, and Eneko Agirre. Learning principled bilingual mappings
   of word embeddings while preserving monolingual invariance. In *Proceedings of the 2016 Conference on Empirical Methods in Natural Language Processing*, pages 2289–2294, Austin,
   Texas, November 2016. Association for Computational Linguistics.
- [3] Mikel Artetxe, Gorka Labaka, and Eneko Agirre. A robust self-learning method for fully unsupervised cross-lingual mappings of word embeddings. In *Proceedings of the 56th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 789–798, Melbourne, Australia, July 2018. Association for Computational Linguistics. https: //github.com/artetxem/vecmap.
- [4] Piotr Bojanowski, Edouard Grave, Armand Joulin, and Tomas Mikolov. Enriching word vectors
   with subword information. *Transactions of the Association for Computational Linguistics*,
   5:135–146, 2017.
- [5] Elia Bruni, Gemma Boleda, Marco Baroni, and Nam-Khanh Tran. Distributional semantics in
   technicolor. In *Proceedings of the 50th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 136–145, 2012.
- [6] Jacques Antonin Chatelon, Donald W Hearn, and Timothy J Lowe. A subgradient algorithm for certain minimax and minisum problems. *Mathematical Programming*, 15(1):130–145, 1978.
- [7] Qian Chen, Xiaodan Zhu, Zhen-Hua Ling, Si Wei, Hui Jiang, and Diana Inkpen. Enhanced
   LSTM for natural language inference. In *Proceedings of the 55th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 1657–1668, Vancouver,
   Canada, July 2017. Association for Computational Linguistics.
- [8] Alexis Conneau, Guillaume Lample, Marc'Aurelio Ranzato, Ludovic Denoyer, and Hervé
   Jégou. Word translation without parallel data, 2018.
- [9] Alexis Conneau, Ruty Rinott, Guillaume Lample, Adina Williams, Samuel Bowman, Holger
   Schwenk, and Veselin Stoyanov. XNLI: Evaluating cross-lingual sentence representations. In
   *Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing*,
   pages 2475–2485, Brussels, Belgium, October-November 2018. Association for Computational
   Linguistics.
- <sup>378</sup> [10] Sunipa Dev, Safia Hassan, and Jeff M Phillips. Absolute orientation for word embedding <sup>379</sup> alignment. *Knowledge and Information Systems*, 2021.
- [11] Haim Dubossarsky, Ivan Vulić, Roi Reichart, and Anna Korhonen. The secret is in the spectra:
   Predicting cross-lingual task performance with spectral similarity measures. In *EMNLP*, 2020.
- [12] James W Eyster, John A White, and Walter W Wierwille. On solving multifacility location
   problems using a hyperboloid approximation procedure. *AIIE Transactions*, 5(1):01–06, 1973.
- [13] Manaal Faruqui and Chris Dyer. Improving vector space word representations using multilingual
   correlation. In *Proceedings of the 14th Conference of the European Chapter of the Association for Computational Linguistics*, pages 462–471, Gothenburg, Sweden, April 2014. Association
   for Computational Linguistics.
- [14] Lev Finkelstein, Evgeniy Gabrilovich, Yossi Matias, Ehud Rivlin, Zach Solan, Gadi Wolfman,
   and Eytan Ruppin. Placing search in context: The concept revisited. In *Proceedings of the 10th international conference on World Wide Web*, pages 406–414, 2001.

- [15] Goran Glavaš, Robert Litschko, Sebastian Ruder, and Ivan Vulić. How to (properly) evaluate
   cross-lingual word embeddings: On strong baselines, comparative analyses, and some miscon ceptions. In *Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics*, pages 710–721, 2019. https://github.com/codogogo/xling-eval?utm\_
   source=catalyzex.com.
- [16] Stephan Gouws, Yoshua Bengio, and Greg Corrado. Bilbowa: Fast bilingual distributed
   representations without word alignments. In Francis Bach and David Blei, editors, *Proceedings* of the 32nd International Conference on Machine Learning, volume 37 of Proceedings of
   Machine Learning Research, pages 748–756, Lille, France, 07–09 Jul 2015. PMLR.
- [17] Jiang Guo, Wanxiang Che, David Yarowsky, Haifeng Wang, and Ting Liu. Cross-lingual
   dependency parsing based on distributed representations. In *Proceedings of the 53rd Annual Meeting of the Association for Computational Linguistics and the 7th International Joint Conference on Natural Language Processing (Volume 1: Long Papers)*, pages 1234–1244,
   Beijing, China, July 2015. Association for Computational Linguistics.
- [18] Guy Halawi, Gideon Dror, Evgeniy Gabrilovich, and Yehuda Koren. Large-scale learning of
   word relatedness with constraints. In *Proceedings of the 18th ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 1406–1414, 2012.
- [19] Karl Moritz Hermann and Phil Blunsom. Multilingual models for compositional distributed
   semantics. In *Proceedings of the 52nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 58–68, Baltimore, Maryland, June 2014. Association
   for Computational Linguistics.

[20] Geert Heyman, Ivan Vulić, and Marie-Francine Moens. Bilingual lexicon induction by learning
to combine word-level and character-level representations. In *Proceedings of the 15th Con- ference of the European Chapter of the Association for Computational Linguistics: Volume 1, Long Papers*, pages 1085–1095, Valencia, Spain, April 2017. Association for Computational
Linguistics.

- [21] Armand Joulin, Piotr Bojanowski, Tomas Mikolov, Hervé Jégou, and Edouard Grave. Loss
  in translation: Learning bilingual word mapping with a retrieval criterion. In *Proceedings*of the 2018 Conference on Empirical Methods in Natural Language Processing, pages 2979–
  2984, Brussels, Belgium, October-November 2018. Association for Computational Linguistics.
  https://github.com/facebookresearch/fastText/tree/master/alignment.
- [22] Mladen Karan, Ivan Vulić, Anna Korhonen, and Goran Glavaš. Classification-based self learning for weakly supervised bilingual lexicon induction. In *Proceedings of the 58th Annual Meeting of the Association for Computational Linguistics*, pages 6915–6922, Online, July 2020.
   Association for Computational Linguistics.
- [23] Alexandre Klementiev, Ivan Titov, and Binod Bhattarai. Inducing crosslingual distributed
   representations of words. In *Proceedings of COLING 2012*, pages 1459–1474, Mumbai, India,
   December 2012. The COLING 2012 Organizing Committee.
- [24] Guillaume Lample, Myle Ott, Alexis Conneau, Ludovic Denoyer, and Marc'Aurelio Ranzato.
   Phrase-based & neural unsupervised machine translation. In *Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing*, pages 5039–5049, Brussels, Belgium,
   October-November 2018. Association for Computational Linguistics.
- 433 [25] Robert F Love, James G Morris, and George O Wesolowsky. Facilities location. 1988.
- [26] Minh-Thang Luong, Richard Socher, and Christopher D Manning. Better word representations
   with recursive neural networks for morphology. In *Proceedings of the seventeenth conference on computational natural language learning*, pages 104–113, 2013.
- [27] EA Maxwell. 100 great problems of elementary mathematics: their history and solution. By
   Heinrich Dorme, translated by David. Ustin Pp. x, 393. \$2.00. 1965.(Dover). *The Mathematical Gazette*, 50(372):213–213, 1966.

- 440 [28] Tomas Mikolov, Kai Chen, Greg Corrado, and Jeffrey Dean. Efficient estimation of word 441 representations in vector space. *arXiv preprint arXiv:1301.3781*, 2013.
- [29] Tomáš Mikolov, Wen-tau Yih, and Geoffrey Zweig. Linguistic regularities in continuous space
   word representations. In *Proceedings of the 2013 conference of the north american chapter of the association for computational linguistics: Human language technologies*, pages 746–751, 2013.
- [30] George A Miller and Walter G Charles. Contextual correlates of semantic similarity. *Language and cognitive processes*, 6(1):1–28, 1991.
- [31] Jiaqi Mu, Suma Bhat, and Pramod Viswanath. All-but-the-top: Simple and effective postpro cessing for word representations. *CoRR*, abs/1702.01417, 2017.
- [32] Michael L Overton. A quadratically convergent method for minimizing a sum of euclidean
   norms. *Mathematical Programming*, 27(1):34–63, 1983.
- [33] Jeffrey Pennington, Richard Socher, and Christopher Manning. GloVe: Global vectors for
   word representation. In *Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing (EMNLP)*, pages 1532–1543, Doha, Qatar, October 2014. Association for
   Computational Linguistics.
- [34] Kira Radinsky, Eugene Agichtein, Evgeniy Gabrilovich, and Shaul Markovitch. A word at a
   time: computing word relatedness using temporal semantic analysis. In *Proceedings of the 20th international conference on World wide web*, pages 337–346, 2011.
- [35] Herbert Rubenstein and John B Goodenough. Contextual correlates of synonymy. *Communica- tions of the ACM*, 8(10):627–633, 1965.
- [36] Sebastian Ruder, Ryan Cotterell, Yova Kementchedjhieva, and Anders Søgaard. A discriminative
   latent-variable model for bilingual lexicon induction. In *Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing*, pages 458–468, Brussels, Belgium,
   October-November 2018. Association for Computational Linguistics. https://github.com/
   sebastianruder/latent-variable-vecmap.
- [37] Vin Sachidananda, Ziyi Yang, and Chenguang Zhu. Filtered Inner Product Projection for
   Multilingual Embedding Alignment. *arXiv e-prints*, page arXiv:2006.03652, June 2020.
- [38] Samuel L. Smith, David H. P. Turban, Steven Hamblin, and Nils Y. Hammerla. Offline bilingual
   word vectors, orthogonal transformations and the inverted softmax. *CoRR*, abs/1702.03859,
   2017.
- [39] Wasi Uddin Ahmad, Zhisong Zhang, Xuezhe Ma, Eduard Hovy, Kai-Wei Chang, and Nanyun
   Peng. On Difficulties of Cross-Lingual Transfer with Order Differences: A Case Study on
   Dependency Parsing. *arXiv e-prints*, page arXiv:1811.00570, November 2018.
- [40] Yehuda Vardi and Cun-Hui Zhang. The multivariate 11-median and associated data depth.
   *Proceedings of the National Academy of Sciences*, 97(4):1423–1426, 2000.
- [41] Ivan Vulić, Goran Glavaš, Roi Reichart, and Anna Korhonen. Do We Really Need Fully
   Unsupervised Cross-Lingual Embeddings? *arXiv e-prints*, page arXiv:1909.01638, September
   2019.
- [42] Ivan Vulic and Marie-Francine Moens. Monolingual and cross-lingual information retrieval
   models based on (bilingual) word embeddings. *Proceedings of the 38th International ACM SIGIR Conference on Research and Development in Information Retrieval*, 2015.
- [43] Zirui Wang, Jiateng Xie, Ruochen Xu, Yiming Yang, Graham Neubig, and Jaime G. Carbonell.
   Cross-lingual alignment vs joint training: A comparative study and A simple unified framework.
   *CoRR*, abs/1910.04708, 2019.

[44] Adina Williams, Nikita Nangia, and Samuel Bowman. A broad-coverage challenge corpus
 for sentence understanding through inference. In *Proceedings of the 2018 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long Papers)*, pages 1112–1122, New Orleans, Louisiana, June 2018.
 Association for Computational Linguistics.

[45] Chao Xing, Dong Wang, Chao Liu, and Yiye Lin. Normalized word embedding and orthogonal
 transform for bilingual word translation. In *Proceedings of the 2015 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Tech- nologies*, pages 1006–1011, Denver, Colorado, May–June 2015. Association for Computational
 Linguistics.

[46] Dongqiang Yang and David Martin Powers. Verb similarity on the taxonomy of WordNet.
 Masaryk University, 2006.

[47] Mozhi Zhang, Keyulu Xu, Ken-ichi Kawarabayashi, Stefanie Jegelka, and Jordan Boyd-Graber.
 Are girls neko or shōjo? cross-lingual alignment of non-isomorphic embeddings with iterative
 normalization. In *Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics*, pages 3180–3189, Florence, Italy, July 2019. Association for Computational Linguistics. https://github.com/zhangmozhi/iternorm?utm\_source=catalyzex.com.

# 502 Checklist

503	1.	For all authors
504 505		(a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
506		(b) Did you describe the limitations of your work? [Yes]
507		(c) Did you discuss any potential negative societal impacts of your work? [Yes]
508 509		(d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
510	2.	If you are including theoretical results
511 512		<ul><li>(a) Did you state the full set of assumptions of all theoretical results? [Yes]</li><li>(b) Did you include complete proofs of all theoretical results? [Yes]</li></ul>
513	3.	If you ran experiments
514 515		(a) Did you include the code, data, and instructions needed to reproduce the main experi- mental results (either in the supplemental material or as a URL)? [Yes]
516 517		(b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes]
518 519		(c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [N/A]
520 521		(d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [Yes]
522	4.	If you are using existing assets (e.g., code, data, models) or curating/releasing new assets
523 524		<ul> <li>(a) If your work uses existing assets, did you cite the creators? [Yes]</li> <li>(b) Did you mention the license of the assets? [N/A]</li> </ul>
525 526		<ul> <li>(c) Did you include any new assets either in the supplemental material or as a URL? [Yes] See code in Appendix E.</li> </ul>
527 528		<ul> <li>(d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [N/A]</li> </ul>
529 530		(e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [N/A]
531	5.	If you used crowdsourcing or conducted research with human subjects
532 533		(a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]

534 (b 535	) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]
536 (c 537	) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? $[N/A]$