Exploring Pedestrian Permeability in Urban Sidewalk Networks

Urban Science, Mobility, Pedestrian Networks, Vector Fields, Continuous Time Random Walks

The spatial structure of cities significantly influences pedestrian mobility, a crucial factor in designing sustainable urban environments. However, while pedestrian movement and walkability have been widely studied, quantitative insights into how urban layouts shape pedestrian flows remain limited. In this work, we study pedestrian diffusion through the lens of discrete vector fields to better understand the effects network structures may impose on walker dynamics.

To spatially characterize these dynamics, we leverage two well-established random walk models: the classical discrete-time random walk (DTRW) and the continuous-time random walk (CTRW) models. The CTRW [1] model introduces a temporal perspective into random walk dynamics by adding a Poisson-distributed random variable with exponentially distributed waiting times to each node, representing the rate at which walkers transition to the next vertex. In our context, to model the transition rates, the waiting times are adjusted based on the duration a walker spends traversing each link (street segment). Within the CTRW framework, this is modeled through a transition matrix R_{ij}, representing the rate between node i and j. With this framework, and given the net flow on edge (i,j) over the time interval dt, $d\omega_{ij} = \left[p_i(t)R_{ij} - p_j(t)R_{ji}\right]dt$, we can characterize the pedestrian vector field as the overall net flow of walkers up to a certain time, $t_{\max}$,

$$\omega_{ij} = \int_0^{t_{\max}} \left[p_i(t)R_{ij} - p_j(t)R_{ji}\right] dt. \tag{1}$$

We highlight that, in the stationary state, there is no net flux in any edge of the network, according to the detailed balance condition $p_iR_{ij} = p_jR_{ji}$, so the sum converges at the mixing time.

We then decompose the resulting field using the Helmholtz-Hodge Decomposition (HHD), see Fig. 1a, which separates the vector field into gradient and solenoidal (cyclic) components, offering insights into pedestrian permeability over the sidewalk network.

By comparing the HHD components generated with those obtained simulating random pedestrians walking at a constant velocity, we demonstrate that CTRWs provide a suitable framework for analysis, as they allow the incorporation of edge lengths and pedestrian speeds at the model. CTRWs capture detailed influences of network geometry on movement patterns that traditional discrete-time random walk models cannot describe (see Fig. 1b). Considering the spatial aspects (see Fig. 1c), experiments on a center-clustered lattice show that the HHD of CTRW flows yields the highest node potentials at the periphery and the lowest within the central cluster. This suggests that random pedestrians move more freely in the central part of the lattice but tend to become trapped in the outer regions. These results indicate that regions with shorter edge lengths and more intricate network structures enhance pedestrian permeability and diffusion, aligning with urban theories on walkability and accessibility, such as those proposed by Jane Jacobs. In contrast, the DTRW results display the opposite pattern (not shown in the figure), evidencing that DTRW are not suitable to model mobility in this context. Theoretically, we highlight that the field is fully characterized by its gradient component, with cyclic components only emerging in simulations due to stochasticity.

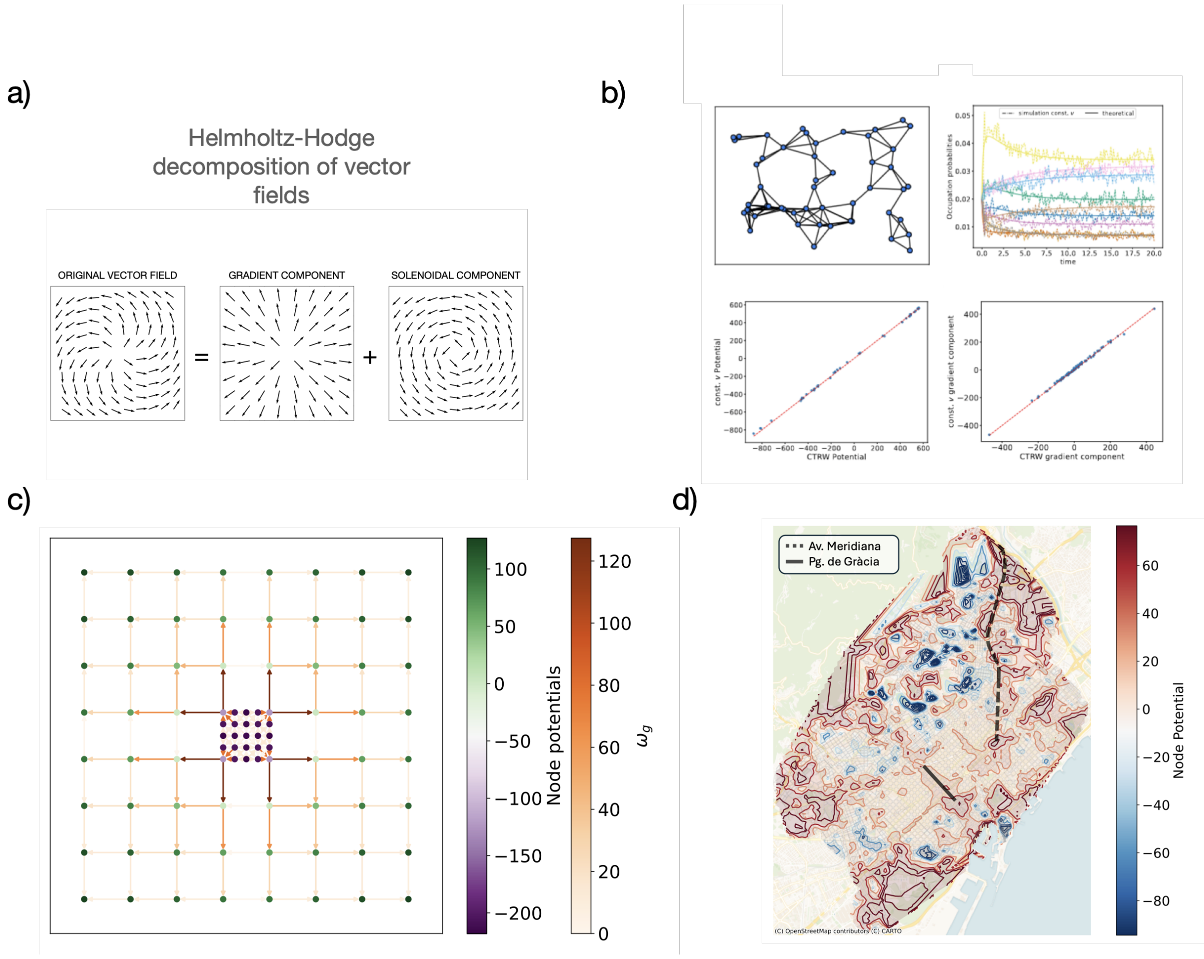

Figure 1: a) Example of the HHD, showing that any well-defined vector field can be decomposed into two patterns. b) Experiments on a random geometric network demonstrating that the CTRW correctly approximates the mobility of constant-velocity pedestrians. c) Node potentials and gradients of the CTRW on a center-clustered lattice. d) Experiments on the sidewalk network of Barcelona, detecting low- and high-permeability regions using the CTRW.

Showcasing a practical application, we analyze the sidewalk networks of Barcelona (Fig. 1d), Paris, and Boston, detailing how variations in local geometry, street density, and connectivity may help to study pedestrian mobility and urban permeability. Overall, our work shows that CTRW framework offers an analytical tool to identify areas that either facilitate or constrain pedestrian movement. We hope our findings will aid in designing pedestrian-friendly cities by highlighting the critical role of structural layout in influencing movement dynamics and urban permeability. A complete version of the work can be found in [2].

References

[1] J. Petit, R. Lambiotte, and T. Carletti, "Classes of random walks on temporal networks with competing timescales," *Applied Network Science*, vol. 4, no. 1, p. 72, 2019.

[2] R. Benassai-Dalmau, J. Borge-Holthoefer, and A. Solé-Ribalta, "Exploring pedestrian permeability in urban sidewalk networks," *Chaos, Solitons & Fractals*, vol. 194, p. 116114, 2025.