
Energy-Based Multimodal VAEs

Anonymous Author(s)

Affiliation

Address

email

Abstract

1 Multimodal VAEs are a promising class of multimodal generative models that
2 constructs a tractable posterior over the latent space given all modalities. Daun-
3 hawer et al. [2022] show that the generative quality of each modality drops as
4 we increase the number of modalities. In this work, we take another direction to
5 address the generative quality of multimodal VAEs by jointly modeling the latent
6 space of unimodal VAEs using energy-based models (EBMs). The role of EBM
7 is to enforce multimodal coherence by learning the correlation among the latent
8 variables. Therefore, our model enjoys the high generative quality of unimodal
9 VAEs while maintaining coherence across different modalities.

10 1 Introduction

11 The real-world data often has multiple modalities such as image, text, and audio, which makes
12 learning from multiple modalities an important task. Recently, promising results have been achieved
13 by multimodal generative models [Ramesh et al., 2021, Saharia et al., 2022]. However, these
14 models are often only generative in one modality while conditioning on the rest. On the other hand,
15 multimodal VAEs are a class of multimodal generative models that are able to generate multiple
16 modalities jointly. To train multimodal VAEs we have to construct a joint posterior over the latent
17 space $z: q(z|\mathbb{X})$, where \mathbb{X} is the set of modalities. To ensure the tractability of the inference network q ,
18 prior work has proposed using a product of experts ($q(z|\mathbb{X}) = \prod_i q(z|X_i)$) [Wu and Goodman, 2018],
19 mixture of experts ($q(z|\mathbb{X}) = \sum_i q(z|X_i)$) [Shi et al., 2019], or in the generalized form, mixture of
20 the product of experts (MoPoE) [Sutter et al., 2021].

21 These models rely on modality subsampling during training to have a better performance on inference
22 with missing modality at the test time. Subsampling of the modalities, as pointed out by Daunhawer
23 et al. [2022], results in a generative discrepancy among modalities. We also observe that conditioning
24 on more modalities often reduces the quality of the generated modality, which happens as a result of
25 using the product of experts for combining the modalities. Product of experts constructs a sharper
26 distribution by adding more components. The sharper the distribution is, the more confident it
27 becomes on the agreeing mode (increases coherence). On the other hand, the resulting distribution
28 becomes very picky and loses its generative quality.

29 To overcome these issues, instead of constructing a joint posterior, we try to explicitly model the joint
30 latent space of individual VAEs: $p_\theta(z_1, z_2, \dots, z_n)$. The joint latent model learns the correlation
31 among the individual latent space without constructing a joint posterior for all modalities. Therefore,
32 it can ensure prediction coherence while maintaining the generative quality. However, as expected,
33 as we increase the number of modalities, the joint latent model becomes more complicated, which
34 requires an appropriate factorization that is a subject of our future work. Nevertheless, conditioning
35 on more modalities results in more accurate marginal distributions, thus increasing the generative
36 quality.

37 **2 EB-MVAE**

38 EBMs have been successfully used for modeling text [Deng et al., 2020] and image [Du and Mordatch,
 39 2019, Song and Ermon, 2019] in the original data space. They also have been used to improve the
 40 performance of VAEs by modeling the latent space [Aneja et al., 2021, Pang et al., 2020]. In
 41 general, deep neural networks are effective in capturing the interaction of the variables, thus making
 42 the EBMs a successful model for joint modeling – EBMs parameterize the energy function of a
 43 Gibbs distribution over all variables using deep neural networks. We utilize this power to jointly
 44 model the latent space of different modalities: $p_\theta(z_1, z_2, \dots, z_n) \propto \exp(E_\theta(z_1, z_2, \dots, z_n))$. We
 45 cannot directly train the parameters θ using methods such as maximum likelihood, but several
 46 alternatives training algorithms have been proposed, including contrastive divergence [Hinton, 2002]
 47 and score-matching [Hyvärinen and Dayan, 2005]. In this work, we use score matching as we found
 48 it more stable and accurate for our setting. In score matching, we directly train the vector field,
 49 $S(\mathbf{z}) = -\nabla_{\mathbf{z}}E(\mathbf{z})$, by minimizing

$$\mathbb{E}_{p(\mathbf{x})}\mathbb{E}_{q(z_1|x_1)}\mathbb{E}_{q(z_2|x_2)}\cdots\mathbb{E}_{q(z_n|x_n)}\left[\text{tr}(\nabla_{\mathbf{z}}S_\theta(\mathbf{z})) + \frac{1}{2}\|S_\theta(\mathbf{z})\|_2^2\right], \quad (1)$$

50 where $q(z_i|x_i)$ is the unimodal posterior over i th modality x_i and is trained by optimizing the individual
 51 ELBO for that modality. We assume all of the modalities are present during training time for
 52 optimizing eq. 1 and we leave training with missing modality for future work. On inference time, any
 53 of the modalities can be missing.

54 **Conditional generation:** We assume at the inference time we have two groups of observed modalities
 55 (indexed by \mathbf{o}) and unobserved modalities (indexed by \mathbf{u}). We define the conditional posterior
 56 distribution for unobserved modalities as:

$$q(\mathbf{z}_{\mathbf{u}}|\mathbf{z}_{\mathbf{o}}, \mathbf{x}_{\mathbf{o}}) = \left[\prod_{i \in \mathbf{o}} q(z_i|x_i)\right] p_\theta(\mathbf{z}_{\mathbf{u}}|\mathbf{z}_{\mathbf{o}}) \quad (2)$$

57 Sampling from $q(\mathbf{z}_{\mathbf{u}}|\mathbf{z}_{\mathbf{o}}, \mathbf{x}_{\mathbf{o}})$ requires samples from unimodal posteriors of given modalities following
 58 by sampling from $p_\theta(\mathbf{z}_{\mathbf{u}}|\mathbf{z}_{\mathbf{o}})$. Knowing that $p_\theta(\mathbf{z}_{\mathbf{u}}|\mathbf{z}_{\mathbf{o}}) \propto \exp(-E_\theta(\mathbf{z}_{\mathbf{u}}, \mathbf{z}_{\mathbf{o}}))$, we sample from
 59 $p_\theta(\mathbf{z}_{\mathbf{u}}|\mathbf{z}_{\mathbf{o}})$ using Langevin dynamics [Welling and Teh, 2011]:

$$\mathbf{z}^{t+1} = \mathbf{z}^t - \frac{\lambda^2}{2}\nabla_{\mathbf{z}}E(\mathbf{z}^t, \mathbf{z}_{\mathbf{o}}) + \lambda\mathcal{N}(\mathbf{0}, \mathbf{I}). \quad (3)$$

60 **3 Experiments**

61 We compare EB-MVAE with different multimodal VAEs, including PoE [Wu and Goodman, 2018],
 62 MoE [Shi et al., 2019], and MoPoE [Sutter et al., 2021] using PolyMNIST dataset [Sutter et al.,
 63 2021]. This dataset consists of five different modalities created by changing the background images
 64 of an MNIST dataset. The encoder and decoder architecture of all methods are the same. We train
 65 the encoders and decoders using β -VAE [Higgins et al., 2016] with β -scheduling. We construct our
 66 energy-based model (EBM) by defining a multi-layer perceptron (MLP) over all five modalities. We
 67 assume all modalities are present during training. To train the EBM, we generate the samples from
 68 the posterior of each modality and minimize eq. 1.

69 Both EBM and VAEs are trained using Adam optimizer [Kingma and Ba, 2014] with a constant
 70 learning rate of 0.001. The VAEs are trained for 300 epochs with $\beta = 0.1$. We run Langevin
 71 dynamics for 40 steps to generate samples from the EBM.

72 We compare all methods on both prediction coherence and generative quality. We measure the
 73 coherence by evaluating the accuracy of the predicted modality based on the digits associated with
 74 the observed modalities. We also measure the generative quality of each modality using the FID
 75 score.

76 To evaluate our method, we first generate samples from the unconditional posterior for both EB-
 77 MVAE and MoPoE. For EB-MVAE since no modality has been observed, the posterior in eq. 2
 78 becomes equal to the joint distribution over all unimodal latent space ($p_\theta(\mathbf{z}_{\mathbf{u}})$). EB-MVAE has
 79 difficulty generating high quality images for modality 1 and modality 5. The main reason is that

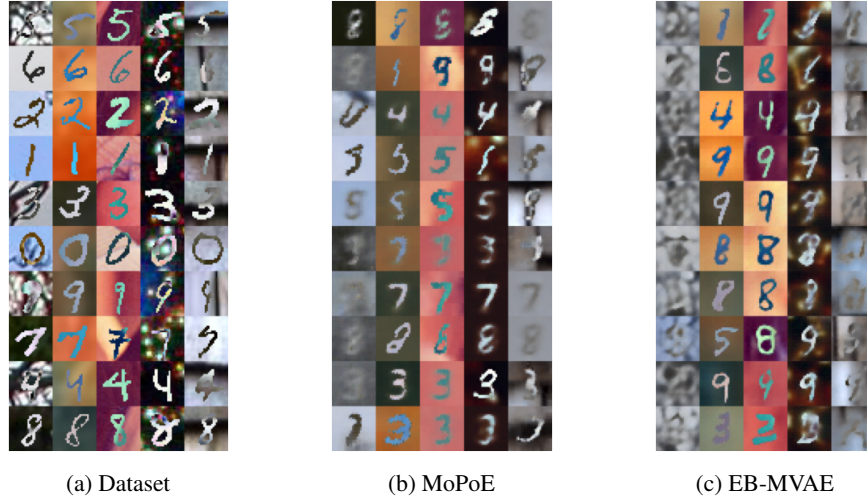


Figure 1: a) Samples from training data. Each column belongs to one modality (from left to right we name it as modality 1 to 5). b) Unconditional samples from MoPoE (no modality is observed). Each column shows the samples for the corresponding modality. c) Unconditional samples from EB-MVAE.

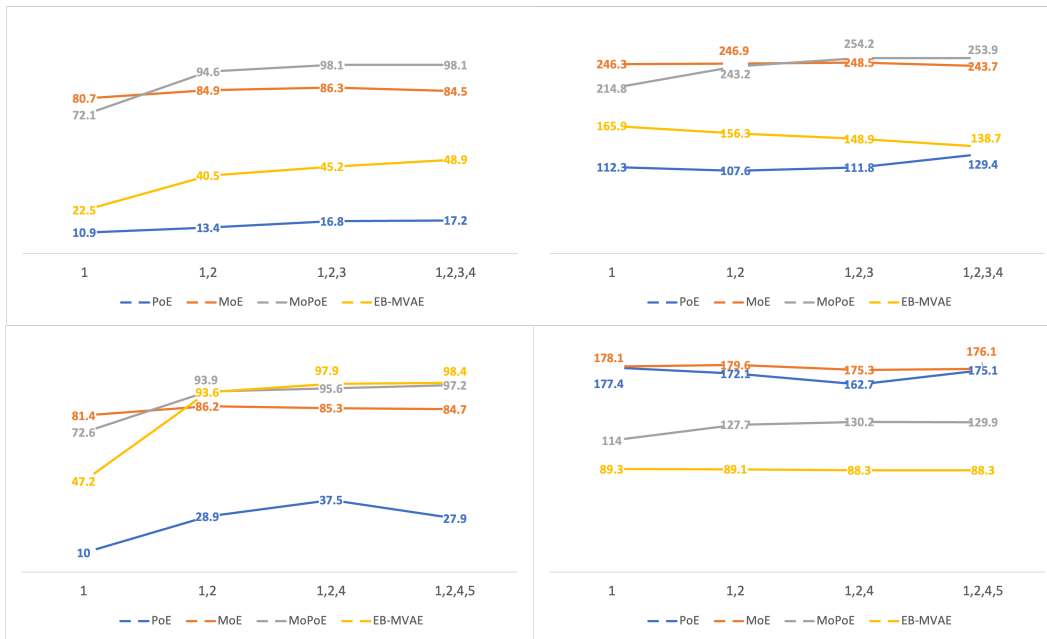


Figure 2: Left: Conditional coherence measured using prediction accuracy. Right: Conditional generative quality measured using FID score. The target modality in the first row is modality 5 and in the second row is modality 3.

80 fitting a joint model to data becomes more difficult as we increase the number of variables (modalities)
 81 and also the digits in these two modalities are more obscured by the background. The unimodal VAE
 82 tries to learn the background pattern as well as the digits and that propagates to the joint EBM model.
 83 MoPoE, on the other hand, tries to learn a common latent space for all modalities, thus emphasizes
 84 more on the common digit rather than the background information.

85 However, we still can expect that we get better conditional performance as we observe more modalities.
 86 To confirm this, we increase the number of observed modalities from 1 to 4 and report the accuracy
 87 and FID score for modality 3 and 5 in Figure 2. For a multimodal generative model, as we condition

88 on more modalities, we expect improvement in both prediction accuracy (coherent cross generation)
 89 and generative quality (synergy) [Shi et al., 2019]. Among PoE, MoE, and MoPoE, only MoE
 90 loosely follow the expected patterns, while MoPoE and PoE only respect coherent cross generation
 91 pattern and violates expected synergy pattern. EB-MVAE, on the other hand, shows better accuracy
 92 as we conditioned on more modality and at the same time its generative quality improves. This
 93 behavior is describable via its joint latent model. As we condition on more modality the marginal
 94 distribution gets closer to the target unimodal distribution. It is worth noting that the PoE, MoE, and
 95 MoPoE either have high quality generative capability (PoE) or high coherence (MoE and MoPoE),
 96 while EB-MVAE has no fundamental limitation (because of its joint modeling of the latent space of
 97 individual modalities) to have both properties. For predicting modality 3 given the rest of modalities,
 98 EB-MVAE has the best accuracy and generative quality among the methods.

99 We also qualitatively compare the conditional posterior of modality 3 given the rest of the modalities
 100 for EB-MVAE and MoPoE. In Figure 3 we draw one generated output using one sample from
 101 $q(z_3|z_1, z_2, z_4, z_5, x_1, x_2, x_4, x_5)$ for five different assignments to x_1, x_2, x_4, x_5 (that has the same
 102 digits) at each row. We also show the generated samples using unimodal posterior $q(z|x_i)$ for different
 103 data points with the same digits (each row). EB-MVAE samples have more variety than MoPoE
 104 samples and better capture the background, and the generative quality of samples is closer to those of
 105 unimodal VAE. This is evidence that the common latent space is more restricted than the joint model
 106 of unimodal latent spaces, which results in lower generative quality.

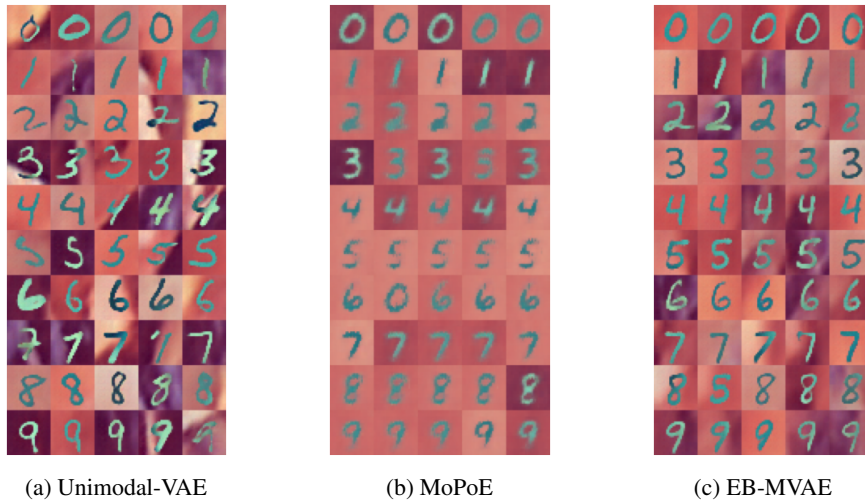


Figure 3: Samples from predicted modality 3. a) Sample from the unimodal VAE using posterior distribution. b) Samples from EB-MVAE conditioned on modalities 1,2,4,5. c) Samples from MoPoE conditioned on modalities 1,2,4,5.

107 4 Conclusion

108 Multimodal VAEs are an important tool for modeling multimodal data. In this paper, we provide
 109 a different multimodal posterior using energy-based models. Our proposed method learns the
 110 correlation of latent spaces of unimodal VAEs using a joint model in contrast to the traditional
 111 multimodal VAE construction that learns a common latent space for all modalities. We show that our
 112 method (EB-MVAE) can generate high quality and coherent samples.

113 References

- 114 Jyoti Aneja, Alex Schwing, Jan Kautz, and Arash Vahdat. {NCP}-{vae}: Variational autoencoders
 115 with noise contrastive priors, 2021. URL <https://openreview.net/forum?id=c1xAGI3nYST>.
- 116 Imant Daunhawer, Thomas M. Sutter, Kieran Chin-Cheong, Emanuele Palumbo, and Julia E Vogt.
 117 On the limitations of multimodal VAEs. In *International Conference on Learning Representations*,
 118 2022. URL <https://openreview.net/forum?id=w-CPUXXrAj>.

- 119 Yuntian Deng, Anton Bakhtin, Myle Ott, Arthur Szlam, and Marc’Aurelio Ranzato. Residual energy-
120 based models for text generation. In *International Conference on Learning Representations*, 2020.
121 URL <https://openreview.net/forum?id=B114SgHKDH>.
- 122 Yilun Du and Igor Mordatch. Implicit generation and modeling with energy based mod-
123 els. In H. Wallach, H. Larochelle, A. Beygelzimer, F. d’Alché-Buc, E. Fox, and R. Gar-
124 nett, editors, *Advances in Neural Information Processing Systems*, volume 32. Curran
125 Associates, Inc., 2019. URL [https://proceedings.neurips.cc/paper/2019/file/
126 378a063b8fdb1db941e34f4bde584c7d-Paper.pdf](https://proceedings.neurips.cc/paper/2019/file/378a063b8fdb1db941e34f4bde584c7d-Paper.pdf).
- 127 Irina Higgins, Loic Matthey, Arka Pal, Christopher Burgess, Xavier Glorot, Matthew Botvinick,
128 Shakir Mohamed, and Alexander Lerchner. beta-vae: Learning basic visual concepts with a
129 constrained variational framework. 2016.
- 130 Geoffrey E Hinton. Training products of experts by minimizing contrastive divergence. *Neural*
131 *Computation*, 14(8):1771–1800, 2002.
- 132 Aapo Hyvärinen and Peter Dayan. Estimation of non-normalized statistical models by score matching.
133 *Journal of Machine Learning Research*, 6(4), 2005.
- 134 Diederik P. Kingma and Jimmy Ba. Adam: A method for stochastic optimization, 2014. URL
135 <https://arxiv.org/abs/1412.6980>.
- 136 Bo Pang, Tian Han, Erik Nijkamp, Song-Chun Zhu, and Ying Nian Wu. Learning latent space
137 energy-based prior model, 2020. URL <https://arxiv.org/abs/2006.08205>.
- 138 Aditya Ramesh, Mikhail Pavlov, Gabriel Goh, Scott Gray, Chelsea Voss, Alec Radford, Mark Chen,
139 and Ilya Sutskever. Zero-shot text-to-image generation, 2021. URL [https://arxiv.org/abs/
140 2102.12092](https://arxiv.org/abs/2102.12092).
- 141 Chitwan Saharia, William Chan, Saurabh Saxena, Lala Li, Jay Whang, Emily Denton, Seyed
142 Kamyar Seyed Ghasemipour, Burcu Karagol Ayan, S. Sara Mahdavi, Rapha Gontijo Lopes, Tim
143 Salimans, Jonathan Ho, David J Fleet, and Mohammad Norouzi. Photorealistic text-to-image
144 diffusion models with deep language understanding, 2022. URL [https://arxiv.org/abs/
145 2205.11487](https://arxiv.org/abs/2205.11487).
- 146 Yuge Shi, Brooks Paige, Philip Torr, et al. Variational mixture-of-experts autoencoders for multi-
147 modal deep generative models. *Advances in Neural Information Processing Systems*, 32, 2019.
- 148 Yang Song and Stefano Ermon. Generative modeling by estimating gradients of the data distribution,
149 2019.
- 150 Thomas M. Sutter, Imant Daunhawer, and Julia E. Vogt. Generalized multimodal elbo, 2021. URL
151 <https://openreview.net/pdf?id=5Y21VORDBV>.
- 152 Max Welling and Yee W Teh. Bayesian learning via stochastic gradient langevin dynamics. In
153 *Proceedings of the 28th international conference on machine learning (ICML-11)*, pages 681–688.
154 Citeseer, 2011.
- 155 Mike Wu and Noah Goodman. Multimodal generative models for scalable weakly-supervised
156 learning. *Advances in Neural Information Processing Systems*, 31, 2018.