A DIFFERENTIABLE ALIGNMENT FRAMEWORK FOR SEQUENCE-TO-SEQUENCE MODELING VIA OPTIMAL TRANSPORT

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ABSTRACT

State-of-the-art end-to-end (E2E) ASR systems, such as the Connectionist Temporal Classification (CTC) and transducer-based models, suffer from peaky behavior and alignment inaccuracies. In this paper, we propose a novel differentiable alignment framework based on one-dimensional optimal transport, enabling the model to learn a single alignment and perform ASR in an E2E manner. We introduce a pseudo-metric, called Sequence Optimal Transport Distance (SOTD), over the sequence space and discuss its theoretical properties. Based on the SOTD, we propose Optimal Temporal Transport Classification (OTTC) loss for ASR and contrast its behavior with CTC. Experimental results on the TIMIT, AMI, and LibriSpeech datasets show that our method considerably improves alignment performance compared to CTC and the more recently proposed Consistency-Regularized CTC, though with a trade-off in ASR performance. We believe this work opens new avenues for seq2seq alignment research, providing a solid foundation for further exploration and development within the community.

1 INTRODUCTION

Sequence-to-sequence (seq2seq) alignment is a fundamental challenge in automatic speech recognition (ASR), where, beyond text prediction, precise alignment of text to the corresponding speech is crucial for many applications. For example, in medical domain, accurate alignment helps speech and language pathologists pinpoint speech segments for analyzing pathological cues, such as stuttering or voice disorders. In real-time subtitling, precise alignment ensures that subtitles are synchronized with spoken words, which is crucial for live broadcasts and streaming content. In language learning tools, ASR systems use alignment to provide feedback on pronunciation and fluency, allowing learners to compare their speech to target pronunciations. In these ASR-driven applications, while word error rate (WER) is an important performance metric, frame-level and word-level alignment accuracy are equally important for improving the system's applicability and responsiveness.

In the literature, two primary approaches to ASR have emerged, i.e., hybrid systems and end-to-end (E2E) models. In hybrid approaches, a deep neural network-hidden Markov model (DNN-HMM) Morgan & Bourlard (1990); Bourlard & Morgan (2012); Young (1996); Povey (2005); Abdel-Hamid et al. (2012); Graves et al. (2013a); Dahl et al. (2012) system is typically trained, where the DNN is optimized by minimizing cross-entropy loss on the forced alignments generated for each frame of audio embeddings from a hidden Markov model-Gaussian mixture model (HMM-GMM). One notable disadvantage of the hybrid approach is that the model cannot be optimized in an E2E manner, which may result in suboptimal performance Hannun (2014). More recently, E2E models for ASR have become very popular due to their superior performance. There are three popular approaches for training an E2E model: (i) attention-based encoder-decoder (AED) models Chan et al. (2015); Radford et al. (2023); Watanabe et al. (2017); Prabhavalkar et al. (2023), (ii) using Connectionist Temporal Classification (CTC) loss Graves et al. (2006); Graves & Jaitly (2014), and (iii) neural Transducer-based models Graves (2012); Kuang et al. (2022); Graves et al. (2013b). AED models use an encoder to convert the input audio sequence into a hidden representation. The decoder, typically auto-regressive, generates the output text sequence by attending to specific parts of the input through an attention mechanism, often referred to as soft alignment Yan et al. (2022) between the audio and text sequences. This design, however, can make it challenging to obtain word-level timestamps and to

do teacher-student training with soft labels. Training AED models also requires a comparatively large amount of data, which can be prohibitive in low-resource setups. In contrast to AED models, CTC and transducer-based models maximize the marginal probability of the correct sequence of tokens (transcript) over all possible valid alignments (paths), often referred to as hard alignment Yan et al. (2022). However, recent research has shown that only a few paths, which are dominated by blank labels, contribute meaningfully to the marginalization, leading to the well-known peaky behavior that can result in suboptimal ASR performance Zeyer et al. (2021). Unfortunately, it is not possible to directly identify these prominent paths, or those that do not disproportionately favor blank labels, in advance within E2E models. This observation serves as the main motivation of our work.

In this paper, we introduce the Optimal Temporal Transport Classification (OTTC) loss function, a novel approach to ASR where our model jointly learns temporal sequence alignment and audio frame classification. OTTC is derived from the Sequence Optimal Transport Distance (SOTD) framework, which is also introduced in this paper and defines a pseudo-metric for finite-length sequences. At the core of this framework is a novel, parameterized, and differentiable alignment model based on one-dimensional optimal transport, offering both simplicity and efficiency, with linear time and space complexity relative to the largest sequence size. This design allows OTTC to be fast and scalable, maximizing the probability of exactly one path, which, as we demonstrate, helps avoid the peaky behavior commonly seen in CTC based models.

To summarize, our contributions are the following:

- We propose a novel, parameterized, and differentiable seq2seq alignment model with linear complexity both in time and space.
- We introduce SOTD, a novel framework for comparing finite-length sequences, with theoretical guarantees on the existence and properties of a minimum.
- We derive a new loss function, i.e., OTTC, specifically designed for ASR tasks.
- Finally, we conduct proof-of-concept experiments on the TIMIT Garofolo et al. (1993), AMI Carletta et al. (2005), and Librispeech Panayotov et al. (2015) datasets, demonstrating that our method mitigates the peaky beahavior, improves alignment performance, and achieves promising results in E2E ASR.

2 Related Work

CTC loss. The CTC criterion Graves et al. (2006) is a versatile method for learning alignments between sequences. This versatility has led to its application across various seq2seq tasks Liu et al. (2020); Chuang et al. (2021); Yan et al. (2022); Gu & Kong (2021); Graves & Schmidhuber (2008); Molchanov et al. (2016). However, despite its widespread use, CTC has numerous limitations that impact its effectiveness in real-world applications. To address issues such as peaky behavior Zeyer et al. (2021), label delay Tian et al. (2023), and alignment drift Sak et al. (2015), researchers have proposed various extensions. These extensions aim to refine the alignment process, ensuring better performance across diverse tasks. Delay-penalized CTC Yao et al. (2023) and blank symbol regularization Yang et al. (2023); Zhao & Bell (2022); Bluche et al. (2015) attempt to mitigate label delay issues. Other works have tried to control alignment through teacher model spikes Ghorbani et al. (2018); Kurata & Audhkhasi (2019) or external supervision Zeyer et al. (2020); Senior et al. (2015); Plantinga & Fosler-Lussier (2019), though this increases complexity. More recently, Bayes Risk CTC Tian et al. (2023) offer customizable, E2E approaches to improve alignment without relying on external supervision. The latest advancement, Consistency-Regularized CTC (CR-CTC) Yao et al. (2024), mitigates extreme peaky behavior by enforcing consistency between CTC distributions obtained from different augmented views of the same audio.

Transducer loss. The transducer loss was introduced to address the conditional independence assumption of CTC by incorporating a predictor network Graves (2012). However, similarly to CTC, transducer models suffer from label delay and peaky behavior Yu et al. (2021). To mitigate these issues, several methods have been proposed, such as e.g., Pruned RNN-T Kuang et al. (2022), which prunes alignment paths before loss computation, FastEmit Yu et al. (2021), which encourages faster symbol emission, delay-penalized transducers Kang et al. (2023), which add a constant delay to all non-blank log-probabilities, and minimum latency training Shinohara & Watanabe (2022), which augments the transducer loss with the expected latency. Further extensions include CIFTransducer for

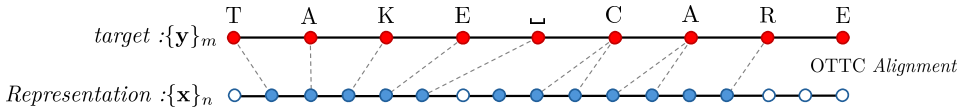

Figure 1: ***Alignment between embeddings of frames and target sequence.*** Red bullets represent the elements of the target sequence $\{\boldsymbol{y}\}_m$, while the blue bullets indicate the frame embeddings $\{\boldsymbol{x}\}_n$. In OTTC, the alignment guides the prediction model F in determining which frames should map to which labels. Additionally, the alignment model has the flexibility to leave some frames unaligned, as represented by the blue-and-white bullets, allowing those frames to be dropped during inference.

efficient alignment Zhang et al. (2024), self-alignment techniques Kim et al. (2021), and lightweight transducer models using CTC forced alignments Wan et al. (2024).

Over the years, the CTC and transducer-based ASR models have achieved state-of-the-art performance. Despite numerous efforts to control alignments and apply path pruning, the fundamental formulation of marginalizing over all valid paths remains unchanged and directly or indirectly contributes to several of the aforementioned limitations. Instead of marginalizing over all valid paths as in CTC and transducer models, we propose a differential alignment framework based on optimal transport, which can jointly learn a single alignment and perform ASR in an E2E manner.

3 Problem Formulation

We define $\mathcal{U}^d_{\leq N} = \bigcup_{n \leq N} \mathcal{U}^d_n$ to be the set of all d-dimensional vector sequences of length at most N. Let us consider a distribution $\mathcal{D}_{\mathcal{U}^d_{\leq N} \times \mathcal{U}^d_{\leq N}}$ and pairs of sequences $(\{\boldsymbol{x}_i\}_{i=1}^n, \{\boldsymbol{y}_i\}_{i=1}^m)$ of length n and m drawn from $\mathcal{D}_{\mathcal{U}^d_{\leq N} \times \mathcal{U}^d_{\leq N}}$. For notational simplicity, the sequences of the pairs $(\{\boldsymbol{x}_i\}_{i=1}^n, \{\boldsymbol{y}_i\}_{i=1}^m)$ will be respectively denoted by $\{\boldsymbol{x}\}_n$ and $\{\boldsymbol{y}\}_m$ in the following. The goal in seq2seq tasks is to train a classifier that can accurately predict the target sequence $\{\boldsymbol{y}\}_m$ from the input sequence $\{\boldsymbol{x}\}_n$, enabling it to generalize to unseen examples. Typically, $n \neq m$, creating challenges for accurate prediction as there is no natural alignment between the two sequences. In this paper, we introduce a framework to address this class of problems, applying it specifically to the ASR domain. In this context, the first sequence $\{\boldsymbol{x}\}_n$ represents an audio signal, where each vector $\boldsymbol{x}_i \in \mathbb{R}^d$ corresponds to a time frame in the acoustic embedding space. The second sequence $\{\boldsymbol{y}\}_m$ is the textual transcription of the audio, where each element $\boldsymbol{y}_i$ belongs to a predefined vocabulary $L = \{l_1, \ldots, l_{|L|}\}$, such that $\{\boldsymbol{y}\}_m \in L^m$, where L^m denotes the set of all m-length sequences formed from the vocabulary L.

4 Optimal Temporal Transport Classification

The core idea is to model the alignment between two sequences as a mapping to be learned along with the frame labels (see Figure 1). As the classification of audio frames improves, inferring the correct alignment becomes easier. Conversely, accurate alignments also improve frame classification. This mutual reinforcement between alignment and classification highlights the benefit of addressing both tasks simultaneously, contrasting with traditional hybrid models that treat them as separate tasks Morgan & Bourlard (1990). To achieve this, we propose the SOTD, a framework for constructing pseudo-metrics over the sequence space $\mathcal{U}^d_{\leq N}$, based on a differentiable, parameterized model that learns to align sequences. Using this framework, we derive the OTTC loss, which allows the model to learn both the alignment and the classification in a unified manner. We denote $[\![1, n]\!] = \{1, \ldots, n\}$.

4.1 Preliminaries

Definition 1. ***Discrete monotonic alignment****. Given two sequences $\{\mathbf{x}\}_n$ and $\{\mathbf{y}\}_m$, and a set of index pairs $\mathbf{A} \subset [\![1, n]\!] \times [\![1, m]\!]$ representing their alignment, we say that $\mathbf{A}$ is a discrete monotonic alignment between the two sequences if:*

- **Complete alignment of $\{\mathbf{y}\}_m$:** Every element of $\{\mathbf{y}\}_m$ is aligned, i.e.,

$$\forall j \in [\![1, m]\!], \exists k \in [\![1, n]\!], \ (k, j) \in \mathbf{A}.$$

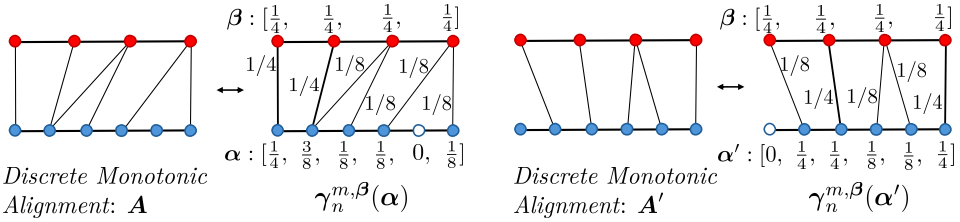

Figure 2: ***Discrete monotonic alignment as 1D OT solution.*** A discrete monotonic alignment represents a temporal alignment between two sequences (target on top, frame embeddings on bottom). It can be modeled by $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}$, as illustrated in the graph. The thickness of the links reflects the amount of mass $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha})_{i,j}$ transported, with thicker links corresponding to higher mass.

- **Monotonicity:** The alignment is monotonic, meaning that for all $(i, j), (k, l) \in \mathbf{A}$
$$i \leq k \;\Rightarrow\; j \leq l.$$

Discrete monotonic alignments model the relationship between temporal sequences, such as those in ASR, by determining which frame should predict which target. The conditions imposed on the target sequence $\{\mathbf{y}\}_m$ ensure that no target element is omitted, while the absence of similar constraints on the source sequence $\{x\}_n$ allows certain audio frames to be considered irrelevant and dropped (see Figure 2). The monotonicity condition preserves the temporal order, ensuring the sequential structure is maintained. In the following sections, we will develop a model capable of differentiating within the space of discrete monotonic alignments.

4.2 Differentiable Temporal Alignment with Optimal Transport

In the following, we introduce 1D OT and define our alignment model. Consider the 1D discrete distributions $\mu[\boldsymbol{\alpha}, n]$ and $\nu[\boldsymbol{\beta}, m]$ expressed as superpositions of δ measures, i.e., a distribution that is zero everywhere except at a single point, where it integrates to 1

$$\mu[\boldsymbol{\alpha}, n] = \sum_{i=1}^{n} \alpha_i \delta_i \quad \text{and} \quad \nu[\boldsymbol{\beta}, m] = \sum_{i=1}^{m} \beta_i \delta_i. \tag{1}$$

The bins of $\mu[\boldsymbol{\alpha}, n]$ and $\nu[\boldsymbol{\beta}, m]$ are $[\![1, n]\!]$ and $[\![1, m]\!]$, respectively, whereas the weights α_i and β_i are components of the vectors $\boldsymbol{\alpha} \in \Delta^n$ and $\boldsymbol{\beta} \in \Delta^m$, with Δ^n the simplex set defined as $\Delta^n = \{\mathbf{v} \in \mathbb{R}^n | 0 \leq v_i \leq 1, \sum_{i=1}^{n} v_i = 1\} \subset \mathbb{R}^n$. OT theory provides an elegant and versatile framework for computing distances between distributions such as $\mu[\boldsymbol{\alpha}, n]$ and $\nu[\boldsymbol{\beta}, m]$, depending on the choice of the cost function Peyré & Cuturi (2019) (chapter 2.4). One such distance is the 2-Wasserstein distance $\mathcal{W}_2$, which measures the minimal cost of transporting the weight of one distribution to match the other. This distance is defined as$_{n,m}$

$$\mathcal{W}_2(\mu[\boldsymbol{\alpha}, n], \nu[\boldsymbol{\beta}, m]) = \min_{\boldsymbol{\gamma} \in \Gamma^{\boldsymbol{\alpha},\boldsymbol{\beta}}} \sum_{i,j=1} \gamma_{i,j} \|i - j\|_2^2, \tag{2}$$

where $\|i - j\|_2^2$ is the cost of moving weight from bin i to bin j and $\gamma_{i,j}$ is the amount of mass moved from i to j. The optimal coupling $\boldsymbol{\gamma}^*$ is searched within the set of valid couplings $\Gamma^{\boldsymbol{\alpha},\boldsymbol{\beta}}$, defined as

$$\Gamma^{\boldsymbol{\alpha},\boldsymbol{\beta}} = \{\boldsymbol{\gamma} \in \mathbb{R}_+^{n \times m} | \boldsymbol{\gamma} \mathbf{1}_m = \boldsymbol{\alpha} \text{ and } \boldsymbol{\gamma}^T \mathbf{1}_n = \boldsymbol{\beta}\}. \tag{3}$$

This constraint ensures that the coupling conserves mass, accurately redistributing all weights between the bins. A key property of optimal transport in 1D is its monotonicity Peyré (2019). Specifically, if there is mass transfer between bins i and j (i.e., $\gamma_{i,j}^* > 0$) and similarly between bins k and l (i.e., $\gamma_{k,l}^* > 0$), then it must hold that $i \leq k \Rightarrow j \leq l$. Consequently, when $\boldsymbol{\beta}$ has no zero components – meaning that every bin from ν is reached by the transport – the set $\{(i, j) \in [\![1, n]\!] \times [\![1, m]\!] \mid \boldsymbol{\gamma}_{i,j}^* > 0\}$ satisfies the conditions of Definition 1, thereby forming a discrete monotonic alignment. This demonstrates that the optimal coupling can effectively model such alignments (see Figure 2).

Parameterized and differentiable temporal alignment. Given any sequences length n and m and $\boldsymbol{\beta}$ with no zero components, we can define the alignment function $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}$

$$\begin{aligned} \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}} :\; & \Delta^n \to \Gamma^{*,\boldsymbol{\beta}}[n] \\ & \boldsymbol{\alpha} \mapsto \boldsymbol{\gamma}^* = \arg\min_{\boldsymbol{\gamma} \in \Gamma} \mathcal{W}(\mu[\boldsymbol{\alpha}, n], \nu[\boldsymbol{\beta}, m]), \end{aligned} \tag{4}$$

where $\Gamma^{*,\boldsymbol{\beta}}[n]$ is the space of all 1D transport solutions between $\mu[\boldsymbol{\alpha}, n]$ and $\nu[\boldsymbol{\beta}, m]$ for any $\boldsymbol{\alpha}$. Differently from $\boldsymbol{\beta}$, $\boldsymbol{\alpha}$ may have zero components, giving the model the flexibility to suppress certain bins, which acts similarly to a blank token in traditional models. In the context of ASR, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ can be referred to as OT weights and label weights, respectively.

Lemma 1: *The function $\boldsymbol{\alpha} \mapsto \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha})$ is bijective from Δ^n to $\Gamma^{*,\boldsymbol{\beta}}[n]$.*

Proof. The proof can be found in Appendix A.2.1.

Proposition 1. ***Discrete Monotonic Alignment Approximation Equivalence.*** *For any $\boldsymbol{\beta}$ that satisfies the condition above, any discrete set of alignments $\boldsymbol{A} \subset [\![1, n]\!] \times [\![1, m]\!]$ between sequences of lengths n and m can be modeled by $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}$ through the appropriate selection of $\boldsymbol{\alpha}$, i.e.,*

$$\forall \mathbf{A}, \exists \boldsymbol{\alpha} \in \Delta^n, (i, j) \in \boldsymbol{A} \iff \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha})_{i,j} > 0. \tag{5}$$

Proof. The proof can be found in Appendix A.2.2.

Thus, we have defined a family of alignment functions $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}$ that are capable of modeling any discrete monotonic alignment, which can be chosen or adapted based on the specific task at hand. The computational cost of these alignment functions is low, as the bins are already sorted, eliminating the need for additional sorting. This results in linear complexity $O(\max(n, m))$ depending on the length of the longest sequence (see Algorithm A.1.1 in the Appendix). Furthermore, these alignments are differentiable, with $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha})_{i,j}$ explicitly expressed in terms of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, allowing direct computation of the derivative $\frac{d\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha})_{i,j}}{d\boldsymbol{\alpha}}$ via its analytical form.

4.2.1 Sequence-to-Sequence Distance

Here, we use the previously designed alignment functions to build a pseudo-metric over sets of sequences $\mathcal{U}_{\leq N}^d$.

Definition 1. ***Sequences Optimal Transport Distance (SOTD).*** *Consider an n-length sequence $\{\boldsymbol{x}\}_n \in \mathcal{U}_{\leq N}^d$, an m-length sequence $\{\boldsymbol{y}\}_m \in \mathcal{U}_{\leq N}^d$, $p = \max(n, m)$, and $q = \min(n, m)$. Let $C : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}_+$, be a differentiable positive cost function. Considering $r \in \mathbb{N}^*$ and a family of vectors $\{\boldsymbol{\beta}\}_N = \{\boldsymbol{\beta}_1 \in \Delta^1, \boldsymbol{\beta}_2 \in \Delta^2, \dots, \boldsymbol{\beta}_N \in \Delta^N\}$ without zero components, we define the SOTD $\mathcal{S}_r$ as*

$$\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{y}\}_m) = \min_{\boldsymbol{\alpha} \in \Delta^n} \Big(\sum_{i,j=1}^{n,m} \boldsymbol{\gamma}_p^{q,\boldsymbol{\beta}_q}(\boldsymbol{\alpha})_{i,j} \cdot C(\boldsymbol{x}_i, \boldsymbol{y}_j)^r \Big)^{1/r}. \tag{6}$$

Note that $\boldsymbol{\beta}_q$ obviously depends on q, but could a priori depend on $\{\boldsymbol{x}\}_n$ and $\{\boldsymbol{y}\}_m$. To simplify the notation, we only denote its dependence on q. However, all the results in this section remain valid under such dependencies, as long as $\boldsymbol{\beta}_q$ components never becomes zero.

Proposition 2. ***Validity of the definition.*** *SOTD is well-defined, meaning that a solution to the problem always exists, although it may not be unique.*

Proof. The proof and the discussion about the non-unicity is conducted in Appendix A.2.3.

Proposition 3. ***SOTD is a Pseudo-Metric.*** *If the cost matrix C is a metric on $\mathbb{R}^d$, then $\mathcal{S}_r$ defines a pseudo-metric over the space sequences with at most N elements $\mathcal{U}_{\leq N}^d$.*

Proof. The proof can be found in Appendix A.2.4.

Since $\mathcal{S}_r$ is a pseudo-metric, there are sequences $\{\boldsymbol{x}\}_n \neq \{\boldsymbol{y}\}_m$ such that $\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{y}\}_m) = 0$. The following proposition describes the conditions when this occurs.

Proposition 4. ***Non-Separation Condition.*** *Let $\mathcal{A}$ be the sequence aggregation operator which removes consecutive duplicates, i.e., $\mathcal{A}(\{\dots, \boldsymbol{x}, \boldsymbol{x}, \dots\}) = \{\dots, \boldsymbol{x}, \dots\}$. Let $\mathcal{P}_{\boldsymbol{\alpha}}$ be the sequence pruning operator which removes any element $\boldsymbol{x}_i$ from sequences corresponding to an $\alpha_i = 0$, i.e., $\mathcal{P}_\alpha(\{\dots, \boldsymbol{x}_{i-1}, \boldsymbol{x}_i, \boldsymbol{x}_{i+1}, \dots\}) = \{\dots, \boldsymbol{x}_{i-1}, \boldsymbol{x}_{i+1}, \dots\}$ iff $\alpha_i = 0$. Further, let us consider $\{\boldsymbol{x}\}_n$ and $\{\boldsymbol{y}\}_m$ such that $\{\boldsymbol{x}\}_n \neq \{\boldsymbol{y}\}_m$. Without loss of generality, we assume that $n \geq m$. Then*

$$\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{y}\}_m) = 0 \text{ iff } \mathcal{A}(\mathcal{P}_{\alpha^*}(\{\boldsymbol{x}\}_n)) = \mathcal{A}(\{\boldsymbol{y}\}_m), \tag{7}$$

where $\boldsymbol{\alpha}^*$ is a minimum for which $\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{y}\}_m) = 0$. It should be noted that this condition holds also when C is neither symmetric nor satisfies the triangular inequality, but is separated (like the cross-entropy for example). (*Proof.* See Appendix A.2.5.)

The consequence of the previous proposition is that we can learn a transformation through gradient descent using a trainable network F which maps input sequences $\{\boldsymbol{x}\}_n$ to target sequences $\{\boldsymbol{y}\}_m$ (with $n \geq m$) by solving the optimization problem

$$\min_F \mathcal{S}_r(F(\{\boldsymbol{x}\}_n), \{\boldsymbol{y}\}_m). \tag{8}$$

We are then guaranteed that a solution $F^*\{\boldsymbol{x}\}_n$ allows us to recover the sequence $\mathcal{A}(\{\boldsymbol{y}\}_m)$. In cases where retrieving repeated elements in $\{\boldsymbol{y}\}_m$ (e.g., double letters) is important, we can intersperse blank labels $\boldsymbol{\phi} \notin L$ between repeated labels as follows: $\{\boldsymbol{y}\}_m = \{\dots, l_i, l_i, \dots\} \rightarrow \{\dots, l_i, \phi, l_i, \dots\}$.

Note on Dynamic Time Warping (DTW): A note on the distinction between our approach and DTW-based methods Itakura (1975) can be found in Appendix A.4.

4.3 Application to ASR: OTTC Loss

In ASR, the target sequences $\{\boldsymbol{y}\}_m$ are d-dimensional one-hot encoding of elements from the set $L \cup \{\phi\}$, where ϕ is a blank label used to separate repeated labels. The encoder F predicts the label probabilities for each audio frame, such that

$$F(\{\boldsymbol{x}\}_n) = \{[p_{l_1}(\boldsymbol{x}_i), \dots, p_{l_{|L|+1}}(\boldsymbol{x}_i)]^T\}_{i=1}^n. \tag{9}$$

The alignment between $F(\{\boldsymbol{x}\}_n)$ and $\{\boldsymbol{y}\}_m$ is parameterized by $\boldsymbol{\alpha}[\{\boldsymbol{x}\}_n, W] \in \Delta^n$, defined as

$$\boldsymbol{\alpha}[\{\boldsymbol{x}\}_n, W] = \mathrm{softmax}(W(\boldsymbol{x}_1), \dots, W(\boldsymbol{x}_n))^T \tag{10}$$

where W is a network that outputs a scalar for each frame $\boldsymbol{x}_i$. Using the framework built in Section 4.2.1 (with $r = 1$ and $C = C_e$, where C_e is the cross-entropy) to predict $\{\boldsymbol{y}\}_m$ from $\{\boldsymbol{x}\}_n$, we train both W and F by minimizing the OTTC objective

$$\mathcal{L}_{OTTC} = -\sum_{i,j=1}^{n,m} \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}_m}(\boldsymbol{\alpha}[\{\boldsymbol{x}\}_n, W])_{i,j} \cdot \log p_{\boldsymbol{y}_j}(\boldsymbol{x}_i). \tag{11}$$

The choice of C_e as cost function arises naturally from the probabilistic encoding of the predicted output of F and the one-hot encoding of the target sequence. Additionally, since C_e is differentiable, it makes the OTTC loss differentiable with respect to F, while the differentiability of the OTTC with respect to W stems from the differentiability of $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}_m}$ with respect to its input $\boldsymbol{\alpha}[\{\boldsymbol{x}\}_n, W]$. Thus, by following the gradient of this loss, we jointly learn both the alignment (via W) and the classification (via F). **Note:** The notation $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}$ in Eq. 11 is valid in the context of ASR since $n \geq m$.

4.4 Link with CTC Loss

In this section, we link the CTC and the proposed OTTC losses. In the context of CTC, we denote by $\mathcal{B}$ the mapping which reduces any sequences by deleting repeated vocabulary (similarly to the previously defined $\mathcal{A}$ mapping in Proposition 5) *and then* deleting the blank token ϕ (e.g., $\mathcal{B}(\{GGOO\phi ODD\}) = \{GOOD\}$). The objective of CTC is to maximise the probability of all possible paths $\{\boldsymbol{\pi}\}_n$ of length n through minimizing

$$-\log \sum_{\{\boldsymbol{\pi}\}_n \in \mathcal{B}^{-1}(\{\boldsymbol{y}\}_m)} p(\{\boldsymbol{\pi}\}_n) = -\log \sum_{\{\boldsymbol{\pi}\}_n \in \mathcal{B}^{-1}(\{\boldsymbol{y}\}_m)} \prod_{i=1}^{n} p(\boldsymbol{\pi}_i), \tag{12}$$

where $\{\boldsymbol{\pi}\} \in L^n$ is an n-length sequence and $\mathcal{B}^{-1}(\{\boldsymbol{y}\}_m)$ is the set of all sequences collapsed by $\mathcal{B}$ into $\{\boldsymbol{y}\}_m$. Let us consider a path $\{\boldsymbol{\pi}\}_n \in \mathcal{B}^{-1}(\{\boldsymbol{y}\}_m)$. Such a path can be seen as an alignment (see Figure 3), where $\{\boldsymbol{x}_i\}$ and $\{\boldsymbol{y}_j\}$ are aligned iff $\boldsymbol{\pi}_i = \boldsymbol{y}_j$. By denoting $\boldsymbol{A}_\pi$ as the corresponding discrete monotonic alignment, one can write

$$-\log p(\{\boldsymbol{\pi}\}_n) = -\sum_{i=1}^{n} \log p_{\boldsymbol{\pi}_i}(\boldsymbol{x}_i) = \sum_{\substack{i,j=1 \\ (i,j)\in \mathbf{A}_{\boldsymbol{\pi}}}}^{n,m} C_e(\boldsymbol{\pi}_j, \boldsymbol{y}_i) \overset{\exists \boldsymbol{\alpha} \in \Delta^n}{=} \sum_{\substack{i,j=1 \\ \boldsymbol{\gamma}_p^{n,\boldsymbol{\beta}_m}(\boldsymbol{\alpha})_{i,j} > 0}}^{n,m} C_e(\boldsymbol{\pi}_j, \boldsymbol{y}_i). \tag{13}$$

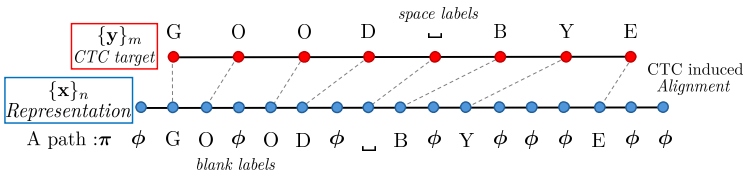

Figure 3: ***A CTC alignment.*** Here, we illustrate one of the valid alignments for CTC. The CTC loss maximizes the marginal probability over all such possible alignments.

with C_e representing the cross-entropy. *The last equality arises from Proposition 1 and the fact that* $\boldsymbol{A}_\pi$ *represents a discrete monotonic alignment.*

The continuous relaxation (i.e., making the problem continuous with respect to alignment) of the last term in this sequence of equalities results in $-\mathcal{L}_{OTTC}$. Therefore, OTTC can be seen as relaxation of the probability associated with a single path, enabling a differentiable path search mechanism. Essentially, OTTC optimization focuses on maximizing the probability of exactly one path, in contrast to CTC, which maximizes the probability across all valid paths.

Additionally, OTTC does not incentivize paths containing many blank tokens, unlike CTC. In CTC, the peaky behavior arises because maximizing the marginal probability over all valid paths can incentivize the model to assign more frames to the blank token Zeyer et al. (2021). In contrast, OTTC does not rely on a blank token to indicate that a frame i should not be classified (blank tokens are only used to separate consecutive tokens). Instead, the model simply sets the corresponding weight α_i to 0 (see Figure 2). This mechanism avoids the peaky behavior exhibited by CTC.

5 Experimental Setup

To demonstrate the viability of the proposed OTTC loss framework, we conduct several proof-of-concept experiments on the ASR task. To this end, we compare alignment quality and ASR performance using the proposed OTTC framework and existing CTC-based models.

Datasets. We conduct our experiments on popular open-source datasets, the TIMIT Garofolo et al. (1993), AMI Carletta et al. (2005), and LibriSpeech Panayotov et al. (2015). TIMIT is a 5 hour English dataset with time-aligned transcriptions, including exact time-frame phoneme transcriptions, making it a standard benchmark for ASR and phoneme segmentation tasks. We report results on the eval set. AMI is an English spontaneous meeting speech corpus that serves as a good benchmark to evaluate our approach in a realistic conversational scenario, due to its spontaneous nature and prior use in alignment evaluation Rastorgueva et al. (2023). For our experiments on this dataset, we train models on the individual head microphone (IHM) split comprising 80 hours of audio, and report results on the official eval set. LibriSpeech is an English read-speech corpus, containing 1000 hours of data. It is a standard benchmark for reporting ASR results. For our experiments, we train models on the official 100, 360, and 960 hour splits, and report results on the two official test sets.

Baselines. We benchmark our performance against the standard CTC. To specifically compare alignment quality, particularly regarding the mitigation of the peaky behavior inherent in CTC-based models, we also include CR-CTC Yao et al. (2024). CR-CTC serves as a strong baseline, chosen for its established effectiveness against such peaky alignments.

Model architectures. We use the 300M parameter Wav2Vec2-large Baevski et al. (2020) as the base model for acoustic embeddings in all the experiments conducted in this work. The Wav2Vec2 is a self-supervised model pre-trained on 60K hours of unlabeled English speech. For the baseline CTC-based models, we stack a dropout layer followed by a linear layer for logits prediction, termed the *logits prediction head*. For the proposed OTTC loss based model, we use a dropout and a linear layer (identical to the baseline) for logits prediction. In addition, as described in Section 4.3, we apply a dropout layer followed by two linear layers on top of the Wav2Vec2-large model for OT weight prediction, with a GeLU Hendrycks & Gimpel (2016) non-linearity in between, termed the *OT weights prediction head*. Note that the output from the Wav2Vec2-large model is used as input for both the logit and OT weight prediction heads, and the entire model is trained using the OTTC loss.

Performance metrics. Alignment quality is assessed using three metrics: peaky behavior, starting frame accuracy, and Intersection Duration Ratio (IDR). Peaky behavior, a common characteristic of

Table 1: Alignment performance of the CTC, CR-CTC, and OTTC based ASR models on TIMIT and AMI datasets. †On TIMIT, we subtract the percentage of real silence, as it is available, unlike AMI.

Model	TIMIT (Phoneme Level)			AMI (Word Level)		
	Peaky†(↓)	F1 Score (↑)	IDR (↑)	Peaky (↓)	F1 Score (↑)	IDR (↑)
CTC	53.51	88.77	26.98	81.93	83.94	16.75
CR-CTC	35.62	88.98	35.82	80.40	84.58	18.20
OTTC	**0.76**	**89.27**	**76.72**	**54.75**	**84.81**	**42.84**

CTC-based models, refers to a large proportion of audio frames being assigned to blank or space symbols (non-alphabet symbols) Zeyer et al. (2021). To quantify this, we compute the average percentage of frames mapped to these symbols. Starting frame accuracy is evaluated using the F1 score, following the methodology proposed in Rastorgueva et al. (2023). It is important to note that this F1 score reflects only the correctness of the predicted token's starting frame and does not fully capture alignment quality. To address this, we introduce IDR, which measures the overlap between predicted and reference word segments, normalized by the reference duration. This provides a finer-grained assessment of temporal alignment. These alignment metrics are computed only on the TIMIT and AMI datasets due to the lack of reliable ground-truth or forced-alignment annotations for LibriSpeech. On TIMIT, where ground-truth alignments are available, we assess alignment at the phoneme level. For AMI, which lacks ground-truth timestamps, we follow the forced-alignment approach in Rastorgueva et al. (2023), but restrict evaluation to word-level timestamps, as they are generally more reliable than phoneme-, letter-, or subword-level annotations. Finally, ASR performance is evaluated using the WER on all considered databases.

Training details. In all our experiments, we use the AdamW optimizer Loshchilov & Hutter (2019) for training. For TIMIT and LibriSpeech, the initial learning rate is set to $lr = 2e^{-4}$, with a linear warm-up for the first 500 steps followed by a linear decay until the end of training. For AMI, the initial learning rate is set to $lr = 1.25e^{-3}$, with a linear warm-up during the first 10% of the steps, also followed by linear decay. We train all considered models for 40 epochs, reporting the test set WER at the final epoch. In our OTTC-based models, both the logits and OT weight prediction heads are trained for the first 30 epochs. During the final 10 epochs, the *OT weight prediction head* is fixed, while training continues on the *logits prediction head*. For experiments on the LibriSpeech (*resp.* TIMIT) dataset, we use character-level (resp. phoneme-level) tokens to encode text. Given the popularity of subword-based units for encoding text Sennrich et al. (2016), we sought to observe the behavior of OTTC-based models when tokens are subword-based, where a token can contain more than one character. For the experiments on the AMI dataset, we use the SentencePiece tokenizer Kudo & Richardson (2018) to train subwords from the training text. Greedy decoding is used for all considered models to generate the hypothesis text.

Choice of label weights ($\boldsymbol{\beta}_q$). To simplify the training setup for our OTTC-based models, we use a fixed and uniform $\boldsymbol{\beta}_q$ (see Sections 4.2 & 4.3), where the length q of $\boldsymbol{\beta}$ is equal to the total number of tokens in the text after augmenting with the blank (ϕ) label between repeating characters.

6 Results and Discussion

Alignment quality. We begin by analyzing the alignment performance of the models on the TIMIT and AMI datasets, with results shown in Table 1. Our proposed OTTC model consistently outperforms the CTC-based models across all alignment metrics on both datasets. A key observation is the significant difference in the percentage of frames assigned to non-alphabet symbols by the CTC-based models, highlighting the peaky behavior inherent in these models. Specifically, the baseline CTC-based models tend to assign a large proportion of frames to blank or space symbols, reflecting a misalignment in predicted word boundaries. In contrast, the OTTC model avoids this issue, preventing extreme peaky behavior observed in CTC-based models. While the OTTC model also outperforms the CTC-based models in F1 score, the margin of improvement is smaller. However, the IDR reveals a substantial advantage for OTTC, with a significant improvement over CTC and CR-CTC. This indicates that CTC-based models often either delay the prediction of word starts or assigns too few frames to non-blank symbols, reinforcing the peaky behavior. Additionally, the performance improvement on the AMI dataset is particularly significant, given its nature of meeting speech. This demonstrates how effectively the OTTC loss adapts to varying speaking rates, showcasing the robustness of our framework in learning alignments despite speech variability.

Table 2: Word Error Rate (WER%) comparison between the baseline CTC model and the proposed OTTC model on all considered datasets. Lower WER is better.

Model	**TIMIT** eval	**AMI** eval	**100h-LibriSpeech** test-clean	**100h-LibriSpeech** test-other	**360h-LibriSpeech** test-clean	**360h-LibriSpeech** test-other	**960h-LibriSpeech** test-clean	**960h-LibriSpeech** test-other
CTC	8.38	11.75	3.36	7.36	2.77	6.58	2.20	5.23
OTTC	8.76	14.27	3.77	8.55	3.00	7.44	2.52	6.16

WER. ASR performance in terms of WER for the CTC model and the proposed OTTC model is depicted in Table 2 for all considered datasets. On the TIMIT dataset, the OTTC model shows a slightly higher WER compared to the CTC model, and while the performance gap is larger on the AMI dataset, it's encouraging to observe consistent performance despite the varied nature of speech. On the LibriSpeech dataset, using the 100-hour training split, the OTTC model achieves a WER of 3.77% on test-clean. As we scale the training dataset (100h $\rightarrow$ 360h $\rightarrow$ 960h), we observe a monotonic improvement in WER for the proposed OTTC-based models, similarly to the CTC-based models. Although the WERs achieved by the OTTC-based models are typically higher than the CTC-based models, the presented results underscore the experimental validity of the SOTD as a metric and demonstrate that learning a single alignment can yield promising results in E2E ASR.

Qualitative alignment comparison. Apart from quantitative alignment comparison (Table 1), we show an alignment from the CTC- and OTTC-based models in Figure 4. For CTC, it can be seen that the best path aligns most frames to the blank token, resulting in peaky behavior Zeyer et al. (2021). In contrast, the OTTC model learns to align all frames to non-blank tokens. This effectively mitigates the peaky behavior observed in the CTC model. Note that OTTC allows dropping frames during alignment (see Section 4.4), however, in practice, we observed that only a few frames are dropped. For additional insights, we plot the evolution of the alignment for the OTTC model during the course of training in Figures 6 & 7. It is evident that the alignment learned early in the training process remains relatively stable as training progresses. The most notable changes occur at the extremities of the predicted label clusters. This observation led us to the decision to freeze the OT weight predictions for the final 10 epochs, otherwise, even subtle changes in alignment could adversely impact the logits predictions because same base model is shared for predicting both the logits and the alignment OT weights.

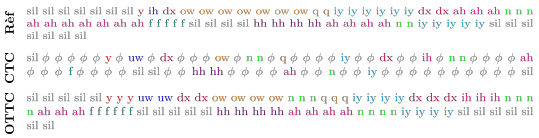

Figure 4: ***CTC and OTTC alignments.*** Phoneme-level transcription of CTC and OTTC, compared to a reference from TIMIT.

In summary, the presented results demonstrate that the proposed OTTC models achieve significant improvements in alignment performance, effectively mitigating the peaky behavior observed in CTC models. Although there is an increase in WER, the improvement in alignment accuracy indicates better temporal modeling. This enhanced alignment could benefit tasks that require precise timing information, such as speech segmentation, event detection, and applications in the medical domain, where accurate temporal alignment is crucial for tasks like clinical transcription or patient monitoring.

7 Conclusion and Future Work

Learning effective sequence-to-sequence alignment has diverse applications across various fields. Building upon our core idea of modeling the alignment between two sequences as a learnable mapping while simultaneously predicting the target sequence, we define a pseudo-metric known as the Sequence Optimal Transport Distance (SOTD) over sequences. Our formulation of SOTD enables the joint optimization of target sequence prediction and alignment, which is achieved through one-dimensional optimal transport. We theoretically show that the SOTD defines a distance with guaranteed existence of a solution, though uniqueness is not assured. We then derive the Optimal Temporal Transport Classification (OTTC) loss for ASR where the task is to map acoustic frames to text. Experiments across multiple datasets demonstrate that our method significantly improves alignment performance while successfully avoiding the peaky behavior commonly observed in CTC-based models. Other sequence-to-sequence tasks could be investigated using the proposed framework, particularly those involving the alignment of multiple sequences, such as audio, video, and text.

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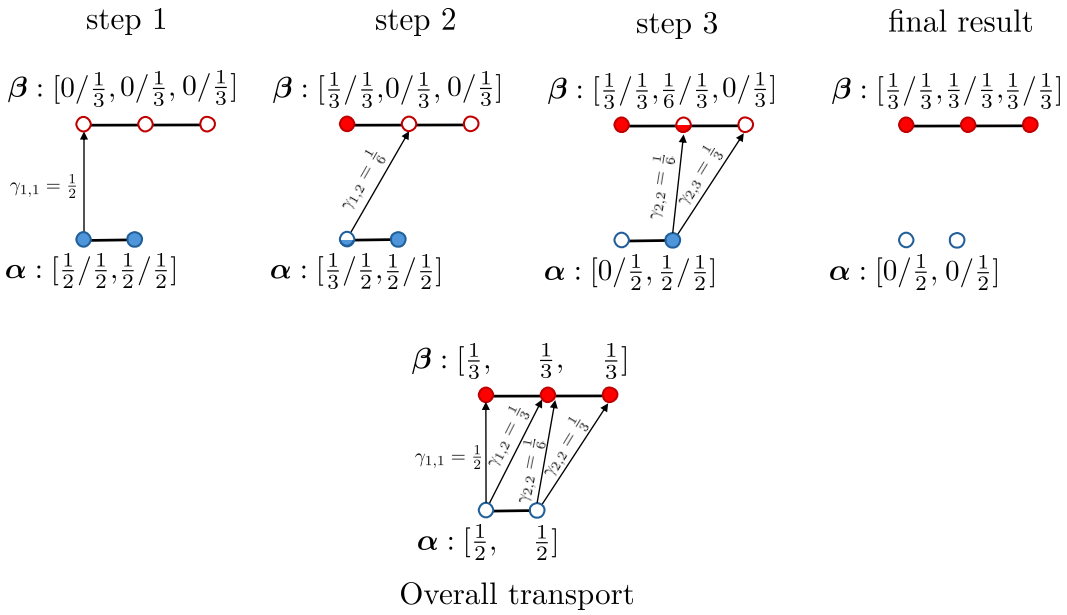

Figure 5: ***1D OT transport computation.*** Illustration of the optimal transport process, computed iteratively by transferring probability mass from the smallest bins to the largest.

A Appendix

A.1 Algorithm and Implementation Details

A.1.1 Alignment Computation

The algorithm to compute $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}$ is given in Algorithm 1. This algorithm computes the 1D optimal transport between $\mu[\boldsymbol{\alpha}, n]$ and $\nu[\boldsymbol{\beta}, m]$, exploiting the monotonicity of transport in this dimension. To do so the first step consist in sorting the bins which has the complexity $O(n \log n) + O(m \log m) = O(\max(n, m) \log \max(n, m))$. Then we transfer the probability mass from one distribution to another, moving from the smallest bins to the largest. A useful way to visualize this process is by imagining that the bins of μ each contain a pot with a volume of a_i filled with water, while the bins of ν each contain an empty pot with a volume of b_j. The goal is to fill the empty pots of ν using the water from the pots of μ. At any given step of the process, we always transfer water from the smallest non-empty pot of μ to the smallest non-full pot of ν. The volume of water transferred from i to j is denoted by $\gamma_{i,j}$. An example of this process is provided in Figure 5.

In the worst case, this process requires $O(n + m)$ comparisons. However, since the bins are already sorted in SOTD, the overall complexity remains $O(n + m) = O(\max(n, m))$. In practice, this algorithm is not directly used in this work, as we never compute optimal transport solely; it is provided here to illustrate that the dependencies of $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}$ on $\boldsymbol{\alpha}$ are explicit, making it differentiable with respect to $\boldsymbol{\alpha}$. An efficient batched implementation version for computing SOTD will be released soon.

A.2 Properties of OTTC

Here can be found proof and more insight about the properties of SOTD, $\mathcal{S}_r$.

A.2.1 Lemma 1 : Bijectivity

Proof of Lemma 1. *Surjectivity*: The surjectivity come from definition of $\Gamma^{*,\boldsymbol{\beta}}[n]$. *Injectivity*: Suppose $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha}) = \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\sigma})$, so $\boldsymbol{\alpha} = [\sum_{j=1}^m \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha})_{i,j}, \ldots, \sum_{j=1}^m \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha})_{i,j}]^T =$

Algorithm 1 : Transport Computation - $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha})$

Ensure: Compute $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha})$.
Require: $\boldsymbol{\alpha} \in \mathbb{R}^n$.
  Set $\boldsymbol{\gamma} \in \mathbb{R}^{n \times m} = \mathbf{0}_{n \times m}$.
  Set $i, j = 0$.
  while T == *True* **do**
    if $\alpha_i < \beta_j$ **then**
      $\boldsymbol{\gamma}_{i,j} = \beta_j - \alpha_i$
      $i = i + 1$
      if $i == n$ **then**
        $T =$ *false*
      end if
      $\beta_j = \beta_j - \alpha_i$
    else
      $\boldsymbol{\gamma}_{i,j} = \alpha_i - \beta_j$
      $j = j + 1$
      if $j == m$ **then**
        $T =$ *false*
      end if
      $\alpha_i = \alpha_i - \beta_j$
    end if
  end while
  Return $\boldsymbol{\gamma}$

$[\sum_{j=1}^m \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\sigma})_{i,j}, \dots, \sum_{j=1}^m \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\sigma})_{i,j}]^T = \boldsymbol{\sigma}$ (because $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha}) \in \Gamma^{\boldsymbol{\alpha},\boldsymbol{\beta}}$ and $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\sigma}) \in \Gamma^{\boldsymbol{\sigma},\boldsymbol{\beta}}$), which conclude the proof.

A.2.2 Proposition 1 : Discrete Monotonic Alignment Approximation Equivalence.

Proof of proposition 1. Let's consider the following proposition $P(k)$:

$$P(k) : \exists \boldsymbol{\alpha}^i \in \Delta^n, \forall i, \forall j \leq k, (i,j) \in \boldsymbol{A} \iff \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha}^i)_{i,j} > 0. \tag{14}$$

Initialisation - $P(1)$**.** $P(1)$ is true. Consider the set $E_1 = \{j \in [\![1, m]\!] \mid (1,j) \in \mathbf{A}\}$, which can be written as $E_1 = \{1, 2, \dots, \max(E_1)\}$ since A is a discrete monotonic alignment. Define $\boldsymbol{\alpha}^1 = [\sum_{j \in E_1} \beta_j, \dots]^T$, where the remaining coefficients are chosen to sum to 1.

Since the alignment $\gamma_n^{m,\boldsymbol{\beta}}$ is computed monotonically (see Appendix A.1.1), $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha}^1)_{1,j} > 0$ if and only if $\alpha_1^1 \leq \beta_1 + \dots + \beta_j$, which corresponds exactly to the set of indices $j \in E_1$, *i.e.*, the aligned indices in $\mathbf{A}$. This proves $P(1)$.

Heredity - $P(k) \Rightarrow P(k+1)$**.** The proof follows similarly to $P(1)$. However two cases need to be considered :

- When $(k+1, \max(E_k)) \in \mathbf{A}$, in this cases we must consider $E_{k+1} = \{j \in [\![1, m]\!] \mid (k+1, j) \in \mathbf{A}\} = \{\max(E_k) = \min(E_{k+1}), \min(E_{k+1})+1, \dots, \max(E_{k+1})\}$ (because $\boldsymbol{\beta}$ has no components) and define $\boldsymbol{\alpha}^{k+1} = [\alpha_1^1, \dots, \alpha_k^k - \frac{\beta_{\max(E_k)}}{2}, \sum_{j \in E_{k+1}} \beta_j - \frac{\beta_{\max(E_k)}}{2}, \dots]^T$, where the remaining parameters are chosen to sum to 1.
- When $(k+1, \max(E_k)) \notin \mathbf{A}$, we must consider $E_{k+1} = \{j \in [\![1, m]\!] \mid (k+1, j) \in \mathbf{A}\} = \{\max(E_k) \neq \min(E_{k+1}), \min(E_{k+1}) + 1, \dots, \max(E_{k+1})\}$ (because $\boldsymbol{\beta}$ has no components) and define $\boldsymbol{\alpha}^{k+1} = [\alpha_1^1, \dots, \alpha_k^k, \sum_{j \in E_{k+1}} \beta_j, \dots]^T$, where the remaining parameters are chosen to sum to 1.

By induction, the proposition holds for all n. Therefore, Proposition 1 (*i.e.*, $P(n)$) is true. An $\boldsymbol{\alpha}$ verifying the condition is :

$$\boldsymbol{\alpha} = [\alpha_1^1, \dots, \alpha_n^n]^T$$

A.2.3 PROPOSITION 2 :VALIDITY OF SOTD DEFINITION

Proof of proposition 2. Since $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}$ is differentiable so continuous, it follows that $\boldsymbol{\alpha} \mapsto \sum_{i,j=1}^{n,m} \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha})_{i,j} \cdot C(\boldsymbol{x}_i, \boldsymbol{y}_j)$ is continuous over Δ^n. Given that Δ^n is a compact set and every continuous function on a compact space is bounded and attains its bounds, the existence of an optimal solution $\boldsymbol{\alpha}^*$ follows.

Non-unicity of the solution. The non unicity come from that if their is a solution $\boldsymbol{\alpha}^*$ and two integer k, l such that $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha}^*)_{k,l} \geq \epsilon > 0$ and $\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha}^*)_{k+1,l} \geq \epsilon > 0$ and $C(\boldsymbol{x}_k, \boldsymbol{y}_l) = C(\boldsymbol{x}_{k+1}, \boldsymbol{y}_l)$, therefore the transport $\hat{\gamma}$ such that :

- $\forall i \in [\![1, n]\!], j, \in [\![1, m]\!], (i, j) \neq (k, l)$, $\hat{\gamma}_{i,j} = \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha}^*)_{i,j}$.
- $\hat{\gamma}_{k,l} = \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha}^*)_{k,l} - \epsilon/2$
- $\hat{\gamma}_{k+1,l} = \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha}^*)_{k+1,l} + \epsilon/2$

provide a distinct solution. Let's denote $\boldsymbol{\sigma} = \{\boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}\}^{-1}(\hat{\gamma}_{i,j})$. First $\boldsymbol{\sigma} \neq \boldsymbol{\alpha}$ because $\sigma_k = \sum_{l=1}^{m} \hat{\gamma}_{k,l} = \sum_{l=1}^{m} \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha}^*)_{k,l} - \epsilon/2 = \alpha_k^* - \epsilon/2$. Second, it's clear that $\sum_{i,j=1}^{n,m} \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha}^*)_{i,j} \cdot C(\boldsymbol{x}_i, \boldsymbol{y}_j) = \sum_{i,j=1}^{n,m} \boldsymbol{\gamma}^{m,\boldsymbol{\beta}_n}(\boldsymbol{\sigma})_{i,j} \cdot C(\boldsymbol{x}_i, \boldsymbol{y}_j)$. Then $\boldsymbol{\sigma}$ is distinct solution.

A.2.4 PROPOSITION 3 : SOTD IS A PSEUDO METRIC

Proof of proposition 3. ***Pseudo-separation.*** It's clear that $\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{x}\}_n) = 0$, this value is attained for $\alpha^* = \boldsymbol{\beta}_n$; where the corresponding alignment $\boldsymbol{\gamma}_n^{n,\boldsymbol{\beta}_n}(\boldsymbol{\alpha}^*)$ corresponds to a one-to-one alignment. Since the two sequences are identical, all the costs are zero.

Symmetry. We have $\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{y}\}_m m) = \mathcal{S}_r(\{\boldsymbol{y}\}_m, \{\boldsymbol{x}\}_n)$ because the expression for $\mathcal{S}_r$ in Eq. 6 is symmetric. Specifically, because C is symmetric as it is a metric.

Triangular inequality. Consider three sequences $\{\boldsymbol{x}\}_n$, $\{\boldsymbol{y}\}_m$ and $\{\boldsymbol{z}\}_o$. Let $p = \max(n, m)$, $q = \min(n, m)$, $u = \max(m, o)$, $v = \min(m, o)$. Define the optimal alignments $\boldsymbol{\gamma}_p^{q,\boldsymbol{\beta}_q}(\boldsymbol{\alpha}^*)$ between $\{\boldsymbol{x}\}_n$ and $\{\boldsymbol{y}\}_m$; and $\boldsymbol{\gamma}_u^{v,\boldsymbol{\beta}_v}(\boldsymbol{\rho}^*)$ between $\{\boldsymbol{y}\}_m$ and $\{\boldsymbol{z}\}_o$. $\forall i \in [\![1, n]\!], \forall j, k \in [\![1, m]\!], \forall l \in [\![1, o]\!]$, we define :

$$\gamma_{i,j}^{xy} = \begin{cases} \boldsymbol{\gamma}_p^{q,\boldsymbol{\beta}_q}(\boldsymbol{\alpha}^*)_{i,j} & \text{if } n \geq m \\ \boldsymbol{\gamma}_p^{q,\boldsymbol{\beta}_q}(\boldsymbol{\alpha}^*)_{j,i} & \text{otherwise.} \end{cases} \tag{15}$$

$$\gamma_{k,l}^{yz} = \begin{cases} \boldsymbol{\gamma}_u^{v,\boldsymbol{\beta}_v}(\boldsymbol{\rho}^*)_{k,l} & \text{if } k \geq l \\ \boldsymbol{\gamma}_u^{v,\boldsymbol{\beta}_v}(\boldsymbol{\rho}^*)_{l,k} & \text{otherwise.} \end{cases} \tag{16}$$

$$\gamma_{j,k}^{yy} = \boldsymbol{\gamma}_p^{q,\boldsymbol{\sigma}^*}(\boldsymbol{\beta}_q)_{j,k} \tag{17}$$

and we define :

$$b_j = \begin{cases} \sum_{i=1}^{n} \gamma_{i,j}^{xy} & \text{if } > 0 \\ 1 & \text{otherwise.} \end{cases} \tag{18}$$

$$c_k = \begin{cases} \sum_{l=1}^{o} \gamma_{k,l}^{yz} & \text{if } > 0 \\ 1 & \text{otherwise.} \end{cases} \tag{19}$$

So γ^{xy} is the optimal transport between $\mu[\boldsymbol{\alpha}^*, p]$ and $\nu[\boldsymbol{\beta}_q, q]$; γ^{yy} is the optimal transport between $\mu[\boldsymbol{\beta}_q, q]$ and $\nu[\boldsymbol{\sigma}^*, u]$ and γ^{yz} is the optimal transport between $\mu[\boldsymbol{\sigma}^*, u]$ and $\nu[\boldsymbol{\beta}_v, v]$, since in 1D optimal transport can be composed, the composition $\frac{\gamma_{i,j}^{xy}\gamma_{j,k}^{yy}\gamma_{k,l}^{yz}}{b_j c_k}$ is an optimal transport between $\mu[\boldsymbol{\alpha}^*, p]$ and $\nu[\boldsymbol{\beta}_v, v]$. Therefore by bijectivity of $\boldsymbol{\gamma}_{\max(p,v)}^{\min(p,v),\boldsymbol{\beta}_{\min(p,v)}}$, there is a $\boldsymbol{\theta} \in \mathbb{R}^{\max(p,v)}$ such that :

$$\boldsymbol{\gamma}_{\max(p,v)}^{\min(p,v),\boldsymbol{\beta}_{\min(p,v)}}(\boldsymbol{\theta}) = \frac{\gamma_{i,j}^{xy}\gamma_{j,k}^{yy}\gamma_{k,l}^{yz}}{b_j c_k} \tag{20}$$

Thus, by the definition of $\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{z}\}_o)$:

$$\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{z}\}_o) \leq \Big(\sum_{i,l=1}^{n,o} \sum_{j,k=1}^{m,m} \boldsymbol{\gamma}_{\max(p,v)}^{\min(p,v),\boldsymbol{\beta}_{\min(p,v)}}(\boldsymbol{\theta}) \cdot C(\boldsymbol{x}_i, \boldsymbol{z}_l)^r \Big)^{1/r} \tag{21}$$

$$\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{z}\}_o) \leq \Big(\sum_{i,l=1}^{n,o} \sum_{j,k=1}^{m,m} \frac{\gamma_{i,j}^{xy}\gamma_{j,k}^{yy}\gamma_{k,l}^{yz}}{b_j c_k} \cdot C(\boldsymbol{x}_i, \boldsymbol{z}_l)^r \Big)^{1/r} \tag{22}$$

$$\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{z}\}_o) \leq \Big(\sum_{i,l=1}^{n,o} \sum_{j,k=1}^{m,m} \frac{\gamma_{i,j}^{xy}\gamma_{j,k}^{yy}\gamma_{k,l}^{yz}}{b_j c_k} \cdot (C(\boldsymbol{x}_i, \boldsymbol{y}_j) + C(\boldsymbol{y}_j, \boldsymbol{y}_k) + C(\boldsymbol{y}_k, \boldsymbol{z}_l))^r \Big)^{1/r} \tag{23}$$

Applying the Minkowski inequality:

$$\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{z}\}_o) \leq \Big(\sum_{i,l=1}^{n,o} \sum_{j,k=1}^{m,m} \frac{\gamma_{i,j}^{xy}\gamma_{j,k}^{yy}\gamma_{k,l}^{yz}}{b_j c_k} \cdot (C(\boldsymbol{x}_i, \boldsymbol{y}_j))^r \Big)^{1/r} + \tag{24}$$

$$\Big(\sum_{i,l=1}^{n,o} \sum_{j,k=1}^{m,m} \frac{\gamma_{i,j}^{xy}\gamma_{j,k}^{yy}\gamma_{k,l}^{yz}}{b_j c_k} \cdot (C(\boldsymbol{y}_j, \boldsymbol{y}_k))^r \Big)^{1/r} + \tag{25}$$

$$\Big(\sum_{i,l=1}^{n,o} \sum_{j,k=1}^{m,m} \frac{\gamma_{i,j}^{xy}\gamma_{j,k}^{yy}\gamma_{k,l}^{yz}}{b_j c_k} \cdot (C(\boldsymbol{y}_k, \boldsymbol{z}_l))^r \Big)^{1/r} \tag{26}$$

Then :

$$\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{z}\}_o) \leq \Big(\sum_{i,j=1}^{n,m} \gamma_{i,j}^{xy} \cdot C(\boldsymbol{x}_i, \boldsymbol{y}_j)^r \Big)^{1/r} + \tag{27}$$

$$\Big(\sum_{j,k=1}^{m,m} \gamma_{j,k}^{yy} \cdot C(\boldsymbol{y}_j, \boldsymbol{y}_k)^r \Big)^{1/r} + \tag{28}$$

$$\Big(\sum_{k,l=1}^{m,o} \gamma_{k,l}^{yz} \cdot C(\boldsymbol{y}_k, \boldsymbol{z}_l)^r \Big)^{1/r} \tag{29}$$

By definition :

$$\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{z}\}_o) \leq \mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{y}\}_m) + \mathcal{S}_r(\{\boldsymbol{y}\}_m, \{\boldsymbol{y}\}_m) + \mathcal{S}_r(\{\boldsymbol{y}\}_m, \{\boldsymbol{z}\}_o) \tag{30}$$

So finally since $\mathcal{S}_r(\{\boldsymbol{y}\}_m, \{\boldsymbol{y}\}_m) = 0$, the triangular inequality holds :

$$\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{z}\}_o) \leq \mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{y}\}_m) + \mathcal{S}_r(\{\boldsymbol{y}\}_m, \{\boldsymbol{z}\}_o). \tag{31}$$

This concludes the proof.

Note: If $\boldsymbol{\beta}$'s depends on $\{\boldsymbol{x}\}_n$, $\{\boldsymbol{y}\}_m$ and $\{\boldsymbol{z}\}_m$, we need to introduce the appropriate γ^{zz} to construct the composition in Equation 20, ensuring the proof remains valid.

A.2.5 PROPOSITION 4 : NON-SEPARATION CONDITION

Proof. Suppose $\mathcal{S}_r(\{\boldsymbol{x}\}_n, \{\boldsymbol{y}\}_m) = 0$, and $\mathcal{A}(\mathcal{P}_{\alpha^*}(\{\boldsymbol{x}\}_n)) \neq \mathcal{A}(\{\boldsymbol{y}\}_n)$. So :

$$\sum_{i,j=1}^{n,m} \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha}^*)_{i,j} \cdot C(\boldsymbol{x}_i, \boldsymbol{y}_j)^r = 0 \tag{32}$$

Let $\mathcal{A}_{\{\boldsymbol{x}\}_n}$ denote the aggregation operator on Δ^n, which groups indices where consecutive elements in $\{\boldsymbol{x}\}_n$ are identical (i.e, $\mathcal{A}([\ldots, \alpha_i, \ldots, \alpha_{i+k}, \ldots]^T) = [\ldots, \alpha_i + \cdots + \alpha_{i+k}, \ldots]^T$ iff $\boldsymbol{x}_i = \cdots = \boldsymbol{x}_{i+k}$). By expanding the right term, we show that; $\forall \boldsymbol{\alpha} \in \Delta^n$:

$$\sum_{i,j=1}^{n,m} \boldsymbol{\gamma}_n^{m,\boldsymbol{\beta}}(\boldsymbol{\alpha})_{i,j} \cdot C(\boldsymbol{x}_i, \boldsymbol{y}_j)^r = \sum_{i,j=1}^{n,m} \boldsymbol{\gamma}_n^{m,\boldsymbol{\mathcal{A}_{\{y\}_m}(\beta)}}(\mathcal{A}_{\{\boldsymbol{x}\}_n}(\boldsymbol{\alpha}))_{i,j} \cdot C(\mathcal{A}(\mathcal{P}_{\boldsymbol{\alpha}}(\{\boldsymbol{x}\}_n)), \mathcal{A}(\{\boldsymbol{y}\}_n))^r \tag{33}$$

Therefore :

$$\sum_{i,j=1}^{n,m} \boldsymbol{\gamma}_n^{m,\boldsymbol{\mathcal{A}_{\{y\}_m}(\beta)}}(\mathcal{A}_{\mathcal{P}_\alpha\{\boldsymbol{x}\}_n}(\boldsymbol{\alpha}^*))_{i,j} \cdot C(\mathcal{A}(\mathcal{P}_{\alpha^*}(\{\boldsymbol{x}\}_n)), \mathcal{A}(\{\boldsymbol{y}\}_n))^r = 0 \tag{34}$$

Since $\mathcal{A}(\mathcal{P}_{\alpha^*}(\{\boldsymbol{x}\}_n)) \neq \mathcal{A}(\{\boldsymbol{y}\}_n)$ their is a $k \in [\![1, m[\![$ such that :

$$\forall k' < k, \mathcal{A}(\{\boldsymbol{x}\}_n)_{k'} = \mathcal{A}(\{\boldsymbol{y}\}_n)_{k'} \quad \textbf{and} \quad \mathcal{A}(\{\boldsymbol{x}\}_n)_k \neq \mathcal{A}(\{\boldsymbol{y}\}_n)_k \tag{35}$$

Because the optimal alignment is monotonous and lead to a 0 cost, necessarily:

$$\forall k' < k, \mathcal{A}_{\mathcal{P}_\alpha(\{\boldsymbol{x}\}_n)}(\boldsymbol{\alpha}^*)_{k'} = \mathcal{A}_{\{\boldsymbol{y}\}_m}(\boldsymbol{\beta})_{k'} \tag{36}$$

which is the only way to have alignment between the k first element which led to 0 cost. Because of the monotonicity of $\boldsymbol{\gamma}_n^{m,\boldsymbol{\mathcal{A}_{\{y\}_m}(\beta)}}(\mathcal{A}_{\mathcal{P}_\alpha\{\boldsymbol{x}\}_n}(\boldsymbol{\alpha}^*))$ the next alignment (s, t) is between the next element with a non zeros weights for both sequences. Since β has non zero component and by the definition of $\mathcal{P}_\alpha$, $s = k$ and $t = k$. Therefore the term $\boldsymbol{\gamma}_n^{m,\boldsymbol{\mathcal{A}_{\{y\}_m}(\beta)}}(\mathcal{A}_{\mathcal{P}_{\alpha^*}(\{\boldsymbol{x}\}_n)}(\boldsymbol{\alpha}^*))_{k,k}$ is non null and the term :

$$\boldsymbol{\gamma}_n^{m,\boldsymbol{\mathcal{A}_{\{y\}_m}(\beta)}}(\mathcal{A}_{\mathcal{P}_\alpha\{\boldsymbol{x}\}_n}(\boldsymbol{\alpha}^*))C(\mathcal{A}(\mathcal{P}_{\alpha^*}(\{\boldsymbol{x}\}_n), \mathcal{A}(\{\boldsymbol{y}\}_n)_k)$$

belong to the sum in depicted in Eq. 34. So $C(\mathcal{A}(\mathcal{P}_{\alpha^*}(\{\boldsymbol{x}\}_n)), \mathcal{A}(\{\boldsymbol{y}\}_n)_k) = 0$ *i.e.*, $\mathcal{A}(\mathcal{P}_{\alpha^*}(\{\boldsymbol{x}\}_n)) = \mathcal{A}(\{\boldsymbol{y}\}_n)_k$ because C is separated. Here a contradiction so we can conclude that:

$$\mathcal{A}(\mathcal{P}_{\alpha^*}(\{\boldsymbol{x}\}_n)) = \mathcal{A}(\{\boldsymbol{y}\}_n).$$

A.3 SUPPLEMENTARY EXPERIMENTAL INSIGHTS

A.4 NOTE ON DYNAMIC TIME WARPING (DTW)

It is important to highlight the distinction between our approach and DTW-based Itakura (1975) alignment methods, particularly the differentiable variations such as soft-DTW Cuturi & Blondel (2018). These methods generally have quadratic complexity Cuturi & Blondel (2018), making them significantly more computationally expensive than ours. Furthermore, in DTW-based methods, the alignment emerges as a consequence of the sequences themselves. When the function F is powerful,

the model can collapse by generating a sequence $F(\{\boldsymbol{x}\}_n)$ that induces a trivial alignment Haresh et al. (2021) (see Appendix A.4.1, where we conducted experiments using soft-DTW for ASR to illustrate this). To mitigate this issue, regularization losses Haresh et al. (2021); Meghanani & Hain (2024) or constraints on the capacity of F Vayer et al. (2022); Zhou & la Torre (2009) are commonly introduced. However, using regularization losses lacks theoretical guarantees and introduces additional hyperparameters. Furthermore, constraining the capacity of F, although more theoretically sound, makes tasks requiring powerful encoders on large datasets impractical. In contrast, our method decouples the computation of the alignment from the transformation function F, offering more flexibility to the model as well as built-in temporal alignment constraints and theoretical guarantees against collapse.

A.4.1 Ablation Studies

This section explores the effects of various design choices and configurations on the performance of the proposed OTTC framework and provides additional insights on its comparison to soft-DTW.

Training with single-path alignment from CTC. A relevant question that arises is whether the gap between the OTTC and CTC models arises from the use of a single alignment in OTTC rather than marginalizing over all possible alignments. To investigate this, we conducted a comparison with a single-path alignment approach. Specifically, we first obtained the best path (forced alignment using the Viterbi algorithm) from a trained CTC-based model on the same dataset. A new model was then trained to learn this single best path using Cross-Entropy. On the 360-hour LibriSpeech setup with Wav2Vec2-large as the pre-trained model, this single-path approach achieved a WER of 7.04% on the test-clean set and 13.03% on the test-other set. In contrast, under the same setup, the OTTC model achieved considerably better results, with a WER of 3.00% on test-clean and 7.44% on test-other (see Table 2). These findings indicate that the OTTC model is effective with learning a single alignment, which may be sufficient for achieving competitive ASR performance.

Fixed OT weights prediction ($\boldsymbol{\alpha}$). We conducted an additional ablation experiment where we replaced the learnable *OT weight prediction head* with fixed and uniform OT weights ($\boldsymbol{\alpha}$). This approach removes the model's ability to search for the best path, assigning instead a frame to the same label during training. Consequently, the model loses the localization of the text-tokens in the audio. For this experiment, we used the 360-hour LibriSpeech setup with Wav2Vec2-large as the pre-trained model. The results show a WER of 3.51% on test-clean, compared to 2.77% for CTC and 3.00% for OTTC with learnable OT weights. On test-other, the WER was 8.24%, compared to 6.58% for CTC and 7.44% for OTTC with learnable OT weights. These results demonstrate that while using fixed OT weights leads to a slight degradation in performance, the localization property is completely lost, highlighting the importance of learnable OT weights for preserving both performance and localization in the OTTC model.

Impact of freezing OT weights prediction head across epochs. In our investigations so far, we arbitrarily selected the number of epochs for which the *OT weights prediction head* ($\boldsymbol{\alpha}$ predictor) remained frozen (see Section 6), as a hyperparameter without any tuning. To further understand its impact, we conducted additional experiments on the 360h-LibriSpeech setup using the Wav2Vec2-large model while freezing the *OT weights prediction head* for the last 5 and 15 epochs. When frozen for the last 5 epochs, we achieve a WER of 3.01%, whereas when frozen for the last 15 epochs, the WER is 3.10%. As shown in the Table 2, freezing the OT head for the last 10 epochs results in a WER of 3.00%. Based on these results, it appears that the model's performance doesn't change considerably when the model is trained for a few more epochs after freezing the alignment part of the OTTC model.

Oracle experiment. We believe that the proposed OTTC framework has the potential to outperform CTC models by making $\boldsymbol{\beta}$ learnable with suitable constraints or by optimizing the choice of static $\boldsymbol{\beta}$. To illustrate this potential, we conduct an oracle experiment where we first force-align audio frames and text tokens using a CTC-based model trained on the same data. This alignment is then used to calculate the $\boldsymbol{\beta}$ values. For example, given the target sentence YES and the best valid path from the Viterbi algorithm ($\phi Y \phi \phi E S$), we re-labeled it to ($\phi Y \phi E S$) and set $\boldsymbol{\beta} = [1/7, 1/7, 2/7, 2/7, 1/7]$. This approach enabled OTTC to learn a uniform distribution for $\boldsymbol{\alpha}$, mimicking CTC's highest probability path. As a result, in both the 100h-LibriSpeech and 360h-LibriSpeech setups, the OTTC model converged much faster and matched the performance of CTC. This experiment underscores

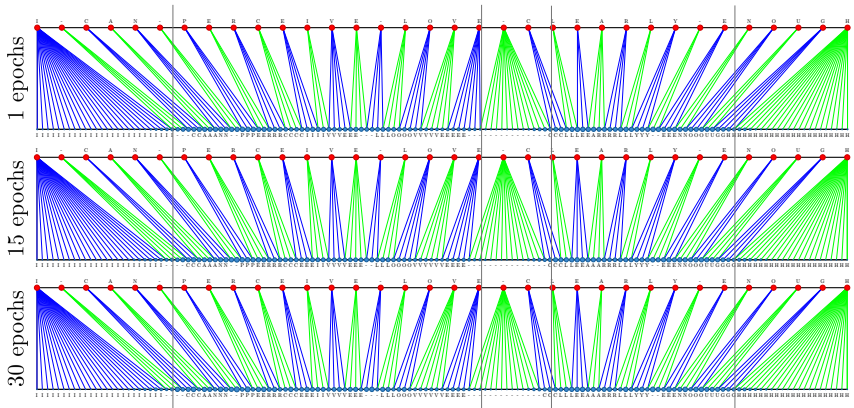

Figure 6: ***Evolution of alignment in the OTTC model during the course of training.*** The red bullets represent elements of the target sequence $\{\boldsymbol{y}\}_m$, while the blue bullets indicate the predicted OT weights for each frame. The size of the blue bullets is proportional to the predicted OT weight.

the critical role of $\boldsymbol{\beta}$, suggesting that a better strategy for its selection or training will lead to further improvements.

Comments on soft-DTW. In soft-DTW, only the first and last elements of sequences are guaranteed to align, while all in-between frames or targets may be ignored; *i.e.*, there is no guarantee that soft-DTW will yield a discrete monotonic alignment. A "powerful" transformation F can map $\mathbf{x}$ to $F(\mathbf{x})$ in such a way that soft-DTW ignores the in-between transformed frames ($F(\mathbf{x})$) and targets ($\mathbf{y}$), which we refer to as a collapse (Section 4.2.1). This is why transformations learned through sequence comparison are typically constrained (e.g., to geometric transformations like rotations) Vayer et al. (2022). Since transformer architectures are powerful, they are susceptible to collapse as demonstrated by the following experiment we conducted using soft-DTW as the loss function. On the 360h-LibriSpeech setup with Wav2Vec2-large model, the best WER achieved using soft-DTW is 39.43%. In comparison, CTC yields 2.77% whereas the proposed OTTC yields 3.00%. A key advantage of our method is that, by construction, such a collapse is not possible.

A.4.2 Alignment Analysis

Temporal evolution of alignment. An example of the evolution of the alignment in the OTTC model during training for 40 epochs without freezing *OT weights prediction head* is shown in Figure 7. Note that during the initial phase of training, there is significant left/right movement of boundary frames for all groups. As training progresses, the movement typically stabilizes to around 1-2 frames. While this can be considered "relatively stable" in terms of alignment, the classification loss (*i.e.*, cross-entropy) in the OTTC framework is still considerably affected by these changes. This change of the loss is what impacts the final performance and the performance difference between freezing or not-freezing the alignments.

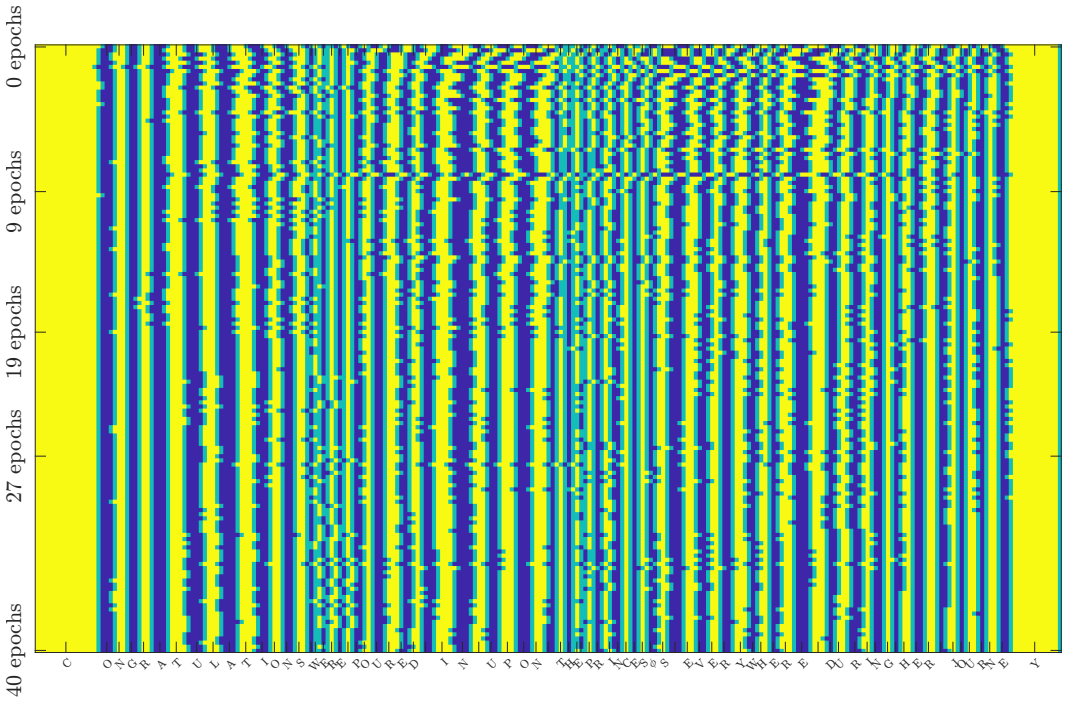

Figure 7: ***Alignment evolution in the OTTC model during training for 40 epochs without freezing OT weights prediction head (α predictor).*** On the x-axis, each pixel corresponds to one audio frame, while the y-axis represents the epoch. Frames grouped by tokens are shown in alternating colors (yellow and dark blue), with the boundaries of each group highlighted in light blue/green. One can note that during the initial phase of training, there is significant left/right movement of boundary frames for all groups. As training progresses, the movement typically stabilizes to around 1-2 frames.