Dense Backpropagation Improves Routing for Sparsely-Gated Mixture-of-Experts

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Abstract

Sparsely-gated Mixture-of-Experts (MoEs) have proven to be more efficient than dense Transformers because they can dynamically activate a subset of their overall parameters by *routing* tokens to selected "experts", allowing practitioners to scale up model parameter counts without significantly increasing total compute. However, current MoE training approaches only update the router with a sparse gradient and suffer from issues such as load imbalance. We propose a new router that can receive a dense gradient update from a sparse forward pass. Our method adds minimal overhead, but improves on the common Top-K routing in both performance and load balance.

1 Introduction

Large-scale pretraining hinges on scaling up the number of parameters in a model, because models with more parameters are more sample-efficient and require less training to reach the same performance as smaller models [Kaplan et al., 2020, Hoffmann et al., 2022]. Most academic research has adopted the dense Transformer architecture [Vaswani et al., 2023] because its performance scales well with parameters and data. However, a sparsely activated Mixture-of-Experts (MoE) Transformer architecture [Shazeer et al., 2017] has been used by many industry deployments [Team et al., 2024, xAI, 2024, Databricks, 2024, Jiang et al., 2024, Snowflake, 2024, DeepSeek-AI et al., 2024] because MoEs have been shown to scale better than dense Transformers [Clark et al., 2022, Du et al., 2022, Lepikhin et al., 2020, Fedus et al., 2022]. This is not unique to dense Transformers; MoEs based on state-space modules rather than dense Transformer blocks outperform networks of only state-space modules [Lieber et al., 2024, Liquid, 2024]. MoEs learn a *routing* function that selectively activates the TopK subset of their parallel MLP modules, or *experts*, most relevant to a given input. This conditionally sparse activation [Jacobs et al., 1991, Jordan and Jacobs, 1994] allows the model parameter count to be increased multiplicatively without significantly increasing the cost of training or inference.

However, this same router that unlocks superior scaling also presents a challenge for MoEs, because the router does not receive a gradient update from experts that it does not activate, and may not learn to route a token to its appropriate expert. One critical issue is the load imbalance problem — where a few experts are over-utilized — which leads to inefficient training and resource usage [Zoph et al., 2022, Zhou et al., 2022]. We propose a new router that can receive a dense gradient update from a sparse forward pass to address the instability issues arising from sparse routing. Our method adds minimal overhead, but improves on the common Top-K routing in both performance and load balance.

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2 Background & Related Work

MoEs. The MoE layer replaces the feedforward networks (FFN) of transformers and consists of two components : **1)** N FFNs (*experts*), $E_0(x), E_1(x), \ldots E_N(x)$ and **2)** a router that assigns tokens to experts. Each input to the MoE layer is processed by K experts where $K < N$ and is thus the source of sparsity in MoEs. The K experts are chosen by the router, which is a learnable component that maps each token to a set of weights over the experts. The router performs a linear transformation $\mathbb{R}^{d_{\text{token}}} \to \mathbb{R}^N$ which produces logits; these are normalized using softmax, resulting in a probability distribution over the experts. With the router's linear transformation parameterized by a matrix W, we can represent the expert weights π in the following way:

$$\pi \in \mathbb{R}^N = \text{Softmax}(Wx) \tag{1}$$

Once we have these expert weights, we apply a routing function to decide which of K experts to route and process this token through.

Top-K routing. A standard method to select K out of N experts given the expert weights is to select the experts corresponding to the K highest weights. Top-K routing [Fedus et al., 2022] passes the token to the K selected experts and averages the expert outputs using these weights to produce the final output. Experts not selected by the Top-K routing function do not process the token, and this introduces sparsity in MoEs. By representing the K chosen experts as the set A, we can express the output of the MoE layer as:

$$y = \Sigma_{i \in A} \pi_i E_i(x) \tag{2}$$

Thus, the expert weights have a dual purpose : They are used by the routing function to decide which of the K experts to process a token through, and also provide the weights for combining the outputs of the expert.

The Top-K routing scheme makes the MoE layer desirable for training large, compute-efficient neural networks. It allows models to be scaled up, by way of increasing the total number of experts, while keeping the compute per token constant (as it is a function of K and not N).

The Router Gradient. Consider the gradient of the MoE layer's output y with respect to the router parameters W. We can express y as a function of W by combining Eq. (1) and Eq. (2). With the chain rule, we can backpropagate through this function by considering the gradient at each respective step:

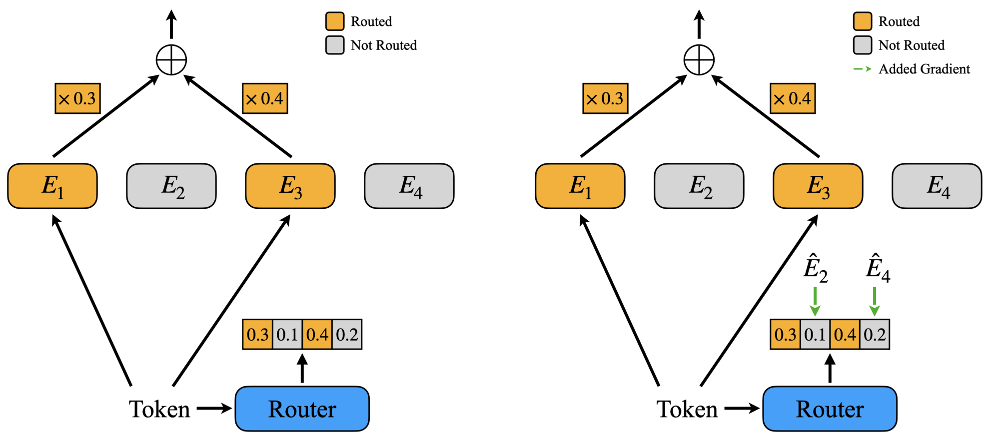
$$\frac{\partial y}{\partial W} = \frac{\partial y}{\partial \pi} \frac{\partial \pi}{\partial W} \tag{3}$$

The steps in Eq. (1) are easily differentiable as they consist of linear operations and activations. Thus, the first term in Eq. (3), $\frac{\partial y}{\partial \pi}$, is straightforward to compute. Eq. (2), however, isn't differentiable because the Top-K expert selection is a discrete function: given the continuous router weights $\pi \in \mathbb{R}^N$, the set of selected experts A is one of $\binom{N}{K}$ combinations. One way to get around backpropagation of nondifferentiable operations is to use the straight-through estimator [Bengio et al., 2013], which treats the operator as the identity function. In this setting, the Top-K routing function is bypassed and Eq. (2) becomes the dot product between π and the vector of all $E_i(x)$ with the following gradient:

$$\frac{\partial y}{\partial \pi} = [E_1(x), \quad E_2(x) \quad \cdots \quad E_N(x)] \tag{4}$$

This gradient requires the output of *all* of the experts for that token. Passing a token through all the experts will destroy the sparsity of the MoE layer. In this work, we develop methods for applying the straight-through estimator while maintaining the sparsity of the MoE layer by *approximating* the output of the experts not selected by Top-K routing.

Related Works. Previous work has tried to address the issue of routing in MoEs. Separate from Top-K is the Sinkhorn routing method [Clark et al., 2022]. Fedus et al. [2022] which



(a) Original Router

(b) Dense Approximation Router (ours)

Figure 1: **Overview of Routing with Dense Approximations**. The original mixture of experts router only receives gradients corresponding to experts the token is routed to, because there is no output from other experts. Our approach provides the router with a complete (dense) gradient, by approximating the activations of experts that a token is not routed to. As indicated by the dashed green arrows, the approximated gradients are not actually connected to the token in the computation graph; instead, they are artificially applied in the backward pass.

proposes an auxiliary loss that encourages load balancing. Dai et al. [2024] propose multiple additional auxiliary loss terms. Recently, Wang et al. [2024] propose learning biases rather than an auxiliary load balancing loss. Even more recently, Phi-3.5-MoE Abdin et al. [2024] uses SparseMixer [Liu et al., 2024, 2023], another estimator for $\partial y/\partial \pi$ not involving straight-through. Our approach is to still use straight-through, but *approximate* these additional expert outputs.

3 Designing a New Routing Method

In this section we design a new router that can receive a dense gradient update while being sparsely activated. In a standard MoE, the embedding corresponding to expert i in the routing layer (i.e. the ith row of the routing weight matrix) receives no gradient update from a token x if x is not routed to expert i. This is because $E_i(x)$ is never computed, so it provides no upstream gradient. This corresponds to experts that are not in the top K being omitted in Eq. (2). We apply an approximation $\hat{E}_i(x)$ as a substitute for the upstream gradient, so that the router can receive some non-zero signal corresponding to this expert. Thus, the router can factor in outputs from all experts when learning to route each token.

3.1 Approximating Expert Activations

To approximate the dense gradient in Eq. (4), we must approximate $E_i(x)$ for every expert i that a token x was not passed to. Although we have no information about what the function E_i looks like for x, when training with large token batch sizes it is very likely that we have outputs of E_i for many other tokens. We develop two general approaches to to develop an estimator $\hat{E}_i(x)$, using the expert outputs of other relevant tokens. *Expert group approximation:* We first apply a single approximation to a large group of tokens that were not routed to expert i. This is efficient, but it may not necessarily be a viable approximation for any specific x. However, we hypothesize that this is a good estimator for the expert output across the entire batch - this is sufficient as we will only need an approximation for the batch gradient to update the router. *Attention approximation:* Our secondary approach produces an expert output approximation specifically for each token (see Appendix A).

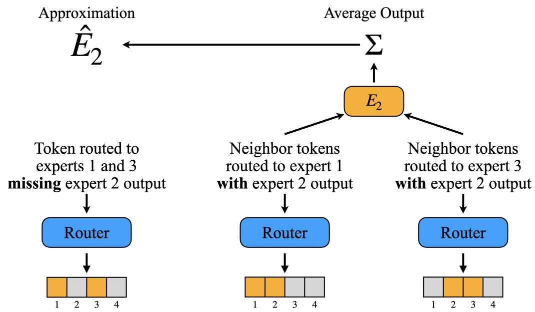


Figure 2: **Architecture of the Expert Group Approximation method.** In this example, we have 4 experts with $K = 2$. Consider all inputs routed to experts 1 and 3, characterized by the routing decision $R = \{1, 3\}$. As described in Figure 1b, we need to approximate these inputs' activations for all other experts. In approximating expert 2, for example, we collect all inputs x' with a routing decision similar to R specifically including expert 2: $R' = \{1, 2\}$ and $R' = \{2, 3\}$. In general there will be K such adjacent groups. The aggregation of these inputs' activations for expert 2 is used to approximate expert 2 for all inputs routed to experts 1 and 3.

3.2 Notation on Expert Routing

Let $R(x)$ be the set of indices corresponding to the K experts that a token x is routed to. This can be thought of as the *routing decision* for x, based on the selected experts A in Eq. (2). For example, in a top-k sparse mixture of experts block with $N = 8$ experts and $K = 2$, x routed to the first and last experts will have $R(x) = \{1, 8\}$. Note $R(x)$ will have $\binom{N}{K}$ possible discrete outputs. We can partition all tokens X based on their routing decisions and denote X_R as the subset of tokens routed to experts indexed by R. In the preceding example, x would belong to the set $X_{\{1,8\}}$. Some of our methods involve denoting whether a token was routed to a set of experts instead of its exact routing decision. We denote tokens routed to expert i along with any other experts as $X_{\{i,\cdot\}}$. For example, $X_{\{1,8\}} = X_{\{1,\cdot\}} \bigcap X_{\{8,\cdot\}}$

3.3 Expert Group Approximation

We primarily consider the case where we approximate the expert output $E_i(x)$ for many tokens at a time. For a token x, we want to approximate outputs of experts that x was not routed to, i.e. $E_i(x)$ where $i \notin R(x)$. We hypothesize that tokens being routed to the same expert is a strong indicator of similarity between the tokens. This is supported by our empirical observations in Appendix B.2. We develop an approximation for $E_i(x)$ by aggregating outputs of E_i for tokens that were routed to both expert i and an expert x was routed to. Formally, we consider an alternate routing decision $R' = \{i, j, \cdot\}, j \in R(x)$ that consists of one expert x is routed to, the expert i we wish to approximate, and any other experts (if $K > 2$). Then, the adjacent token space $X_{R'}$ will consist of tokens that are very similar to x by virtue of having similar routing decisions (see Fig. 9). Moreover, they will be routed to expert i, and we hypothesize that their outputs $\sum_{x' \in X_{R'}} E_i(x')$ will approximately represent $E_i(x)$. We can aggregate such outputs over all possible routing decisions:

$$\forall x \in X_R : \hat{E}_i(x) = \frac{1}{K} \sum_{j \in R} \frac{1}{|X_{\{i,j,\cdot\}}|} \sum_{x' \in X_{\{i,j,\cdot\}}} E_i(x') \tag{5}$$

We apply a single aggregate approximation for each routing decision to all tokens with that routing decision. Note that we only compute N^2 individual sums as that is the number of possible combinations $\{i, j, \cdot\}$. In Fig. 2 we visualize this method for $K = 2$.

4 Evaluation

4.1 Evaluation

Main Result. Our main result compares the Expert Group Approximation, which performs a dense update of the router weights by approximating the dense gradient, to baseline Top-K routing. Details on model training are provided in Appendix C. In Table 1 we find that our lightweight approximation method improves performance by a similar amount as activating an additional expert (that is, going from $K = 2$ to $K = 3$), without the additional computational overhead during training and inference of actually needing to use the parameters of a third expert. The choice of $K = 2$ follows Zoph et al. [2022].

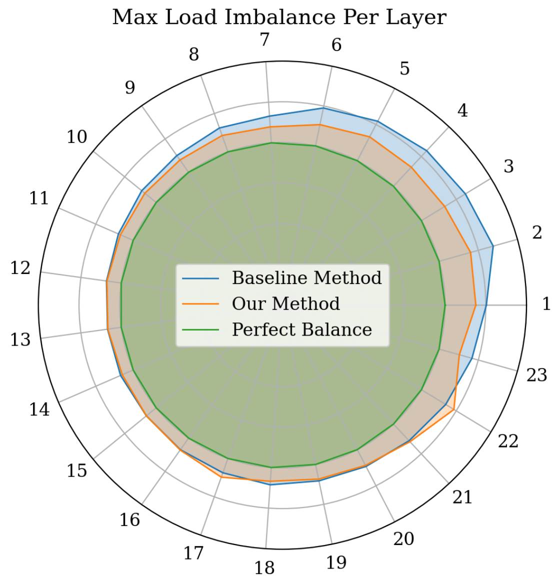


Figure 3: **Load Balancing using Expert Group Approximation**. We define maximum load imbalance at each layer as $\max_i(N \cdot f_i)$ where f_i is the fraction of tokens routed to each of N experts. The green ring indicates perfect balance, where each expert receives $1/N$ fraction of tokens; the outer ring indicates a maximum imbalance of 1.5. We record maximum imbalance after training on 3 billion tokens. By sending a complete gradient signal to the router, our model has better distribution of load than the baseline.

Load Balance. Our method improves over the baseline in perplexity, but the reason for this is in how it improves the routing distribution. Without gradient signal for unactivated experts, top-K routing may not be able to learn a balanced distribution across experts. This would lead to many more tokens being routed to some experts than others. In Fig. 3 we validate that the baseline top-K ($K = 2$) routing has an "imbalanced load", as measured by the proportion of tokens being routed to different experts (labeled by color) relative to the baseline (dotted red line) of an even distribution of tokens across experts. Our method

Table 1: Our expert group approximation obtains the best validation perplexity after 20B tokens, achieving the same performance as $K = 3$ without activating an additional expert.

Activated Experts	**Routing Method**	**Validation Perplexity**
$K = 1$	Baseline	19.61
$K = 2$	Baseline	18.92
$K = 3$	Baseline	18.56
$K = 2$	Expert Group Approx. (Ours)	**18.55**

improves load balance, which may be one cause for improved performance and is of independent interest on its own because it will lead to greater efficiency during inference.

Ablations. We conduct further ablations on design choices and efficiency in Appendix C.1.

5 Discussion

We propose a training method for MoEs to improve load balancing and language modeling performance. By approximating the signal of a dense mixture-of-experts layer, the MoE router is able to learn a better distribution of routing inputs to different experts. This approximated dense signal unlocks the possibility for more sparse MoEs at training and inference time. Whereas typical TopK routing would provide too sparse of a signal to learn a stable routing distribution, our method demonstrates significant improvements in load balancing and perplexity in very sparse configurations.

Limitations. The scope of our evaluation is limited; we only train models for at most 20B tokens, and the largest MoE we train has fewer than 1B active parameters. Furthermore, we only report the validation perplexity on a held-out subset of the training dataset and do not report any benchmark scores. The scope of problems caused by routers includes load balance and inability to handle distribution shifts during finetuning, but we only analyze the impact of our method on load balance and do not know whether it actually makes it easier to finetune MoEs. We plan to address these limitations in a future version of this work.

Future Work. Our methods are somewhat unique in that they scale with the token batch size per GPU, and improvements in memory efficiency therefore are critical. Developing and integrating kernels to reduce the memory requirements of the MoE itself will allow us to use larger microbatches. Another avenue for future work is developing entirely custom kernels using our methods in order to reduce the computational overhead of approximating the dense router gradient.

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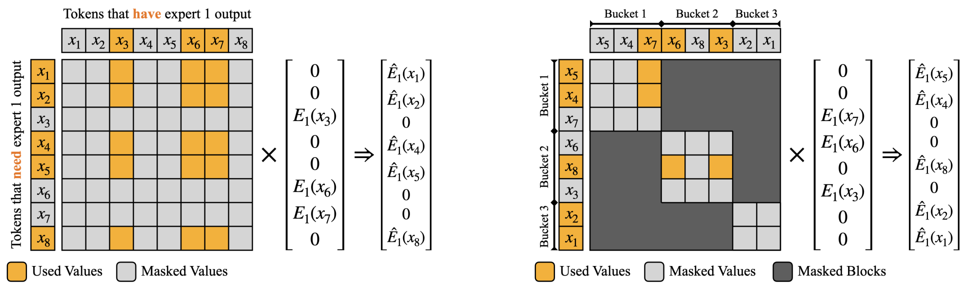


(a) Direct Attention

(b) Attention w/ LSH

Figure 4: **Attention scores of direct and LSH attention methods**. For each expert, we define an attention head which uses queries corresponding to inputs not routed to the expert, and keys corresponding to inputs routed to the expert. Grey entries denote queries and keys which do not meet this criteria, and whose attention scores are masked out. The attention scores of each head will then be multiplied by the values, corresponding to expert outputs of tokens routed to that expert. This implementation is common to both the direct and LSH attention method. In the latter, we further optimize the attention calculation by sorting inputs into buckets based on cosine similarity. This creates a block-sparse attention map, allowing kernels to entirely skip a majority of the attention computation.

A Token-specific Approximations Using Attention

A.1 Global Attention

Our Expert Group Approximation computes an approximation, for each expert, for all tokens routed to it from each other expert, and in this manner computes N^2 approximations. However, we may want to actually compute an approximation for specific tokens. Consider tokens belonging to the set $x \in X^C_{\{i,\cdot\}}$, i.e. tokens *not* routed to expert i. We want to approximate $E_i(x)$ for such x. At a high level, we want to search for similar tokens to x, select their expert outputs $E_i(x_j)$, and aggregate these outputs as a weighted linear combination. A well-known approach to this problem is attention. We want to query with all tokens *not* routed to expert i, i.e. $X^C_{\{i,\cdot\}}$. The keys will correspond to tokens that *were* routed to expert i, i.e. $X_{\{i,\cdot\}}$. And the values will be the expert outputs of these relevant inputs. Fig. 4a (left) outlines how we compute an approximation using multi-head attention, where each head corresponds to approximating for a single expert.

A.2 Sparse Attention using LSH

Computing attention across all tokens on an accelerator is computationally expensive, and we do not need attention scores for *all* the tokens to compute the approximation, just for the most similar tokens to x. With a block-sparse attention mask, we can greatly reduce the attention computation especially when most of the computed scores would be redundant. In Fig. 4b we outline our attention approximation that uses locality-sensitive hashing (LSH) to group tokens into buckets, with a high probability that the nearest neighbors to a token will lie in the same bucket. The attention mask now has an additional condition: the query index q and key index k must correspond to tokens in the same bucket. We sort the QKV into groups based on their assigned buckets to encourage a block-diagonal attention mask, and verify that this sparsity reduces the runtime of our attention approximation. Note that as exemplified in Fig. 4b, it is possible that some tokens receive no approximation because there are no keys to query in the bucket. In this case, we set the approximation to 0.

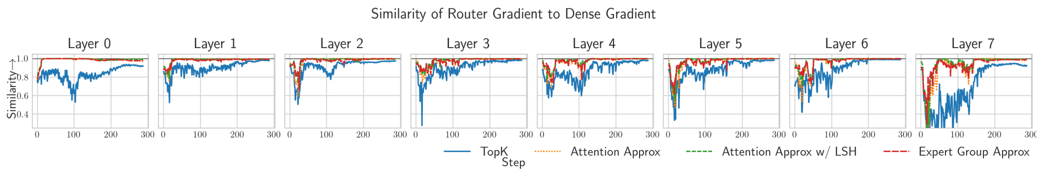


Figure 5: **Accuracy in approximating the dense router gradient for each approach**. This is recorded using a model with 8 experts and $K = 2$. The *dense gradient* of the output with respect to the router weights is artificially computed at each step by passing inputs through a dense mixture of experts layer, where all experts are selected. This is done independently from the actual forward pass computation, while using the same set of MoE parameters. The similarity between this dense gradient and the actual gradient propagated to the router indicates how well the router is learning from all experts. We plot this similarity using a standard TopK router, along with using each of our proposed router modifications. Our approaches are much more accurate and stable in approximating the dense router gradient.

B Approximation Statistics

B.1 Approximation Fidelity

We verify that our method is indeed faithfully approximating the dense router gradient i.e. the gradient to the router if all experts were activated. We track the dense gradient by routing to all experts and backpropagating only on the MoE output (independent of the full forward pass). This dense gradient is compared to the actual router gradient for each of our approaches in Fig. 5. We also observe a major difference between the gradient of the standard Top-K router and our approach.

The differences in our approaches become clear as we scale the model to become more sparse. We expand to $N = 32$ experts while maintaining $K = 2$ in Fig. 6 and find that it is more difficult to approximate the true dense router gradient. While all of our approaches sufficiently approximate the dense gradient with $N = 8$ experts, the performance gap between them is apparent with $N = 32$. The expert group approximation and LSH attention methods are significantly better than the direct attention method, and this is also consistent with our validation results in Table 1. This is likely due to the heuristics we apply to restrict our approximation to only the most relevant tokens: the expert group approximation requires inputs to have an expert in common, and LSH requires inputs to be similar. Moreover, the gap between our methods and Top-K is wider with 32 experts. We believe that in larger models with even more experts, our method will yield increasingly significant improvements over Top-K routing.

In Fig. 7 we reproduce the gradient similarity plots with SparseMixer [Liu et al., 2023]. Surprisingly, we find that SparseMixer is the worst approximation of the dense gradient across the board. Initial experiments also validate that SparseMixer does not outperform any of the other methods.

In Fig. 8 we provide an additional analysis of the gradient norm of our approximation compared to the dense gradient. We include statistics for SparseMixer as well. This logging is also done with $N = 8$ and $K = 2$. The Top-K gradient has significantly lower norm than the dense gradient, and the SparseMixer is an order of magnitude lower in many cases. Our methods closely approximate the dense gradient norm consistently; replicating both the direction and magnitude suggests that we are sufficiently approximating the dense gradient entirely.

B.2 Empirical Observations on Input Similarity

Our methods operate on the assumption that expert outputs for an input can be approximated by taking outputs from other similar inputs. We observe this during training by

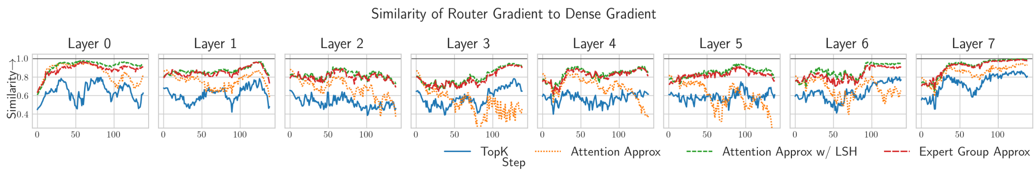


Figure 6: **Dense router gradient approximation accuracy with fine-grained experts**. We implement fine-grained experts as in DeepSeekMoE [DeepSeek-AI et al., 2024] to observe the behavior of our approximation methods across more experts while keeping parameter count fixed. In this example, the model now has 32 experts with $K = 2$. With more experts, it becomes increasingly hard to approximate the dense gradient, and the difference between our methods and the Top-K router is more apparent. Moreover, we can clearly compare the efficacy of each method and see that the attention approximation with LSH is the best. Note the average number of tokens per expert also decreases by a factor of 4 as well, and we would expect even better performance in our approximations by scaling the train batch size.

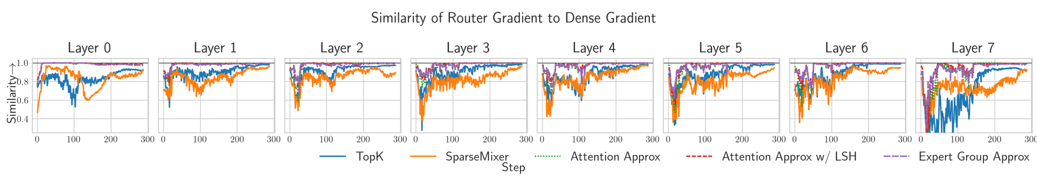


Figure 7: **Comparison of gradient approximation with our methods and SparseMixer**. We provide additional results showing how our gradient approximations compare against SparseMixer, another method to estimate the dense gradient.

partitioning each batch of inputs based on the experts that they are routed to. For each expert, we compute cosine similarity among all possible pairs of inputs routed to the expert and cosine similarity between expert outputs of the corresponding pairs. We specifically track the expert output similarity when these inputs have a cosine similarity > 0.75. In Fig. 10 we demonstrate that when inputs are very similar, they tend to have very similar expert outputs on average. Thus, we can approximate a missing expert output for a token by taking a nearby token's expert output.

Moreover, our expert group approximation method specifically assumes that being routed to the same expert is a proxy indicating similarity. For this method to work, it must be the case that two inputs that share experts in common are similar on average. Another desirable property is that these inputs have a similarity above some threshold (> 0.75) with a very high probability. Then, when approximating an expert output for a token, it suffices to use the output for another token routed to one of the same experts. We demonstrate the two above properties empirically in Fig. 9. Inputs with one expert in common are not only very similar on average, they are also very similar with a high probability. This suggests that our gradient estimator using the expert group approximation method is both accurate and consistent.

C Experimental Setup

Model Architecture. We train an MoE with 24 blocks, a hidden dimension of 1024, and 8 experts, for a total of 2B parameters, 780M of which are activated when we use the standard $K = 2$ top-K routing. We use SwiGLU [Shazeer, 2020] MLPs following Llama [Touvron et al., 2023], using an expansion factor such that the intermediate size of the MLP is 2816, 16 attention heads with dimension is 64, LayerNorm [Ba et al., 2016] and RoPE [Su et al., 2023].

Dataset. We train on FineWeb [Penedo et al., 2024] with the Llama3 tokenizer [Dubey et al., 2024]. We split it into train, validation, and test splits and report the validation perplexity.

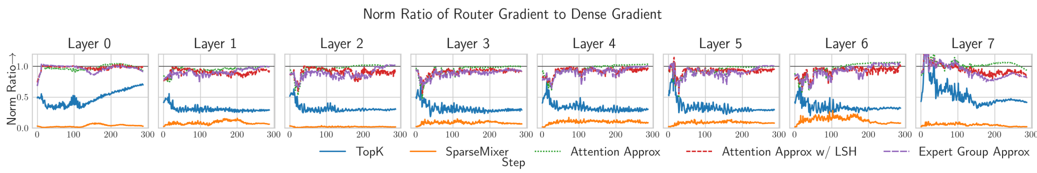
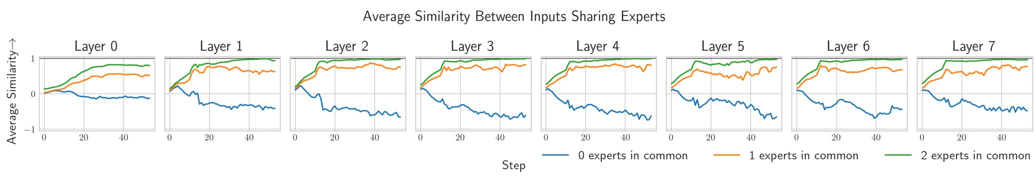
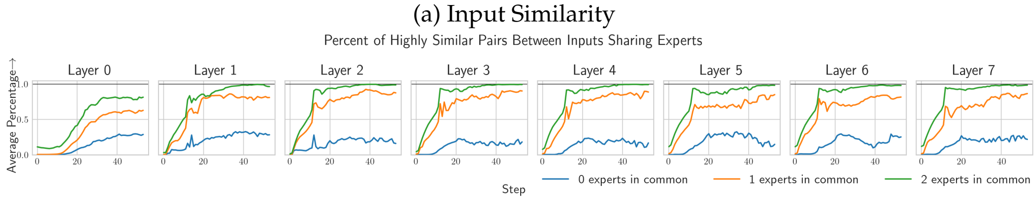


Figure 8: **Comparison of gradient norms relative to the dense gradient**. When computing the dense gradient, we also record its L_2 norm and log the ratio of this to the L_2 norms of the actual router gradients during training. Our methods produce router gradients with approximately the same magnitude. Along with the results showing strong cosine similarity, this suggests that we are almost perfectly approximating the dense gradient.



(a) Input Similarity



(b) Proportion of Highly Similar Inputs

Figure 9: **Similarity of Inputs Routed to Same Experts**. In a model with $N = 8$ experts and $K = 2$, we consider three distinct groups of input pairs: those were not routed to any of the same experts, those that have exactly one expert in common, and those that have both experts in common. Fig. 9a denotes the cosine similarity, on average, between inputs of each group. As expected, we see that inputs that are routed to both the same experts become highly similar, especially as training progresses. We also see inputs with no experts in common diverge in terms of similarity. However, inputs that have just one expert in common are still very similar, regardless of the other expert they are routed to. Moreover, Fig. 9b shows that a high percentage of inputs are highly similar — we define "highly similar" as having a cosine similarity > 0.75. This suggests that having at least one expert in common is a consistent indicator of similarity across groups of inputs.

Hyperparameters. We use the AdamW optimizer [Loshchilov and Hutter, 2019]. We use the modified cosine learning rate schedule from Ibrahim et al. [2024]. We set the minimum learning rate to 6×10^{-5}, the max learning rate to 6×10^{-4}, and the number of warmup iterations to 1000. We use a sequence length of 2048 and a global batch size of 1024, resulting in a global token batch size of 2^{21}. The total number of iterations is $10,000$ so that we train on $20B$ tokens, roughly following the compute-optimal [Hoffmann et al., 2022] number of training tokens for a $1B$ dense model. We set the auxiliary loss [Fedus et al., 2022] to 0.01.

Implementation. We train with the gpt-neox library [Andonian et al., 2023] integrated with Megablocks [Gale et al., 2022]. The TFLOPS vary depending on the method and the number of experts chosen; for simplicity, we do not account for the router or the number of experts activated when reporting the TFLOPS, so that the number of flops we count in a forward and backward pass is the same as a dense model.

We plot validation results throughout training in Fig. 11.

C.1 Ablating Design Choices

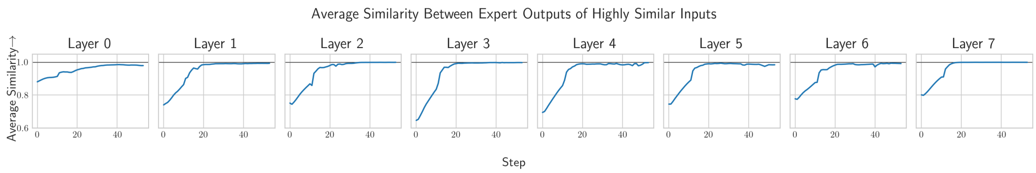


Figure 10: **Similarity of Expert Outputs With Similar Inputs.** For each expert in a model with $N = 8$ experts and $K = 2$, we consider the similarity of expert outputs when the inputs are "highly similar" i.e. with cosine similarity > 0.75. After a few training steps, the average similarity is very high and approaches the maximum value of 1. This supports our assumption about the expert networks being Lipschitz continuous, as similar inputs indeed produce very similar expert outputs.

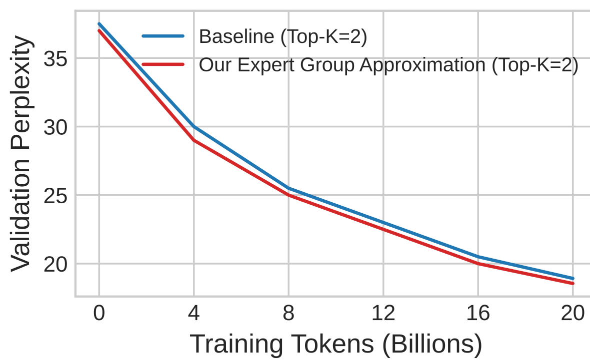


Figure 11: We plot the validation perplexity on FineWeb for the baseline Top-K router and our expert group approximation router. Without incurring significant overhead, we improve over the baseline.

We now present additional results on ablating our main design choices.

Expert Group Approximation. We consider two variations on the expert group approximation method. As a reminder, in this method we construct a mask of shape $experts, experts$ for each token. The row is the expert that token was routed to, and the column is the expert we want an approximation for. When we take the product of this mask and the router scores, we can weight each row by the probability corresponding to the expert we want an approximation for, or weight each approximation by the probability for the expert we're using the approximation for. The former should give us more "accurate" approximations, because it will prioritize tokens that are more likely to be routed to the expert we want an approximation for. The latter should give us more "viable" approximations, because it weights by closeness to the space we're using the approximation for. We compare these methods to the baseline in Table 2. Neither method improves over the baseline, but we think this may warrant further investigation.

Routing Method	Validation Perplexity
Expert Group Approx.	20.81
"Accurate"	20.97
"Viable"	21.14

Table 2: Ablating design choices in the expert group approximation method. Validation perplexity is reported after 12B tokens.

Comparing Different Approximation Methods. We use the Expert Group Approximation method for our main results because it is lightweight, easy to implement, and provides good performance. However, the other two methods we consider also outperform the top-K ($K = 2$) baseline. Indeed, as we showed in Fig. 5, the Attention+LSH method seems to obtain a better approximation of the true dense gradient. The primary reason why we report our main results with Expert Grouping is because the Expert Group Approximation method requires no additional memory overhead. This allows us to use larger microbatches, and therefore there are more tokens on each GPU that we can use for the approximation. In Table 3 we find that even with a microbatchsize 4× smaller than that of the Expert Group method, the Attention+LSH method is competitive.

Routing Method	Microbatchsize	Validation Perplexity
Attention	4	18.72
Attention+LSH	4	18.64
Expert Group	16	18.55
Baseline	16	18.92

Table 3: Comparison of activated experts, routing methods, and validation perplexity after training on 20B tokens.

Method Overhead. We have already outlined the implementation of the Expert Group Approximation method, which only requires materializing two additional tensors of size $experts, experts$ and $experts, micro\_batch\_size \times sequence\_length$. In Table 4 we compare the throughput of our method to the baseline, and find that even with an unoptimized method, we achieve 97.7% of the throughput of the baseline. We anticipate that we can further close this gap by directly modifying the gradient in the backward pass, rather than performing the approximation in the forward pass as we currently do and letting PyTorch's autograd compute the gradient.

Routing Method	TFLOPS
Top-K (K=2)	73.4
Expert Group Approx.	71.7

Table 4: Comparing the throughput of the baseline and our method.