Quantum Preferential Attachment

quantum communication, preferential attachment, complex networks, graph theory, phase transitions

Extended Abstract

Although the quantum internet is a rapidly developing technological reality [1, 2], it is yet to be formulated what kind of quantum network structures will eventually emerge. Current assumptions span from grid-like meshes [3] to scale-free networks, resembling the classical internet. Here, we propose a simple model for the growth of optical fiber-based quantum networks and show that it results in a rich class of quantum complex networks, different from the networks emerging from standard preferential attachment [4]. Just like in the classical case, in the quantum preferential attachment (QPA) model each new incoming node aims to communicate with a desired target node according to (nonlinear) preferential attachment [5], however, it might not connect to the target directly. Indirect quantum communication through an intermediate node is already a technological reality that preserves the absolute security of the communication channel [6]. Therefore, in the quantum scenario, incoming nodes can utilize this flexibility to overcome the unique challenges in quantum communication by connecting to an arbitrary node within a given range of the target node, including, but not restricted to the target node. As a result, existing nodes are not only attractive due to their direct utility but also showcase an indirect utility due to their network neighborhood, leading to qualitative changes in the emergent network properties compared to standard preferential attachment. As our main finding, we demonstrate that quantum preferential attachment leads to two distinct classes of network architectures, both of which are small-world networks with highly heterogeneous node degrees. Our framework also connects the quantum model to several classical extensions of preferential attachment as limiting cases of a unified phase diagram, allowing us to obtain rigorous results throughout the phase diagram.

References

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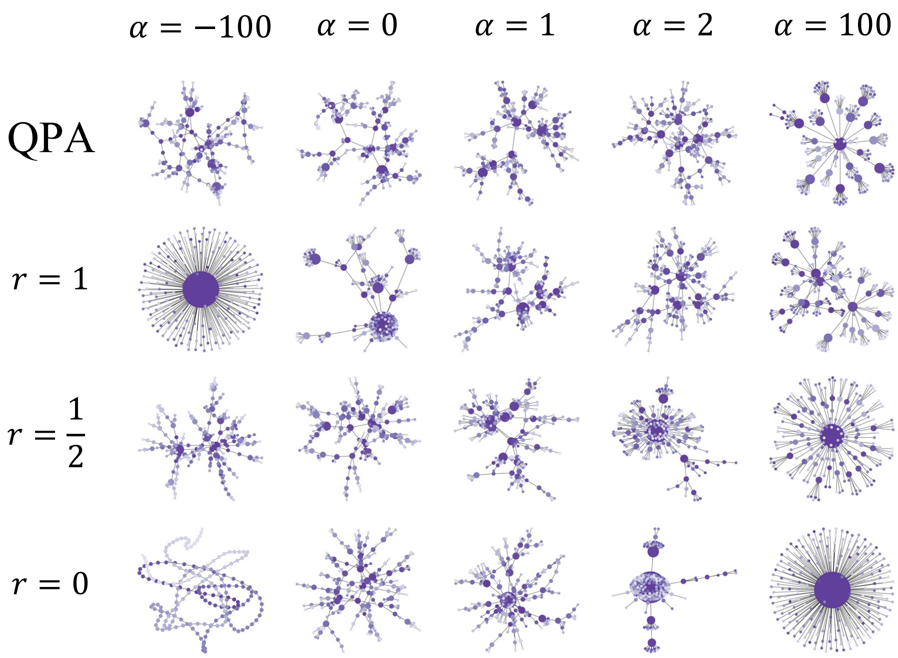

Figure 1: **Phase diagram of the constant redirection model and QPA.** Redirection happens either with a probability $r \in [0,1]$, or $r_i = d_i/(d_i+1)$ in the QPA model. Before redirection, a target node is first selected according to nonlinear preferential attachment with probability $\propto d_i^{\alpha}$. Node size reflects degree (larger nodes have higher connectivity) and color indicates temporal ordering (earlier nodes have darker shades). All simulations begin from the same initial condition: three nodes connected in a linear chain.

Table 1: **Summary of the numerical and analytic results.** For $r = 1/2$ we indicate the generic results for $0 < r < 1$ if known. Earlier results from literature are denoted with (*). BA stands for the Barabási-Albert model, UA for uniform attachment, RFT for the random friend tree, while k2 is an indirect model that grows with the second neighbor degree.

	Property	$\alpha \to -\infty$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha \to \infty$
QPA	architecture	Scale-free, $\gamma \approx 3.5$	Scale-free, $\gamma \approx 3.5$	Scale-free, $\gamma \approx 3.5$	k2, hierarchical	Rich club
	$d_{\max}$	$O(N^{0.4})$	$O(N^{0.4})$	$O(N^{0.4})$	$O(N^{2/3})$	$O(\sqrt{N})$
	leaves	$\frac{1}{2}N$	$O(N)$, $\approx 0.59N$	$O(N)$, $\approx 0.61N$	$O(N)$	$N - O(\sqrt{N})$
	diameter	$O(\log N)$	$O(\log N)$	$O(\log N)$	$O(1)$	$O(1)$
$r = 1$	architecture	Star	RFT, scale-free, $\gamma \approx 1.6$	BA, scale-free, $\gamma = 3$	k2, hierarchical	Rich club
	$d_{\max}$	N	$O(N)$	$O(\sqrt{N})$	$O(N^{2/3})$	$O(\sqrt{N})$
	leaves	$N - 1$	$N - O(N^{\delta})$, $\delta \approx 0.6$	$\frac{2}{3}N$	$N - O(N^{2/3})$	$N - O(\sqrt{N})$
	diameter	2	$O(\log N)$	$O(\log N)$	$O(1)$	$O(1)$
$r = 1/2$	architecture	Scale-free, $\gamma \approx 3.5$	Scale-free $\gamma = 4.3$	BA, scale-free, $\gamma = 3$	Hub and spokes	Star-like
	$d_{\max}$	$O(N^{0.4})$	$O(N^{0.3})$	$O(\sqrt{N})$	$O(N)$	$O(N)$
	leaves	rN	$\approx 0.61N$	$\frac{2}{3}N$	$\approx 0.75N$	$(1 - r + r^2)N$
	diameter	$O(\log N)$	$O(\log N)$	$O(\log N)$	$O(\log N)$	$O(1)$
$r = 0$	architecture	Chain-like	UA, exponential	BA, scale-free, $\gamma = 3$	Hub and spokes	Star
	$d_{\max}$	$O(1)$	$O(\log N)$	$O(\sqrt{N})$	$O(N)$	N
	leaves	$O(1)$	$\frac{1}{2}N$	$\frac{2}{3}N$	$O(N)$	$N - 1$
	diameter	$O(N)$	$O(\log N)$	$O(\log N)$	$O(1)$	2