# ALIGNMENT-AWARE MODEL EXTRACTION ATTACKS ON LARGE LANGUAGE MODELS

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# ABSTRACT

Model extraction attacks (MEAs) on large language models (LLMs) have received increasing attention in recent research. However, existing attack methods typically adapt the extraction strategies originally developed for deep neural networks (DNNs). They neglect the underlying inconsistency between the training tasks of MEA and LLM alignment, leading to suboptimal attack performance. To tackle this issue, we propose Locality Reinforced Distillation (LoRD), a novel model extraction algorithm specifically designed for LLMs. In particular, LoRD employs a newly defined policy-gradient-style training task that utilizes the responses of victim model as the signal to guide the crafting of preference for the local model. Theoretical analyses demonstrate that I) The convergence procedure of LoRD in model extraction is consistent with the alignment procedure of LLMs, and II) LoRD can reduce query complexity while mitigating watermark protection through exploration-based stealing. Extensive experiments on domain-specific extractions validate the superiority of our method in extracting various state-of-the-art commercial LLMs. Our code is available at: https://anonymous.4open.science/r/LoRD-MEA-1EF2/.

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# 1 INTRODUCTION

028 In recent years, we have witnessed the remark-029 able success of large language models (LLMs) such as ChatGPT (cha, 2024), Gemini (Anil 031 et al., 2024), and Claude (cla, 2024), which are now widely employed in various consumer and 033 industrial applications. Despite their success, 034 these models may suffer from model extraction attacks (MEAs) (Krishna et al., 2020; Rafi et al., 2022; Xu et al., 2022; Li et al., 2023b), where their knowledge could be at risk of being 037 stolen by an adversary through a *local model* that learns on the data collected from the victim model. Besides of some "open-source" LLMs 040 (e.g., Alpaca (Taori et al., 2023)), which are 041 trained on the chat history of GPT-4, cases of 042 commercial model theft among companies have 043 also been reported recently (Heath, 2023).

Under such a real-world threat, instead of focusing on MEAs against conventional DNNs, which have been extensively studied theoretically (Saad & Solla, 1995; Tian, 2020; Zhou et al., 2021) and empirically (Jagielski et al., 2020; Tramèr et al., 2016; Papernot et al., 2017), a few recent works turn to explore model extractional data in the second statement of the second statement





Figure 1: Comparison between vanilla MEAs on conventional DNNs (left) and MEAs on LLMs with alignments (right).

 tion algorithms and theorems for LLMs. For example, Wallace et al. (2020) propose a monolingualquery-based imitation attack framework to steal machine translation knowledge from generative language models such as GPT-2. Li et al. (2023b) investigate threats of stealing the code-related knowledge from LLMs. However, these studies inherit those MEA algorithms from traditional fields, such as computer vision (Tramèr et al., 2016; Papernot et al., 2017), and train the local model via
supervised learning like maximum likelihood estimation (MLE) (Bengio et al., 2000; Myung, 2003),
while neglecting the inconsistency of training tasks between MEAs and the alignments (Ouyang et al., 2022; Glaese et al., 2022; Bai et al., 2022a;b; Perez et al., 2023) of modern LLMs. As shown in Figure 1, modern LLMs typically employ alignments using reinforced learning, which is missing in the local model training of conventional MEAs. As a result, these attacks usually suffer from poor performance.

In this paper, we challenge the effectiveness of MLE in stealing a reinforcement-learning-aligned
 LLM, by analyzing its potential drawbacks as follows:

Low query efficiency. Current LLM-oriented MEAs suffer from unacceptably significant query times because they must collect enough generated responses, which entails exponential complexity in terms of generated tokens, resulting in low query efficiency.

Vulnerability against defenses. Directly learning from the responses of victim models can cause
local models to inadvertently incorporate those *watermarks* (Cong et al., 2022; He et al., 2022;
Zhao et al., 2022; He et al., 2021) embedded in the output of victim models. The residue of such
watermarks makes the extraction process less stealthy and even serves as provenance evidence of
model theft.

072 Motivated by these limitations, we propose Locality Rein-forced Distillation (LoRD), a queryefficient and watermark-resistant model extraction attack under a training paradigm with LLM's 073 alignments. Stealing LLMs via reinforcement learning paradigms is challenging. The main reason is 074 that the key component in the alignment procedure of LLMs, reinforcement learning with human 075 feedback (RLHF) (Bai et al., 2022a;b; Perez et al., 2023), heavily relies on the feedback signal of 076 *human annotators*, which is difficult to reproduce directly in the context of MEAs. To tackle this 077 challenge, we develop a policy-gradient-style extraction procedure. This approach regards the *locality direction* between the generations of local models and victim models as the implicit reward signal. It 079 can thus achieve a *human-feedback-free* reinforcement learning for our extraction attack. From the theoretical perspective, we show why those existing MEAs using MLE and knowledge distillation 081 (KD) are inconsistent with the optimization procedure in LLMs' alignments. Along this way, we also demonstrate why LoRD can achieve stronger watermark resistance and higher query efficiency.

Extensive experiments on five downstream NLP tasks and ten datasets demonstrate that it is feasible to steal a commercial LLM with 175 billion parameters by a pre-trained local model with only 8 billion parameters under a given domain. The resulting local model performs statistically similar to the victim model for tasks not requiring extra knowledge (e.g., data-to-text), and only  $0 \sim 3$ percentage lower for tasks requiring it (e.g., translation and QAs). This result poses an immediate threat of task-specific extraction on commercial LLMs. To further draw the capability boundary of such a threat, we also illustrate the "spectrum" in difficulties and upper bounds for extracting LLMs.

To summarize, the contributions of our paper are as follows:

New Perspective of LLM Alignment for MEAs. We present LoRD, a novel model extraction attack
 algorithm for LLMs. To our best knowledge, it is the first effective and realistic extraction algorithm
 that is compatible with the alignment procedure of LLMs.

Theoretical Guarantee. We theoretically prove that the convergence procedure of LoRD in MEAs
 is consistent with the alignments of LLMs. Furthermore, we demonstrate that LoRD can reduce
 query complexity while mitigating watermark protection through exploration-based stealing.

Systematical Evaluation. Extensive experiments on domain-specific extractions demonstrate that
 our method outperforms current extraction strategies across different downstream NLP tasks.

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  - 2 BACKGROUND
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104 2.1 POLICY GRADIENT MODELS 105

Policy gradient models (PGM) are commonly used in reinforcement learning (RL) algorithms to optimize the agents based on the decided *action* of RL agents. Represented by TRPO (Schulman et al., 2015) and PPO (Schulman et al., 2017), policy gradient models minimize the the following

<sup>108</sup> objective function:

$$\mathcal{L}_{pg,j} = -\hat{\mathbb{E}}_j[p_j^r(\theta)A_j],\tag{1}$$

where at each decision step j,  $p_j^r(\theta) = \frac{\pi_{\theta}(a_j|s_j)}{\pi_{\theta_{old}}(a_j|s_j)}$  refers to the probability ratio defined by the optimized policy  $\pi_{\theta}(a_j|s_j)$  and the initial policy  $\pi_{\theta_{old}}(a_j|s_j)$ ,  $s_j$  denotes the *state* of the environment,  $a_j$  denotes the decided *action* of  $\pi_{\theta}$ , and  $A_j$  is the *de-biased reward* of  $a_j$ .  $A_j$  is estimated by the Q-value minus the V-value, i.e.,

$$A_{j}(s_{j}, a_{j}) = Q(s_{j}, a_{j}) - V(s_{j}).$$
<sup>(2)</sup>

Intuitively, Q-value refers to the *reward* if employing action  $a_j$  at the given environment state  $s_j$ , which can be seen as the label of policy's decision. V-value represents the estimation of the expected reward at  $s_j$ . Consequently,  $A_j$  denotes the *surprise* when taking action  $a_j$ .

To alleviate the *off the cliff* phenomenon that a large bad gradient update occurred from Equation 1, PGMs, such as PPO and TRPO, add some regularization terms to avoid large gradients. Specifically, TRPO constrains the distribution between  $\pi_{\theta}$  and  $\pi_{\theta_{old}}$  with KL divergence, and PPO warps a "clip" function to constrain the bounds of  $p_j^r(\theta)$ .

### 125 2.2 LANGUAGE MODELING

**Supervised Training (SFT).** Given a pre-trained model with parameters  $\theta$ , supervised training is essentially the *maximum likelihood estimation (MLE)* task (Bengio et al., 2000; Myung, 2003), which fine-tunes  $\theta$  on the labeled dataset  $\mathcal{D}_{tr}^s = \{(\mathbf{x}_i, \mathbf{y}_i) | i = 1, 2, ..., N_{trs}\}$  by minimizing the following objective function:

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$$\mathcal{L}_{mle} = -\prod_{i}^{N_{trs}} P_{\theta}(\mathbf{y}_{i}|\mathbf{x}_{i}) = -\prod_{i}^{N_{trs}} \prod_{j}^{N} P_{\theta}(y_{i,j}|\mathbf{x}_{i},\mathbf{y}_{i,(3)$$

where N denotes the sequence length of  $\mathbf{y}_i$ ,  $y_{i,j}$  denotes the j-th token in  $\mathbf{y}_i$ , and  $\mathbf{y}_{i,<j} = \{y_{i,0}, ..., y_{i,j-1}\}$ . The logarithmic formula of Equation 3 can also be seen as a *joint cross-entropy* loss function:

$$\mathcal{L}_{ce} = -\sum_{i}^{N_{trs}} \log P_{\theta}(\mathbf{y}_{i} | \mathbf{x}_{i}) = -\sum_{i}^{N_{trs}} \sum_{j}^{N} \log P_{\theta}(y_{i,j} | \mathbf{x}_{i}, \mathbf{y}_{i,(4)$$

Equation 4 is extensively utilized in LLM's pre-training and fine-tuning procedures. For instance, it can be applied to *instruction-following supervised fine-tuning (SFT)* with the training set  $\mathcal{D}_{tr}$ , wherein  $\mathbf{x}_i$  encompasses the instruction and the task input, while  $\mathbf{y}_i$  denotes the reference response.

Aligning LLMs merely by SFT is not always practical, as MLE tends to align the model with the one-hot distribution of y, making it challenging to draw a sufficient variety of examples due to the *"exponential explosion"* of tokens (see Section 4 for more details). Moreover, providing standard answers for LLMs can sometimes be daunting for annotators, which further slows down and even degrades the alignment process through direct training.

Therefore, instead of "learning from answers" as in Equation 4, learning from *preferences* is proposed, which only requires the annotators to select a better response from a pair of texts generated by LLMs.

Aligning from Preferences. Employing reinforcement learning in LLMs typically consists of three stages. First, the annotators construct a preference dataset  $\mathcal{D}^{pref} = \{(\mathbf{x}_i, \mathbf{y}_i^+, \mathbf{y}_i^-)\}$  by chatting with LLMs and rating their responses, where  $\mathbf{y}_i^+$  and  $\mathbf{y}_i^-$  denote the rated positive and negative responses of the dialogue context  $\mathbf{x}_i$ , respectively. Then, a *reward model*  $R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y}) \rightarrow \mathbf{r}$  is trained based on  $\mathcal{D}^{pref}$  to simulate the environment and predict the reward values of tokens in given texts. It is trained with a pair-wise loss,

$$\mathcal{L}_{r} = -\sum_{(\mathbf{x}, \mathbf{y}^{+}, \mathbf{y}^{-}) \sim \mathcal{D}^{pref}} \sigma(R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y}^{+}) - R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y}^{-})),$$
(5)

where  $\sigma(\cdot)$  denotes the sigmoid function. Based on the reward model  $R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y})$ , we can finally train the language models  $P_{\theta}$  by maximizing its reward, i.e.,

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$$\max_{\theta} \sum_{\mathbf{x} \sim \mathcal{D}_q} R_{\theta_{\phi}}(\mathbf{x}, \hat{\mathbf{y}}) - \beta \mathbb{D}_{KL}[P_{\theta}(\hat{\mathbf{y}}|\mathbf{x})||P_{\theta_{init}}(\hat{\mathbf{y}}|\mathbf{x})], \tag{6}$$

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Figure 2: The stealing procedure of LoRD.

175 where  $\mathcal{D}_q$  denotes the dataset of text inputs,  $\hat{\mathbf{y}} \sim P_{\theta}(\mathbf{y}|\mathbf{x})$  denotes the sampled sequence of 176 the training model, and  $\theta_{init}$  is the initialized parameters of the model, e.g. the parameters after SFT. The Kullback-Leibler (KL) divergence term,  $\beta \mathbb{D}_{KL}[P_{\theta}(\mathbf{y}|\mathbf{x})||P_{\theta_{init}}(\mathbf{y}|\mathbf{x})]$ , introduced by 177 TRPO (Schulman et al., 2015), is incorporated to constrain the shift of distribution in generated texts 178  $\hat{\mathbf{y}}$ , where  $\beta$  is the hyperparameter. 179

Consequently, SFT shown in Equation 4 fine-tunes the pre-trained model with parameters  $\theta_{pre}$  into an aligned model  $\theta_{sft}$  through MLE, and RLHF outlined in Equation 6, further aligns  $\theta_{sft}$  towards 182 the target model  $\theta_{vic}$ . As this procedure is not consistent with the conventional training framework of DNNs, it remains unclear whether current MEAs (detailed in Appendix C.2) are effective and 183 efficient in stealing a LLM. Specifically, we will first put forward a new stealing method in Section 3, and compare it with current MEAs in Section 4. 185

#### 3 LORD: LOCALITY REINFORCED DISTILLATION

#### 3.1 OVERVIEW

191 In this subsection, we delve into the details 192 of our model extraction framework, LoRD 193 (Locality Reinforced Distillation). As described 194 in Algorithm 1, LoRD follows a reinforcement learning paradigm, that is, it consists of sev-196 eral periods, and in each period, the model will 197 learn to explore new responses and attempt to enhance the model trained in the last period. However, different from LLMs' alignments, the 199 agent can neither obtain the reward from the 200 reward model directly, nor label positive and 201 negative responses manually. This motivates us 202 to design a new RL method which can implicitly 203 measure the reward for generated tokens under 204 the guidance of victim model's responses. 205

Illustrated by Figure 2, LoRD first requires the 206 model to sample two sentences randomly at pe-207 riod t - 1, which are denoted as  $\mathbf{y}_{t-1}^+$  and  $\mathbf{y}_{t-1}^-$ , 208



Figure 3: Determination of the positive and negative samples in LoRD. We sample  $\mathbf{y}_{t-1}^+$  and  $\mathbf{y}_{t-1}^-$  from  $P_{\theta_{t-1}}(\cdot|\mathbf{x})$ , and compute their conditional probabilities. The response with a higher probability increment on  $\theta_t$ is selected as the positive sample.

respectively. In a new period t, it first computes the changes of likelihoods for these two sentences, 209 among the old model  $P_{\theta_{t-1}}$  and the current model  $P_{\theta_t}$ . These changes of likelihoods, denoted as 210  $\Delta_t^+$  and  $\Delta_t^-$ , indicate whether a selected sentence is locally *isotropic* ( $\Delta > 0$ ) to the optimization 211 direction with victim model's response  $\mathbf{y}_{vic}$  or not ( $\Delta \leq 0$ ), which can be seen as the feedback signal 212 for  $P_{\theta_t}$  in the current optimization step. For convenience, we may swap  $\mathbf{y}_{t-1}^+$  with  $\mathbf{y}_{t-1}^-$  to make 213 sure that  $\Delta_t^+ > \Delta_t^-$  always holds. In this way, for pairs  $(\mathbf{x}, \mathbf{y}_{vic})$  we can take  $\mathbf{y}_{t-1}^+$  as a *locality* 214 *neighborhood* of  $\mathbf{y}_{vic}$  and  $\mathbf{y}_{t-1}^{-}$  as the negative sample, all of which can be utilized in the training of 215  $P_{\theta_t}$ . Figure 3 illustrates this procedure. Additionally, LoRD takes  $\mathbf{y}_{t-1}^+$  as the positive label under

the current scope only when  $\Delta^+$  or  $P_{\theta_t}(\mathbf{y}_{t-1}^+|\mathbf{x})$  exceed their respective fixed thresholds  $\tau_1$  and  $\tau_2$ . If these conditions are not met, it will use  $\mathbf{y}_{vic}$  as a substitute for  $\mathbf{y}_{t-1}^+$  to enable a cold start.

Based on  $\mathbf{y}_{vic}$ ,  $\mathbf{y}_{t-1}^+$ , and  $\mathbf{y}_{t-1}^-$ , we now design LoRD's loss function.

**221 3.2 DESIGN OF LOSS FUNCTIONS** 

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From Section 2.1, we know that the loss function of a policy gradient model can be expressed as an *objective function* to maximize the rewards of decisions (see Equation 1) and a *regularization term* to ensure the stability of training. Following this paradigm, the loss function of LoRD could be

$$\mathcal{L}_{\text{LoRD}} = \mathcal{L}_{obj} + \mathcal{L}_{reg}.$$
(7)

**Objective function**  $\mathcal{L}_{obj}$ . Inspired by the reward model  $R_{\theta_{\phi}}$  existed in Equation 6, which is trained to distinguish between positive and negative samples, we propose utilizing the logarithmic proportion of positive to negative samples as the means of achieving a de-biased reward, i.e.,

$$\mathcal{L}_{obj} = -\sum_{\mathbf{x}\in\mathcal{D}_q} \log[\frac{P_{\theta_t}(\mathbf{y}_{t-1}^+|\mathbf{x})}{P_{\theta_t}(\mathbf{y}_{t-1}^-|\mathbf{x})}] = -\sum_{\mathbf{x}\in\mathcal{D}_q} [\log P_{\theta_t}(\mathbf{y}_{t-1}^+|\mathbf{x}) - \log P_{\theta_t}(\mathbf{y}_{t-1}^-|\mathbf{x})].$$
(8)

Equation 8 exhibits similarities to previous studies on RL-enhanced LLM (Peters & Schaal, 2007;
Peng et al., 2019; Go et al., 2023; Korbak et al., 2022; Rafailov et al., 2023). We provide a theoretical
explanation for its consistency with the learning procedure of RLHF and the deduction procedure, as
detailed in Section 4 and Appendix B.1.

However, training the local model merely by  $\mathcal{L}_{obj}$  is ineffective due to two reasons: *i*) when  $\mathcal{L}_{LoRD}$  :=  $\mathcal{L}_{obj}$ , no information from the victim model's responses is incorporated into the selection of  $\mathbf{y}_{t-1}^+$ beyond the cold start phase, resulting in a meaningless *self-reward-based learning* loop for the stealing procedure; *ii*) the convergence of the local model's training cannot be guaranteed.

To address these two issues simultaneously, we design the regularization term as follows.

**Regularization loss**  $\mathcal{L}_{reg}$ . Different from LLM's RLHF (Schulman et al., 2015; Rafailov et al., 2023; Bai et al., 2022a) that typically constrain  $\theta_t$  with initial model's generating distribution  $P_{\theta_{init}}(\cdot|\mathbf{x})$  in RLHF, LoRD aims to directly constrain  $\theta_t$  with victim model's distribution  $P_{\theta_{vic}}(\cdot|\mathbf{x})$ .

Unfortunately,  $P_{\theta_{vic}}(\cdot|\mathbf{x})$  is typically **inaccessible** within the APIs of commercial LLMs and is not feasible for our black-box scenarios. Consequently, we incorporate the regularization techniques employed in PPO and TRPO but tailor our regularization as a bounded contrastive term between the likelihood of  $\theta_t$  under the victim model's response and the negative sample, i.e.,

$$\mathcal{L}_{reg} = -\sum_{\mathbf{x}\in\mathcal{D}_q} clip(\log[\frac{P_{\theta_t}(\mathbf{y}_{vic}|\mathbf{x})}{P_{\theta_t}(\mathbf{y}_{t-1}^-|\mathbf{x})}]) = -\sum_{\mathbf{x}\in\mathcal{D}_q} clip(\log P_{\theta_t}(\mathbf{y}_{vic}|\mathbf{x}) - \log P_{\theta_t}(\mathbf{y}_{t-1}^-|\mathbf{x})).$$
(9)

252 In Equation 9, we utilize PPO's  $clip(\cdot)$  function to limit the value of the regularization term, as 253 we expect the regularization term could only be used to avoid the off the cliff problem (Schulman 254 et al., 2017; 2015) in RL's convergence. Besides, our contrastive term can be seen as a streamlined 255 black-box variant of the KL divergence in TRPO. This simplification offers two advantages: i) it 256 alleviates the necessity of loading the initial model's weights, leading to a substantial reduction 257 in GPU memory usage; *ii*) it eliminates the need for  $P_{\theta_t}(\cdot|\mathbf{x})$ , which would otherwise necessitate 258 an additional exponential operation of  $\log P_{\theta_t}(\cdot|\mathbf{x})$  that would slow down the forward computation 259 process and increase extra consumption.<sup>1</sup>

Incorporating Equation 8 with Equation 9, we can reshape the loss function of LoRD as

$$\mathcal{L}_{\text{LoRD}} = \mathcal{L}_{obj} + \mathcal{L}_{reg} = \sum_{\mathbf{x} \in \mathcal{D}_q} \log[\frac{P_{\theta_t}(\mathbf{y}_{t-1}^{-}|\mathbf{x})}{P_{\theta_t}(\mathbf{y}_{t-1}^{+}|\mathbf{x})}] + clip(\log[\frac{P_{\theta_t}(\mathbf{y}_{t-1}^{-}|\mathbf{x})}{P_{\theta_t}(\mathbf{y}_{vic}|\mathbf{x})})].$$
(10)

Finally, we wrap  $\mathcal{L}_{\text{LoRD}}$  with a sigmoid function  $\sigma(\cdot)$  to normalize the loss to the interval (0, 1), which is

$$\mathcal{L} = \sum_{\mathbf{x} \sim \mathcal{D}_q} \sigma(\log[\frac{P_{\theta_t}(\mathbf{y}_{t-1}^- | \mathbf{x})}{P_{\theta_t}(\mathbf{y}_{t-1}^+ | \mathbf{x})}] + clip(\log[\frac{P_{\theta_t}(\mathbf{y}_{t-1}^- | \mathbf{x})}{P_{\theta_t}(\mathbf{y}_{vic} | \mathbf{x})}])).$$
(11)

<sup>&</sup>lt;sup>1</sup>logsoftmax is preferred in the implementation of deep learning frameworks (tor), as the exponential operation in *softmax* and the logarithmic operation in *cross-entropy* can be canceled out by each other.

# 270 4 THEORETICAL ANALYSIS 271

This section will compare LoRD with current model extraction methods from a theoretical perspective.
We will first reveal the underlying inconsistency between the optimization procedure of LLMs, which
typically involves RL-based alignments, and the previous model extraction approaches utilizing *MLE*and *knowledge distillation (KD)*. Subsequently, we will demonstrate in theory the reasons why LoRD
can achieve stronger watermark resistance and higher query efficiency than existing methods.

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4.1 CONSISTENCY ANALYSIS REGARDING DIFFERENT LEARNING TASKS

Based on the analysis of the four objective functions for MLE, KD, RLHF and LoRD, we reach the
proposition 1, and illustrate their convergence procedure exhibited in Figure 4. A detailed proof can
be found in Appendix B.1.

Proposition 1 (Consistency in Stealing Procedure). The learning procedure for LLMs' alignments is consistent with the stealing procedure of LoRD, i.e., they both attempt to maximize the difference between the probabilities of positive and negative samples. Conversely, they are inconsistent with either MLE or KD. In MLE, the objective is maximizing the label probability, while KD aims to minimize the distance among all dimensions.

Albeit the inconsistency in their *training proce- dures*, we put forward Proposition 2 to demonstrate that *with enough samples*, all these methods will reach the same distribution results.

292 **Proposition 2** (Equivalence when Converged). Ideally, for any loss value of Equations 4, 5, 293 6, 10, or 11 converging to 0, we have  $\mathbf{y}^+ \equiv$ 294  $\mathbf{y}_{vic}$ . Meanwhile, the local model's distribu-295 tion  $P_{\theta}(\cdot|\mathbf{x})$  will approach that of the victim 296 model  $P_{\theta_{wie}}(\cdot|\mathbf{x})$  on MEAs from all three dis-297 cussed MEA methods, including LoRD, MLE, 298 and KD. 299

300 Proposition 2 ensures that the local model will 301 converge to the victim model regardless of 302 the choice of MEA methods. So what is the benefit of LoRD? In Section 4.2, we will show 303 that LoRD outperforms current MEAs with 304 two aspects: the query time reduction (i.e., 305 size of the query set  $\mathcal{D}_q$ ), and the watermark 306 resistance of the learned local model. 307



Figure 4: Illustrations for the converging procedure of probability distributions regarding four methods, namely MLE (a), KD (b), RLHF (c), and LoRD (d). Arrows indicate the expected optimization direction. We mark the distribution dimensions learned with labels in *blue*, and employ *pink* and *yellow* components to indicate the probabilities of positive and negative tokens, respectively.

4.2 COMPARATIVE ANALYSIS ON MODEL STEALING

**Query Efficiency.** Let  $N_Q$  and  $N_R$  denote the sequence lengths of the query text and the response text, respectively. For MLE, the *ideal* query numbers to populate the entire text space are given by  $\mathcal{O}(V^{N_Q} \cdot V^{N_R})$ , where V represents the size of the vocabulary. In contrast, LoRD possesses the capability to automatically explore the generation token space, thereby significantly reducing the query requirements about generation candidates to a constant level. Specifically, the complexity of LoRD's query requirements is  $\mathcal{O}(V^{N_Q} \cdot C)$ , where C is a constant that correlates with the capability of local models.

Based on the above analysis, a straightforward concern with employing MLE in LLMs' extraction is that, given the limited query times in real-world practices, it may suffer from incomplete learning, especially for text generation tasks. Consequently, the local model may tend to memorize some specific responses instead of achieving a broad understanding and generation. We call such a phenomenon *preference overfitting (PO)*, which indicates that the local model is only effective on a limited set of explored samples, and yet does not generalize well to unseen scenarios. In such cases, the local model usually exhibits a more "rugged" decision surface, which appears to *overfit* the preference sentences in  $\mathcal{D}_{tr}$ , as shown in Figure 11 (b). Figure 10 provides a visualization of it. 347

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		BL	EU			BERTScore			Rouge-L	
	1	2	3	4	Pre.	Rec.	F1.	Pre.	Rec.	F1.
			Text to SQL:	WikiSQL (Zho	ong et al., 2017	7) with 64 quer	y samples			
Victim Model	54.1	41.4	32.1	24.4	86.9	93.5	90.1	58.9	62.1	59.7
Local Model	$20.2 \pm 0.2$	$14.5 \pm 0.2$	$10.9 \pm 0.1$	$8.1 \pm 0.1$	$82.5 \pm 0.0$	$92.4 \pm 0.1$	$87.1 \pm 0.0$	$22.6 \pm 0.3$	$66.4 \pm 0.4$	$33.2 \pm 0.3$
+MLE	$54.0 \pm 1.6$	$37.5 \pm 2.1$	$26.4 \pm 2.0$	$18.8 \pm 1.8$	$83.1 \pm 0.2$	$92.9 \pm 0.2$	$87.7 \pm 0.2$	$56.2 \pm 1.5$	$56.1 \pm 0.9$	$55.8 \pm 1.2$
+LoRD	$55.1 \pm 2.3$	$39.0 \pm 3.6$	$28.0 \pm 4.0$	$20.4 \pm 3.9$	$83.4 \pm 0.4$	$92.9 \pm 0.3$	$87.9 \pm 0.4$	$57.7 \pm 2.2$	$56.3 \pm 2.0$	$56.7 \pm 2.1$
Text to SQL: Spider (Zhong et al., 2017) with 64 qu					with 64 query	samples				
Victim Model	9.4	3.9	2.1	1.1	77.7	84.1	80.6	17.1	36.3	21.8
Local Model	$6.4 \pm 0.2$	$2.1 \pm 0.1$	$0.9 \pm 0.1$	$0.5 \pm 0.0$	$80.0 \pm 0.1$	$82.6 \pm 0.1$	$81.2 \pm 0.1$	$10.0 \pm 0.3$	$21.5 \pm 0.6$	$12.7 \pm 0.4$
+MLE	$6.2 \pm 0.9$	$1.3 \pm 0.5$	$0.6 \pm 0.3$	$0.2 \pm 0.2$	$76.4 \pm 0.7$	$81.8 \pm 0.4$	$78.9 \pm 0.6$	$12.7 \pm 1.6$	$18.3 \pm 1.6$	$14.3 \pm 1.6$
+LoRD	$9.1 \pm 0.9$	$2.8 \pm 0.5$	$1.3 \pm 0.4$	$0.6 \pm 0.2$	$77.7 \pm 0.4$	$83.1 \pm 0.5$	$80.2 \pm 0.3$	$16.9 \pm 0.1$	$24.1 \pm 0.2$	$18.8 \pm 0.1$
Data to Text: E2E NLG (Dušek et al., 2020) with 64 query samples										
Victim Model	51.8	27.0	26.8	19.1	93.9	94.6	94.2	49.6	54.6	51.4
Local Model	$31.1 \pm 0.1$	$20.1 \pm 0.2$	$13.5 \pm 0.2$	$8.9 \pm 0.3$	$86.1 \pm 0.1$	$92.4 \pm 0.1$	$89.1 \pm 0.1$	$29.0 \pm 0.3$	$49.4 \pm 0.4$	$35.9 \pm 0.3$
+MLE	$53.0 \pm 0.9$	$38.0 \pm 0.6$	$27.5 \pm 0.5$	$19.9 \pm 0.4$	$89.1 \pm 0.0$	$94.5 \pm 0.0$	$91.8\pm0.0$	$48.3 \pm 0.5$	$54.2 \pm 1.4$	$50.4 \pm 0.9$
+LoRD	$53.1 \pm 1.1$	$38.2 \pm 0.9$	$27.8 \pm 0.7$	$20.2 \pm 0.5$	$89.1 \pm 0.1$	$94.5 \pm 0.1$	$91.7 \pm 0.1$	$48.3 \pm 0.7$	$53.5 \pm 1.4$	$50.2 \pm 0.9$
	•		Data to Text:	CommonGen (	(Lin et al., 202	0) with 64 que	ry samples			
Victim Model	33.3	18.5	11.1	6.9	91.3	92.1	91.7	33.6	40.7	36.1
Local Model	$12.2 \pm 0.0$	$6.5 \pm 0.1$	$3.8 \pm 0.0$	$2.3 \pm 0.0$	$83.0 \pm 0.0$	$89.7 \pm 0.0$	$86.2 \pm 0.0$	$14.6 \pm 0.1$	$46.2 \pm 0.2$	$21.6 \pm 0.0$
+MLE	$32.4 \pm 2.0$	$18.3 \pm 1.3$	$10.9 \pm 1.0$	$6.6 \pm 0.7$	$84.2 \pm 0.1$	$91.7 \pm 0.0$	$87.8 \pm 0.0$	$31.7 \pm 2.4$	$41.1 \pm 0.4$	$35.1 \pm 1.6$
+LoRD	$32.1 \pm 1.3$	$18.0 \pm 0.9$	$10.7 \pm 0.5$	$6.4 \pm 0.3$	$84.1 \pm 0.0$	$91.6 \pm 0.1$	$87.7 \pm 0.0$	$31.4 \pm 1.1$	$40.3\pm0.9$	$34.6 \pm 0.9$
			Summarizat	ion: TLDR (Ki	rk et al., 2023	al., 2023) with 64 query samples				
Victim Model	11.9	5.0	2.6	1.5	85.9	88.4	87.1	13.4	30.9	18.4
Local Model	$6.9 \pm 0.0$	$3.2 \pm 0.1$	$1.7 \pm 0.0$	$1.0 \pm 0.0$	$81.0 \pm 0.1$	$87.6 \pm 0.0$	$84.1 \pm 0.0$	$10.5 \pm 0.1$	$41.1 \pm 0.1$	$16.4 \pm 0.1$
+MLE	$10.6 \pm 0.5$	$4.8 \pm 0.2$	$2.6 \pm 0.1$	$1.6 \pm 1.1$	$83.6 \pm 0.7$	$88.4 \pm 0.2$	$85.9 \pm 0.5$	$14.3 \pm 0.5$	$32.7 \pm 1.1$	$18.9 \pm 0.4$
+LoRD	$10.2 \pm 0.3$	$4.5 \pm 0.1$	$2.4 \pm 0.1$	$1.4 \pm 0.0$	$84.1 \pm 0.1$	$88.3 \pm 0.1$	$86.2 \pm 0.1$	$12.8 \pm 0.3$	$33.2 \pm 0.9$	$18.0 \pm 0.2$
		Sumn	narization: CN	N Daily Mail	(Hermann et a	l., 2015) with (	54 query samp	les		
Victim Model	20.4	10.8	6.4	4.1	86.4	87.8	87.1	22.4	40.8	28.2
Local Model	$4.9 \pm 0.0$	$3.6 \pm 0.0$	$2.7 \pm 0.0$	$2.1 \pm 0.0$	$80.5 \pm 0.0$	$88.3 \pm 0.0$	$84.2 \pm 0.0$	$10.9 \pm 0.0$	$79.1 \pm 0.1$	$18.8 \pm 0.0$
+MLE	$5.1 \pm 0.5$	$3.7 \pm 0.0$	$2.8 \pm 0.0$	$2.2 \pm 0.0$	$80.6 \pm 0.0$	$88.3 \pm 0.0$	$84.3\pm0.0$	$11.3 \pm 0.1$	$78.6 \pm 0.1$	$19.3 \pm 0.1$
+LoRD	$5.3 \pm 0.0$	$3.9 \pm 0.0$	$2.9 \pm 0.0$	$2.3 \pm 0.0$	$80.6 \pm 0.0$	$88.4 \pm 0.0$	$84.3\pm0.0$	$11.3 \pm 0.1$	$78.6 \pm 0.2$	$19.1 \pm 0.1$
			Summarization	n: Samsum (G	liwa et al., 201	19) with 64 que	ery samples			
Victim Model	20.7	11.4	6.9	4.4	88.1	91.7	89.8	24.2	50.5	31.6
Local Model	$8.9 \pm 0.2$	$5.2 \pm 0.1$	$3.3 \pm 0.1$	$2.1 \pm 0.1$	$80.9 \pm 0.2$	$90.1 \pm 0.1$	$85.2 \pm 0.2$	$17.0 \pm 0.3$	$61.8 \pm 0.5$	$25.5 \pm 0.4$
+MLE	$16.9 \pm 1.1$	$9.4 \pm 0.7$	$5.8 \pm 0.4$	$3.7 \pm 0.3$	$83.9 \pm 0.9$	$90.9 \pm 0.6$	$87.3 \pm 0.8$	$25.2 \pm 0.8$	$49.8 \pm 2.5$	$31.0 \pm 1.7$
+LoRD	$18.4 \pm 0.7$	$10.1 \pm 0.3$	$6.0 \pm 0.2$	$3.7 \pm 0.1$	$84.9 \pm 0.1$	$91.5\pm0.1$	$88.1\pm0.1$	$23.2\pm0.8$	$49.7 \pm 1.5$	$30.2 \pm 0.6$

Table 1: MEA comparison on three tasks, including structured text generation, data to text, and summarization. We use GPT-3.5-turbo as the victim model, and Llama3-8B (lla, 2024) as the local initial model. The *intensity* of the red or blue color corresponds to the degree of underperformance or outperformance relative to the victim model. More experiments are in Table 2 and Table 6.

351 Watermark Resistance. Another limitation of prevalent objective functions, such as MLE and KD, 352 is their susceptibility to watermarks (Cong et al., 2022; He et al., 2022; 2021; Kirchenbauer et al., 353 2023) of output contents, i.e., while stealing knowledge from LLMs via responses  $y_{vic}$ , watermarks 354 within them will also been passively inherited by the local model. Consequently, the generated 355 sentences of the local model may possess some *residual* of watermarks, which might be detected as evidence of stealing. 356

357 Despite introducing current watermark removal techniques, we indicate that LoRD can mitigate the 358 influences of watermarks naturally, as it does not learn the likelihood of victim models' responses 359  $\mathbf{y}_{vic} \sim \mathcal{D}_{tr}$  directly, but relies on  $\mathbf{y}_{vic}$  to determine positive and negative labels from responses 360 generated by the local model.

361 As depicted in Equation 8, LoRD guides the local model to learn the likelihood of  $y_{t-1}^+$  instead 362 of  $y_{vic}$ , which means that it will not been influenced by watermarks contained in  $y_{vic}$  explicitly. 363 However, the regularization term  $\mathcal{L}_{reg}$ , as well as the replacement  $\mathbf{y}_{t-1}^+ \leftarrow \mathbf{y}_{vic}$  for a cold start, will 364 indeed introduce watermarks from  $\mathbf{y}_{vic}$ . To address this, we can reshape Equation 11 into a convex 365 combination of the objective function and the regularization, i.e.,

366  $\mathcal{L} = \mathbb{E}[(1 - \lambda_1) \cdot (\log P_{\theta_t}(\mathbf{y}_{t-1}^+ | \mathbf{x}) - \log P_{\theta_t}(\mathbf{y}_{t-1}^- | \mathbf{x})) + \lambda_1 \cdot clip(\log P_{\theta_t}(\mathbf{y}_{vic} | \mathbf{x}) - \log P_{\theta_t}(\mathbf{y}_{t-1}^- | \mathbf{x}))],$ 367 where  $0 \le \lambda_1 \le 1$  is the hyperparameter. 368

369 When  $\lambda_1$  is small, the convergence of LoRD will substantially focus on maximizing  $P_{\theta_t}(\mathbf{y}_{t-1}^+|\mathbf{x})/P_{\theta_t}(\mathbf{y}_{t-1}^-|\mathbf{x})$ , with which the local model will exhibit a strong watermark resistance 370 ability. When  $\lambda_1$  increases, LoRD will tend to rely more on the guidance of  $\mathbf{y}_{vic}$ , resulting in a higher 371 risk of introducing watermarks. In the case of  $\lambda_1 = 1$ , the local model will converge to the victim 372 model without any exploration and watermark resistance, which might suffer from the same level of 373 defense by watermarks. 374

375 From a global perspective,  $\mathcal{L}_{obj}$  represents the exploration and the locality learning ability of LoRD, which can mitigate the influences of watermarks. On the other hand,  $\mathcal{L}_{reg}$  ensures the stability of 376 the training procedure. Therefore,  $\mathcal{L}$  characterizes a trade-off via  $\lambda_1$  between the stability and the 377 diversity during stealing, and Equation 11 can be seen as a special case of  $\mathcal{L}$  with  $\lambda_1 = 0.5$ .

#### 378 5 **EXPERIMENTS** 379

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# 5.1 Settings

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**Datasets.** We evaluate MEAs on five mainstream natural language generation (NLG) tasks, including machine translation, text summarization, question answering, structured text generation, and data-to-384 text. We select ten representative datasets: WMT16 (Bojar et al., 2016), TLDR (Kirk et al., 2023), 385 CNN Daily Mail (Hermann et al., 2015), Samsum (Gliwa et al., 2019), WikiSQL (Zhong et al., 2017), 386 Spider (Yu et al., 2018), E2E-NLG (Dušek et al., 2020), CommonGen (Lin et al., 2020), PIQA (Bisk et al., 2020), and TruthfulQA (Lin et al., 2021) as benchmarks for our domain-specific evaluation. 388 These datasets cover most of the downstream tasks in natural language generation. We compare not 389 only the stealing efficacy of different MEA methods, but also the stealing difficulty across different 390 downstream tasks. Table 5 lists all datasets and backbones used in the paper.

391 **Baselines.** As described in Section 2.2 and 4.1, we compare LoRD with two types of model extraction 392 methods: maximum likelihood estimation (MLE) and knowledge distillation (KD). For MLE and 393 LoRD, we conduct MEAs under pure **black-box attack settings** (see Appendix D for more details of 394 the threat model). For KD, the predicted distributions are used specifically under grey-box settings. 395

Metrics. For text generation tasks, we evaluate extracted models with a semantic-level and two 396 lexical-level metrics, BERTScore (Zhang et al., 2020), BLEU (Papineni et al., 2002), and Rouge-397 L (Lin, 2004), all of which are commonly used in the NLG evaluation. Regarding reasoning tasks 398 (e.g., QA), we use Precision, Recall, Accuracy, and F1 score as their evaluation metrics. 399

**Implementation Details.** We use Llama3-8B as the local model to learn the outputs generated by 400 victim models. We set sequence length varying 128 to 4096 depending on the selected tasks, and 401 learning rate  $3 \times 10^{-5}$ . Our experiments run on  $2 \times 80$ GB Nvidia Tesla A100. We execute each 402 training five times and record the mean values and standard variances in the following sections. For 403 LoRD, we set  $\tau_1$  and  $\tau_2$  to 0.8 and -0.1, respectively. Besides, we set the period number  $N_t$  to 404 512, and use  $\lambda_1 = 0.5$  as the default formation of the loss function. The victim model's response, 405  $y_{vic}$ , is generated by token sampling with a temperature of 1, the default setting for current LLM 406 APIs. The local model also uses token sampling, but with a temperature of 0.8 and Top-P probability 407 clipping (Holtzman et al., 2019) at 0.98. We use this setting to enhance the stability of generation 408 in local models. Note that we have not incorporated sampling strategies with their corresponding 409 hyperparameters into the design of LoRD. We believe that MEAs considering sampling strategies could inspire more powerful MEA methods, and we leave these improvements for future work. 410

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# 5.2 STEALING DOMAIN-SPECIFIC KNOWLEDGE

414 We first select GPT-3.5-turbo, a checkpoint of ChatGPT, as the basic victim model. This is because 415 its API provides *probabilities* of candidate words when generating responses. We employ Llama3-416 8B (Ila, 2024), a small LLM with only a 4.5% fraction of parameters than the victim model as our 417 initial local model. Though this LaViSH (Large-Victim-Small-Heist) setting contradicts previous 418 assumptions (Tramèr et al., 2016; Papernot et al., 2017; Jagielski et al., 2020) in MEA that the copy 419 model should usually be "wider" or "larger" than the victim model to contain its knowledge, we 420 believe this setting is more applicable in real world scenarios (Li et al., 2023b). Appendix D provides 421 more detail for this setting. Besides, the number of query times selected in this section is less than 422 100, a significant degradation compared to previous studies (Li et al., 2023b). This is because, in our experiments, copy models can easily learn the knowledge with a few training samples and then exhibit 423 only slight improvements afterward. More discussions on query times can be found in Appendix 424 A.1.1. 425

426 Fidelity and limits on stealing. We first examine the fidelity and limits of a small LLM to steal 427 commercial LLMs. As shown in Table 1, we list the performance of the victim model and the local 428 model on three tasks, and provide two MEA methods, local model fine-tuned with MLE (+MLE) and 429 LoRD (+LoRD), respectively. In Table 1, cells highlighted in *red* indicate poorer outcomes compared to the victim model, whereas *blue* signifies results that are on par or potentially superior to the victim 430 model. The *intensity* of the red or blue color corresponds to the degree of underperformance or 431 outperformance relative to the victim model.

432 We can see that the original performance of the local model is significantly lower than the victim 433 model, i.e., with a 50% decrease in BLEU-4 or  $10 \sim 25$  decrease in Rouge-L. Once we employ 434 MEAs in the local model, its performance rapidly boosts to nearly the same as the victim model, with 435  $0 \sim 40\%$  points of gaps in BERTScore. These gaps are negligible (e.g. < 1% in summarization) 436 in some tasks, but remain eminent in other tasks such as reasoning, structured text generation, and machine translation. This phenomenon indicates that domain-specific model extractions can 437 effectively learn domain-specific abilities from victim models but may perform poorly if downstream 438 tasks require extra knowledge, such as machine translation and QA. We provide a stealing comparison 439 among different local models in Table 9. 440

# 441 Comparison among stealing methods. Tables 442 1, 6, and 2 compare the stealing efficacy be443 tween MLE and our LoRD. The results consis-

tently show that LoRD significantly outperforms 444 MLE under the same MEA settings. Besides, for 445 challenging tasks such as reasoning and transla-446 tion, LoRD exhibits much higher improvements, 447 which demonstrates that it can address the pref-448 erence overfitting problem discussed in Section 449 4.2 and do enable the local model to learn the 450 task ability from victim models. However, we 451 also observe that for some tasks (e.g., summa-452 rization), LoRD shows no statistical difference 453 from MLE, probably because these tasks are relatively simple, where merely MLE has already 454 achieved comparable results to victim models. 455

Tasks difficulties comparison. Based on previous analysis, we observe that the performance and limitations of MEA depend on the category of tasks. Additionally, sometimes datasets in the same task exhibit significant differences in stealing. We put forward two metrics to measure task difficulties: the *fidelity* that measures extraction

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Figure 5: Spectrum of the fidelity and performanceup on extracting different downstream tasks. Current datasets can be divided into three groups: high fidelity and high performance-up (HFHP), high fidelity but low performance-up (HFLP), and low fidelity but high performance-up (LFHP).

efficacy compared to victim models, and the *performance-up*, which assesses the performance gain before and after stealing for a given local model. Formally, given a test set  $\mathcal{D}_{te} = \{(\mathbf{x}, \mathbf{y})\}$  and a corresponding metric  $\mathcal{M}(hypothesis, reference)$ , the fidelity (F) and performance-up (P) of the local model  $\theta_{N_t}$  can be defined as:

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$$P = \frac{\sum_{\mathbf{x}, \mathbf{y} \in \mathcal{D}_{te}} \mathcal{M}(\mathbf{y}_{N_t}, \mathbf{y})}{\sum_{\mathbf{x}, \mathbf{y} \in \mathcal{D}_{te}} \mathcal{M}(\mathbf{y}_{vic}, \mathbf{y})}, P = \frac{\sum_{\mathbf{x}, \mathbf{y} \in \mathcal{D}_{te}} \mathcal{M}(\mathbf{y}_{N_t}, \mathbf{y})}{\sum_{\mathbf{x}, \mathbf{y} \in \mathcal{D}_{te}} \mathcal{M}(\mathbf{y}_0, \mathbf{y})},$$
(12)

where  $\mathbf{y}_{N_t} \sim P_{\theta_{N_t}}(\cdot|\mathbf{x})$ ,  $\mathbf{y}_0 \sim P_{\theta_0}(\cdot|\mathbf{x})$ , and  $\mathbf{y}_{vic} \sim P_{\theta_{vic}}(\cdot|\mathbf{x})$  denote the sampled responses from the trained local model ( $\theta_{N_t}$ ), the initial local model ( $\theta_0$ ), and the victim model ( $\theta_{vic}$ ), respectively. In Figure 5, we illustrate a "spectrum" of extracting various downstream tasks based on these two metrics defined in Equation 12. The figure can assist in recognizing and defending commercial LLM's knowledge.

From Figure 5, we observe five tasks forming the following three scenario groups and datasets coming from the same tasks are mostly in the same group:

- High fidelity and high performance-up (HFHP). These tasks are challenging for a pre-trained model but can be effectively learned with the guidance of victim models. This group includes two tasks: data-to-text and structured text generation.
- High fidelity but low performance-up (HFLP). The initial local model already achieves a comparable performance to the victim model. QAs and summarization are in this group.
- Low fidelity but high performance-up (LFHP). While MEAs significantly improve the local model's performance, gaps between the local and victim models remain difficult to bridge with domain-specific extraction alone. Machine translation is a representative task whose reasons are explained in Section 5.2.

# 486 5.3 RESISTANCE TO WATERMARKS

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Current LLM watermarking methods have been shown (Kirchenbauer et al., 2023) to be robust against 489 commonly used erasing strategies (e.g., rephrasing), making watermark removal a distinct challenge. 490 In this section, we validate the inherent resistance of LoRD to watermarks, suggesting that LoRD is 491 preliminarily resistant to text watermarking. As described in Section 4, we highlight that LoRD can 492 extract the victim models' knowledge with two terms: the straightforward likelihood learning term 493  $\log P_{\theta_t}(\mathbf{y}_{t-1}|\mathbf{x}) - \log P_{\theta_t}(\mathbf{y}_{t-1}^-|\mathbf{x})$  and the exploration term  $\log P_{\theta_t}(\mathbf{y}_{t-1}^+|\mathbf{x}) - \log P_{\theta_t}(\mathbf{y}_{t-1}^-|\mathbf{x})$ , where 494 we can tune the hyperparameter  $\lambda_1$  as shown in  $\mathcal{L}$  to trade off the exploration and the convergence 495 speed. Typically, a lower  $\lambda_1$  encourages the model for conducting a slower but more diverse and 496 localized exploration from its own generated text  $\mathbf{y}_{t-1}^+$ , potentially enhancing watermark resistance. 497 In this subsection, we evaluate this analysis empirically.

498 Watermarking Details. Unlike previous exper-499 imental settings in Section 5, here we cannot 500 utilize commercial LLMs as victim models due 501 to the inability to control token sampling inside 502 LLMs. Instead, we employ Llama3-70B as the victim model and watermark its outputs based on "green" tokens selection. Following prior re-504 search (Kirchenbauer et al., 2023), we separate 505 the predicted vocabulary into a green word set 506 and a *red word set*, assigning them randomly 507 with the seed derived from the hash of generated tokens at the last generation step. Subsequently, 509 we sample the next token exclusively from the 510 green set, determined by a certain probability. 511

In this way, given the hypothesis  $H_0$  that *texts* are generated without the knowledge of the green word set, we can estimate the probability  $H_0$  occurs (*P*-value) and the *Z*-score of it for these texts. A high P-value, among with a low



Figure 6: Comparison of watermarks resistance.

Z-score, indicates stronger watermark resistance for MEA algorithms.

**Result Analysis.** As depicted in Figure 6, we evaluate the watermark resistance for both MLE and 518 LoRD, and demonstrate how LoRD's performance varies with different values of  $\lambda_1$ . The Z-score 519 of LoRD witnesses a consistent increase as  $\lambda 1$  arises, indicating that the "confidence" in rejecting 520 the hypothesis, i.e., the risk to be suspected, arises when  $\lambda_1$  increases. This finding coincides with 521 the analysis in Section 4. However,  $\lambda_1 = 0$  is a *abnormal* point in WMT (de-en), which might be 522 because it disables the regularization term of LoRD's loss function. For tasks the local model does 523 not own enough enough knowledge, it will lead to a significant performance degradation. Besides, 524 we observe that the P-values of LoRD are generally higher than those of MLE when  $\lambda_1$  is below 0.8, indicating that LoRD typically exhibits stronger watermarking resistance than MLE in most situations. It is noteworthy that this enhanced resistance seems not a "tax" of MEAs efficacy, as the Rouge-L (F1) scores of LoRD consistently surpass those of MLE and do not exhibit a significant 527 negative correlation with their P-values. 528

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# 6 CONCLUSION

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In this paper, we have focused on the extraction problem of commercial large language models.
 We proposed LoRD, a practical and realistic extraction algorithm which is consistent with the alignment procedure of large language models. Our analysis proved that LoRD can reduce the query time significantly and mitigate the certification of current watermarks naturally, surpassing existing MEA algorithms' capabilities. Extensive experiments on domain-specific stealing demonstrated the superiority of our method.

#### 540 7 ETHICAL CONSIDERATIONS

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As discussed in Section 1, MEAs are becoming increasingly prevalent in industrial settings and have 543 already been executed, yet there remains a critical gap in understanding which specific tasks are 544 more susceptible and what capabilities are necessary for effective executions. This lack of knowledge exacerbates the challenges faced by LLM maintainers in safeguarding their systems. Our research can 546 contribute to that. Besides, the theoretical problem we address (as shown in Section 4) offers a novel and insightful perspective on the nature of this threat. Based on these two points, we believe the 547 548 benefits of our paper outweigh potential harms, which aligns with the principles of the *Menlo Report* (Bailey et al., 2012) on ethics. Additionally, we have submitted an anonymous version of 549 the paper to the maintainers of the victim models used in our study to assist in improving their model 550 security. 551

552 It is important to acknowledge, however, that the algorithms we propose could inadvertently enhance the efficiency of illicit extraction efforts by adversaries. To mitigate this risk, we have introduced 553 and analyzed two defensive strategies in Appendix 8, assessing both their effectiveness and potential 554 vulnerabilities under adaptive attack scenarios. This ensures a comprehensive approach to bolstering 555 the security of LLMs. 556

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#### 8 POTENTIAL DEFENSES

Query Detection. One approach to effectively prevent the attack of LoRD is by detecting the distribution of query texts. This is because LoRD, similar to current MEA algorithms, makes 561 no improvements to query samples, indicating that it can be detected by analyzing the statistical 562 information of the adversary's queries, such as the number of queries, distribution of query contents, 563 and so on. However, this defense is usually resource-consuming, as it requires the LLM provider to store all query texts of each user. Besides, the potential for false positives could adversely affect the 565 user experience. 566

567 More Powerful Watermarks. While we highlight the watermark resistance of LoRD, watermarking remains one of the most effective solutions to mitigate MEAs. For example, some model-level 568 watermarks, such as backdoor-based watermarking (Jia et al., 2021; Lv et al., 2024), can effectively 569 certify the theft of DNNs. While model-level (e.g. backdoor-based) watermarks on pre-trained 570 models raised increasing concerns recently (Peng et al., 2023a; Gu et al., 2022; Li et al., 2023a), 571 model-level watermarking on LLMs remains preliminary. Besides, this technique might not work 572 when the adversary only steals a subset of knowledge in which no backdoor is embedded. 573

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Figure 7: Comparison of query efficiency between MLE and LoRD on PiQA, where the green *horizontal line* represents the performance of the initialized local model. We increase query times for each method until reaching their bottlenecks. It can be found that the model extracted by LoRD typically performs a higher accuracy than MLE under the same number of queries. At the same time, LoRD reaches bottlenecks significantly earlier, reducing about 87% query cost compared with MLE.

#### SUPPLEMENTAL EXPERIMENTS А

#### A.1 SCALING THE STEALING

In this subsection, we explore essential capac-937 ities to steal domain-specific knowledge from 938 LLMs. We first analyze the influence of query 939 times for the adversary, then compare the effi-940 cacy when utilizing different sizes of the local 941 model, and finally compare the fidelity among 942 different victim and local models. 943

**QUERY TIMES** A.1.1 945

946 We first investigate the influence of query num-947 bers on MEAs. Specifically, we sample query examples randomly from the query dataset, start-948 ing from 4, and incrementally increase it until 949 the performance of the learned model stabilizes. 950 Figure 7 illustrates the stealing efficacy of LoRD 951 and MLE on PiQA. 952

We observe that the scores of MLE and LoRD

consistently increase as the query number rises,

Model/Metric	BLEU-1	BLEU-4	Rouge-L	BERTScore					
	Czech to English with 16 query samples								
Victim Model	0.611	0.604	0.957						
Local Model	0.255	0.105	0.348	0.868					
+MLE	$0.535 \pm 0.01$	$0.245 \pm 0.01$	$0.526 \pm 0.01$	$0.899 \pm 0.00$					
+LoRD	$0.545 \pm 0.01$	$0.249 \pm 0.00$	$0.538 \pm 0.01$	$0.906 \pm 0.00$					
	German to English with 16 query sample								
Victim Model	0.661	0.377	0.652	0.965					
Local Model	0.276	0.130	0.359	0.877					
+MLE	$0.578 \pm 0.02$	$0.302 \pm 0.01$	$0.573 \pm 0.02$	$0.904 \pm 0.01$					
+LoRD	$0.587 \pm 0.00$	$0.308 \pm 0.00$	$0.589 \pm 0.00$	$0.917 \pm 0.00$					
	Finnish to Er	ıglish with 16 qı	ery samples						
Victim Model	0.558	0.252	0.557	0.953					
Local Model	0.242	0.085	0.320	0.866					
+MLE	$0.444 \pm 0.03$	$0.173 \pm 0.02$	$0.449 \pm 0.03$	$0.905 \pm 0.00$					
+LoRD	$0.498 \pm 0.01$	$0.196 \pm 0.00$	$0.485 \pm 0.01$	$0.905 \pm 0.00$					

Table 2: MEA comparison on WMT16 (Bojar et al., 2016) among MLE and our LoRD methods, where we use GPT-3.5-turbo as the victim model, and Llama3-8B (lla, 2024) as the local initial model.

showing that a larger query number can improve 955 stealing efficacy steadily until reaching their empirical upper bounds. Additionally, LoRD typically 956 obtains a higher score than MLE with the same number of queries, and reaches bottlenecks earlier, 957 which can reduce the required query numbers by 87% compared to MLE. Moreover, in Figure 7, 958 the performance of LoRD exhibits a relatively lower standard variance than MLE, indicating a more 959 stable training procedure.

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# A.1.2 SCALES OF LOCAL MODELS

As shown in our threat model (see Appendix D), we assume the adversary is stealing existing 963 commercial LLMs with a small local model. This raises the question of selecting an appropriate 964 interval of the local model's size. To address this concern, we illustrate the correlation between the 965 local model's size and extraction efficacy on two machine translation tasks, Russian-to-English (ru-en) 966 and German-to-English (de-en), as shown in Figure 8. Here, we employ seven OPT models (Zhang 967 et al., 2022) as local models, with parameters ranging from 125 million to 30 billion, to minimize the 968 interruptions of factors other than model size. 969

Figure 8 shows a sharp distinction between two machine translation tasks. In the de-en task, the 970 performance of the local model increases steadily with model size, while this trend is not evident 971 in the ru-en task with model size smaller than 30 billion. Nevertheless, the performance of a



Figure 8: Experiments varying different model parameter scales.

989 30 billion parameter learned local model in ru-en cannot even be comparable to that of a 1.3 billion parameter local model in the de-en task. This phenomenon suggests that for tasks requiring 990 commonsense knowledge, such as machine translation, the local model should at least possess 991 foundational knowledge of the task (e.g., pre-trained on Russian texts) to learn from victim models 992 effectively. Besides, experiments in BERTScore (F1) show that sometimes LoRD may underperform 993 MLE when the local model has fewer than 1 billion parameters, demonstrating that it is challenging 994 to bootstrap LoRD's exploration with a very small local model. By summarizing the increase in 995 LoRD's curves, a model with 2.7 billion appears sufficient to steal domain-specific knowledge from 996 commercial LLMs. 997

### A.1.3 FIDELITY UNDER DIFFERENT VICTIM AND LOCAL MODELS

We then evaluate the fidelity of extracting different victim models using various pre-trained local models. Specifically, we select GPT-3.5, GPT-4, and GPT-40 as victim models, and employ five state-of-the-art open-source models, Phi-3 (3.8B), OPT (6.7B), Qwen-2 (7B), Mistral-V3 (7B), and Llama-3 (8B), as local models, as shown in Figure 9.

Horizontally, while GPT-4 exhibits a consis-1008 tently lower extracted fidelity compared to the 1009 other two victim models, vulnerabilities of the 1010 three victim models are generally similar. Ver-1011 tically, fidelity of different local models can be 1012 significantly impacted by their performance. For 1013 instance, OPT (6.7B) shows a noticeably lower 1014 score compared to the other four models, which 1015 indicates that the initial performance of the lo-1016 cal model will affect the performance of MEAs. 1017 Besides, Phi-3 (3.8B) achieves a comparable fi-



Figure 9: Fidelity of extracted models with different victim models (GPT-3.5-turbo, GPT-4, and GPT-4o) and different local models (Phi-3, OPT, Qwen2, MistralV3, and Llama3).

delity to larger models like Llama-3 (8B), demonstrating that the size of a local model does not
 influence final fidelity in domain-specific stealing after 2.7 billion, which corroborates the observation
 in Appendix A.1.2.

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### 022 A.2 VISUALIZATION OF DISTRIBUTIONS

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We also investigate the *probability distributions* in the generation procedure among different extraction methods. Specifically, we visualize these distributions for four models, the victim model (GPT-3.5-turbo), the initial local model (Ilama3-8B), and the learned local models with MLE and LoRD.

1026			DiaSafety					SafeRLHF			
1027	Model	Toxicity	Insult	Profanity	Severe Toxity	Threat	Toxicity	Insult	Profanity	Severe Toxity	Threat
	Llama3-8B (initial)	1420	7.94	8.35	1.58	2.29	7.92	2.71	2.80	0.30	1.49
1028	+MLE	8.31	3.69	4.31	0.83	1.50	4.87	1.98	1.66	0.16	1.02
1029	+LoRD	6.45	2.81	3.56	0.71	1.34	3.55	1.15	2.84	0.38	0.79

Table 4: Comparison on safety alignment extraction tasks.

As plotted in Figure 10, each row in the subfigures refers to the distribution when generating the *i*-th token, with each column element indicating the *probability* predicted for the corresponding token index. We limit the visualization to no more than five token probabilities as currently only GPT-3.5-turbo provides the token prediction probabilities during generation, with a maximum of 5 candidate tokens (ope).

From Figure 10, we can see that both MLE 1039 and LoRD successfully redistribute the genera-1040 tion of the initial local model into a distribution 1041 similar to the victim model's, where probabil-1042 ities, especially Top-1 tokens, have been well 1043 inherited in the extraction. This phenomenon 1044 supports our analysis in Proposition 2. How-1045 ever, distributions of MLE extracted models are 1046 consistently sharper than LoRD's, which aligns with our analysis in Section 4.2, where we claim 1047 that MLE leads local models to overfit to the 1048 preferred sentences (i.e., Top-1 tokens), namely 1049 PO, and thus to disrupt the original distributions, 1050 leveraging unusual low probabilities for other 1051 token indexes. The reason why LoRD can be 1052 resistant watermarks, i.e., tokens in Top-1, can 1053 also be derived from this discovery. 1054

To compare MLE and LoRD accurately, we quantize the *entropy* of these distributions, and compute the *KL divergence*  $(\mathbb{D}_{KL})$ , and the



Figure 10: Token generation distributions of four models, namely the victim model, the (initial) local model, and the local model learned through LoRD and MLE, respectively. We visualize their logarithmic probability on examples sampled from the train set and test set, where a deeper color indicates a higher probability.

Spearman Correlation (Spear. Corr.) with respect to the victim and initial local model. As shown in Table 3, while the MLE extracted model exhibits a lower KL divergence (i.e., high distribution similarity) with the victim model than LoRD's on the training dataset, its KL divergence becomes comparable to LoRD's on the test set. Meanwhile, its Spearman correlation significantly decreases from 0.78 to 0.27, which shows that MLE cannot effectively imitate prediction behaviors of the victim model when encountering data beyond the training dataset.

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# 1064 A.3 STEALING SAFETY ALIGNMENTS

1066 Besides of the domain-specific model extraction, we also propose the safety alignment extrac-1067 tion. Specifically, we select two popular safety 1068 alignment datasets for the experiments, namely 1069 SafeRLHF (Ji et al., 2024) and DiaSafety (Sun 1070 et al., 2022), to assess the safety of the gener-1071 ated responses. We employed PerspectiveAPI<sup>2</sup> 1072 to automatically evaluate the safety of the re-1073 sponses. We select five key aspects of safety 1074 probabilities: Toxicity, Insult, Profanity, Severe 1075 Toxicity, and Threat. In these categories, a lower 1076 score indicates better safety performance. For

Models/Matrice	Entropy	To V	ictim Model	To Initial Local Model			
wouciswiences	Enuopy	$\mathbb{D}_{KL}\downarrow$	Spear. Corr.↑	$\mathbb{D}_{KL}$	Spear. Corr.		
On training dataset							
Initial Local Model	0.395	0.503	0.620	-	-		
+ LoRD	0.209	0.051	0.880	0.169	0.680		
+ MLE	0.271	0.029	0.780	0.051	0.540		
		On the tes	st dataset				
Initial Local Model	0.269	0.471	0.680	-	-		
+ LoRD	0.122	0.033	0.640	0.046	0.720		
+MLE	0.275	0.032	0.274	0.001	0.740		

Table 3: Quantization analysis on distributions. A low KL divergence or a high Spearman correlation indicates a high similarity.

the LoRD model, we have retained the same hyper-parameters as those used in our domain-specificexperiments to ensure consistency.

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<sup>&</sup>lt;sup>2</sup>https://perspectiveapi.com/



Figure 11: Comparison of learned joint *prediction distributions* among the victim model (a), local models are learned with MLE (b) and LoRD (c). Simply obtaining the tokens from the victim model (solid black squares), MLE may only memorize specific responses and build a complicated decision surface, resulting in *preference overfitting*. In contrast, LoRD further explores the candidate generation paths (dashed arrows and squares) under the guidance of the victim's generation, which is expected to better approximate the victim model in terms of generalization ability, especially under a limited query budget.

As shown in Table 4, we can see that both MLE and LoRD significantly reduce the harmful information after the stealing procedure. However, LoRD consistantly outperforms MLE on most of the indicators, suggesting that it can achieve better performance in the alignment task.

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1104 B PROOFS

## 1106 B.1 PROOFS OF PROPOSITION 1

As we described in Section 2, both existing methods and LoRD are learned from the victim model's response  $\mathbf{y}_{vic}$  and the corresponding probability distribution  $P_{\theta_{vic}}(\cdot|\mathbf{x}) \in \mathbb{R}^V$ , where V denotes the vocabulary size. Therefore, we first investigate how the local model is learned to emulate the distribution of the victim model,  $P_{\theta_{vic}}(\cdot|\mathbf{x})$ , under the following three stealing strategies.

Expected Distribution of MLE. We can first reshape the MLE loss into a special formation ofKullback-Leibler divergence with labels of one-hot distributions, that is,

$$\mathcal{L}_{ce} = -\sum_{\mathbf{x}, \mathbf{y} \sim \mathcal{D}_{tr}} \log P_{\theta}(\mathbf{y}_{vic} | \mathbf{x}) = \sum_{\mathbf{x}, \mathbf{y} \sim \mathcal{D}_{tr}} \sum_{j}^{N} \mathbb{D}_{KL}[\mathbf{1}_{y_{vic,j}} || P_{\theta}(\cdot | \mathbf{x}, \mathbf{y}_{vic, < j})], \quad (13)$$

where  $\mathbf{1}_{y_{vic,j}}$  is a one-hot vector in which only  $\mathbf{1}_{y_{vic,j}}[y_{vic,j}] = 1$  and all the other elements are 0. Equation 13 demonstrates that MLE learns to maximize the probability of  $\mathbf{y}_{vic,j}$ , without explicit constraints on probabilities across other dimensions.

**Expected Distribution of KD.** Following a previous work (Hinton et al., 2015), the objective function of KD is 1122 of KD is 1122 C D (H) (H)

$$\mathcal{L}_{kd} = \mathbb{D}_{KL}[P_{\theta_{vic}}(\cdot|\mathbf{x})||P_{\theta}(\cdot|\mathbf{x})] + T^2 \cdot \mathbb{D}_{KL}[\mathrm{SM}(P_{\theta_{vic}}(\cdot|\mathbf{x})/T)||\mathrm{SM}(P_{\theta}(\cdot|\mathbf{x})/T)], \quad (14)$$

where SM(·) represents the *softmax function*, and T > 1 denotes the temperature to smooth the targeted distribution  $P_{\theta_{vic}}(\cdot|\mathbf{x})$  in both the original and the smoothed probability across all dimensions, which is exceptionally comprehensive among these methods.

**Expected Distribution of Alignments.** Replacing Equation 6 with Equation 5, we can merge the optimization target of LLMs' alignments as

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$$\min_{\theta*} - \sum_{(\mathbf{x}, \mathbf{y}^+, \mathbf{y}^-) \sim \mathcal{D}^{pref}} \sigma \left( \log \frac{P_{\theta*}(\mathbf{y}^+ | \mathbf{x}) / P_{\theta*}(\mathbf{y}^- | \mathbf{x})}{P_{\theta_{init}}(\mathbf{y}^+ | \mathbf{x}) / P_{\theta_{init}}(\mathbf{y}^- | \mathbf{x})} \right)$$
(15)

1132  

$$\Rightarrow \max_{\theta*} \sum_{(\mathbf{x}, \mathbf{y}^+, \mathbf{y}^-) \sim \mathcal{D}^{pref}} \log P_{\theta*}(\mathbf{y}^+ | \mathbf{x}) - \log P_{\theta*}(\mathbf{y}^- | \mathbf{x}),$$

where  $\theta$  and the expected parameters of the models as

$$P_{\theta*}(\mathbf{y}|\mathbf{x}) = \frac{1}{Z(x)} P_{\theta_{init}}(\mathbf{y}|\mathbf{x}) \cdot e^{\frac{1}{\beta}R_{\phi}(\mathbf{x},\mathbf{y})}.$$
(16)

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1139 We provide a detailed derivation for Equation 16 in Appendix B.2. By replacing Equation 15 with 1140 Equation 16, the expected distribution can be represented as  $\mathbf{r}_{i,j} \cdot P_{\theta_{init}}(\cdot|\mathbf{x})$ , in which  $\mathbf{r}_{i,j}$  indicates the 1141 wrapped distribution gain. This distortion aims to maximize the ratio  $P_{\theta}(y_j^+|\mathbf{x}, \mathbf{y}_{<j}^+)/P_{\theta}(y_j^-|\mathbf{x}, \mathbf{y}_{<j}^-)$ , 1142 and leave the probabilities in other dimensions unconstrained directly.

1143 **Expected Distribution of LoRD.** Similar to alignments, the expected converging procedure by the ob-1144 jective function  $\mathcal{L}_{obj}$  is also intended to maximize the ratio between positive samples and negative sam-1145 ples, i.e.,  $P_{\theta_t}(\mathbf{y}_{t-1}^+|\mathbf{x})/P_{\theta_t}(\mathbf{y}_{t-1}^-|\mathbf{x})$ . Meanwhile, the regularization term  $P_{\theta_t}(\mathbf{y}_{vic}|\mathbf{x})/P_{\theta_t}(\mathbf{y}_{t-1}^-|\mathbf{x})$ 1146 will guide the models to maximize the ratio between  $\mathbf{y}_{vic}$  and  $\mathbf{y}_{t-1}^-$ . As the "standard response" to 1147 be learned,  $\mathbf{y}_{vic}$  can be viewed sufficiently as a positive example. Therefore, we can derive that 1148 the optimization target of LoRD is consistent with RLHF's optimization, i.e., both encourage local 1149 models to maximize the probability proportion between positive and negative samples.

Similar to Equation 16 in which the optimized model can be seen as the distortion of the original model  $P_{\theta_{init}}$ , in LoRD the optimized model can be regarded as the distortion of the local model  $P_{\theta_0}$ , with  $P_{\theta_t}(\cdot|\mathbf{x}) = \mathbf{r}_{i,j}^t P_{\theta_{t-1}}(\cdot|\mathbf{x})$  at each step t, where the distortion term  $\mathbf{r}_{i,j}^t$  is intended to jointly maximize  $P_{\theta_t}(\mathbf{y}_{t-1}^+|\mathbf{x})/P_{\theta_t}(\mathbf{y}_{t-1}^-|\mathbf{x})$  and  $P_{\theta_t}(\mathbf{y}_{vic}|\mathbf{x})/P_{\theta_t}(\mathbf{y}_{t-1}^-|\mathbf{x})$ , while leaving the probabilities in other dimensions unconstrained directly.

# 1156B.2The Deduction of Equation 16 in Proposition 1

From Equation 6, we can get that

$$\max_{\theta} \sum_{\mathbf{x} \sim \mathcal{D}_{q}} R_{\theta_{\phi}}(\mathbf{x}, \hat{\mathbf{y}}) - \beta \mathbb{D}_{KL}[P_{\theta}(\mathbf{y}|\mathbf{x})||P_{\theta_{init}}(\mathbf{y}|\mathbf{x})]$$

$$\max_{\theta} \sum_{\mathbf{x} \sim \mathcal{D}_{q}} \sum_{\mathbf{y} \sim P_{\theta}(\cdot|\mathbf{x})} R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y}) - \beta[\log P_{\theta}(\mathbf{y}|\mathbf{x}) - \log P_{\theta_{init}}(\mathbf{y}|\mathbf{x})]$$

$$\approx \max_{\theta} \sum_{\mathbf{x} \sim \mathcal{D}_{q}} \sum_{\mathbf{y} \sim P_{\theta}(\cdot|\mathbf{x})} \frac{1}{\beta} R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y}) + \log \frac{P_{\theta}(\mathbf{y}|\mathbf{x})}{P_{\theta_{init}}(\mathbf{y}|\mathbf{x})}$$

$$\approx \min_{\theta} \sum_{\mathbf{x} \sim \mathcal{D}_{q}} \sum_{\mathbf{y} \sim P_{\theta}(\cdot|\mathbf{x})} -\log(\exp(\frac{1}{\beta} R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y}))) + \log \frac{P_{\theta}(\mathbf{y}|\mathbf{x})}{P_{\theta_{init}}(\mathbf{y}|\mathbf{x})}$$

$$\approx \min_{\theta} \sum_{\mathbf{x} \sim \mathcal{D}_{q}} \sum_{\mathbf{y} \sim P_{\theta}(\cdot|\mathbf{x})} \log \frac{P_{\theta}(\mathbf{y}|\mathbf{x})}{\exp(\frac{1}{\beta} R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y})) \cdot P_{\theta_{init}}(\mathbf{y}|\mathbf{x})}.$$

$$\approx \min_{\theta} \sum_{\mathbf{x} \sim \mathcal{D}_{q}} \sum_{\mathbf{y} \sim P_{\theta}(\cdot|\mathbf{x})} \log \frac{P_{\theta}(\mathbf{y}|\mathbf{x})}{\exp(\frac{1}{\beta} R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y})) \cdot P_{\theta_{init}}(\mathbf{y}|\mathbf{x})}.$$

1173 If we define a partition function  $Z(\mathbf{x})$  with the formation of

$$Z(\mathbf{x}) = \sum_{\mathbf{y}} P_{init}(\mathbf{y}|\mathbf{x}) \exp(\frac{1}{\beta} R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y})),$$
(17)

1178 we can reformat the optimization target as

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$$\min_{\theta} \sum_{\mathbf{x} \sim \mathcal{D}_q} \sum_{\mathbf{y} \sim P_{\theta}(\cdot | \mathbf{x})} \log \frac{P_{\theta}(\mathbf{y} | \mathbf{x})}{\exp(\frac{1}{\beta} R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y})) \cdot P_{\theta_{init}}(\mathbf{y} | \mathbf{x})}$$

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$$\Rightarrow \min_{\theta} \sum_{\mathbf{x}} \log \frac{Z(\mathbf{x}) \cdot P_{\theta}(\mathbf{y}|\mathbf{x})}{\exp(\frac{1}{2}R_{\theta}(\mathbf{x},\mathbf{y})) \cdot P_{\theta}(\mathbf{y}|\mathbf{x})}$$

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$$= \min_{\theta} \sum_{\mathbf{x} \sim \mathcal{D}_q} \sum_{\mathbf{y} \sim P_{\theta}(\cdot | \mathbf{x})} \exp(\frac{1}{\beta} R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y})) \cdot P_{\theta_{init}}(\mathbf{y} | \mathbf{x})$$

$$= \log Z(\mathbf{x}).$$

If we mark  $\frac{1}{Z(\mathbf{x})} \exp(\frac{1}{\beta} R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y})) \cdot P_{\theta_{init}}(\mathbf{y} | \mathbf{x})$  as  $P_{\theta^*}(\mathbf{y} | \mathbf{x})$ , then we have

$$\min_{\theta} \sum_{\mathbf{x} \sim \mathcal{D}_{\theta}} \sum_{\mathbf{y} \sim P_{\theta}(\cdot | \mathbf{x})} \log \frac{Z(\mathbf{x}) \cdot P_{\theta}(\mathbf{y} | \mathbf{x})}{\exp(\frac{1}{\beta} R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y})) \cdot P_{\theta_{init}}(\mathbf{y} | \mathbf{x})} - \log Z(\mathbf{x})$$

 $\Rightarrow \min_{\theta} \sum_{\mathbf{x} \sim \mathcal{D}_q} \sum_{\mathbf{y} \sim P_{\theta}(\cdot | \mathbf{x})} \log \frac{P_{\theta}(\mathbf{y} | \mathbf{x})}{P_{\theta*}(\mathbf{y} | \mathbf{x})} - \log Z(\mathbf{x}).$ 

Because  $Z(\mathbf{x})$  is independent to  $\mathbf{y}$ , we can deduct that

$$\min_{\theta} \sum_{\mathbf{x} \sim \mathcal{D}_q} \sum_{\mathbf{y} \sim P_{\theta}(\cdot | \mathbf{x})} \log \frac{P_{\theta}(\mathbf{y} | \mathbf{x})}{P_{\theta*}(\mathbf{y} | \mathbf{x})} - \log Z(\mathbf{x})$$
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$$\Rightarrow \min_{\theta} \sum_{\mathbf{x} \sim \mathcal{D}_q} \left[ \sum_{\mathbf{y} \sim P_{\theta}(\cdot | \mathbf{x})} \log \frac{P_{\theta}(\mathbf{y} | \mathbf{x})}{P_{\theta*}(\mathbf{y} | \mathbf{x})} \right] - \log Z(\mathbf{x})$$

$$1203$$

$$\Rightarrow \min_{\theta} \sum_{\mathbf{x} \sim \mathcal{D}_q} \mathbb{D}_{KL}[P_{\theta}(\mathbf{y}|\mathbf{x})||P_{\theta*}(\mathbf{y}|\mathbf{x})] - \log Z(\mathbf{x}).$$
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As we know that  $Z(\mathbf{x})$  does not contain  $\theta$ , the above optimization target actually minimizes the KL-divergence between the distribution of  $P_{\theta}$  and  $P_{\theta*}$ , demonstrating that  $\theta*$  is the optimal value of  $\theta$  that satisfies 

$$P_{\theta*}(\mathbf{y}|\mathbf{x}) = \frac{1}{Z(\mathbf{x})} \exp(\frac{1}{\beta} R_{\theta_{\phi}}(\mathbf{x}, \mathbf{y})) \cdot P_{\theta_{init}}(\mathbf{y}|\mathbf{x}).$$
(19)

(18)

Based on equation 19, we can see that the optimal distribution of  $\theta$  is built upon  $P_{\theta_{init}}$  with a distortion, as we discussed in Section 4.1. 

#### **B.3** PROOFS OF PROPOSITION 2

**Guarantee of MLE.** From Equation 13 we can obtain that when  $\mathcal{L}_{ce}$  decreases to 0, the KL diver-gence between  $P_{\theta}(\cdot|\mathbf{x})$  and  $P_{\theta_{vic}}(\cdot|\mathbf{x})$  decreases to 0, indicating that  $P_{\theta}(\cdot|\mathbf{x})$  equals to  $P_{\theta_{vic}}(\cdot|\mathbf{x})$ . 

**Guarantee of KD.** As we know,  $\mathbb{D}_{KL}(p,q) \ge 0 \forall p$  and q. Therefore, if  $\mathcal{L}_{kd}$  shown in Equation 14 equals to 0, then both  $\mathbb{D}_{KL}[P_{\theta}(\cdot|\mathbf{x})||P_{\theta_{vic}}(\cdot|\mathbf{x})]$  and  $\mathbb{D}_{KL}[SM(P_{\theta}(\cdot|\mathbf{x})/T)||SM(P_{\theta_{vic}}(\cdot|\mathbf{x})/T)]$ equal to 0. For the latter one, we have 

$$\begin{split} & \mathbb{D}_{KL}[\mathrm{SM}(P_{\theta_{vic}}(\cdot|\mathbf{x})/T)||\mathrm{SM}(P_{\theta}(\cdot|\mathbf{x})/T)] \\ &= \mathbb{E}_{\mathbf{y} \sim P_{\theta_{vic}}(\cdot|\mathbf{x})} \mathbb{E}_{y \in \mathbf{y}} \left[ \log \frac{\exp(P_{\theta}(y|\mathbf{x},\mathbf{y}_p)/T) / \sum_{y' \in \mathbf{y}} \exp(P_{\theta_{vic}}(y|\mathbf{x},\mathbf{y}_p)/T)}{\exp(P_{\theta_{vic}}(y|\mathbf{x},\mathbf{y}_p)/T) / \sum_{y' \in \mathbf{y}} \exp(P_{\theta}(y|\mathbf{x},\mathbf{y}_p)/T)} \right] \\ &= \mathbb{E}_{\mathbf{y} \sim P_{\theta_{vic}}(\cdot|\mathbf{x})} \mathbb{E}_{y \in \mathbf{y}} \left[ \log \frac{\exp((P_{\theta}(y|\mathbf{x},\mathbf{y}_p) - P_{\theta_{vic}}(y|\mathbf{x},\mathbf{y}_p))/T)}{\sum_{y' \in \mathbf{y}} \exp(P_{\theta_{vic}}(y|\mathbf{x},\mathbf{y}_p)/T) / \sum_{y' \in \mathbf{y}} \exp(P_{\theta}(y|\mathbf{x},\mathbf{y}_p)/T)} \right] \end{split}$$

where we can observe that only when  $P_{\theta}(\cdot|\mathbf{x})$  equals to  $P_{\theta_{vic}}(\cdot|\mathbf{x})$  can this term reduce to 0. Inte-grating the analysis of these two terms, we can obtain that  $\mathcal{L}_{kd} = 0$  represents the local model's distribution converge to that of the victim model. 

Guarantee of LoRD. When  $\mathcal{L}$  shown in Equation 11 equals to 0, the proportion of  $P_{\theta_t}(\mathbf{y}_{vic}|\mathbf{x})/P_{\theta_t}(\mathbf{y}_{t-1}^-|\mathbf{x})$  and  $P_{\theta_t}(\mathbf{y}_{t-1}^+|\mathbf{x})/P_{\theta_t}(\mathbf{y}_{t-1}^-|\mathbf{x})$  should limit to  $-\infty$ . As we know that *i*) in a distribution  $\sum P_{\theta_t}(\cdot|\mathbf{x}) = 1$  and *ii*)  $\mathbf{y}_{t-1}^+$  is a dynamic positive response generated at each period, we can deduct that when  $\mathcal{L} = 0$  there must be  $\mathbf{y}_{vic} = \mathbf{y}_{t-1}^+$ , i.e.,  $P_{\theta_t}(\mathbf{y}_{vic}|\mathbf{x}) = P_{\theta_t}(\mathbf{y}_{t-1}^+|\mathbf{x}) = 1$  and  $P_{\theta_t}(\mathbf{y}_{t-1}^-|\mathbf{x}) = 0$ . Note that this is merely a theoretical limit that cannot be reached, because  $\mathbf{y}_{t-1}^-$ will not be sampled if its probability is 0, and  $\mathbf{y}_{t-1}^+$  usually doesn't exhibit a significant distinction to  $\mathbf{y}_{t-1}^{-}$  when sampling.

1242	Datasets\ Models	Links
1243	PIQA	https://huggingface.co/datasets/piqa
1244	TruthfulQA	https://huggingface.co/datasets/truthful_qa
1015	WMT16	https://huggingface.co/datasets/wmt16
1245	E2E NLG	https://huggingface.co/datasets/e2e_nlg
1246	CommonGen	https://huggingface.co/datasets/allenai/common_gen
1247	WikiSQL	https://huggingface.co/datasets/wikisql
1010	Spider	https://huggingface.co/datasets/spider
1248	TLDR	https://huggingface.co/datasets/UCL-DARK/openai-tldr-filtered
1249	SamSUM	https://huggingface.co/datasets/samsum
1250	CNN Daily Mail	https://huggingface.co/datasets/cnn_dailymail
1051	Llama3-8B	https://huggingface.co/meta-llama/Meta-Llama-3-8B-Instruct
1251	Llama3-70B	https://huggingface.co/meta-llama/Meta-Llama-3-70B-Instruct
1252	Phi3-3.8B	https://huggingface.co/microsoft/Phi-3-mini-4k-instruct
1253	OPT-6.7B	https://huggingface.co/facebook/opt-6.7b
1054	Qwen2-7B	https://huggingface.co/Qwen/Qwen2-7B-Instruct
1234	MistralV3-7B	https://huggingface.co/mistralai/Mistral-7B-Instruct-v0.3

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Table 5: Datasets and pre-trained model checkpoints used in the paper.

# 1259 C SUPPLEMENTAL RELATED WORKS

# 1261 C.1 HUMAN-FEEDBACK-FREE ALIGNMENTS

There are several alternatives to the standard RLHF approach. Lee et al. (2023) propose reinforcement learning with AI feedback (RLAIF) as a means to diminish the annotation burden associated with the preference assessments. Besides, there are some approaches, such as direct preference optimization (DPO) (Rafailov et al., 2023), that conceptualize the language model itself as the reward model and thus consolidate Equation 5 and Equation 6 into a unified supervised and preference-based training task. Since they do not change the primary targets (i.e., maximizing rewards) and optimization strategies of LLM's alignments, we only consider the standard formation of alignments for simplicity in our theoretical analysis.

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# 1272 C.2 LANGUAGE MODELS EXTRACTION

Studies to steal language models originated from the natural language understanding (NLU) models, such as BERT(Devlin et al., 2019), and then evolved to generative language models, especially large language models recently.

Krishna et al. (2020) highlights early recognition of model extraction threats in language models. By
constructing text inputs with randomly vocabulary sampling, they successfully extract the weights
from BERT-based APIs. Besides, Rafi et al. (2022) investigate the feasibility of side-channel model
extraction attacks, revealing that by analyzing extra signals from GPU kernels, one could accurately
steal the model architecture and its parameters. Subsequent research (Xu et al., 2022) has thoroughly
investigated the strategy of ensembling victim models to train a competitor model that surpasses its
teachers.

The exploration of generative language model extraction is still in its infant stage, with only a handful 1284 of studies thus far. Wallace et al. (2020) investigate imitation attacks on natural language models. By 1285 designing monolingual query texts and collecting responses, they successfully extract the knowledge 1286 from a simulated machine translation model under the black-box settings. This research exhibits 1287 that slight architectural differences will not influence the extraction between language models. Li 1288 et al. (2023b) also explores the potential risks of stealing the code-generation abilities of LLMs into 1289 smaller downstream models. Unlike previous research (Wallace et al., 2020), this is the first study 1290 that selects LLMs as targets. By collecting large-scale domain-specific samples, they fine-tune a 1291 7-billion local pre-trained model with them and show the similarity between the victim and local models in both performances and adversarial samples. However, these two studies employ the MLE 1293 loss (Equation 3) as the MEA method, neither considering whether MLE is compatible with LLMs's training, especially the alignment procedure shown in Section 2.2, nor addressing optimizations 1294 related to query efficiency and the watermark resistance. Besides, the scope of these studies is limited 1295 to stealing specific knowledge in a few downstream domains. At the same time, most of the critical

1296	Model/Metric	Accuracy	Precision	Recall	F1 Score
1297		PIQA (Bisk et al.	, 2020) with 64 q	uery samples	
1298	Victim Model	0.828	0.828	0.827	0.827
1000	Local Model	0.622	0.638	0.621	0.609
1299	+MLE (baseline)	$0.760 \pm 0.02$	$0.771 \pm 0.01$	$0.760\pm0.02$	$0.757 \pm 0.03$
1300	+KD (gre-box)	$0.759 \pm 0.02$	$0.760 \pm 0.02$	$0.759 \pm 0.02$	$0.759 \pm 0.02$
1301	+LoRD (ours)	$0.785 \pm 0.01$	$0.795 \pm 0.01$	$0.785 \pm 0.01$	$0.783 \pm 0.02$
1202	Tr	uthfulQA (Lin et	al., 2021) with 64	4 query samples	
1502	Victim Model	0.414	0.500	0.207	0.293
1303	Local Model	0.391	0.500	0.195	0.281
1304	+MLE (baseline)	$0.381 \pm 0.17$	$0.500\pm0.00$	$0.190\pm0.09$	$0.266 \pm 0.09$
1305	+KD (gre-box)	$0.463 \pm 0.03$	$0.500\pm0.00$	$0.232\pm0.01$	$0.316\pm0.01$
1306	+LoRD (ours)	$0.408 \pm 0.05$	$0.500\pm0.00$	$0.204 \pm 0.03$	$0.289 \pm 0.03$
1300					

Table 6: MEA comparison on QA tasks among MLE and our LoRD methods, where we use GPT-3.5turbo as the victim model, and Llama3-8B (lla, 2024) as the local initial model.

aspects of LLMs and the required extraction capabilities, such as query numbers and local model scales, remain unresolved.

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# 1314 C.3 TEXT WATERMARKS

1315 In contrast to stealing LLMs, IP protection methods have received considerable attention recently. 1316 By sampling a stealthy but representative "greed word set" on the vocabulary distribution, these 1317 methods (Cong et al., 2022; He et al., 2022; 2021; Kirchenbauer et al., 2023) can remap the generated 1318 words into their synonyms or add the "watermarked" token automatically, and thus effectively certify 1319 the output. Besides, strategies such as integrating embeddings into the representation as the back-1320 door (Peng et al., 2023b) or manipulating the probabilities with crafted sinusoidal noises (Zhao et al., 2022; 2023) are also proposed. However, these approaches often presume more stringent conditions 1321 regarding the victim and the suspected models. This paper will further assess the effectiveness of 1322 LoRD and current MEAs in evading these black-box watermarking strategies. 1323

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# 1325 D A DETAILED THREAT MODEL

Adversary's Objective. The adversary's objective is to steal the targeted knowledge from LLMs.
 Specifically, we select machine translation, reasoning, data-to-text, structured text generation, and
 summarization as the downstream domain-specific tasks. The adversary aims to develop a *query-efficient* MEA algorithm, since the amount of input and generated tokens will be counted as the costs.
 Additionally, the MEA methods are expected to be *watermark-resistant*, i.e., they are highly desired to reduce the risks of exposure to unauthorized stealing.

**Targeted Models.** We select Llama3-70B, GPT-3.5-turbo, and GPT-4o as the victim models in this paper. Unlike previous works that only deployed simulated local victim models (e.g., OPT (Zhang et al., 2022)), our selections aim to expose the stealing threat on realistic AI services. Besides, our target models are specifically constrained to LLMs fine-tuned with alignment methods (e.g., RLHF) since they are not only state-of-the-art solutions now but also more valuable due to their human-based alignments.

Adversary's Capabilities. In accordance with the LLM-based AI service APIs, we identify two attack scenarios: black-box and grey-box attacks. In the black-box scenario, only textual responses the adversary is allowed to obtain. At the same time, all other information, such as the temperature, sampling strategies, and the hidden states of LLMs, are unseen and inaccessible. On the contrary, a grey-box attack allows the adversary to access the generation probabilities distribution of tokens. Notice that both MLE and our LoRD method are under black-box settings, and we only adopt grey-box settings on some particular stealing methods, such as knowledge distillation.

Besides, this paper posits that the adversary usually has worse training conditions than the victims.
Specifically, query times and the scale of the local model available to the adversary are much smaller
than the victims' training datasets and model parameters. This setting has been adopted in previous
LLMs' extraction (Li et al., 2023b). We call it a LaViSH (Large-Victim-Small-Heist) framework, which allows us to estimate the upper bound of MEA risks empirically. For adversaries with more

1350 Task Instruction WMT16 1351 Please translate the sentence from [source language] to English. PiQA & TruthfulQA Please select the correct answer for the "Question" of Users. Question: 1352 [question] Selection 1: [Selection1] Selection 2:[Selection2]. 1353 E2E NLG Please translate the information to a sentence in natural language. CommonGen Please generate a sentence based on the words provided by Users. 1354 WikiSQL& Spider Please return to me the SQL sentence based on the text (i.e., Question) 1355 and the table information (i.e., Table) provided by the User. 1356 TLDR& SamSUM Please \*\*summarize\*\* the content given by the user. Please \*\*summarize\*\* the content given by the user. CNN Daily Mail 1357 1358 Table 7: Instructions used in the different downstream datasets. 1359 1360 substantial resources, they can train more powerful MEA-based LLMs by leveraging MEA algorithms 1361 under our LaViSH settings. 1362 1363 1364 LIMITATIONS AND FUTURE WORKS E 1365 MEAs on Multi-modal Models. While this paper delves into MEAs for large language models, it 1367 acknowledges the oversight of the multi-modal attribution of current commercial models (Anil et al., 1368 2024; Achiam et al., 2024) that integrate various forms of data such as text, images, voice, and so on. 1369 The challenge of extending MEA algorithms to accommodate these models, which requires extra considerations on the unified representation of concepts, remains unexplored. Future work could 1370 focus on developing MEA methodologies sensitive to multi-modal data nuances. 1371 1372 **Capacities beyond LaViSH Settings.** We utilize the LaViSH setting to describe the model capacity 1373 of adversaries in our threat model (see Appendix D). However, sometimes, the adversary might 1374 possess comparable or superior training resources to the victims. Though this paper posits that our 1375 MEA algorithms and theoretical analysis are still compatible with such conditions, we concede that concrete experimental validation and results beyond LaViSH settings are not presented here. 1376 1377 Lower-level Extractions. This study evaluates MEAs at the performance level, i.e., it measures 1378 the extraction effectiveness simply through task performance metrics, or the similarity of learned 1379 distributions to the victim model. This setting is justified, as performance metrics are essential for 1380 evaluating task-related knowledge and the practical application of LLMs. However, it does not 1381 consider the lower-level similarities between the victim and local models. Can we achieve neuronlevel alignments in LLM's MEAs? How does a LaViSH setting hurt LLM's MEAs? Is it compatible 1382 to extract a MoE (Mix-of-the-Expert) (Shazeer et al., 2017) victim model with a dense local model? 1383 These questions are not addressed in this research. 1384 1385 1386 1387 1388 1389 1390 1391 1392 1393 1394 1399 1400 1401 1402

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1404 1405 1406 1407 1408 1409 1410 Algorithm 1 LoRD Algorithm 1411 1: Input: Query dataset  $\mathcal{D}_q$ , local language model  $\theta_{init}$ , interface of the victim model  $P_{\theta_{vic}}(\cdot|\cdot)$ , 1412 train period number  $N_t$ , threshold values  $\tau_1$  and  $\tau_2$ . 1413 2: // Initialization. 1414 3:  $\theta_0 \leftarrow \theta_{init}, \mathcal{D}_{tr} \leftarrow \emptyset, \mathcal{D}_0^+ \leftarrow \emptyset, \mathcal{D}_0^- \leftarrow \emptyset, t \leftarrow 0;$ 1415 4: // Query the victim model. 1416 5: for  $\mathbf{x} \sim \mathcal{D}_q$  do  $\begin{aligned} \mathbf{y}_{vic} &\leftarrow \stackrel{\uparrow}{P}_{\theta_{vic}}(\cdot|\mathbf{x}); \\ \mathcal{D}_{tr} &\leftarrow \mathcal{D}_{tr} \cup \{(\mathbf{x}, \ \mathbf{y}_{vic}, \ P_{\theta_{vic}}(\mathbf{y}_{vic}|\mathbf{x}))\}; \end{aligned}$ 1417 6: 1418 7: 8: **end for** 1419 9: // Train local model. 1420 10: // Initialize the positive and negative datasets. 11:  $\mathcal{D}_0^+ \leftarrow \mathcal{D}_q;$ 1421 1422 12: for  $(\mathbf{x}, \mathbf{y}_{vic}, P_{\theta_{vic}}(\mathbf{y}_{vic} | \mathbf{x})) \sim \mathcal{D}_{tr}$  do 1423  $\mathbf{y}_0^- \sim P_{\theta_t}(\cdot | \mathbf{x});$ 13: 1424  $\mathcal{D}_0^- \leftarrow \mathcal{D}_0^- \cup \{(\mathbf{x}, \mathbf{y}_0^-, P_{\theta_0}(\mathbf{y}_0^- | \mathbf{x}))\};$ 14: 1425 15: end for 1426 16: while  $t < N_t$  do 1427 17:  $t \leftarrow t + 1;$ 1428  $\theta_t \leftarrow \theta_{t-1};$ 18: 1429 19: for  $(\mathbf{x}, \mathbf{y}_{vic}, P_{\theta_{vic}}(\mathbf{y}_{vic} | \mathbf{x})) \sim \mathcal{D}_{tr}$  do 1430 20:  $\mathbf{y}_t^+, \mathbf{y}_t^- \sim P_{\theta_t}(\cdot | \mathbf{x});$ 1431  $\mathcal{D}_t^+ \leftarrow \mathcal{D}_t^+ \cup \{(\mathbf{x}, \mathbf{y}_t^+)\};$ 21:  $\mathcal{D}_t^{-} \leftarrow \mathcal{D}_t^{-} \cup \{(\mathbf{x}, \mathbf{y}_t^{-})\};$ 1432 22: 1433 23: end for 1434 24: // Forward. 25: for  $\mathbf{x}, \mathbf{y}_{vic}, \mathbf{y}_{t-1}^+, \mathbf{y}_{t-1}^- \sim (\mathcal{D}_{tr}, \mathcal{D}_{t-1}^+, \mathcal{D}_{t-1}^-)$  do 1435  $\Delta^+ \leftarrow \log P_{\theta_t}(\mathbf{y}_{t-1}^+|x) - \log P_{\theta_{t-1}}(\mathbf{y}_{t-1}^+|x);$ 1436 26: 1437  $\Delta^{-} \leftarrow \log P_{\theta_t}(\mathbf{y}_{t-1}^{-}|x) - \log P_{\theta_{t-1}}(\mathbf{y}_{t-1}^{-}|x);$ 27: 1438 28: if  $\Delta^+ < \Delta^-$  then 1439 29:  $swap(y_{t-1}^+, y_{t-1}^-);$ 1440 30: swap $(\Delta^+, \Delta^-)$ ; 1441 31: end if if  $P_{\theta_t}(\mathbf{y}_{t-1}^+|x) < \tau_1$  &&  $\Delta^+ < \tau_2$  then 1442 32:  $\mathbf{y}_{t-1}^+ \leftarrow \mathbf{y}_{vic};$ end if 1443 33: 34: 1444 // Compute loss with Equation 10 or 11. 35: 1445  $\mathcal{L} \leftarrow \log[\frac{P_{\theta_t}(\mathbf{y}_{t-1}^{-}|\mathbf{x})}{P_{\theta_t}(\mathbf{y}_{t-1}^{+}|\mathbf{x})}] + clip(\log[\frac{P_{\theta_t}(\mathbf{y}_{t-1}^{-}|\mathbf{x})}{P_{\theta_t}(\mathbf{y}_{t-1}|\mathbf{x})})];$ 1446 36: 1447 37: // Backward. 1448  $\theta_t \leftarrow \text{stepUpdate}(\theta_t, \mathcal{L});$ 38: 1449 39: end for 1450 40: end while 1451 41: return  $\theta_t$ 1452 1453 1454

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