SIRS processes competing on simplicial complexes

epidemic spreading, higher-order interactions, simplicial complexes, competition, SIRS processes

Extended Abstract

Contagion processes occur in diverse contexts, from rumour and knowledge diffusion to epidemics. They do not occur in isolation: competing contagions can contend such that a host supports only one strain at a time. While most models assume pairwise transmission, real systems often involve higher-order group interactions–naturally represented by simplicial complexes [1]–which can generate qualitatively distinct dynamics [2]. Network epidemiological frameworks representing competition and higher-order interactions have focused on SIS (susceptible-infected-susceptible) or SIR (susceptible-infected-recovered) processes, which cannot describe all stages of many diseases, including Zika, Malaria, and Dengue [3].

To address these limitations, we propose a stochastic compartmental model for susceptible-infected-recovered-susceptible (SIRS) processes competing via higher-order interactions on simplicial complexes of size N. Each node occupies one of five states: (1) susceptible; (2/3) infected with epidemic A/B; or (4/5) recovered from and resistant to epidemic A/B. For analytic tractability, we derive both a deterministic $3N$-dimensional Microscopic Markov Chain (MMC) [4] and a three-dimensional mean field (MF) [2] approximation of the stochastic model. We analyse the system's fixed points and, via the next-generation method and eigenvalue analysis, determine stability conditions.

Combining SIRS dynamics with competition and higher order interactions leads to rich phenomena: bistability in which the final steady state depends on initial condition (see Fig. 1); limit cycles whose existence can be highly sensitive to parameter choices (see Fig. 2); and discontinuous phase transitions with critical mass effects in which steady state infection is zero (substantially larger than zero) below (above) a critical infection rate (see Fig. 3). We also compare the approximations in terms of their ability to reproduce the eight classes of dependence of final state on initial condition observed in stochastic simulations. These include: extinction of both epidemics; dominance of one over the other; simultaneous outbreak of both epidemics from any nonzero seeding; bistability; and an interesting case exhibiting three steady states, two stable and one unstable (see inset of Fig. 1). This comparison reveals that MMC captures all eight of these classes, while MF achieves only seven (see Fig. 4). Our work extends contagion modelling by integrating higher-order interactions, competition, and the full SIRS cycle, and illustrates how microscopic approaches like MMC can capture the richness of the resultant dynamics.

References

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[4] Joan T Matamalas, Sergio Gómez, and Alex Arenas. "Abrupt phase transition of epidemic spreading in simplicial complexes". In: *Phys. Rev. Res.* 2.1 (2020), p. 012049.

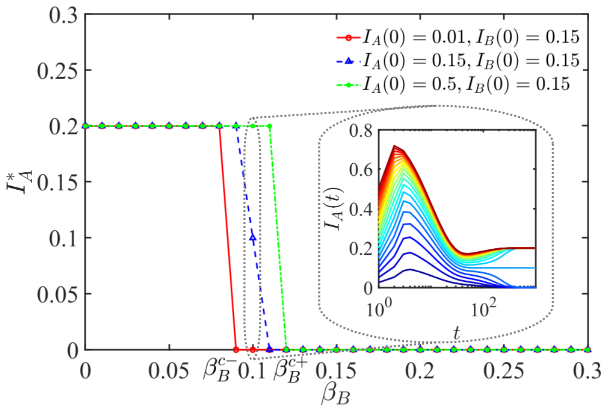

Figure 1: Bistability under MMC. Variation with 1-simplex infection rate β_B of steady state levels of disease. The inset shows the time evolution of epidemic A for different initial conditions when $\beta_B = 0.1$. The range $[\beta_B^{c-}, \beta_B^{c+}]$ is the interval of bistability, in which multiple stable states exist. Different colours represent different initial conditions.

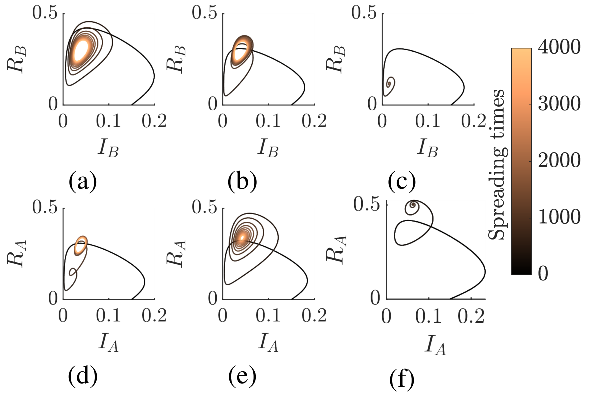

Figure 2: Steady oscillations in epidemic A and B under MMC. $I_A(t)$ ($I_B(t)$) versus $R_A(t)$ ($R_B(t)$) as spreading time t varies. Different colours represent different times. The 1-simplex infection rate β_A takes values (a) 0.0; (b) 0.00599; (c) 0.006; (d) 0.00601; (e) 0.0065; (f) 0.01.

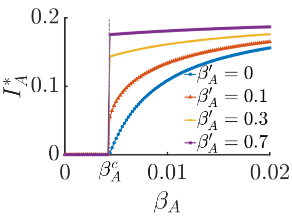

Figure 3: Discontinuous phase transitions and critical mass effects under MMC. Variation with 1-simplex infection rate β_A of steady state level of infection I_A^*. Each coloured line corresponds to a different 2-simplex infection rate β_A'. The gray dash-dotted line presents the critical value β_A^c.

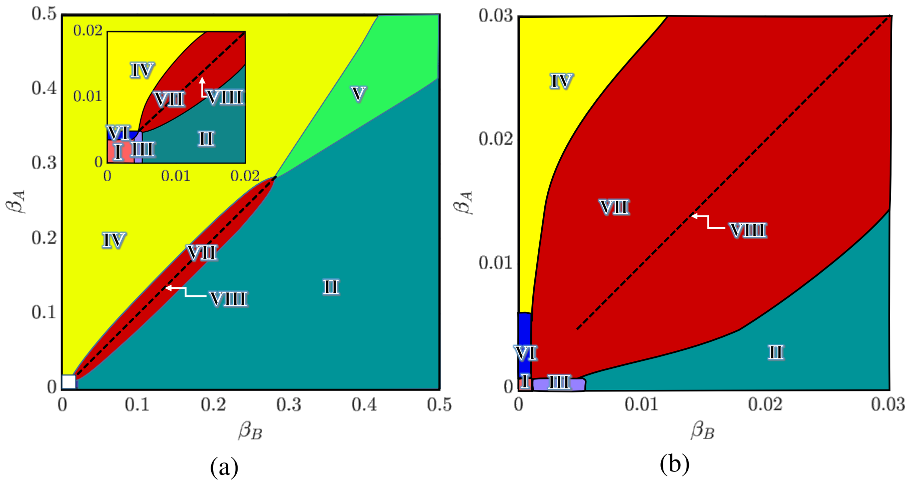

Figure 4: The MMC approach captures more distinct classes of dependence of steady state on initial condition. Phase diagram mapping each choice of 1-simplex infection rates β_A and β_B to one of eight dynamical classes (I-VIII) observed in the stochastic model. (a) Eight cases observed under MMC. The inset shows the seven classes observed for the smallest values of β_A and β_B. (b) Seven classes observed under MF, with Class V, corresponding to coexistence of epidemics A and B, missing.