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# Contrastive Mixture of Posteriors for Counterfactual Inference, Data Integration and Fairness

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## Abstract

1 Learning meaningful representations of data that can address challenges such as  
2 batch effect correction, data integration and counterfactual inference is a central  
3 problem in many domains including computational biology. Adopting a Condi-  
4 tional VAE framework, we identify the mathematical principle that unites these  
5 challenges: learning a representation that is marginally independent of a condition  
6 variable. We therefore propose the Contrastive Mixture of Posteriors (CoMP)  
7 method that uses a novel misalignment penalty to enforce this independence. This  
8 penalty is defined in terms of mixtures of the variational posteriors themselves,  
9 unlike prior work which uses external discrepancy measures such as MMD to  
10 ensure independence in latent space. We show that CoMP has attractive theoretical  
11 properties compared to previous approaches, especially when there is complex  
12 global structure in latent space. We further demonstrate state of the art performance  
13 on a number of real-world problems, including the challenging tasks of aligning  
14 human tumour samples with cancer cell-lines and performing counterfactual infer-  
15 ence on single-cell RNA sequencing data. Incidentally, we find parallels with  
16 the fair representation learning literature, and demonstrate CoMP has competitive  
17 performance in learning fair yet expressive latent representations.

## 18 1 Introduction

19 Large scale datasets describing the molecular properties of cells, tissues and organs in a state of health  
20 and disease are commonplace in computational biology. Referred to collectively as ‘omics data,  
21 thousands of features are measured per sample and, as single-cell methodologies have developed, it  
22 is now typical to measure such features across  $10^5$ – $10^6$  samples [1, 2]. Given these two properties of  
23 ‘omics data, the need for scalable algorithms to learn meaningful low-dimensional representations  
24 that capture the variability of the data has grown. As such, Variational Autoencoders (VAEs) [3, 4]  
25 have become an important tool for solving a range of modelling problems in the biological sciences  
26 [5, 6, 7, 8, 9, 10]. One such problem is utilising representations for counterfactual inference,  
27 e.g. predicting how a certain cell or cell-type, observed only in the control, would have behaved when  
28 exposed to a drug [9, 10, 11]. Another key problem is removing batch effects—spurious shifts in  
29 observations due to differing experimental conditions—from data in order to integrate or compare  
30 multiple datasets [5, 12, 13, 14, 15].

31 We present a formal account of these challenges and show that, to a great extent, they can be seen  
32 as different aspects of a the same underlying problem, namely, that of learning a representation  
33 that is marginally independent of a condition variable (e.g. experimental batch, stimulated vs.  
34 control). Figure 1 [CoMP] illustrates what this looks like in practice: the complete overlap of the cell  
35 populations from different conditions in the latent space. This directly addresses batch correction, and  
36 in the case where we also have a generative model that maps from latent space back to the original

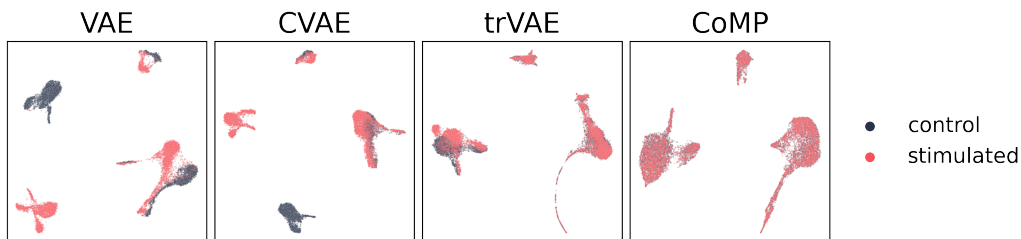


Figure 1: Latent representations of a single-cell gene expression dataset under two conditions (see Section 6.2). From fully disjointed (VAE) to a well-mixed pair of distributions (CoMP).

37 data space, methods that solve this problem can also be applied to counterfactual inference [10].  
 38 This same mathematical requirement for independence also occurs in the fair representation learning  
 39 literature, in which we seek a representation that removes a sensitive attribute, e.g. gender.

40 Neither the VAE nor the conditional VAE (CVAE) [16] are typically successful at learning repre-  
 41 sentations that achieve this desired independence, as shown in Figure 1. Despite the CVAE being  
 42 theoretically able to remove batch effects, there is no constraint that prevents it from separating  
 43 different conditions in latent space. Existing methods use a penalty to encourage the CVAE to learn  
 44 representations that overlap correctly in latent space, with Maximum Mean Discrepancy (MMD) [17]  
 45 being the most common penalty, applied in the VFAE [18] and the more recent trVAE [10]. These  
 46 methods, however, suffer from a number of drawbacks: conceptually, they introduce an extraneous  
 47 discrepancy measure that is not a part of the variational inference framework; practically, they re-  
 48 quire the choice of, and hyperparameter tuning for, an MMD kernel; empirically, whilst trVAE is a  
 49 significant improvement over an unconstrained CVAE, Figure 1 [trVAE] shows that it can still fail to  
 50 exactly align different conditions in latent space.

51 To overcome these difficulties, we introduce *Contrastive Mixture of Posteriors (CoMP)*, a new  
 52 method for learning aligned representations in a CVAE framework. Our method features the novel  
 53 CoMP misalignment penalty that forces the CVAE to remove batch effects. Inspired by contrastive  
 54 learning [19, 20], the penalty encourages representations from different conditions to be close, whilst  
 55 representations from the same condition should be spread out. To achieve this, we approximate the  
 56 requisite marginal distributions using mixtures of the variational posteriors themselves, leading to a  
 57 penalty that does not require an extraneous discrepancy measure or a separately tuned kernel. We  
 58 prove that the CoMP penalty is a stochastic upper bound on a weighted sum of KL divergences, so  
 59 minimising the penalty minimises a well-established statistical divergence measure. We analyse the  
 60 training gradients of the CoMP and MMD penalties, finding key differences that help explain why  
 61 CoMP gradients are generally more stable and better suited to datasets with complex global structure.

62 As shown in Figure 1 [CoMP], our method can achieve visually perfect alignment on a number of  
 63 real-world biological datasets. We apply CoMP to two challenging biological problems: 1) aligning  
 64 gene expression profiles between tumours and their corresponding cell-lines, as tackled in [21] and  
 65 2) estimating the gene expression profile of an unperturbed cell as if it *had* been treated with a  
 66 chemical perturbation (counterfactual inference) [9]. We show that CoMP outperforms existing  
 67 methods, achieving state-of-the-art performance on both tasks. Finally, given the connections to fair  
 68 representation learning, we apply CoMP to the problem of learning a representation that is independent  
 69 of gender in the UCI Adult Income dataset [22], showing that we can learn a representation that is  
 70 fully independent of the protected attribute whilst maintaining useful information for other prediction  
 71 tasks. CoMP represents a conceptually simple and empirically powerful method for learning aligned  
 72 representation, opening the door to answering high-value questions in biology and beyond.

## 73 2 Background

### 74 2.1 Variational Autoencoders and extensions

75 We begin by assuming that we have  $n$  observations  $\mathbf{x}_1, \dots, \mathbf{x}_n$  of an underlying data distribution.  
 76 Variational autoencoders (VAEs) [3, 4] explain the high-dimensional observations  $\mathbf{x}_i$  using low  
 77 dimensional representations  $\mathbf{z}_i$ . The standard VAE places a standard normal prior  $\mathbf{z} \sim p(\mathbf{z})$  on

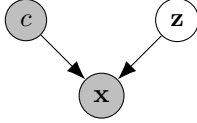


Figure 2: Structural Equation Model for observation  $\mathbf{x}$  under known condition  $c$  with unobserved latent variable  $\mathbf{z}$ . In this model,  $\mathbf{z}$  and  $c$  are independent in the prior.

78 the latent variable, and learns a generative model  $p_\theta(\mathbf{x}|\mathbf{z})$  that reconstructs  $\mathbf{x}$  using  $\mathbf{z}$ , alongside  
 79 an inference network  $q_\phi(\mathbf{z}|\mathbf{x})$  that encodes  $\mathbf{x}$  to  $\mathbf{z}$ . Both  $\theta$  and  $\phi$  are trained jointly by maximising  
 80 the ELBO, a lower bound on marginal likelihood given by  $\log p_\theta(\mathbf{x}) \geq \mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})} [\log p_\theta(\mathbf{x}|\mathbf{z})] -$   
 81  $\text{KL} [q_\phi(\mathbf{z}|\mathbf{x})||p(\mathbf{z})]$ . This can be maximised using stochastic optimisers [23, 3]. Various extensions  
 82 of the VAE have been proposed, such as the  $\beta$ -VAE [24], which scales the KL term of the ELBO  
 83 by a hyperparameter  $\beta$ . Because the isotropic normal prior may limit the expressivity of the model  
 84 [25], various authors have considered alternative priors. For example, [26] proposed the Variational  
 85 Mixture of Posteriors (VaMP) prior, that replaces the isotropic Gaussian with a mixture of posteriors  
 86 from the encoder network itself, evaluated at a number of learned pseudo-inputs.

87 So far, we have assumed that the only data available are the observations  $\mathbf{x}_1, \dots, \mathbf{x}_n$ , but in many  
 88 practical applications we may have additional information such as a condition label for each ob-  
 89 servation. For example, in gene knock-out studies, we have information about which gene was  
 90 targeted for deletion in each cell; in multi-batch experiments we have information about which exper-  
 91 imental batch each samples was collected in. Thus, we augment our data by considering data pairs  
 92  $(\mathbf{x}_1, c_1), \dots, (\mathbf{x}_n, c_n)$  where  $\mathbf{x}$  is the same high-dimensional observation, and  $c$  is a label indicating  
 93 the condition or experimental batch that  $\mathbf{x}$  was collected under.

94 Whilst VAEs are theoretically able to model the pairs  $(\mathbf{x}_i, c_i)$ , it makes sense to build a model that  
 95 explicitly distinguishes between the  $\mathbf{x}$  and  $c$ . The simplest model is the Conditional VAE (CVAE)  
 96 [16]. In this model, a conditional generative model  $p_\theta(\mathbf{x}|\mathbf{z}, c)$  and a conditional inference network  
 97  $q_\phi(\mathbf{z}|\mathbf{x}, c)$  are trained using a modified ELBO. A key observation for our work is that the CVAE has  
 98 many different ways to model the data. For example, it can completely ignore the condition  $c$  in  $p_\theta$   
 99 and  $q_\phi$ , reducing to the original VAE. Assuming that  $\mathbf{x}$  is not independent of  $c$ , this failure mode of  
 100 the CVAE would be apparent on a visualization of the representations. For example, different values  
 101 of  $c$  might be visible as separate latent clusters, as shown in Figure 1 [CVAE].

## 102 2.2 Counterfactual inference

103 If  $(\mathbf{x}_i, c_i)$  represents an RNA transcript and the gene knock-out applied to the cell, a natural question  
 104 to ask is “How would the transcript have differed if a different knock-out  $c'$  had been applied?” In  
 105 general, *counterfactual inference* attempts to answer questions of the form “How would the data have  
 106 changed if  $c_i$  had been replaced by  $c'$ ?” Answering counterfactual questions is a notoriously difficult  
 107 task, because they naturally refer to unobservable data [27]. A principled approach to such questions  
 108 is to adopt the framework of Structural Equation Models [28, 27]. For example, we could assume that  
 109 the data generating process is given as in Figure 2. If this model is correct, counterfactual inference  
 110 in the Pearl framework [27] can then be performed by: 1. *abduction*: inferring the latent  $\mathbf{z}$  from  $\mathbf{x}$   
 111 and  $c$  using  $p(\mathbf{z}|\mathbf{x}, c)$ , 2. *action*: swap  $c$  for  $c'$ , 3. *prediction*: use  $p(\mathbf{x}|\mathbf{z}, c')$  to obtain a predictive  
 112 distribution for the counterfactual. Thus, the counterfactual distribution of  $\mathbf{x}_i$  observed with condition  
 113  $c_i$  but predicted for condition  $c'$  is given by

$$p(\mathbf{x}_{c=c'}|\mathbf{x}_i, c_i) = \int p(\mathbf{z}|\mathbf{x}_i, c_i)p(\mathbf{x}|\mathbf{z}, c') dz. \quad (1)$$

114 In order to make use of this relationship, we must fit a latent variable model [29] such as a CVAE that  
 115 will estimate the encoding distribution  $p(\mathbf{z}|\mathbf{x}_i, c_i)$  and the generative distribution  $p(\mathbf{x}|\mathbf{z}, c')$ .

## 116 3 Unifying counterfactual inference, data integration and fairness

117 We have seen that batch effect correction, data integration and counterfactual inference are central  
 118 problems of interest for the application of latent variable models in computational biology.

119 For counterfactual inference, latent variable models such as the CVAE are increasingly popular  
 120 choices [29]. The failure mode in which different values of  $c$  form separate latent clusters, however,  
 121 can be catastrophic for this application. When this happens, simply switching  $c_i$  to  $c'$  is not correct,  
 122 we have to account for the shift between clusters [9]. Mathematically, the latent space clustering  
 123 phenomenon violates the assumption  $\mathbf{z} \perp\!\!\!\perp c$  that is required by the model in Figure 2. Thus, whilst it  
 124 is not always possible to know when we have found the right causal model [30], we can immediately  
 125 say that a model in which  $\mathbf{z}$  and  $c$  are dependent is not correct.

126 Another key challenge for computational biology is data integration. Suppose our data  
 127  $(\mathbf{x}_1, c_1), \dots, (\mathbf{x}_n, c_n)$  in which  $c_i$  indicates the experimental batch, exhibits *batch effects*—these are  
 128 changes in the observation  $\mathbf{x}_i$  due to the experimental conditions rather than true changes in the  
 129 underlying biology. One approach to dataset integration is to create a representation  $\mathbf{z} = \mathbf{z}(\mathbf{x}, c)$   
 130 that ‘subtracts’ the batch effects. Downstream tasks can then work with  $\mathbf{z}$  in place of  $\mathbf{x}$  without  
 131 learning signal based on misleading batch effects. To know when we have successfully subtracted  
 132 batch effects, we might assume that there are no population-level differences between batches. In  
 133 other words, the marginal distribution of  $\mathbf{z}$  should be the same for each value of the condition  $c$ .

134 Thirdly, this same notion of building a representation that cannot be used to recover  $c$  has been studied  
 135 widely in recent literature on fairness [31, 18, 32, 33]. In particular, if we wish to make a predictive  
 136 rule based on  $\mathbf{x}$  that does not discriminate between individuals in different conditions  $c$ , we can use  
 137 a fair representation  $\mathbf{z}$ , one which cannot be used to recover  $c$ , as an intermediate feature and train  
 138 our model using  $\mathbf{z}$ . Such a representation clearly needs to contain information from  $\mathbf{x}$ , but without  
 139 containing any information that could be used to recover  $c$ .

140 To connect these three notions of ‘alignment in representation space’ we recall the key components  
 141 of the CVAE—the encoder  $q_\phi(\mathbf{z}|\mathbf{x}, c)$  and decoder  $p_\theta(\mathbf{x}|\mathbf{z}, c)$ —and we now drop the  $\theta, \phi$  subscripts  
 142 for conciseness. The marginal distribution of representations within condition  $c \in \mathcal{C}$  is  $q(\mathbf{z}|c) =$   
 143  $\mathbb{E}_{p(\mathbf{x}|c)} [q(\mathbf{z}|\mathbf{x}, c)]$ , and the marginal distribution of over all conditions not equal to  $c$  is denoted

$$q(\mathbf{z}|\neg c) = \frac{\sum_{c' \in \mathcal{C}, c' \neq c} p(c') q(\mathbf{z}|c')}{\sum_{c' \in \mathcal{C}, c' \neq c} p(c')}. \quad (2)$$

144 The following Theorem brings together key notions in counterfactual inference, data integration and  
 145 fair representation learning. See Appendix B for the proof.

146 **Theorem 1.** *The following are equivalent: 1)  $\mathbf{z} \perp\!\!\!\perp c$  under distribution  $q$ , 2) for every  $c, c' \in \mathcal{C}$ ,  
 147  $q(\mathbf{z}|c) = q(\mathbf{z}|c')$ , 3) for every  $c \in \mathcal{C}$ ,  $q(\mathbf{z}|c) = q(\mathbf{z}|\neg c)$ , 4) the mutual information  $I(\mathbf{z}, c) = 0$  under  
 148 distribution  $q$ , 5)  $\mathbf{z}$  cannot predict  $c$  better than random guessing.*

## 149 4 Contrastive Mixture of Posteriors

150 We have seen that counterfactual inference, data integration and fair representation learning can be  
 151 understood through the unified concept of learning a representation such that the latent variable  $\mathbf{z}$  is  
 152 independent of the condition  $c$  under the distribution  $q$ , so that the latent clusters with different values  
 153 of  $c$  are perfectly aligned. Building off the CVAE, which rarely achieves this in practice, a number of  
 154 authors have attempted to use a penalty term to reduce the dependence of  $\mathbf{z}$  upon  $c$  during training.  
 155 The most successful methods, such as trVAE [10], are based on a Maximum Mean Discrepancy  
 156 (MMD) [17]. We discuss this and other common methods in Section 5. Whilst trVAE and related  
 157 methods can work well, they require an MMD kernel, not a part of the original model, to be specified  
 158 and its parameters to be carefully tuned. Experimentally, we observe that MMD-based methods  
 159 can often struggle when there is complex global structure in the latent space. We also analyse the  
 160 gradients of MMD penalties, showing that they have several undesirable properties.

161 We propose a novel method to ensure the conditions of Theorem 1 do hold in a CVAE model. Our  
 162 penalty is based on posterior distributions obtained from the model encoder itself. That is, we do not  
 163 introduce any external discrepancy measure, rather we propose a penalty term that arises naturally  
 164 from the model itself. Taking our inspiration from contrastive learning [19, 20] and the VaMP prior  
 165 [26], we suggest a novel penalty to enforce equation condition 3) of Theorem 1. This equation  
 166 requires the equality of the marginal distribution  $q(\mathbf{z}|c)$  and  $q(\mathbf{z}|\neg c)$  for each  $c \in \mathcal{C}$ . In practice,  
 167 these marginal distributions can be approximated by finite *mixtures*. To encourage greater overlap  
 168 between  $q(\mathbf{z}|c)$  and  $q(\mathbf{z}|\neg c)$ , we can encourage points with the condition  $c$  to be in areas of high  
 169 density under the representation distribution for *other* conditions, i.e. areas in which  $q(\mathbf{z}|\neg c)$  is also

170 high. To encourage this, we can add the penalty term  $\mathcal{P}_0(\mathbf{z}_i, c_i) = -\log q(\mathbf{z}_i | \neg c_i)$  to the objective  
 171 for the data pair  $(\mathbf{x}_i, c_i)$ . When we minimise  $\mathcal{P}_0$ , this brings the representations of samples under  
 172 condition  $c_i$  towards regions of high density under  $q(\mathbf{z} | \neg c)$ . Since the density  $q(\mathbf{z} | \neg c)$  is not known  
 173 in closed form, we approximate  $q(\mathbf{z} | \neg c)$  using other points in the same training batch as  $(\mathbf{x}_i, c_i)$ .  
 174 Indeed, suppose we have a batch  $(\mathbf{x}_1, c_1), \dots, (\mathbf{x}_B, c_B)$ . We let  $I_c$  denote the subset of indices for  
 175 which  $c_j = c$  and  $I_{\neg c}$  denote its complement. We use the approximation

$$\log q(\mathbf{z}_i | \neg c_i) \approx \log \left( \frac{1}{|I_{\neg c_i}|} \sum_{j \in I_{\neg c_i}} q(\mathbf{z}_i | \mathbf{x}_j, c_j) \right) \quad (3)$$

176 and we will show in Theorem 2, this approximation in fact leads to a valid stochastic bound.

177 It may happen that the penalty  $\mathcal{P}_0$  causes points to become too tightly clustered. Indeed, the  
 178 penalty encourages latent variables to gravitate towards high density regions of  $q(\mathbf{z} | \neg c_i)$ . Inspired  
 179 by contrastive learning, we include a second term which promotes higher entropy of the marginal,  
 180 thereby avoiding tight clusters of points. Combined with  $\mathcal{P}_0$ , this leads us to a second penalty  
 181  $\mathcal{P}_1(\mathbf{z}_i, c_i) = \log q(\mathbf{z}_i | c_i) - \log q(\mathbf{z}_i | \neg c_i)$ . Again, the density  $q(\mathbf{z} | c)$  is not known in closed form,  
 182 but we can approximate it using points within the same training batch in a similar fashion to (3).  
 183 Combining both approximations and taking the mean over the batch gives our *Contrastive Mixture of*  
 184 *Posteriors (CoMP) misalignment penalty*

$$\text{CoMP penalty} = \frac{1}{B} \sum_{i=1}^B \log \left( \frac{1}{|I_{c_i}|} \sum_{j \in I_{c_i}} q(\mathbf{z}_i | \mathbf{x}_j, c_i) \right) - \log \left( \frac{1}{|I_{\neg c_i}|} \sum_{j \in I_{\neg c_i}} q(\mathbf{z}_i | \mathbf{x}_j, c_j) \right). \quad (4)$$

185 where  $\mathbf{x}_{1:B}, c_{1:B}, \mathbf{z}_{1:B} \sim \prod_{i=1}^B p(\mathbf{x}_i, c_i) q(\mathbf{z}_i | \mathbf{x}_i, c_i)$  is a random training batch of size  $B$ ,  $I_c$  denotes  
 186 the subset of  $\{1, \dots, B\}$  with condition  $c$  and  $I_{\neg c} = \{1, \dots, B\} \setminus I_c$ . Our method therefore utilises  
 187 a training penalty for CVAE-type models that encourages the conditions of Theorem 1 to hold by  
 188 using mixtures of the variational posteriors themselves to approximate  $q(\mathbf{z} | c)$  and  $q(\mathbf{z} | \neg c)$ . We do  
 189 not introduce an additional kernel or hyperparameter-heavy discrepancy measures.

190 As hinted at by the definition of  $\mathcal{P}_1$ , CoMP can be seen as approximating a symmetrised KL-  
 191 divergence between the distributions  $q(\mathbf{z} | c)$  and  $q(\mathbf{z} | \neg c)$ . In fact, the following theorem shows that  
 192 the CoMP misalignment penalty is a *stochastic upper bound on a weighted sum of KL-divergences*.

193 **Theorem 2.** *The CoMP misalignment penalty satisfies*

$$\begin{aligned} & \mathbb{E}_{\prod_{i=1}^B p(\mathbf{x}_i, c_i) q(\mathbf{z}_i | \mathbf{x}_i, c_i)} \left[ \frac{1}{B} \sum_{i=1}^B \log \left( \frac{1}{|I_{c_i}|} \sum_{j \in I_{c_i}} q(\mathbf{z}_i | \mathbf{x}_j, c_i) \right) - \log \left( \frac{1}{|I_{\neg c_i}|} \sum_{j \in I_{\neg c_i}} q(\mathbf{z}_i | \mathbf{x}_j, c_j) \right) \right] \\ & \geq \sum_{c \in \mathcal{C}} p(c) \text{KL} [q(\mathbf{z} | c) || q(\mathbf{z} | \neg c)] \end{aligned}$$

194 and the bound becomes tight as  $B \rightarrow \infty$ .

195 The proof is presented in Appendix B. Our result reveals that our new penalty directly enforces  
 196 condition 3) of Theorem 1 by reducing the KL divergence between each pair  $q(\mathbf{z} | c), q(\mathbf{z} | \neg c)$  weighted  
 197 by  $p(c)$ . As with standard contrastive learning, our method benefits from larger batch sizes. We  
 198 add the CoMP misalignment penalty to the familiar  $\beta$ -VAE objective to give our *complete training*  
 199 *objective* for a batch of size  $B$  as

$$\mathcal{L}_B^{\text{CoMP}} = \frac{1}{B} \sum_{i=1}^B \left[ \log p(\mathbf{x}_i | \mathbf{z}_i, c_i) + \beta \log \frac{p(\mathbf{z}_i)}{q(\mathbf{z}_i | \mathbf{x}_i, c_i)} - \gamma \log \left( \frac{\frac{1}{|I_c|} \sum_{j \in I_c} q(\mathbf{z}_i | \mathbf{x}_j, c_i)}{\frac{1}{|I_{\neg c}|} \sum_{j \in I_{\neg c}} q(\mathbf{z}_i | \mathbf{x}_j, c_j)} \right) \right] \quad (5)$$

200 with one new hyperparameter  $\gamma$  that controls the strength of the regularisation we apply to enforce  
 201 the requirements  $\mathbf{z} \perp\!\!\!\perp c$ . Theorem 2 shows that, if  $\mathcal{L}_B^\beta$  is the standard  $\beta$ -VAE objective, then we are  
 202 maximising  $\mathbb{E} [\mathcal{L}_B^{\text{CoMP}}] \leq \mathbb{E} [\mathcal{L}_B^\beta] - \gamma \sum_{c \in \mathcal{C}} p(c) \text{KL} [q(\mathbf{z} | c) || q(\mathbf{z} | \neg c)]$ .

#### 203 4.1 Analysing CoMP gradients

204 Before presenting empirical results on the performance of CoMP, we attempt to understand how it  
 205 differs from existing penalties in the literature. Specifically, we compare CoMP with a Gaussian

206 posterior family with MMD using a Radial Basis Kernel [34]. In Appendix C, we show that both  
 207 methods can be interpreted as applying a penalty to each element  $\mathbf{z}_i, c_i$  of the training batch. We  
 208 show further that, under certain conditions, the gradient of the MMD penalty for  $\mathbf{z}_i, c_i$  takes the form

$$\nabla_{\mathbf{z}_i} \mathcal{P}_{\text{MMD}}(\mathbf{z}_i, c_i) = \frac{2}{|I_{c_i}|^2} \sum_{j \in I_{c_i}} e^{-\|\mathbf{z}_i - \mathbf{z}_j\|^2} (\mathbf{z}_j - \mathbf{z}_i) - \frac{4}{|I_{\neg c_i}| |I_{c_i}|} \sum_{j \in I_{\neg c_i}} e^{-\|\mathbf{z}_i - \mathbf{z}_j\|^2} (\mathbf{z}_j - \mathbf{z}_i), \quad (6)$$

209 whilst the CoMP penalty gradient takes the form

$$\nabla_{\mathbf{z}_i} \mathcal{P}_{\text{CoMP}}(\mathbf{z}_i, c_i) = \frac{2 \sum_{j \in I_{c_i}} e^{-\|\mathbf{z}_i - \mu_{\mathbf{z}_j}\|^2} (\mu_{\mathbf{z}_j} - \mathbf{z}_i)}{B \sum_{j \in I_{c_i}} e^{-\|\mathbf{z}_i - \mu_{\mathbf{z}_j}\|^2}} - \frac{2 \sum_{j \in I_{\neg c_i}} e^{-\|\mathbf{z}_i - \mu_{\mathbf{z}_j}\|^2} (\mu_{\mathbf{z}_j} - \mathbf{z}_i)}{B \sum_{j \in I_{\neg c_i}} e^{-\|\mathbf{z}_i - \mu_{\mathbf{z}_j}\|^2}} \quad (7)$$

210 where  $\mu_{\mathbf{z}_j}$  is the variational mean for  $\mathbf{z}_j$ . One important feature of the MMD gradients is that, if  
 211  $\|\mathbf{z}_i - \mathbf{z}_j\|^2$  is large for all  $j \neq i$ , for instance when the point  $\mathbf{z}_i$  is part of an isolated cluster, then  
 212 the gradient to update the representation  $\mathbf{z}_i$  will be small. So if  $\mathbf{z}_i$  is already very isolated from the  
 213 distribution  $q(\mathbf{z} | \neg c_i)$ , then the gradients bringing it closer to points with condition  $\neg c_i$  will be small.  
 214 In comparison to the MMD gradient, it can be seen that gradients for CoMP are *self-normalised*. This  
 215 means that the gradient through  $\mathbf{z}_i$  will be large, even when  $\mathbf{z}_i$  is very far away from any points with  
 216 condition  $\neg c_i$ . This, in turn, suggests that that CoMP is likely to be preferable to MMD when we have  
 217 a number of isolated clusters or interesting global structure in latent space, something which often  
 218 occurs with biological data. The CoMP approach also bears a relationship with nearest-neighbour  
 219 approaches [35]. Indeed, for a Gaussian posterior as  $\sigma \rightarrow 0$ , the  $\neg c_i$  term of the gradient places all  
 220 its weight on the nearest element of the batch under condition  $\neg c_i$ .

## 221 5 Related Work

222 The problem of batch correction in data integration has been addressed using linear [12, 13] and  
 223 nonlinear methods [14, 15] that perform transformations of the original feature space. In both cases,  
 224 the goal is to transform the feature space so that information related to the scientific question of  
 225 interest is retained while dependence on the batch (or nuisance covariate) is reduced. Methods based  
 226 on representation learning learn a low-dimensional representation,  $\mathbf{z} = q(\mathbf{x})$ , which is independent  
 227 of nuisance factors while also being a faithful representation of the original data [18, 5, 36, 10, 37].  
 228 Of these, the work that is most similar to ours are the VFAE [18], in which the authors introduce an  
 229 MMD [17] penalty to encourage the marginal distributions of  $\mathbf{z}$  under different values of  $c$  to be close,  
 230 and the trVAE [10], where the MMD penalty is applied to the output of the first layer of the decoder,  
 231 rather than to  $\mathbf{z}$  directly. Representation learning algorithms for counterfactual inference have been  
 232 shown to benefit from a penalty enforcing distributional similarity between the representations of the  
 233 treated and untreated samples [12]. Elsewhere, authors have applied the variational autoencoder to  
 234 inference on causal graphs [38, 39, 40].

## 235 6 Experiments

236 We perform experiments on three datasets; 1) Tumour / Cell Line: bulk expression profiles of  
 237 tumours and cancer cell-lines across 39 different cancer types; 2) Single-cell PBMCs: single-cell  
 238 gene expression (scRNA-seq) profiles of interferon (IFN)- $\beta$  stimulated and untreated peripheral blood  
 239 mononuclear cells (PBMCs) [41]; 3) UCI Adult Income: personal information relating to education,  
 240 marriage status, ethnicity, self-reported gender of census participants and a binary high / low income  
 241 label (\$50,000 threshold) [22]. All experiments used a 90/10 training/validation split.

242 The two broad objectives across our experiments are 1) to demonstrate the extent to which the two  
 243 random variables  $\mathbf{z}_i$  and  $c_i$  are independent, and 2) to quantify useful information retained in  $\mathbf{z}_i$ . To  
 244 benchmark CoMP on the first objective, we use the following pair of  $k$  nearest-neighbor metrics:  
 245 kBET $_{k,\alpha}$  [42], the metric used to evaluate batch correction methods in biology, and a local Silhouette  
 246 Coefficient [43]  $s_{k,c}$ . In both cases a low value close to zero would indicate good local mixing of  
 247 sample representations. As for the second objective, if we assume the existence of an additional  
 248 discrete label  $d_i$  that represents information one wishes to preserve – in the Tumour / Cell Line  
 249 case,  $d_i$  is the cancer type, while for the PBMC experiment, it refers to cell type – then we calculate  
 250 kBET and  $s$  separately for every fixed- $d_i$  subpopulation and take the mean. We refer to these as the  
 251 *mean Silhouette Coefficient*  $\bar{s}_{k,c}$  and the *mean kBET* metric m-kBET respectively. Full details of the  
 252 datasets and metrics are given in Appendix D.

Table 1: Tumour / Cell Line experiment results, with  $k = 100$ ,  $c = \text{Cell Line}$ , and  $\alpha = 0.01$ .  $s_{k,c}$  and  $\tilde{s}_{k,c}$  are the two Silhouette Coefficient variants (see Section 6). The top scores are in **bold**.

	Accuracy	$s_{k,c}$	$\text{kBET}_{k,\alpha}$	$\tilde{s}_{k,c}$	$\text{m-kBET}_{k,\alpha}$
VAE	0.209	0.658	0.974	0.803	0.581
CVAE	0.328	0.554	0.931	0.684	0.571
VFAE	<b>0.585</b>	0.168	0.258	0.198	0.188
trVAE	<b>0.585</b>	0.096	0.163	0.138	0.123
Celligner	0.578	0.082	0.525	0.568	0.226
<i>CoMP (ours)</i>	0.579	<b>0.023</b>	<b>0.160</b>	<b>0.094</b>	<b>0.101</b>

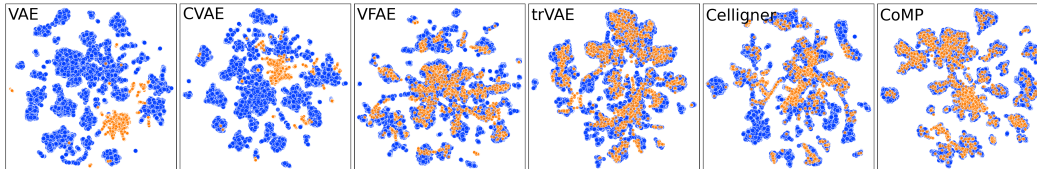


Figure 3: 2D UMAP projection of posterior means of  $\mathbf{z}_i$  from Tumour / Cell Line data. Tumours (blue) and cell lines (orange).

## 253 6.1 Alignment of tumour and cell-line samples

254 Despite their widespread use in pre-clinical cancer studies, cancer cell-lines are known to have  
 255 significantly different gene expression profiles compared to their corresponding tumour samples.  
 256 Here we evaluate the ability of CoMP to factorise out the tumour / cell line condition from its latent  
 257 representations. This can be seen as both a dataset integration and batch effect correction task. In  
 258 addition to the set of  $k$  nearest neighbor-based mixing evaluations, we train a Random Forest model  
 259 on the representations of the tumour samples and their cancer-type labels and assess the prediction  
 260 accuracy on held-out cell lines. To match the results from [21], the evaluations are performed on the  
 261 2D UMAP projections, The results are presented in Table 1.

262 As expected, both the VAE and CVAE baselines fail at the mixing task; the three explicitly penalised  
 263 VAE models and, to a lesser extent, the *Celligner* method have good mixing performances, with  
 264 CoMP outperforming the benchmark models by a significant margin on the silhouette coefficient  
 265 and kBET metric, while successfully maintaining a high accuracy in the cancer-type prediction  
 266 task. We also see from Figure 3 that CoMP representations have the fewest instances of isolated  
 267 tumour-only clusters. Finally, from our evaluation on the  $\tilde{s}$  and m-kBET metrics, we can deduce that  
 268 the occurrence of cell lines of one cancer type erroneously clustering around tumours of a different  
 269 type is less frequent for CoMP compared to the other models. In Appendix D we qualitatively validate  
 270 this for several example clusters.

## 271 6.2 Interventions

272 Obtaining molecular measurements from biological tissues typically requires destructive sampling.  
 273 For example, to obtain scRNA-seq data, each cell is lysed so that the RNA molecules contained  
 274 within it can be extracted and sequenced. This process destroys each cell, meaning that we are  
 275 unable to study the gene expression profile of the same cell over time or under multiple experimental  
 276 conditions. As we discussed in Section 2.2, counterfactual inference can be used to predict how the  
 277 molecular status of a destroyed biological sample would have differed if it were measured under  
 278 different experimental conditions, such as applications of different drugs.

279 To assess CoMP’s utility in counterfactual inference, we trained it on scRNA-seq data from PBMCs  
 280 that were either stimulated with IFN- $\beta$  or left untreated (control) [41]. It is clear from Figure 4 that  
 281 IFN- $\beta$  stimulation causes clear shifts in the latent space between stimulated and control cells from the  
 282 same cell type. Noticeably, the CD14 and CD16 monocyte and dendritic cell (DC) populations see  
 283 greater shifts in their gene expression after stimulation. CVAE fails to align these particular cell types  
 284 in the latent space, while trVAE, VFAE and CoMP perform better. However, stimulated and control

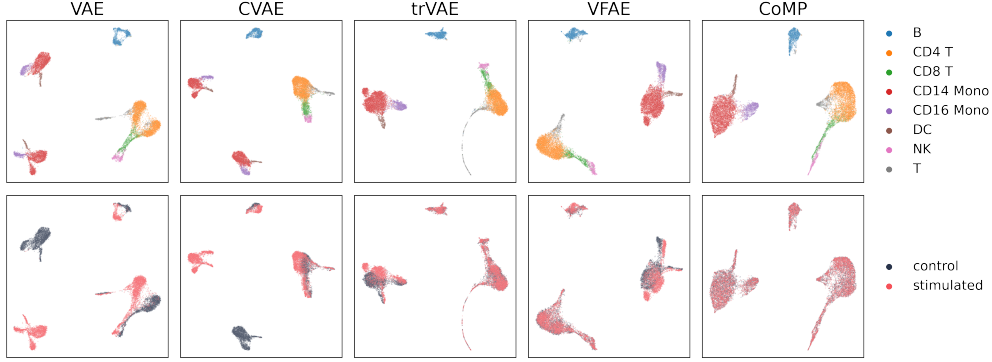


Figure 4: 2D UMAP projections of posterior means of  $\mathbf{z}_i$  derived from stimulated and control PBMC scRNA-seq data. Top row: colours indicate immune cell types, bottom row: colours indicate condition (IFN- $\beta$  stimulation or control).

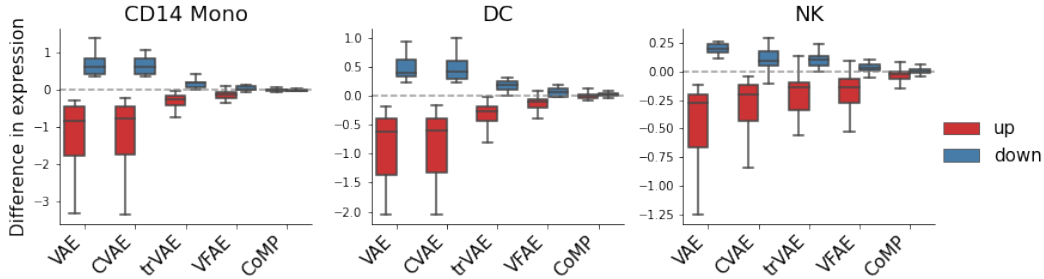


Figure 5: The difference in gene expression values for the top 50 differentially expressed genes (up-regulated: red, down-regulated: blue) between IFN- $\beta$  stimulated cells and counterfactually stimulated control cells for CD14 monocytes, dendritic cells (DC) and natural killer (NK) cells. See Appendix D for further details.

285 cells are better mixed in the latent space derived from CoMP than those from the other models (see  
 286 metrics presented in Appendix D).

287 Next we perform a counterfactual prediction task under a IFN- $\beta$  control-to-stimulation variable  
 288 swap, i.e. the gene expression profiles for control cells were reconstructed through the decoder with  
 289 the condition,  $c \rightarrow$  stimulated. This means we utilise equation (1) with our encoder  $q_\phi(\mathbf{z}|\mathbf{x}, c)$  and  
 290 decoder  $p_\theta(\mathbf{x}|\mathbf{z}, c')$  in place of  $p(\mathbf{z}|\mathbf{x}, c)$  and  $p(\mathbf{x}|\mathbf{z}, c')$ . The degree to which the models respect the  
 291 requirement  $\mathbf{z} \perp\!\!\!\perp c$  will influence the quality of predictions. Figure 5 shows how the profiles of  
 292 (actual) stimulated cells differ from the counterfactual predictions for a selection of cell types (see  
 293 Appendix D for the complete set of results). We see that baseline models tend to systematically  
 294 underestimate the expression of genes up-regulated by stimulation and overestimate those down-  
 295 regulated. CoMP outperforms all other models by accurately predicting the expression alterations  
 296 brought about by stimulation.

### 297 6.3 Fair Classification

298 The goal for this fair classification task is to learn a representation on the Adult Income dataset that is  
 299 not predictive of an individual’s gender whilst still being predictive of their income. We compute a  
 300 baseline by predicting gender and income labels directly from the input data and compare our method  
 301 to the published results for the VFAE [18] and the trVAE. We also include results for a standard  
 302 VAE and CVAE. Unlike in [18], where the representations  $\mathbf{z}$  are sampled from the posterior before  
 303 classification, our experiments used the posterior means to avoid the noise from sampling acting to  
 304 mask the inclusion of predictive information about gender in the encodings.

305 CoMP achieves a gender accuracy that is close to random (67.5%), tying with the VFAE results from  
 306 [18] whilst also remaining competitive with the other methods on income accuracy (Table 2). CoMP



Table 2: UCI Adult Income experiment results with  $k = 1000$ ,  $c = \text{Male}$  for  $s_{k,c}$ , and  $k = 100$ ,  $\alpha = 0.01$  for  $\text{kBET}_{k,\alpha}$ . A lower gender prediction accuracy is better; 0.675 is the lowest achievable.

	Gender Acc.	Income Acc.	$s_{k,c}$	$\text{kBET}_{k,\alpha}$
Original data	0.796	<b>0.849</b>	0.067	0.786
VAE	0.764	0.812	0.054	0.748
CVAE	0.778	0.819	0.054	0.724
VFAE (sampled) [18]	0.680	0.815	-	-
VFAE (mean)	0.789	0.805	0.046	0.571
trVAE	0.698	0.808	0.066	0.731
<i>CoMP (ours)</i>	<b>0.679</b>	0.805	<b>0.011</b>	<b>0.451</b>

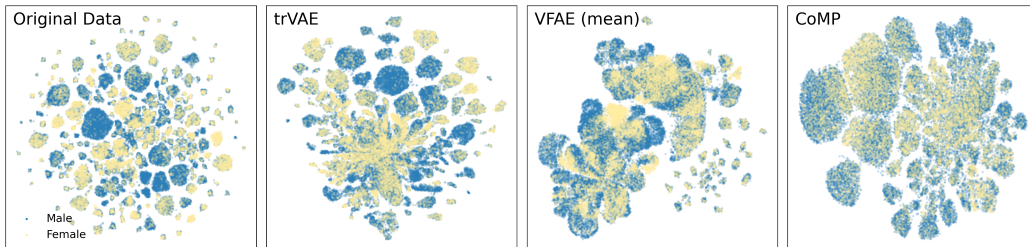


Figure 6: UMAP projections for the UCI Adult Income dataset, coloured by gender. Showing the original data and latents for trVAE, VFAE and CoMP. Male (blue) and female (yellow).

307 also outperforms all methods on the nearest neighbour and silhouette metrics (Table 2). Latent space  
 308 mixing between males and females can be seen qualitatively in the 2D UMAP projection (Figure 6).

## 309 7 Conclusion

310 **Limitations** We presented Contrastive Mixture of Posteriors (CoMP) as an effective means to  
 311 perform batch correction, data integration, counterfactual inference and fair representation learning  
 312 in a CVAE framework. Whilst CoMP covers the majority of common use-cases for these tasks, there  
 313 are several limitations that are avenues of future research. For example, in scRNA-seq analysis, there  
 314 is often the need to integrate more than two datasets together, or to adjust for continuous condition  
 315 variables. Mathematically, CoMP is applicable to any number of discrete conditions, and it would be  
 316 interesting to apply it to a setting with  $> 2$  conditions. Extensions of CoMP could tackle the case  
 317 of a continuous condition variable. Additionally, CoMP requires the condition variable  $c$  to be fully  
 318 observed: future work might attempt to generalise to the partially observed case.

319 **Summary** We identified marginal independence between the representation  $\mathbf{z}$  and condition  $c$  as  
 320 the mathematical thread linking data integration, counterfactual inference and fairness. We proposed  
 321 CoMP, a novel method to enforce this independence requirement in practice. We saw that CoMP  
 322 has several attractive theoretic properties. First, CoMP only uses the variational posteriors, requiring  
 323 no additional discrepancy measures such as MMD. Second, we proved that the CoMP penalty can  
 324 be interpreted as an upper-bound on a weighted sum of KL divergences, connecting it to a well-  
 325 founded divergence measure. Third, we demonstrated that, unlike MMD, CoMP gradients have a  
 326 self-normalising property, allowing one to obtain strong gradients for distant points in a latent space  
 327 with complex global structure. Empirically, we demonstrated CoMP’s performance when applied to  
 328 two biological and one fair representation learning dataset. These biological datasets are of critical  
 329 importance in drug discovery, for example matching cell-lines to tumours for effective pre-clinical  
 330 assay development of anti-cancer compounds. Overall, CoMP has the best in class performance on  
 331 all tasks across a range of metrics that measure either latent space mixing or fairness.

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## 457 Checklist

- 458 1. For all authors...
- 459 (a) Do the main claims made in the abstract and introduction accurately reflect the paper’s  
 460 contributions and scope? [Yes]
- 461 (b) Did you describe the limitations of your work? [Yes] See Section 7
- 462 (c) Did you discuss any potential negative societal impacts of your work? [N/A]
- 463 (d) Have you read the ethics review guidelines and ensured that your paper conforms to  
 464 them? [Yes]
- 465 2. If you are including theoretical results...
- 466 (a) Did you state the full set of assumptions of all theoretical results? [Yes]
- 467 (b) Did you include complete proofs of all theoretical results? [Yes] See Appendix B
- 468 3. If you ran experiments...
- 469 (a) Did you include the code, data, and instructions needed to reproduce the main experi-  
 470 mental results (either in the supplemental material or as a URL)? [Yes] See Appendix  
 471 D
- 472 (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they  
 473 were chosen)? [Yes] See Appendix D
- 474 (c) Did you report error bars (e.g., with respect to the random seed after running experi-  
 475 ments multiple times)? [Yes] See Appendix D
- 476 (d) Did you include the total amount of compute and the type of resources used (e.g., type  
 477 of GPUs, internal cluster, or cloud provider)? [Yes] See Appendix D
- 478 4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
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- 483 (d) Did you discuss whether and how consent was obtained from people whose data you’re  
 484 using/curating? [N/A]
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 486 information or offensive content? [N/A]
- 487 5. If you used crowdsourcing or conducted research with human subjects...
- 488 (a) Did you include the full text of instructions given to participants and screenshots, if  
 489 applicable? [N/A]
- 490 (b) Did you describe any potential participant risks, with links to Institutional Review  
 491 Board (IRB) approvals, if applicable? [N/A]
- 492 (c) Did you include the estimated hourly wage paid to participants and the total amount  
 493 spent on participant compensation? [N/A]