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Critical threshold for synchronizability of high-dimensional Kuramoto oscillators under higher-order interactions

Keywords: Kuramoto model, higher-order interactions, synchronizability, critical slowing down, synchronization

Extended Abstract

Collective synchronization of the Kuramoto model has been extensively studied in diverse scientific disciplines, and since then, it has been generalized in various perspectives. As the Kuramoto model itself is defined on the unit circle as a one-dimensional object, it however would not be suitable to model high-dimensional behaviors appropriately. Moreover, another limitation is that it is restricted to two-body interactions. In this presentation based on [1], we focus on the high-dimensional Kuramoto model particularly together with three-body interactions. For this model, we find a critical threshold for complete synchronizability in terms of interaction strengths. Precisely, by denoting κ_1 and κ_2 for two- and three-body interaction strengths, respectively, we show that if $\kappa_1 + \kappa_2 > 0$, then complete synchronization can emerge, whereas if $\kappa_1 + \kappa_2 < 0$, then complete synchronization cannot occur. For the critical case $\kappa_1 + \kappa_2 = 0$, we show that the emergence of complete synchronization crucially depends on the sign of κ_1 and particularly that critical slowing down is observed at critical transition. It should be mentioned that the line $\kappa_1 + \kappa_2 = 0$ was mentioned in [2] where the readers can easily notice that the line seems critical. Our theoretical results are supplemented by numerical experiments which also provide qualitative insight not captured in the theoretical analysis.

References

[1] Huh, H. and Kim, D., *Critical threshold for synchronizability of high-dimensional Kuramoto oscillators under higher-order interactions.*, *Chaos* **24** (2024) 123119.

[2] Kovalenko, K., Dai, X., Alfaro-Bittner, K., Raigorodskii, A. M., Perc, M. and Boccaletti, S., *Contrarians synchronize beyond the limit of pairwise interactions.*, *Phys. Rev. Lett.* **127** (2021) 258301.

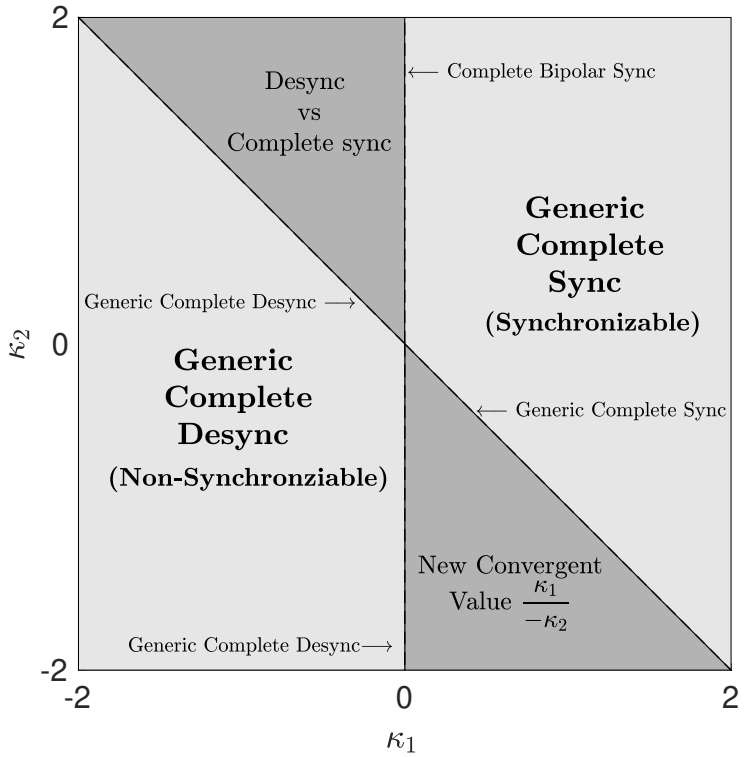

Figure 1: **Main result.** We show that $\kappa_1 + \kappa_2 = 0$ is the critical line for synchronizability for $N \geq 2$. In addition, critical slowing down is observed on the critical line $\kappa_1 + \kappa_2 = 0$ with $\kappa_1 > 0$.