Automatic Generation of In-Context Math Examples Using Multi-Modal **Consistency**

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⁰⁰¹ Abstract

 Large Language Models (LLMs) have ad- vanced Natural Language Processing (NLP) tasks but limited in mathematical reasoning. To address this, few-shot examples are used in prompts for in-context learning. However, existing methods require annotated datasets, re- sulting in higher computational costs and lower quality examples. To mitigate these limitations, we propose APMath, a framework that automat- ically generates high-quality in-context exam- ples to enhance LLMs' mathematical reasoning. APMath ensures consistency across different modalities (e.g., Chain-of-Thought (CoT), code snippets, and equations) by generating and se- lecting mutations that improve response consis- tency. Evaluated on four math problem datasets, APMath outperforms six baselines, with LLM accuracy ranging from 87.0% to 99.3%. It sur- passes the state-of-the-art in-context example retrieval method in three of the four datasets by 1.9% to 4.4%, without relying on an external annotated dataset.

024 1 Introduction

 LLMs have achieved state-of-the-art performances in many NLP applications [\(Min et al.,](#page-9-0) [2024\)](#page-9-0). How- ever, they exhibit limited proficiency in solving [m](#page-9-2)athematical problems [\(Rae et al.,](#page-9-1) [2021;](#page-9-1) [Srivas-](#page-9-2) [tava et al.,](#page-9-2) [2022\)](#page-9-2). This limitation arises due to the fact that math tasks require understanding complex multi-step reasoning to solve the problems. To overcome the deficiency in math-solving capability, in-context learning has been proposed [\(Wei et al.,](#page-10-0) [2022;](#page-10-0) [Zhang et al.,](#page-10-1) [2023\)](#page-10-1). These approaches lever- age few-shot examples, each consisting of math problem and its explanation, embedding the exam- ples into prompts to facilitate learning within the context towards improved performance.

 However, existing in-context learning for math tasks has limitations. The generation of in-context examples requires extensive resources and often de-[p](#page-10-0)ends on large, externally annotated datasets [\(Wei](#page-10-0)

[et al.,](#page-10-0) [2022;](#page-10-0) [Zhang et al.,](#page-10-1) [2023\)](#page-10-1). This process could **043** be labor-intensive, involving manual curation of ex- **044** amples, and computationally expensive, relying on **045** sophisticated retrieval models to find appropriate **046** examples from the external datasets. Furthermore, **047** the scale of the external dataset may be constrained, **048** limiting the search space for identifying suitable **049** math problems and their explanations for the target **050** problem. These limitations hinder the automatic **051** generation of appropriate in-context examples, lim- **052** iting the practicality and scalability of in-context **053** learning for math tasks. 054

To overcome these limitations, we identify two **055** challenges. First, it is crucial to generate math prob- **056** lems relevant to the target problem for in-context **057** learning. The relevant problems for in-context **058** learning provides appropriate reasoning algorithms **059** to solve the target problem, and these algorithms en- **060** sure accurate reasoning explanations. However, the 061 creation of such examples requires substantial costs **062** due to manual curation and extensive search within **063** annotated datasets. Second, the retrieval of high- **064** quality in-context examples is challenging. The **065** existing method evaluates the relevancy of the in- **066** context example with the target problem by measur- **067** [i](#page-10-1)ng the semantic similarity between them [\(Zhang](#page-10-1) **068** [et al.,](#page-10-1) [2023\)](#page-10-1). However, this approach does not al- **069** ways guarantee that the retrieved examples contain **070** comprehensive or high-quality explanations that **071** can assist in solving the target problem. Conse- **072** quently, the quality of the LLM's response to the **073** target problem may depend on the quality of ex- **074** planations provided by the in-context examples. In **075** scenarios where explanations are evaluated solely 076 by manual assessment, automating the evaluation **077** of these explanations remains a challenging. **078**

To address these challenges, we employ a multi- **079** modal technique for the generation and retrieval of **080** in-context examples. Multi-modal learning, which **081** integrates information from diverse sources such as **082** text, images, and videos, has demonstrated poten- **083**

 tial in improving model comprehension. Prior stud- ies have shown that models trained on multi-modal data can attain a deeper understanding of the con- [t](#page-8-0)ent, consistently across different modalities [\(Lin](#page-8-0) [and Parikh,](#page-8-0) [2015;](#page-8-0) [Su et al.,](#page-10-2) [2020;](#page-10-2) [Radford et al.,](#page-9-3) [2021\)](#page-9-3). Similarly, LLMs also possess the ability to produce diverse forms of responses to the same mathematical problem, known as modality, such as generating CoT, composing code snippets, or for- [m](#page-8-1)ulating complex mathematical equations [\(Kojima](#page-8-1) [et al.,](#page-8-1) [2022;](#page-8-1) [Wang et al.,](#page-10-3) [2023b;](#page-10-3) [Imani et al.,](#page-8-2) [2023\)](#page-8-2). The consistency of LLM responses across these modalities can act as an indicator for evaluating the confidence in LLM predictions. Consistency, defined as the degree of agreement among model predictions, has been a focus of prior research as a method for measuring the reliability of responses, leading to accurate answers [\(Wang et al.,](#page-10-3) [2023b;](#page-10-3) [Imani et al.,](#page-8-2) [2023\)](#page-8-2). The convergence of the con- sistency across independent modalities suggests a lower likelihood of systematic bias or errors being present only in a single modality. Consequently, this aids in estimating the accuracy of LLM re- sponses. Accordingly, the key insight of our work lies in leveraging consistency across modalities, combined with few-shot learning techniques, to improve model performance.

 In this work, we present an automated in-context prompting approach for math problem, referred to as APMath, that addresses the above challenges with the aid of mutation and consistency over modalities. APMath operates by initially gener- ating a collection of mutated math problems and 117 their corresponding LLM responses across various prompt modalities. This procedure ensures that the mutation maintains the same reasoning algorithm utilized for solving the target problem, resulting in potentially the most relevant in-context exam- ples, addressing the first challenge. Subsequently, APMath selects a subset of mutated examples that improves consistency of responses across modali- ties for the target math problem. This tackles the second challenge by evaluating LLM responses of mutations through consistency. By doing so, it el- evates the confidence level of the LLM, thereby leading to a correct answer. Our experimental eval- uation shows that APMath produces higher accura- cies on four popular arithmetic reasoning datasets over OpenAI GPT large language models, includ- ing ASDiv (97.1%), SVAMP (87.0%), GSM8k (83.8%), and MultiArith (99.3%). These accuracies outperforms not only the single zero-shot prompt

baselines for all four datasets but also the state-of- **136** the-art in-context example retrieval method in three **137** out of the four datasets by 1.9% to 4.4%, without **138** relying on an external annotated dataset. **139**

2 Motivation **¹⁴⁰**

In this section, we present responses to an arith- **141** metic math problem using in-context examples to **142** motivate the development of APMath. Figure [1](#page-2-0) **143** illustrates an example from the SVAMP, a widely **144** [u](#page-9-4)sed dataset for arithmetic reasoning problems [\(Pa-](#page-9-4) **145** [tel et al.,](#page-9-4) [2021\)](#page-9-4). This demonstrates the diverse **146** responses of an LLM (GPT-3.5) to the same math **147** problem under different settings. At the top, it **148** presents the answers to the math problem across **149** different modalities (CoT, code, and equation) in **150** zero-shot setting, revealing that the LLM responses **151** are inconsistent and the confidence level in the **152** answers is low. For example, CoT response incor- **153** rectly concludes that $\mathscr{I}I7 + \mathscr{I}7 = \mathscr{I}24$ " (step 5), leading to the erroneous answer *"the answer is \$24"* **155** (step 6). In contrast, the bottom of the figure shows **156** in-context examples retrieved from the target prob- **157** lem by altering the numerical values (the yellow **158** box). In this setting, the LLM's answers across **159** all modalities are consistent, resulting in a high **160** confidence level and correct answers. Specifically, **161** the accuracy of the CoT response is attributed to **162** the underlying algorithm, represented by $''x - 65 + 163$ $39 = 67$ ", which is also applied to solve the target 164 problem, $x - 17 + 10 = 7$ " (step 4).

Overall, our study is motivated by the exam- **166** ples presented in Figure [1](#page-2-0) with regard to two key **167** facets. First, altering the numerical values in a **168** math problem does not change the underlying rea- **169** soning algorithm used to solve it. Therefore, us- **170** ing the reasoning algorithm for in-context learn- **171** ing can provide relevant instructions that enable **172** LLMs to accurately solve the problem. This in- **173** sight facilitates the automated generation of math 174 problems that are assumed to operate under the **175** same reasoning algorithm. Second, the quality of **176** LLM responses can vary across different modal- **177** ities. Figure [1](#page-2-0) demonstrates that answers to the **178** math problem can differ across various modalities. **179** This shows the effectiveness of using consistency **180** across modalities to enhance the reliability of LLM **181** responses. It motivates us to leverage the degree **182** of the consistency as a metric for evaluating the **183** confidence level of the responses for the automatic **184** selection of high-quality in-context examples. **185**

Figure 1: Responses of GPT-3.5 to an arithmetic reasoning problem derived from the SVAMP dataset [\(Patel et al.,](#page-9-4) [2021\)](#page-9-4). The top presents the responses across CoT, Python code, and Mathematical Equation in a zero-shot setting. The bottom shows the responses in the CoT modality utilizing in-context examples retrieved from the mutation.

¹⁸⁶ 3 Approach

 We have developed APMath, an automated in- context example generation framework. APMath is structured with two main goals: 1. we employ a mutation technique on the target math problem for the generation of relevant in-context examples, con- sidering these mutations as potential candidates for in-context examples. 2. we incorporate a genetic algorithm that selects mutations by maximizing the consistency of the responses to the target question to ensure the correct reasoning path in the prompt. Figure [2](#page-3-0) presents an overview of APMath. The *Ini- tial Consistency Computation Phase* first collects the preliminary responses of an LLM to the target problem across modalities. Next, the consistency of these responses is evaluated. If this consistency reaches its maximum value (i.e., all the answers are the same), then it returns the answer from the responses as the output. The inputs of this phase include the target problem in text and an LLM. In the scenarios where the consistency does not reach its maximum value, both the target problem and the LLM proceed to the *Target Problem Muta- tion Phase*. This phase mutates the target problem by altering the numerical values in the problem. Furthermore, we use the LLM to process these mu- tated problems and obtain their corresponding re- sponses across modalities. A mutation is accepted if its responses are consistent across the modali-ties. This phase is crucial as it addresses the first

goal, which is to obtain the relevant problems for **216** in-context learning. Additionally, we employ a *Mu-* **217** *tation Selection by Consistency Optimization Phase* **218** to achieve the second goal, which is the retrieval of **219** high-quality in-context examples. In this phase, the **220** LLM responses to the target problem are collected **221** for each mutation, with the mutation prepended as **222** an in-context example. The consistency of these **223** responses is then evaluated. If this consistency **224** reaches its maximum value, the response is used as **225** the output. Otherwise, we further evaluate whether **226** the new consistency score surpasses the previous **227** score without the mutation or if the most consistent **228** answer using the mutation differs from the previ- **229** ous one. If either condition is met, we update the **230** prompts across modalities, the consistency score, **231** and the most consistent answer with the new mu- **232** tation as an in-context example. This process is **233** repeated for all mutations, ultimately yielding the **234** most consistent answer as the output. **235**

3.1 Initial Consistency Computation **236**

Given a target problem and an LLM, we obtain 237 top-K responses for each modality. The answers **238** are then extracted from these LLM responses, and **239** their consistency is evaluated across the modali- **240** ties. Specifically, given the top- K answers for a 241 specific modality mod , ANS_{mod} , and its collec- 242 tion of LLM answers across modalities, ANS = **243** ${ANS_{mod}}$ $|ANS_{mod} = LM (q_{tqt}, p_{mod})$ $\cap p_{mod} \in$ 244 $P_{MOD} \cap mod \in MOD\}$, where LM is an LLM, 245

Figure 2: Overview of APMath.

 q_{tqt} is a target problem, and P_{MOD} is a set of **247** prompt across modalities MOD. We define con-**248** sistency across the modalities in Equation [3:](#page-3-1)

$$
freq\left(a, ANS\right) = \sum_{ans \in ANS} \delta\left(a = ans\right) \tag{1}
$$

250

252

$$
SC(a, ANS) = \frac{freq(a, ANS)}{|ANS|} \tag{2}
$$

$$
C(a, ANS, w) = \sum_{mod \in MOD} w_{mod} \cdot SC(a, ANS_{mod})
$$
\n(3)

 where freq(a, ANS) in Equation [1](#page-3-2) represents the number of occurrences of a specific answer a within the answer set ANS. In Equation [2,](#page-3-3) 257 257 257 257 SC denotes self-consistency score of the a for the ANS [\(Wang et al.,](#page-10-3) [2023b\)](#page-10-3). The consistency score across the MOD, C(a, ANS, w) in Equation [3,](#page-3-1) is the weighted sum of SC for a unique answer a across the MOD. The modality belief weight wmod represents the degree of empirical confidence of a specific modality and is set as a hyperparameter of APMath. An answer with a higher consistency score C for a specific answer indicates a higher level of confidence in the answer across modalities, while a lower score indicates lower confidence. If the consistency score of answer to the target prob- lem fails to reach the maximum value, we proceed to the mutation phase, described in Section [3.2.](#page-3-4)

271 3.2 Target Problem Mutation

 This phase generates a pool of problems that can potentially provide relevant knowledge for LLM to solve the target problem. This is achieved by mutating the target problem, resulting in a set of mutated problems. We operate under the assump- tion that a problem identical to the original one, but with different numerical values, follows the same reasoning path in solving the original problem. The **279** process first identifies the numerical values present **280** in the target problem. These identified values are **281** then randomly mutated; the original values are re- **282** placed with their mutated values, and responses **283** are obtained from the LLM to these mutated prob- **284** lems across modalities. The validity of these muta- **285** tions is verified by estimating the accuracy of the **286** LLM's responses to the mutated problems through **287** response consistency across modalities. A muta- **288** tion is considered acceptable if responses are con- **289** sistent across modalities; otherwise, it is rejected. **290** By repeatedly applying this mutation process to **291** the target problem, this phase generates pairs of **292** mutated problems and their corresponding LLM **293** responses across modalities. **294**

3.3 Mutation Selection by Consistency **295** Optimization **296**

Although the mutation phase is capable of produc- **297** ing numerous mutations, it is important to evaluate **298** the quality of these mutations for their utility as **299** in-context examples to accurately solve the target **300** problem. To address this challenge, we have devel- **301** oped an optimization strategy aimed at improving **302** consistency as defined in the Equation [3.](#page-3-1) **303**

Algorithm [1](#page-4-0) shows how the optimization strat- **304** egy identifies the in-context examples. It takes the **305** LLM, the target problem, prompts across modal- **306** ities, initial answer and consistency score to the **307** target problem measured in the phase descreibed **308** in Section [3.1,](#page-2-1) a pool of mutations and the maxi- **309** mum number of in-context examples to be used as **310** inputs. If the algorithm attains a maximum consis- **311** tency value, which means that all answers are the **312** same across modalities, it returns this answer as 313

Algorithm 1 Consistency optimization algorithm.

1: Input: a large language model LM , target question q_{tgt} , prompts over modalities \widetilde{P}_{MOD} , initial the most consistent answer of the target problem ans₀, consistency score of the ans₀ mc₀, mutations $M_{q_{tgt}}$, maximum number of in-context examples $N_{example}$, belief weights across modalities W

Algorithm 2 get_answer algorithm.

```
1: Input: responses of the target question over modalities R, belief weights
     across modalities W
2: Output: final answer final\_ans, consistency score of the final answer
     final cs
3: unique\_answers = set()4: cs, sc = dict(), dict()5: for each mod from R.modalities do<br>6: Ans_{topk} = extract\_answers6: Ans_{topk} = extract\_answers(R[mod])<br>7: for each ans from unq_ans do
7: for each ans from ung\_ans do<br>8: sc(mod,ans] = self\_con8: scl/mol, ans] = self\_consistency(ans, Ans_{topk})<br>9: unique answers.add(ung ans)
         unique\_answers.add(unq\_ans)10: for each ans from unique_answers do<br>11: cs[ans] = 0.011: cs[ans] = 0.0<br>12: for each mod
12: for each mod from R.modalities do<br>13: cs[ans] \text{ +} \text{W}_{mod} * \text{SC} [mod, a]cs[ans] \coloneqq W_{mod} * SC[mod,ans]14: final\_ans, final\_cs = get\_highest\_score\_answer(cs)15: return final ans, final cs
```
 the final answer (lines [6-](#page-4-1)[7\)](#page-4-2). Otherwise, the algo- rithm incorporates each mutation as an in-context example in prompts (lines [9-](#page-4-3)[18\)](#page-4-4). Subsequently, LLM responses using the mutation across modali- ties are obtained, and the consistency is calculated based on these responses using the $get_answer()$ algorithm (line [18\)](#page-4-4). Algorithm [2](#page-4-5) elucidates the get_answer() function. This function first gathers the self-consistency score for each unique answer across modalities (lines [7-](#page-4-6)[8\)](#page-4-7). Then it computes the consistency score, as defined in Equation [3](#page-3-1) (lines [11-](#page-4-8)[13\)](#page-4-9). Finally, it returns the highest con- sistency score and its corresponding answer. Re- turning to Algorithm [1,](#page-4-0) the algorithm then eval- uates the difference in the consistency scores be- fore and after the addition of the mutation to the modality prompts. Mutations are selected as in-

Table 1: Prompts for large language model over different modals.

Modality	Prompt				
CoT	"{QUESTION} Let's think step by step and end your response with 'the answer is {answer}"'				
Code	"I want you to act like a mathematician. I will type mathematical question and you will respond with a function named with 'func' in python code that returns the answer of the question. the function should have no arguments. I want you to answer only with the final python code and nothing else. Do not write explanations: {QUESTION}"				
Equation	"{QUESTION} Write a wolframalpha mathemati- cal equation with no explanations and no units to the numbers in the equation. Generate the answer format starting with 'Answer $=$ "'				

context examples in prompts if there is an increase **331** in modal-consistency following their addition, or if **332** the LLM generates a different answer than the pre- **333** vious one (line [19\)](#page-4-10). Upon selection of a mutation, **334** prompts across modalities, answer for the target **335** problem, and their respective consistency values **336** are updated to facilitate the search for additional **337** mutations (lines [20-](#page-4-11)[24\)](#page-4-12). This process iterates un- **338** til either the number of selected mutations or the **339** consistency reaches its maximum, or until no muta- **340** tions remain (lines [5,](#page-4-13) [6](#page-4-1) and [10,](#page-4-14) respectively). The fi- **341** nal answer for the target problem, obtained through **342** the iterative optimization process, is returned. **343**

4 Experiments **³⁴⁴**

4.1 Experimental setup 345

Dataset. We assess the performance of APMath **346** on the following widely used public arithmetic **347** reasoning benchmarks: the Math Word Problem **348** Repository MultiArith [\(Roy and Roth,](#page-9-5) [2016\)](#page-9-5), AS- **349** Div [\(Miao et al.,](#page-9-6) [2020\)](#page-9-6), SVAMP [\(Patel et al.,](#page-9-4) [2021\)](#page-9-4), **350** and GSM8k [\(Cobbe et al.,](#page-8-3) [2021\)](#page-8-3), a recently pub- **351** lished benchmark of grade-school-math problems. **352** Large Language Models. We evaluate the AP- **353** Math using GPT-3.5 [\(OpenAI,](#page-9-7) [2023\)](#page-9-7). It is a **354** transformer-based architecture with 175 billion pa- **355** rameters. Specificlly, we utilize the public engine **356** *gpt-3.5-turbo* from the OpenAI models. **357**

Prompts over modalities. Inspired by the prompts **358** presented in [\(Akin,](#page-8-4) [2022\)](#page-8-4), we manually crafted **359** the prompts detailed in Table [1](#page-4-15) over three modal- **360** ities. The first and second columns of Table [1](#page-4-15) **361** represent the modality type and the correspond- **362** ing prompt text, respectively, with the placeholder **363** *"{QUESTION}"* used to represent the input ques- **364** tion. The goal of the prompt design is to segregate **365** the explanation from the corresponding final an- **366** swer, thereby facilitating the automatic parsing of 367

 the answer from the LLM responses. The prompt for the CoT modality generates a reasoning path. The phrase within the prompt, *"Let's think step by step"*, facilitates step-by-step thinking before providing an answer. The instruction *"end your response with 'the answer is {answer}' "* prompts the LLM to conclude its response with the phrase *'the answer is {answer}'*, where *{answer}* repre- sents the ultimate answer to the question. For the code and equation modalities, we obtain the gen- erated executable Python code and WolframAlpha mathematical equation from the LLM with no ad- ditional explanation provided. We then execute the code and equation using the Python command and WolframAlpha API [\(WolframAlpha,](#page-10-4) [2023\)](#page-10-4), respec- tively. Finally, we consider the returned value as the answer for the respective modality.

 Evaluation Metric. We compare accuracy of LLM responses, defined as the ratio of the number of correctly predicted answers to the number of arith-metic math questions in the test datasets.

 Baselines. We evaluate APMath by assessing its accuracy on the datasets compared to baseline methods. It aims to demonstrate APMath's abil- ity to generate relevant examples in zero-shot con- texts. We also show the effectiveness of consistency across modalities by comparing it to APMath's per- formance without this feature. Furthermore, we compare an existing state-of-the-art method for re- trieving in-context examples with APMath to high- light the effectiveness of mutations for in-context examples over those from external datasets.

 • Zero-shot with a specific Modality: This prompt solely uses a specific modality without any in-context examples. In this experiment, we utilize the CoT, Code, and Equation modal- ities, denoted as CoTPrompt, CodePrompt and EqnPrompt, respectively. For each modal- ity, the final answer is determined by selecting the most frequently occurring answers from the top three responses [\(Wang et al.,](#page-10-3) [2023b\)](#page-10-3). The prompt used is identical to the correspond-ing modality prompt in Table [1.](#page-4-15)

- **411** Majority voting of answers across modal-**412** ities (MajVotModals): This method deter-**413** mines the final answer by majority voting of **414** answers across the three modalities.
- **415** APMath w/o consistency over modalities **416** (APMath w/o modalities): For a specific **417** modality, we employ a subset of mutations **418** that improve the self-consistency, as defined in

Equation [2,](#page-3-3) of the LLM's top-K responses to **419** a target problem, utilizing these as in-context **420** examples. Specifically, we extract the top-3 **421** responses of the LLM for the CoT modality. **422**

• In-context example retrieval method (Auto- **423** CoT): AutoCoT [\(Zhang et al.,](#page-10-1) [2023\)](#page-10-1) is im- **424** plemented. It clusters the embedding vec- **425** tors of retrieval examples using Sentence- **426** BERT [\(Reimers and Gurevych,](#page-9-8) [2019\)](#page-9-8) into K 427 clusters. Next, for each clustered examples, **428** the embedding vector of the target question **429** is compared with them, and the closest exam- **430** ple is selected. These K examples are then **431** utilized as in-context examples. In this exper- **432** iment, we construct 8 clusters, providing 8 433 in-context examples for each target question. **434**

Implementation Details and Hardware Environ- **435** ment. We utilized the OpenAI API to run GPT-3.5. **436** We applied temperature with $T = 0.7$ and truncated 437 at the top-3 responses. Due to the limited resources, **438** we generated 20 mutated questions for each orig- **439** inal question and obtained their LLM responses **440** to identify relevant in-context examples. In addi- **441** tion, The modality belief weight for the aggregation **442** across the modalities is set to 0.4 for CoT modal- **443** ity and 0.3 for both code and equation modalities, **444** reflecting the more important role of CoT modal- **445** ity in reasoning and logical flow [\(Wei et al.,](#page-10-0) [2022;](#page-10-0) 446 [Chowdhery et al.,](#page-8-5) [2023\)](#page-8-5). All experiments were con- **447** ducted on a Ubuntu 14.04 server with three Intel **448** Xeon E5-2660 v3 CPUs @2.60GHz, eight Nvidia **449** 1080Ti GPUs, and 500GB of RAM. **450**

4.2 Results **451**

Comparison of APMath with Zero-Shot Base- **452** lines. We first report the experimental results of **453** APMath to demonstrate the effectiveness of AP- **454** Math in the zero-shot setting. Table [2](#page-6-0) shows the **455** accuracies of the LLM on different math problem **456** datasets. The first and second columns denotes **457** the names of the datasets and the number of math **458** problems for each dataset used in the experiment, **459** respectively. Columns 3-6 show the accuracies of **460** LLMs achieved using baseline methods. The last **461** column shows the LLM accuracy attained with AP- **462** Math. The results show that *APMath outperforms* **463** *all the baselines on all four datasets*. APMath **464** achieved better accuracy than the baselines by 1% **465** to 7.3% on ASDiv, 3.8% to 15.4% on SVAMP, 4% **466** to 22.4% on GSM8k, and 1% to 62.1% on Multi- **467** Arith. This result indicates that APMath exhibits **468**

Table 2: Accuracies of GPT-3.5 across different math problem datasets in the zero-shot setting.

				Dataset #Data CoTPrompt [%] CodePrompt [%] EqnPrompt [%] MajVotModals [%] APMath [%]	
		95.0	89.8	96.1	97.1
	ASDiv 1218 95.2 SVAMP 1000 79.5	79.7	71.6	83.2	87.0
GSM8k \parallel 1319	77.5	69.4	61.4	79.8	83.8
MultiArith $\ $ 600	96.0	98.3	37.2	97.0	99.3

Table 3: Accuracies on GPT-3.5 with in-context examples across different math problem datasets.

Dataset	AutoCoT $\lceil \% \rceil$	APMath w/o modalities $\lceil \% \rceil$	APMath $\lceil \% \rceil$
ASDiv	97.4	96.1	97.1
SVAMP	82.6	85.4	87.0
GSM8k	81.4	86.9	83.8
MultiArith	97.2	96.8	99.3

Table 4: Results of manual study to evaluate the correctness of GPT-3.5 responses to mutated math problems. The number of mutations and correct mutations used as in-context examples are denoted as MutUsed and CorrectMutUsed, respectively.

 significant improvements in enhancing accuracy across various mathematical problem datasets in zero-shot setting. It suggests that the additional contextual processing enabled by APMath's muta- tion selection phase is crucial for handling more mathematical queries that may not be as effectively addressed through standard zero-shot methodolo- gies. In Appendix [A.1,](#page-10-5) we provide samples of APMath generated in-context examples for each of the four datasets.

 Effectiveness of Mutation as In-context Exam- ples. The comparison of LLM accuracies using APMath with the other baselines are shown in Table [3.](#page-6-1) The first column lists the names of the mathematical problem datasets. The second and third columns report the accuracies achieved by APMath without consistency over modalities and by the in-context example retrieval method, Au- toCoT, respectively. Comparing to AutoCoT, AP- Math improves accuracy by 4.4%, 2.4%, and 2.1% for the SVAMP, GSM8k, and MultiArith, respec- tively. For the ASDiv dataset, APMath exhibits a slight decrease in accuracy by 0.3%. Overall, *AP- Math's in-context examples is more effective than those produced by current retrieval-based methods in the absence of external retrieval datasets and* **495** *models*.

496 Effectiveness of Combination Across Modalities. **497** Comparing to APMath w/o modalities (third column in Table [3\)](#page-6-1), APMath exhibits a decreased ac- **498** curacy by 3.1% for the GSM8k dataset. Out of the **499** 79 problems correctly answered by APMath but in- **500** correctly by APMath w/o modalities, APMath fails **501** to find in-context examples for 53 problems, while **502** APMath w/o modalities succeeds. For the remain- **503** ing 26 problems, the discrepancies are attributed to **504** the randomness induced by the temperature setting **505** of the LLM. However, APMath improves accuracy **506** by 1%, 1.6%, and 2.5% for the ASDiv, SVAMP, **507** and MultiArith datasets, respectively. This result **508** suggests that the idea of increasing consistency 509 over various modalities is effective. Additionally, **510** we conduct a manual study to assess the consis- **511** tency across modalities as a metric of evaluating **512** the correctness of LLM responses to mutated prob- **513** lems used as in-context examples. The correctness **514** is determined by manually verifying the LLM re- **515** sponses with the expected outcomes derived from **516** the ground truth data, ensuring that the model accu- **517** rately interprets and solves the mutated problems. **518** The results of this manual study are presented in Ta- **519** ble [4.](#page-6-2) The second column shows the number of mu- **520** tations used as in-context examples for target prob- **521** lems in each dataset. The third column shows the **522** number of correct mutations used as in-context ex- **523** amples for the target math problems. Table [4](#page-6-2) shows **524** that APMath provides accurate LLM responses to **525** the mutated math problems, with accuracy ranging **526** from 73.3% (132 out of 180) for SVAMP to 100% **527** (24 out of 24) for MultiArith. The variance in ac- **528** curacy across datasets is attributed to the different **529** complexities of the problems in each dataset. Com- **530** plexity arises from the structure of the problems, **531** the steps required to solve them, and the mathe- **532** matical concepts involved. Consequently, more **533** complex problems present greater challenges for **534** LLMs. Despite this variance, these results suggests **535** that consistency across modalities plays a crucial **536** role in ensuring the correctness of the responses, **537** thereby enhancing the effectiveness of evaluating **538** LLM responses across different modalities. **539**

⁵⁴⁰ 5 Related Work

 In-context Learning. There has been recent advancement in in-context learning. Saunshi *et al.* [\(Saunshi et al.,](#page-9-9) [2021\)](#page-9-9) suggests that downstream tasks can be solved linearly by conditioning on a prompting words following an input text. Xie *et al.* [\(Xie et al.,](#page-10-6) [2022\)](#page-10-6) suggests that the language model can infer in-context shared latent concept [a](#page-8-6)mong examples in a prompt. Levine *et al.* [\(Levine](#page-8-6) [et al.,](#page-8-6) [2022\)](#page-8-6) establishes that the information within in-context examples gives more improvements. In addition, Wei *et al.* [\(Wei et al.,](#page-10-0) [2022\)](#page-10-0) has imple- mented manually hand-crafted the few-shot exam- ples for improving quality of CoT explanation that LLM generates. However, to tackle the need for manually hand-crafted few-shot examples, recent studies have developed a retriever to select analogy examples for demonstration [\(Zhang et al.,](#page-10-1) [2023;](#page-10-1) [Rubin et al.,](#page-9-10) [2022;](#page-9-10) [Su et al.,](#page-9-11) [2023;](#page-9-11) [Wang et al.,](#page-10-7) [2023a;](#page-10-7) [Luo et al.,](#page-8-7) [2023\)](#page-8-7). These studies differ from ours in that they require a substantial amount of fully annotated data to train models and retrieve in-context examples, whereas APMath generates in-context examples automatically through muta-tion and consistency optimization.

 Consistency in LLM. Prior research has suggested that language models may experience inconsistency in natural language conversation [\(Adiwardana et al.,](#page-8-8) [2020\)](#page-8-8), and factual knowledge extraction [\(Elazar](#page-8-9) [et al.,](#page-8-9) [2021\)](#page-8-9). Wang *et al.* [\(Wang et al.,](#page-10-3) [2023b\)](#page-10-3) uti- lize answer consistency across various reasoning **paths within top-K responses to enhance accuracy.** Camburu *et al.* [\(Camburu et al.,](#page-8-10) [2020\)](#page-8-10) introduced an adversarial framework aimed at verifying lan- guage models' coherence in generating natural lan- guage explanations. Moreover, recent studies have tackled the issue of inconsistency in the long-form creative writing generated by LLMs through tech- niques like prompt chaining [\(Mirowski et al.,](#page-9-12) [2022\)](#page-9-12) and editing to rectify long-range factual inconsis- tencies within story passages [\(Yang et al.,](#page-10-8) [2022\)](#page-10-8). In this paper, we concentrate on quantifying the consistency of answers across various modalities and leveraging this metric to estimate the accuracy of LLM responses by incorporating mutations as in-context examples.

 Prompt Optimization. Our research also inter- sects with prompt optimization. Research work improves hard prompts via an iterative local edit and gradient-free search [\(Prasad et al.,](#page-9-13) [2023\)](#page-9-13) or gradient-based optimization [\(Sun et al.,](#page-10-9) [2023\)](#page-10-9). Yang *et al.* [\(Yang et al.,](#page-10-10) [2023\)](#page-10-10) describes the op- 591 timization task in natural language and feeds it to **592** the large language model as a prompt and then **593** generates new prompt. Compared with them, AP- **594** Math automatically optimizes in-context examples **595** across modalities, rather than relying on a single **596** modality to improve the robustness of evaluation **597** of LLM behavior. In addition, prior research work **598** have optimized a small continuous vector for down- **599** [s](#page-8-11)tream tasks, leaving LLM parameters frozen [\(Li](#page-8-11) 600 [and Liang,](#page-8-11) [2021;](#page-8-11) [Zhong et al.,](#page-10-11) [2021;](#page-10-11) [Sun et al.,](#page-10-12) **601** [2022b,](#page-10-12)[a;](#page-10-13) [Chen et al.,](#page-8-12) [2023\)](#page-8-12). Diao *et al.* [\(Diao et al.,](#page-8-13) **602** [2023\)](#page-8-13) applies a policy gradient to estimate the gra- **603** dients of the parameters of the categorical distri- **604** bution of each discrete prompt. However, they are **605** limited to the white-box setting, requiring access- **606** ing the parameters of a pre-trained model while **607** APMath is in black-box optimization by the consis- **608** tency of LLM responses across modalities. In addi- **609** tion, Mishra *et al.* [\(Khashabi et al.,](#page-8-14) [2022\)](#page-8-14) studies **610** advantages of prompt tuning, but it requires manual **611** efforts. Zhou *et al.* [\(Zhou et al.,](#page-10-14) [2023\)](#page-10-14) automate **612** the generation of instructions and select the most **613** suitable instruction based on computed evaluation 614 scores. However, their focus lies on instruction **615** induction tasks rather than math problem-solving **616** tasks. **617**

6 Conclusions **⁶¹⁸**

This paper introduces APMath, a novel tool that **619** automates the generation of relevant in-context ex- **620** amples to enhance the arithmetic problem-solving **621** capabilities of LLMs. APMath automates the mu- **622** tation of target math problems, generating variants **623** that use the same solving algorithm. It also em- **624** ploys a consistency check across various LLM re- **625** sponse modalities to evaluate answer confidence **626** and estimate accuracy for both original and altered **627** problems. Additionally, it identifies mutations for **628** in-context examples that improve response con- **629** sistency. Evaluations show APMath significantly **630** enhances answer accuracy for mutated math prob- **631** lems, demonstrating efficient generation of relevant **632** in-context examples without manual annotations or **633** external datasets. The proposed consistency check **634** method increases LLM response confidence and **635** correctness. 636

7 Limitation **⁶³⁷**

This work has several limitations. First, APMath re- **638** lies on the random mutation of the identified numer- **639** ical values. The effectiveness of such random muta- **640**

 tions is limited in generating realistic mutated math problems. We leave it as our future work to ex- plore context-aware mutation mutation techniques for the target math problems. Second, APMath demonstrates evident advantages only for solving arithmetic math problems through mutation while maintaining the same reasoning algorithm. Future work will aim to extend generating mutations for a broader range of reasoning problems.

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A Appendix **⁹⁵⁸**

A.1 Qualitative Analysis of APMath **959**

Table [5](#page-11-0) to [8](#page-14-0) presents four illustrative examples from **960** the datasets employed in our experiments to eval- **961** uate APMath. In each table, the first row displays **962** in-context examples generated by APMath. The **963** second row shows the target mathematical prob- **964** lem to be solved. Furthermore, the first and sec- **965** ond columns in the third row present GPT-3.5's **966** responses to the target problem across modalities, **967** using the zero-shot setting and APMath, respec- **968** tively. **969**

These examples demonstrate that in the zero- **970** shot setting, GPT-3.5's reasoning is incorrect in at **971** least one modality across all datasets. Conversely, **972** the in-context examples generated by APMath cor- **973** rectly instruct GPT-3.5 to solve the target problem, **974** resulting in accurate answers by following the same **975** reasoning implemented in the in-context examples. **976**

(In-context 1)

Q: Olivia had 69 math problems to complete . She spent about 15 minutes on each problem and then she spent 42 minutes checking her answers . How many minutes did she spend total?

[CoT] A: To find the total time Olivia spent, we need to calculate the time she spent on solving the math problems and the time she spent checking her answers separately. First, let's calculate the time Olivia spent on solving the math problems. Since she spent about 15 minutes on each problem, we can multiply the number of problems by 15: 69 problems * 15 minutes/problem = 1035 minutes Next, let's calculate the time Olivia spent checking her answers, which is given as 42 minutes. Now, let's add the time spent on solving the problems and checking the answers together: 1035 minutes $+ 42$ minutes = 1077 minutes Therefore, Olivia spent a total of 1077 minutes. The answer is 1077 minutes.

[Code] A: def func(): return $69 * 15 + 42$

[Eqn] A: Answer = $69*15 + 42$

(In-context 2)

Q: Olivia had 52 math problems to complete . She spent about 10 minutes on each problem and then she spent 50 minutes checking her answers . How many minutes did she spend total?

[CoT] A: To find the total number of minutes Olivia spent, we need to add the time she spent solving the math problems to the time she spent checking her answers. First, we need to calculate the time Olivia spent solving the math problems. Since she spent about 10 minutes on each problem, we can multiply the number of problems by 10: 52 problems * 10 minutes/problem = 520 minutes. Next, we need to calculate the time Olivia spent checking her answers, which is given as 50 minutes. Finally, we can find the total time by adding the time spent solving the problems to the time spent checking the answers: 520 minutes $+ 50$ minutes $= 570$ minutes. Therefore, the answer is 570 minutes.

[Code] A: def func(): return $52 * 10 + 50$

[Eqn] A: Answer = $52*10 + 50$

Target Q: Olivia had 7 math problems to complete. She spent about 4 minutes on each problem and then she spent 3 minutes checking her answers. How many minutes did she spend total?

(In-context 1)

Q: Edward spent \$8 . Then he received \$12 from his friend . Now he has \$19 . How much did Edward have before he spent his money?

[CoT]A: Step 1: Edward spent \$8. Step 2: He received \$12 from his friend. Step 3: Now he has \$19. Step 4: To find out how much Edward had before he spent his money, we can use the equation: $x - 8 +$ $12 = 19$, where x represents the amount he had before spending his money. Step 5: Simplifying the equation, we get $x + 4 = 19$. Step 6: Subtracting 4 from both sides, we get $x = 15$. Step 7: Therefore, Edward had \$15 before he spent his money. The answer is \$15.

[Code]A: def func(): return $(19 - 12 + 8)$

 $[Eqn]A: Answer = x - 8 + 12 = 19$

(In-context 2)

Q: Olivia had 52 math problems to complete . She spent about 10 minutes on each problem and then she spent 50 minutes checking her answers . How many minutes did she spend total?

[CoT]A: Step 1: Edward spent \$65. Step 2: He received \$39 from his friend. Step 3: Now he has \$67. Step 4: To find out how much Edward had before he spent his money, we can use the equation: x $-65 + 39 = 67$, where x represents the amount he had before spending his money. Step 5: Simplifying the equation, we get $x - 26 = 67$. Step 6: Adding 26 to both sides, we get $x = 93$. Step 7: Therefore, the answer is 93.

 $[CodeIA: def func(): return (67 - 39) + 65$

[Eqn]A: Answer = $x - 65 + 39 = 67$

Target Q: Edward spent \$17. Then he received \$10 from his friend. Now he has \$7. How much did Edward have before he spent his money?

Table 7: APMath generated in-context examples for MultiArith.

(In-context 1)

Q: Gwen and her friends were recycling paper for their class . For every 1 pounds they recycled they earned 38 point . If Gwen recycled 15 pounds and her friends recycled 36 pounds, how many points did they earn?

[CoT]A: First, we need to calculate the number of points Gwen earned for recycling 15 pounds. Since they earned 38 points for every 1 pound, Gwen earned 15 * 38 = 570 points. Next, we need to calculate the number of points Gwen's friends earned for recycling 36 pounds. Following the same logic, her friends earned 36 * 38 = 1368 points. Finally, we need to find the total number of points earned by Gwen and her friends. Adding Gwen's points to her friends' points, we get 570 + 1368 = 1938 points. Therefore, the answer is 1938.

 $[Code] A: def func(): gwen_pounds = 15 friends_pounds = 36 points_perr_pound = 38 total_points =$ (gwen_pounds + friends_pounds) * points_per_pound return total_points

 $[Eqn]A: Answer = 15*38 + 36*38$

Target Q: Gwen and her friends were recycling paper for their class. For every 3 pounds they recycled they earned one point. If Gwen recycled 5 pounds and her friends recycled 13 pounds, how many points did they earn?

Table 8: APMath generated in-context examples for GSM8k.

(In-context 1)

Q: George has 46% more pears than bananas . If George has 90 bananas, how many fruits does George have?

[CoT]A: If George has 90 bananas, then he has $46/100 * 90 = «46/100*90 = 41.4 \times 41.4$ more pears than bananas. This means George has $90 + 41.4 = \sqrt{90+41.4} = 131.4 \times 131.4$ pears. In total, George has $90 + 131.4 = (90 + 131.4 = 221.4 \times 221.4$ fruits. The answer is 221.4.

 $[Code]$ A: def func(): bananas = 90 pears = bananas $*$ 1.46 total fruits = bananas + pears return total_fruits

[Eqn]A: Answer = $90 + 1.46 * 90$

(In-context 2)

Q: George has 56% more pears than bananas . If George has 74 bananas, how many fruits does George have?

[CoT]A: If George has 74 bananas, then he has $56/100 * 74 = \frac{66}{100 * 74} = 41.44 \times 41.44$ more pears than bananas. This means George has $74 + 41.44 = \sqrt{74 + 41.44} = 115.44 \times 115.44$ pears. Therefore, George has a total of 74 + 115.44 = «74+115.44=189.44»189.44 fruits. The answer is 189.44.

 $[Code] A: def func(): bananas = 74 pears = bananas + (bananas * 56 / 100) return bananas + pears$ [Eqn]A: Answer = $74 + 1.56 * 74$

Target Q: George has 45% more pears than bananas. If George has 200 bananas, how many fruits does George have?

