

000 001 002 003 004 005 RELDIFF: RELATIONAL DATA GENERATIVE MODELING 006 WITH GRAPH-BASED DIFFUSION MODELS 007

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009 Paper under double-blind review
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011 ABSTRACT 012

013 Real-world databases are predominantly relational, comprising multiple interlinked
014 tables that contain complex structural and statistical dependencies. Learning gener-
015 ative models on relational data has shown great promise for producing synthetic
016 data, which can unlock access to previously underutilized information and support
017 the training of powerful foundation models. However, existing methods often strug-
018 gle to capture their complexity, typically reducing relational data to conditionally
019 generated flat tables and imposing limiting structural assumptions. To address
020 these limitations, we introduce RELDIFF, a novel diffusion generative model that
021 synthesizes relational databases by explicitly modeling their foreign key graph
022 structure. RELDIFF combines a joint graph-conditioned diffusion process across
023 all tables for attribute synthesis and a $D2K$ +SBM graph generator based on the
024 stochastic block model for structure generation. The decomposition of graph struc-
025 ture and relational attributes ensures both high fidelity and referential integrity, both
026 of which are crucial aspects of synthetic relational database generation. RELDIFF
027 achieves state-of-the-art performance in generating synthetic relational databases
028 on 11 benchmark datasets.

029 1 INTRODUCTION 030

031 Relational databases, which organize information into multiple interconnected tables governed by
032 foreign key references, underpin over 70% of global data management systems (DB-Engines, 2024)
033 and form the foundation for much of today’s digital infrastructure. However, unlocking access to
034 high-quality real-world datasets is often limited by fairness and privacy concerns (Ntoutsi et al.,
035 2020; Hernandez et al., 2022; van Breugel & van der Schaar, 2023), particularly in sensitive domains
036 like healthcare (Appenzeller et al., 2022; Gonzales et al., 2023) and finance (Assefa et al., 2020;
037 Potluru et al., 2024). Thus, synthetic data generation emerges as a promising solution, offering a
038 way to preserve crucial statistical properties while effectively mitigating privacy risks (Raghunathan,
039 2021). Moreover, synthetic data can unlock access to valuable enterprise and healthcare databases,
040 facilitating the creation of powerful relational and tabular foundation models (van Breugel & van der
041 Schaar, 2024) and has shown promise in missing value imputation (You et al., 2020; Zhang et al.,
042 2025) and data augmentation (Fonseca & Bacao, 2023).

043 Unlike image data, which comprises pure continuous pixel values with local spatial correlations, or
044 text data, which comprises tokens that share the same vocabulary, tabular data includes **heterogeneous**
045 and **often imbalanced** distributions (Xu et al., 2019), making it challenging to learn joint probabilities
046 across multiple columns. Moreover, relational databases exhibit complex structural hierarchies
047 and statistical dependencies across their interconnected tables, often stored at varying levels of
048 normalization (Codd, 1970; Delobel, 1978), which further intensifies these inherent difficulties.

049 A common simplification involves flattening relational schemas into single tables (Ge et al., 2021;
050 Ghazi et al., 2023), but this approach quickly becomes impractical for large-scale and complex
051 schemas characteristic of real-world databases (Pang et al., 2024). More recent methods (Cai
052 et al., 2023; Xu et al., 2023; Pang et al., 2024) instead model relational databases as a sequence of
053 conditionally generated tables. This design requires a pre-specified ordering of tables and typically
054 relies on strong independence assumptions, which constrain modeling flexibility. As a result, current
055 methods have struggled with two key limitations: (1) the inability to generate arbitrary relational
056 schemas and (2) the failure to effectively preserve inter-table correlations and long-range dependencies
057 between attributes linked by foreign key relationships (Hudovernik et al., 2024).

In this paper, we propose RELDIFF, a principled generative framework for relational databases. RELDIFF enables the synthesis of arbitrary relational schemas by explicitly modeling the underlying database structure with graphs. Instead of simply decomposing the relational into conditionally independent tables, RELDIFF first utilizes a specifically designed $D2K+SBM$ graph generator to preserve the cardinality of foreign key relationships and the hierarchical dependencies inherent in the relational structure. Built upon this faithful structural representation, we further define a joint graph-based diffusion model for attribute synthesis across interconnected tables, built upon a unified diffusion process modeling both continuous and discrete features. The resulting model can explicitly capture both inter- and intra-table dependencies powered by graph neural networks (GNNs), and can model the heterogeneous numerical and categorical features within each table.

The unique advantages of RELDIFF come from the key innovations in our approach, including: (1) A principled formulation for generating foreign key structures in relational databases, incorporating hard constraints to ensure *referential integrity* through a novel application of Bayesian stochastic blockmodels; (2) A joint diffusion model for synthesizing *mixed-type* attributes, conditioned on graph structure using GNNs, to better capture global dependencies; (3) We define our diffusion model in data space and explicitly consider dimension tables, a fundamental component of real-world databases, as a distinct data type. Furthermore, all the developments are general and data agnostic, without any predefined assumption of the relational data structures. These innovations allow us to model relational databases with arbitrarily complex schemas and preserve both statistical and structural dependencies of the data.

We conduct comprehensive experiments to justify the effectiveness of our proposed method. Empirical results across two benchmarks, covering 11 datasets and 8 metrics, demonstrate that RELDIFF can consistently outperform previous methods, with up to 80% improvement over the state-of-the-art in preserving column correlation between connected tables. The significant improvements highlight the superior generative capacity of our approach on relational data.

2 RELATED WORK

Relational Database Synthesis. Patki et al. (2016) were the first to propose a learning-based method for relational database synthesis - the Synthetic Data Vault (SDV). Recent methods broadly fall into neural network-based (prioritizing fidelity) and marginal-based (focusing on differential privacy) approaches. While marginal-based methods are established for single-table synthesis (Zhang et al., 2017; McKenna et al., 2022), their extension to relational data is more recent, with PrivLava (Cai et al., 2023) and several newer methods emerging (Alimohammadi et al., 2025; Kapenekakis et al., 2024; Cai et al., 2025).

We focus on neural network-based relational database synthesis, preserving fidelity and utility for arbitrary schemas. Graph variational autoencoder-based methods (Mami et al., 2022) have been investigated but encounter scalability issues with real-world databases. Generative adversarial network (GAN)-based methods, such as RCTGAN (Gueye et al., 2023) and IRG (Li et al., 2024a), extend the successful CTGAN (Xu et al., 2019) architecture from single-table synthesis to relational data. However, recent advancements have demonstrated the superior performance of diffusion models over GANs in various generative tasks. Autoregressive approaches, leveraging language models (Solatorio & Dupriez, 2023) and any-order networks (Tiwald et al., 2025), have also been investigated, but their autoregressive nature makes them better suited for simpler relational structures, particularly those with single-parent schemas. In contrast, the work of Xu et al. (2023) proposes a method for generating many-to-many datasets using bipartite $2K$ random graphs. Similar to our approach, they decouple the generation of database structure and attribute synthesis. However, their graph generation method does not capture hierarchical relational structures and primarily focuses on many-to-many relationships, generating tables sequentially. Building on the success of diffusion models for single-table data generation (Kotelnikov et al., 2023; Zhang et al., 2024; Shi et al., 2025a), two diffusion-based methods for relational data have emerged: ClavaDDPM (Pang et al., 2024) and RGCLD (Hudovernik, 2024). Both of these approaches largely treat relational database synthesis as a series of conditional single-table generation tasks, relying on a pre-specified table ordering and introducing limiting assumptions about the relational dependencies. A detailed overview of related work is provided in Appendix A.

108 **Graph Structure Generation.** Realistic and efficient graph generation models originate from the
 109 degree sequence problem. dK -random graphs (Mahadevan et al., 2006) preserve the degrees of nodes
 110 in d -sized connected subgraphs. $0K$ graphs preserve: the density (by convention), $1K$ graphs: the
 111 degree distribution, $2K$ graphs: the joint degree distribution of neighbors, $3K$ graphs: the degrees
 112 of connected triplets of nodes. While there exist efficient methods to generate up to $2K$ -graphs,
 113 generating $3K$ graphs is NP-hard. Therefore, the $2K+$ graph construction framework (Tillman et al.,
 114 2019) proposes heuristic approaches for targeting additional properties such as connected components
 115 and clustering. We extend the $2K+$ framework by preserving (not only targeting) hierarchical and any
 116 other modular organization present in relational databases. We employ the Stochastic Block Model
 117 (SBM) (Holland et al., 1983) where nodes are partitioned into blocks which define their connectivity
 118 (blocks are subsets of relational tables). The degree-corrected SBM (Dasgupta et al., 2004) accounts
 119 for the variance in node degrees, while the microcanonical version enforces hard constraints on the
 120 edges (Peixoto, 2017). A hierarchy of nested SBMs reduces the minimum detectable size of blocks
 121 from $\mathcal{O}(\sqrt{n})$ down to $\mathcal{O}(\log n)$, where n is the number of nodes (Peixoto, 2014).

122 Deep generative models for graphs, such as diffusion (Liu et al., 2019; Vignac et al., 2023) and
 123 autoregressive models (You et al., 2018; Liao et al., 2019), exist but often assume dense representation,
 124 scaling poorly to large relational database graphs (Jang et al., 2024; Li et al., 2024b). Unlike our
 125 model, they typically do not enforce hard structural constraints relevant to our work.

127 3 PRELIMINARIES

129 **Notation.** We begin by introducing a formal definition of relational databases, following the RDL
 130 framework (Fey et al., 2024), which provides a principled abstraction of relational data as heteroge-
 131 neous graphs. This formulation enables us to decouple the generative process into two components:
 132 (1) the generation of database structure via a schema-consistent relational entity graph, and (2) the
 133 joint synthesis of entity-level attributes conditioned on this structure and local neighborhoods.

134 A relational database $(\mathcal{T}, \mathcal{L})$ consists of a collection of tables $\mathcal{T} = \{T_1, \dots, T_n\}$, and links between
 135 tables $\mathcal{L} \subseteq \mathcal{T} \times \mathcal{T}$. A link $L = (T_{\text{fkey}}, T_{\text{pkey}})$ exists if a foreign key column in T_{fkey} references a
 136 primary key column in T_{pkey} . Each table is a set $T = \{v_1, \dots, v_{n_T}\}$, whose elements $v_i \in T$ are
 137 called *rows* or *entities*. Each entity $v \in T$ is defined as a tuple: $v = (p_v, K_v, x_v)$ where: p_v is the
 138 **primary key**, uniquely identifying the entity v , $K_v \subseteq \{p_{v'} : v' \in T' \text{ and } (T, T') \in \mathcal{L}\}$ is the set of
 139 **foreign keys**, establishing links from $v \in T$ to entities $v' \in T'$, where $p_{v'}$ is the primary key of v' in
 140 table T' and x_v contains the **attributes**, representing the informational content of the entity.

141 The two central objects of RDL as defined by Fey et al. (2024) are the *schema graph* and *relational*
 142 *entity graph*. The schema graph defines the table-level structure of data. Given a relational database
 143 $(\mathcal{T}, \mathcal{L})$ and the inverse set of links as $\mathcal{L}^{-1} = \{(T_{\text{pkey}}, T_{\text{fkey}}) \mid (T_{\text{fkey}}, T_{\text{pkey}}) \in \mathcal{L}\}$, the schema graph is
 144 the graph $(\mathcal{T}, \mathcal{R})$ with node set \mathcal{T} and edge set $\mathcal{R} = \mathcal{L} \cup \mathcal{L}^{-1}$. The nodes of the schema graph serve
 145 as type definitions for the heterogeneous relational entity graph.

147 The relational entity graph is a heterogeneous graph $G = (\mathcal{V}, \mathcal{E}, \phi, \psi)$, with node set \mathcal{V} and edge set
 148 $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and type mapping functions $\phi : \mathcal{V} \rightarrow \mathcal{T}$ and $\psi : \mathcal{E} \rightarrow \mathcal{R}$, where each node $v \in \mathcal{V}$ belongs
 149 to a node type $\phi(v) \in \mathcal{T}$ and each edge $e \in \mathcal{E}$ belongs to an edge type $\psi(e) \in \mathcal{R}$. Specifically, the
 150 sets \mathcal{T} and \mathcal{R} from the schema graph define the node and edge types of our relational entity graph.

151 Real-world relational databases contain diverse data types. Following prior work, we focus on
 152 numerical, categorical, and datetime attributes, representing each as either a continuous or discrete
 153 random variable for unified modeling. We handle dimension tables explicitly as fixed-size vocabulary
 154 lookups to ensure schema consistency and improve sample quality.

155 **Gaussian Diffusion.** For a numerical attribute z , the forward diffusion process gradually perturbs
 156 the data with increasing Gaussian noise: $q_{\text{num}}(z_t \mid z_0) = \mathcal{N}(z_0, (\sigma^{\text{num}}(t))^2)$, where $\sigma^{\text{num}}(t) :
 157 [0, 1] \rightarrow \mathbb{R}^+$ is an increasing function that governs the cumulative noise level over time. The
 158 marginal distribution $p(z_0)$ at time $t = 0$ corresponds to the data distribution, while $p(z_1)$ converges
 159 to the know noise distribution $\mathcal{N}(0, (\sigma^{\text{num}}(1))^2)$ from which we can easily sample. Following the
 160 framework of Karras et al. (2022), the true reverse distribution $q_{\text{num}}(z_s \mid z_t)$ can be formulated by the
 161 solutions to the ordinary differential equation (ODE) $dz^{\text{num}} = -[\frac{d}{dt} \sigma^{\text{num}}(t)] \sigma^{\text{num}}(t) \nabla_z \log p_t(z) dt$,
 where $0 \leq s < t \leq 1$. To learn the generative model, we approximate the true score function

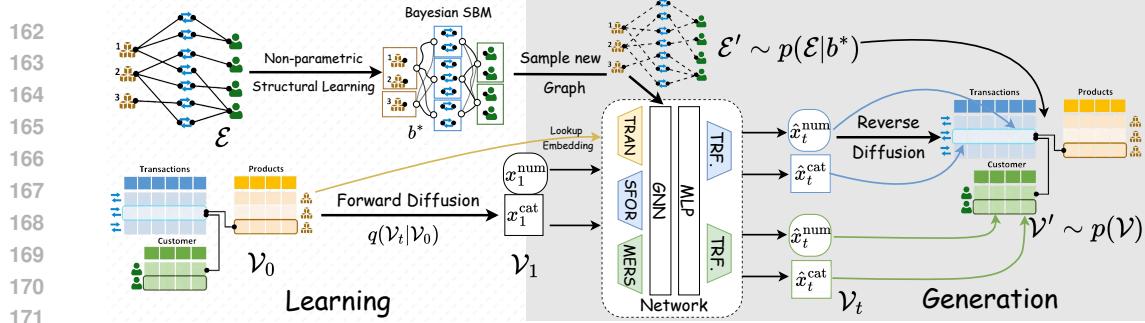


Figure 1: **RELDIFF framework overview.** RELDIFF applies forward diffusion to mixed-type attributes within each relational table and performs joint reverse denoising across tables, conditioned on the relational entity graph and node neighborhoods. Learnable embeddings handle dimension tables (e.g., *Products*), and a sampled synthetic entity graph guides the generation process.

$\nabla_z \log p_t(z^{\text{num}})$ using a neural network μ_θ^{num} , which is trained via the following denoising loss:

$$\mathcal{L}_{\text{num}}(\theta) = \mathbb{E}_{z_0 \sim p(z_0)} \mathbb{E}_{t \sim U[0,1]} \mathbb{E}_{\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \|\mu_\theta^{\text{num}}(z_t, t) - \epsilon\|_2^2. \quad (1)$$

Masked Diffusion. For a categorical attribute c with K categories, we introduce an additional $(K + 1)^{\text{th}}$ to represent the special [MASK] state and denote it as $\mathbf{m} = (0, \dots, 1) \in \{0, 1\}^K$ using the one-hot representation. Let $\text{cat}(\cdot; \pi)$ denote the categorical distribution over K classes, parameterized by the probability vector $\pi \in \Delta^K$. The forward diffusion process operates in continuous-time by gradually masking the original values: $q_{\text{cat}}(c_t | c_0) = \text{cat}(c_t; \alpha_t c_0 + (1 - \alpha_t)\mathbf{m})$, where $\alpha_t = \exp(-\sigma^{\text{cat}}(t))$ is a decreasing function between 0 and 1 controlling the masking rate. Following Sahoo et al. (2024) and Shi et al. (2024), the true reverse transition distribution $q(c_s | c_t)$ is given as:

$$q_{\text{cat}}(c_s | c_t) = \begin{cases} \text{cat}(c_s; c_t) & c_t \neq \mathbf{m}, \\ \text{cat}\left(c_s; \frac{(1 - \alpha_s)\mathbf{m} + (\alpha_s - \alpha_t)q_{\text{cat}}(c_0 | c_t)}{1 - \alpha_t}\right) & c_t = \mathbf{m}. \end{cases} \quad (2)$$

To model this generative process, we train a neural network μ_θ^{cat} to predict the original category c_0 from the masked input c_t , i.e. to estimate $q_{\text{cat}}(c_0 | c_t)$. The model is optimized using a cross-entropy loss under the continuous-time limit:

$$\mathcal{L}_{\text{cat}}(\theta) = \mathbb{E}_q \int_{t=0}^{t=1} \frac{\alpha'_t}{1 - \alpha_t} \mathbb{1}_{\{c_t = \mathbf{m}\}} \log \langle \mu_\theta^{\text{cat}}(c_t, t), c_0^{\text{cat}} \rangle dt. \quad (3)$$

4 METHOD

This section details **RELDIFF**, our framework for generating synthetic relational databases. We provide a high-level overview of our generative model in Section 4.1. First, we describe how we model the relational graph structure using Bayesian stochastic blockmodels, ensuring referential integrity and preserving relationship cardinalities and hierarchical dependencies (Section 4.2). Next, we present our joint relational diffusion model for synthesizing attributes across the database schema, parameterized by a single heterogeneous graph neural network with tabular transformer-based encoders and decoders (Section 4.3). Finally, we detail the training and sampling procedures (Section 4.4).

4.1 OVERVIEW

In line with Probabilistic Relational Models (PRMs) (Friedman et al., 1999; Getoor et al., 2001), we decompose the generative process into modeling the relational graph structure and the attributes of the tables. We treat the relational entity graph $G = (\mathcal{V}, \mathcal{E})$ as a single sample from some unknown joint distribution $p(\mathcal{V}, \mathcal{E})$. Our objective is to sample from this distribution, ensuring adherence to referential integrity constraints and the preservation of statistical dependencies. We formalize this joint sampling through the following conditional decomposition $p(\mathcal{V}, \mathcal{E}) = p(\mathcal{E})p(\mathcal{V} | \mathcal{E})$. This decomposition enables efficient modeling even for complex schemas. We provide a discussion of alternative formulations in Appendix F. See also Xu et al. (2023) for a detailed discussion for many-to-many schemas.

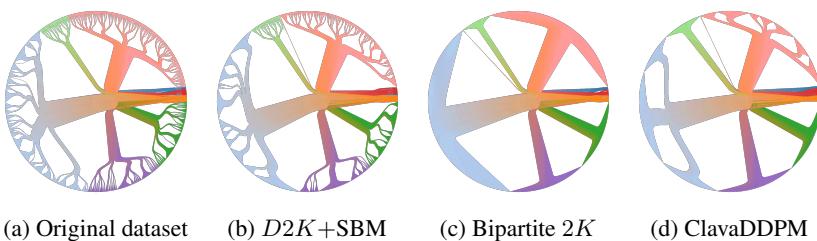


Figure 2: **Hierarchical structure** of the F1 dataset. Our SBM-based method preserves the F1 dataset’s foreign key graph’s joint degree distribution and hierarchy. In contrast, the bipartite $2K$ -graph approach (Xu et al., 2023) loses this structure despite matching degree distributions, and ClavaDDPM, by implicitly modeling the structure, retains some hierarchy but not the degrees.

To model graph structure $p(\mathcal{E})$, we present a novel approach based on established models from graph theory. Within our joint diffusion framework, we formulate $p(\mathcal{V} \mid \mathcal{E})$ to explicitly model the dependencies between nodes representing entities across different tables during generation.

4.2 GRAPH STRUCTURE GENERATION

To generate realistic synthetic relational structures, we focus on sampling graphs that preserve the original database size, enforce referential integrity, and respect exact table and relationship cardinalities. Modern deep generative approaches are not applicable here due to their scalability limitations with large, sparse graphs (Jang et al., 2024). Instead, we base our approach on classical random graph models, which provide principled, efficient mechanisms for sampling large, structured graphs while preserving referential integrity by design.

The graph structure of a relational database is a heterogeneous entity graph $G = (\mathcal{V}, \mathcal{E}, \phi, \psi)$ defined above. Our objective is to learn a distribution $p(\mathcal{E})$ over such graphs, conditioned on a fixed set of nodes $v \in \mathcal{V}$, the number of edges $m_r = |\mathcal{E}_r|$ of each type $r \in \mathcal{R}$ and the node degree sequence $k_v^r = |v \in \mathcal{E}_r|$, where $\sum_{v \in \mathcal{V}} k_v^r = 2m_r$. This corresponds to fixed row counts, and exact entity and relationship cardinalities. **We can opt to preprocess the entity graph by converting nodes with two parents and no children into many-to-many edges to improve computational efficiency (see Appendix E.1).** This one-to-one transformation is reverted after sampling the new edge set.

To generate samples from $p(\mathcal{E})$, we utilize nonparametric Bayesian SBMs (Peixoto, 2019) as a model of graphs with the above structural constraints. The microcanonical degree-corrected SBM (Peixoto, 2017) defines the distribution $p(\mathcal{E} \mid b, m, k)$, where $b : \mathcal{V} \rightarrow \mathbb{Z}$ is a partition of nodes into some latent (disjoint) blocks. By setting $b = \phi$, and generating the subgraph corresponding to each foreign-key relationship independently, we obtain a $D2K$ graph generator (Tillman et al., 2017) of individual relationships. $2K$ -random graphs preserve the node degree sequence and the degree correlations of neighbouring nodes (Mahadevan et al., 2006). To preserve also a global hierarchical and other modular organization of the database, we use the maximum likelihood partition b^* that minimizes the description length of a nested hierarchy of SBMs (Peixoto, 2014). Note that we constrain the partition b^* by node types ϕ such that $b^*(v) = b^*(u)$ implies $\phi(v) = \phi(u)$ for all $v, u \in \mathcal{V}$. We refer to this model as $D2K+SBM$ graphs, consistent with the literature (Tillman et al., 2017; 2019).

The generation process proceeds in three stages.

1. We employ an efficient Markov Chain Monte Carlo approach (Peixoto, 2014) to infer the most likely partition b^* of the edge set \mathcal{E} .
2. For each relationship $r \in \mathcal{R}$, we sample a new edge set \mathcal{S}_r independently from $p(\mathcal{E}_r \mid b_r^*, m_r, k^r)$, where b_r^* is the partition of nodes induced by \mathcal{E}_r . When \mathcal{E}_r induces a simple graph, we ensure to sample \mathcal{S}_r only from simple graphs (unless stated otherwise).
3. The final generated graph is induced by the edge set $\cup_{r \in \mathcal{R}} \mathcal{S}_r$.

Our approach preserves the joint degree distribution of individual relationships by construction and, as illustrated in Figure 2, retains the hierarchical structure of the relational entity graph.

270 4.3 JOINT MULTI-RELATIONAL DIFFUSION
271

272 To model the distribution over node attributes, we define a hybrid diffusion process that applies
273 forward noise independently across entries and independently across attribute types (numerical and
274 categorical) within each entry. Let \mathcal{V}_t denote the set of all entities at time t , and x_t^v the attribute of a
275 single entry v . This forward process is given by:

$$276 \quad q(\mathcal{V}_t \mid \mathcal{V}_0) = \prod_{v_t \in \mathcal{V}_t} q_{\text{num}}^{\phi(v)}(x_t^{\text{num}} \mid x_0^{\text{num}}) \cdot q_{\text{cat}}^{\phi(v)}(x_t^{\text{cat}} \mid x_0^{\text{cat}}), x_t \in v_t. \quad (4)$$

278 The true reverse process is then modeled as the joint posterior:
279

$$280 \quad q(\mathcal{V}_s \mid G_t) = \prod_{v_s \in \mathcal{V}_s} q_{\text{num}}^{\phi(v_s)}(x_s^{\text{num}} \mid x_t^{\text{num}}, G_t) \cdot q_{\text{cat}}^{\phi(v_s)}(x_s^{\text{cat}} \mid x_t^{\text{cat}}, G_t), x_s \in v_s, \quad (5)$$

282 which factorizes over entities but allows each denoising step to condition on the full database context
283 G_t at a given timestep t . Note that to simplify notation, we omit defining a common space on which
284 q is defined, but make a distinction between $q^{\phi(v)}$, which are defined on subsets of the joint space.
285

286 We formulate the learning objective as a graph-based denoising task, where the goal is to train a
287 model $p_{\theta}(x_s^v \mid x_t^v, G_t)$ that reconstruct clean attributes from noisy inputs. However, conditioning
288 on the full graph at every step can be computationally prohibitive, as real-world databases typically
289 contain millions of entries.

290 To improve efficiency while still capturing the interactions between attributes across the connected
291 tables, we assume conditional independence of each node $v \in \mathcal{V}$ given its noisy k -hop neighborhood at
292 timestep t , denoted $\mathcal{N}_k(v)_t$. Under this assumption, the model is approximated as $p_{\theta}(x_s^v \mid x_t^v, \mathcal{N}_k(v)_t)$
293 and parameterized using a GNN that operates locally over the neighborhood $\mathcal{N}_k(v)_t$.

294 By plugging in the objective functions corresponding to the masked and Gaussian diffusion processes
295 (defined by eqs. (1) and (3)), we end up with the following optimization objective with two weight
296 terms λ_{num} and λ_{cat} :

$$297 \quad \mathcal{L}_{\text{RELDIFF}}(\theta) = \sum_{T_i \in \mathcal{T}} (\lambda_{\text{num}} \mathcal{L}_{\text{num}}(\theta) + \lambda_{\text{cat}} \mathcal{L}_{\text{cat}}(\theta)) = \mathbb{E}_{t \sim U(0,1)} \sum_{T_i \in \mathcal{T}} \mathbb{E}_{x_t^v \sim q^{T_i}(x_t^v, x_0^v)} \\ 298 \quad \left(\lambda_{\text{num}} \|\mu_{\theta}^{\text{num}}(x_t^v, t, \mathcal{N}_k(v))^{T_i} - \epsilon_{\text{num}}\|_2^2 + \sum_{c_t \in x_t^v, \text{cat}} \frac{\lambda_{\text{cat}} \alpha_t'}{1 - \alpha_t} \mathbb{1}_{\{c_t = \mathbf{m}\}} \langle \mu_{\theta}^{\text{cat}}(x_t^v, t, \mathcal{N}_k(v))^{T_i}, c_0^{\text{cat}} \rangle \right) \\ 299 \quad 300 \quad 301 \quad 302 \quad 303 \quad 304 \quad 305 \quad 306 \quad 307 \quad 308 \quad 309 \quad 310 \quad 311 \quad 312 \quad 313 \quad 314 \quad 315 \quad 316 \quad 317 \quad 318 \quad 319 \quad 320 \quad 321 \quad 322 \quad 323 \quad 324 \quad 325 \quad 326 \quad 327 \quad 328 \quad 329 \quad 330 \quad 331 \quad 332 \quad 333 \quad 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324 computational perspective, since it provides complete access to the computational graph and enables
 325 loss computation across all nodes. However, it would constrain us to sampling a single noise level
 326 per training iteration, as the entire graph must be denoised simultaneously. In contrast, minibatch
 327 training provides a practical trade-off: it reduces memory consumption and enables sampling across
 328 different noise levels, at the cost of reusing only a portion of the computation graph per step.

329 **Subgraph Sampling for Efficient Training.** For databases organized into multiple disjoint subgraphs
 330 (e.g., those following a snowflake schema), minibatch construction is straightforward. We sample n
 331 subgraphs independently at each training step and compute the loss across all nodes within them. In
 332 more general settings, where foreign key relationships form a connected network rather than isolated
 333 components, we adapt the neighbor sampling procedure of [Hamilton et al. \(2017\)](#). Specifically, we
 334 begin by selecting a set of seed nodes for each table, then sample their k -hop neighborhoods. The
 335 resulting subgraphs are merged to form a single, connected minibatch subgraph used for training.

336 **Sampling** During the sampling process, we first generate a new relational entity graph using our
 337 $D2K + \text{SBM}$ graph generator, capturing the structural properties of the original database. To initiate
 338 the reverse diffusion, we sample initial values for numerical attributes from a standard Gaussian prior
 339 distribution and set all categorical attributes to the masked state. At each subsequent denoising step
 340 of the reverse diffusion, we jointly denoise the attributes of all tables using our learned diffusion
 341 model, allowing for simultaneous refinement and capturing inter-table dependencies. For the reverse
 342 diffusion process, we utilize a stochastic sampler, similar to the one proposed by [Shi et al. \(2025a\)](#), to
 343 introduce stochasticity and diversity into the generated samples.

344 After completing the denoising process, we transform the generated graph entities back into a tabular
 345 format, reconstructing the table structure. Finally, we add foreign keys to the tables based on the
 346 edges of the generated relational entity graph, ensuring the generated data adheres to the structural
 347 relationships defined by the schema.

349 5 EXPERIMENTS

350 We evaluate RELDIFF by comparing it against 6 generative methods using two relational database
 351 generation benchmarks consisting of 11 real-world datasets totaling 67 tables and 64 foreign key
 352 relationships. We focus on multi-table fidelity and downstream task performance. Additional results
 353 on single-table fidelity, privacy, and ablation studies are provided in Appendix D. In Appendix E, we
 354 analyze the efficiency of our method by reporting training and sampling runtimes and by scaling to
 355 two large relational databases—an order of magnitude larger in the number of rows and tables than
 356 those used in prior work—demonstrating RELDIFF’s superior scalability.

358 5.1 EXPERIMENTAL SETUP

359 **Real-world datasets.** We conduct our evaluation on eleven real-world relational databases from two
 360 benchmarks from related work. The datasets include: *Biodegradability*, *Berka*, a relational version
 361 of the *Cora* dataset, *Walmart Recruiting - Store Sales Forecasting*, *Airbnb Bookings*, *Rossmann*
 362 *Store Sales*, *CCS*, *Instacart 05*, and *F1*. These datasets vary in the number of tables, the maximum
 363 depth, the number of foreign-key relationships, and structural complexity. Details can be found in
 364 Appendix B.1.

365 **Baselines.** We compare our method with state-of-the-art methods from each benchmark. These
 366 include ClavaDDPM ([Pang et al., 2024](#)), RCTGAN ([Gueye et al., 2023](#)), RealTabFormer ([Solatorio &](#)
 367 [Dupriez, 2023](#)), SDV ([Patti et al., 2016](#)) and TabularARGN ([Tiwald et al., 2025](#)) on the SyntheRela
 368 benchmark and ClavaDDPM, SDV and PrivLava ([Cai et al., 2023](#)) on the ClavaDDPM benchmark.

369 **Evaluation metrics.** We follow the protocols of [Jurković et al. \(2025\)](#) and [Pang et al. \(2024\)](#) and
 370 use the same evaluation metrics: 1) Fidelity: Shape, Trend, C2ST, C2ST-Agg, k -hop correlation
 371 and cardinality similarity assess how well the synthetic data can faithfully recover the ground-truth
 372 data distribution; 2) Downstream tasks: machine learning efficiency using RDL utility evaluates the
 373 models’ potential to power downstream tasks; 3) Privacy: The distance to closest record (DCR) score
 374 evaluates the level of privacy protection by measuring how close the synthetic data is to the training
 375 data and membership inference attacks (MIA) evaluate how much of the training data can be detected
 376 by an attacker given a sample of held-out and synthetic data. We provide detailed descriptions of all
 377 metrics in Appendix B.2.

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Table 1: **Multi-table results** on the SyntheRela benchmark. For each dataset we report the average detection accuracy for C2ST-Agg (lower is better) and k-hop correlation similarity (higher is better). The number of k-hop results is determined by maximum depth of the dataset. We report the best result in **bold**. DNC denotes *Did Not Converge*. (Baselines from [Jurkovič et al. \(2025\)](#).)

	Metric	TabARGN	ClavaDDPM	RCTGAN	REALTABF.	SDV	RelDiff	Improv.
Airbnb	C2ST-Agg (↓)	63.47 \pm 0.88	\approx 100.0	98.22 \pm 0.08	99.13 \pm 0.02	99.94 \pm 0.01	55.68 \pm 0.66	12.26
	Cardinality (↑)	98.59 \pm 0.32	99.65 \pm 0.06	95.45 \pm 0.62	76.38 \pm 0.47	26.36 \pm 0.03	100.0	0.35
	1-HOP (↑)	79.66 \pm 0.36	86.69 \pm 0.14	68.78 \pm 0.54	33.99 \pm 5.76	24.58 \pm 0.03	89.37 \pm 0.38	3.10
Rossmann	C2ST-Agg (↓)	60.43 \pm 0.63	85.77 \pm 0.07	86.11 \pm 1.01	85.90 \pm 1.33	98.37 \pm 0.23	51.06 \pm 1.39	15.51
	Cardinality (↑)	94.17 \pm 1.84	99.19 \pm 0.29	82.69 \pm 1.95	41.82 \pm 10.29	99.16 \pm 0.15	100.0	0.81
	1-HOP (↑)	92.95 \pm 0.78	82.81 \pm 0.47	87.02 \pm 0.17	80.25 \pm 0.84	73.84 \pm 0.34	96.73 \pm 0.18	4.06
Walmart	C2ST-Agg (↓)	94.81 \pm 1.68	73.33 \pm 2.92	94.81 \pm 1.68	90.0 \pm 0.91	88.52 \pm 1.60	66.30 \pm 1.68	9.60
	Cardinality (↑)	65.93 \pm 1.98	93.33 \pm 2.28	88.15 \pm 1.51	85.56 \pm 4.57	86.30 \pm 1.09	100.0	7.14
	1-HOP (↑)	75.40 \pm 1.49	86.40 \pm 1.73	79.02 \pm 0.15	74.99 \pm 0.20	76.64 \pm 1.07	91.87 \pm 0.42	6.34
Berka	C2ST-Agg (↓)	80.56 \pm 1.86	69.12 \pm 0.63	76.86 \pm 2.22		77.43 \pm 0.14	55.69 \pm 0.82	19.43
	Cardinality (↑)	85.17 \pm 0.84	96.43 \pm 0.36	81.28 \pm 1.07		80.53 \pm 0.72	100.0	3.70
	1-HOP (↑)	72.82 \pm 0.38	87.92 \pm 1.66	78.87 \pm 0.91	-	59.09 \pm 0.49	96.88 \pm 0.06	10.20
	2-HOP (↑)	65.51 \pm 0.31	84.41 \pm 2.46	77.98 \pm 0.95		23.09 \pm 0.21	95.79 \pm 0.02	13.49
F1	3-HOP (↑)	59.34 \pm 0.62	80.67 \pm 2.18	78.65 \pm 0.69		58.23 \pm 0.58	90.19 \pm 0.22	11.81
	C2ST-Agg (↓)	95.90 \pm 0.94	82.52 \pm 0.25	91.23 \pm 0.39		94.55 \pm 0.24	64.85 \pm 0.11	21.41
	Cardinality (↑)	58.17 \pm 3.71	88.45 \pm 3.05	56.82 \pm 1.55	-	71.88 \pm 0.12	100.0	13.06
	1-HOP (↑)	77.37 \pm 0.26	79.35 \pm 0.03	79.14 \pm 0.72	-	68.45 \pm 0.20	93.46 \pm 0.10	17.78
IMDB	2-HOP (↑)	76.25 \pm 0.32	84.18 \pm 0.12	83.50 \pm 0.82		76.93 \pm 0.24	95.91 \pm 0.03	13.94
	C2ST-Agg (↓)	73.76 \pm 1.78	65.0 \pm 0.34	81.56 \pm 2.0			53.29 \pm 0.45	18.01
	Cardinality (↑)	81.19 \pm 0.80	98.95 \pm 0.03	79.53 \pm 1.27	-	DNC	100.0	1.06
Biodeg.	1-HOP (↑)	88.64 \pm 0.70	91.57 \pm 1.25	81.76 \pm 0.20			94.84 \pm 0.35	3.57
	C2ST-Agg (↓)	88.86 \pm 0.26		83.82 \pm 3.35		98.02 \pm 0.06	47.04 \pm 0.27	43.88
	Cardinality (↑)	79.53 \pm 0.24		85.22 \pm 0.50		61.17 \pm 0.36	100.0	17.35
Cora	1-HOP (↑)	61.36 \pm 0.47	-	75.80 \pm 1.46	-	49.09 \pm 0.59	89.37 \pm 3.76	17.90
	2-HOP (↑)	60.54 \pm 0.44		77.04 \pm 1.96		47.80 \pm 2.16	86.59 \pm 5.02	12.40
Cora	C2ST-Agg (↓)	68.80 \pm 0.67		73.74 \pm 0.47		99.59 \pm 0.03	69.30 \pm 0.52	0.0
	Cardinality (↑)	96.27 \pm 0.13	-	90.48 \pm 2.16	-	68.82 \pm 0.29	100.0	3.87
	1-HOP (↑)	80.42 \pm 0.34		68.39 \pm 0.08		4.95 \pm 0.12	72.16 \pm 1.19	0.0

Implementation Details. All reported experiment results are the average of 3 randomly sampled synthetic data samples. Additional implementation details, such as the hardware and software information as well as hyperparameter settings, are in Appendix C.

5.2 MULTI-TABLE FIDELITY

We first evaluate multi-table fidelity on the SyntheRela benchmark datasets. We focus on two key metrics: C2ST-Agg, which probes the preservation of higher-order interactions and aggregations across connected tables, and k-hop similarity, which evaluates correlations between columns of paired tables at varying depths within the database schema. In line with previous work, we also report cardinality similarity. However, our approach consistently achieves a perfect score as it preserves the degree distributions. Table 1 shows that RELDIFF consistently outperforms all baselines, securing second-best performance in all but three experiments. Notably, RELDIFF exhibits a significantly smaller performance drop when transitioning from single-table (Table 6) to multi-table C2ST evaluation. Figure 3 illustrates that this degradation is $7\times$ lower than that of the closest competitor, ClavaDDPM, highlighting RELDIFF’s superior ability to maintain data fidelity in relational contexts. Additional single-table fidelity and privacy results are in Appendices D.1 and D.3.

Next, we evaluate the results on the ClavaDDPM (Pang et al., 2024) benchmark. Following the original evaluation protocol, we report single-table metrics (Trend and Shape) and multi-table metrics (cardinality and k-hop similarity). We omit the MovieLens and Berka datasets as they are already included in the SyntheRela benchmark. Results in Table 2 show that RELDIFF is the best on all multi-table fidelity metrics, with an average improvement of **25.3%** in preserving k-hop correlations. RELDIFF outperforms other methods on all but two single-table evaluations.

To better visualize the fidelity metrics, we plot column-wise marginal densities in appendix D.2.

5.3 PERFORMANCE ON DOWNSTREAM TASKS

High-quality synthetic data offers the key advantage of replacing real data for analysis and effective learning on downstream tasks like classification and regression. We evaluate this capacity using Machine Learning Efficiency (ML-E) on RDL tasks (Robinson et al., 2024). According to the RDL

432
 433 **Table 2: End-to-end results on the ClavaDDPM benchmark.** We follow the evaluation protocol
 434 by Pang et al. (2024) and report the cardinality similarity, column shapes, trend scores and correlations
 435 between columns in connected tables. DNC denotes *Did Not Converge*.

	Metric	PrivLava	SDV	ClavaDDPM	RelDiff	Improv.
California	Cardinality	99.90 \pm 0.03	71.45 \pm 0.0	99.19 \pm 0.29	100.0	0.10
	Shape	99.71 \pm 0.02	72.32 \pm 0.0	98.77 \pm 0.02	99.52 \pm 0.02	0.0
	Trend	98.49 \pm 0.05	50.23 \pm 0.0	97.65 \pm 0.05	98.72 \pm 0.01	0.24
	1-HOP	97.46 \pm 0.12	54.89 \pm 0.0	95.16 \pm 0.39	98.72 \pm 0.0	1.29
	AVG 2-WAY	97.97 \pm 0.09	52.56 \pm 0.0	96.41 \pm 0.20	98.72 \pm 0.01	0.77
Instacart 05	Cardinality			95.30 \pm 0.79	100.0	4.93
	Shape			89.84 \pm 0.29	96.85 \pm 0.85	7.80
	Trend		DNC	99.62 \pm 0.04	95.71 \pm 0.42	0.0
	1-HOP			76.42 \pm 0.39	85.83 \pm 1.20	12.31
	2-HOP			39.29 \pm 3.38	70.74 \pm 0.14	80.05
CCS	AVG 2-WAY			76.02 \pm 0.78	85.78 \pm 0.81	12.84
	Cardinality			74.36 \pm 8.40	100.0	0.76
	Shape		DNC	69.04 \pm 4.38	92.37 \pm 2.30	6.41
	Trend			94.84 \pm 1.0	98.47 \pm 0.79	0.28
	1-HOP			21.74 \pm 9.62	83.15 \pm 4.22	7.62
	AVG 2-WAY			41.68 \pm 6.73	87.33 \pm 3.12	92.01 \pm 2.95
						5.36

449 results presented in Table 3 RELDIFF achieves the best performance on all datasets — sharing the best
 450 results on the two datasets with the simplest schema and substantially outperforming other methods
 451 on datasets with more complex schemas. This demonstrates our method’s competitive capacity to
 452 capture and replicate key features of the real data that are most relevant to learning downstream
 453 machine learning tasks. We observe that methods with lower performance on data fidelity sometimes
 454 outperform stronger methods on utility, highlighting that fidelity and utility are two distinct aspects
 455 of synthetic data quality (Hansen et al., 2023). Despite this nuance, RELDIFF consistently achieves
 456 strong performance across both aspects.

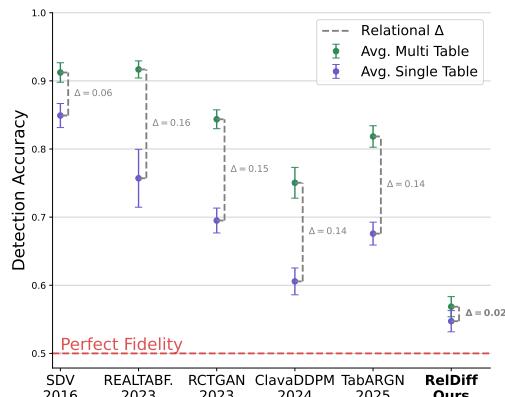
457 **Table 3: RDL-utility results.** We report ROC-AUC (higher is better) for classification and MAE
 458 (lower is better) for regression tasks. We report the naive baseline scores (mean or majority class) in
 459 parentheses. “-” denotes that the utility pipeline could not be used. We **highlight** the best results for
 460 each dataset and report the mean and standard error for each metric.

Dataset	Metric	ORIG.	TabRGN	ClavaDDPM	RCTGAN	REALTABF.	SDV	RelDiff	Improv.
Rossmann	MAE (↓)	178 (324)	229 \pm 3	193 \pm 0.70	224 \pm 4	256 \pm 35	3428 \pm 0.01	193 \pm 1	0.0
Walmart	MAE (↓)	9531 (14.7k)	13848 \pm 14	11426 \pm 1052	13435 \pm 416	13862 \pm 300	13679 \pm 87	10475 \pm 1379	8.32
Airbnb	AUC (↑)	0.69 (0.5)	0.66 \pm 0.01	0.51 \pm 0.03	0.63 \pm 0.01	-	0.57 \pm 0.00	0.66 \pm 0.01	0.0
Berka	AUC (↑)	0.81 (0.5)	0.59 \pm 0.23	0.52 \pm 0.16	-	-	-	0.84 \pm 0.02	42.4
Fl	AUC (↑)	0.77 (0.5)	0.38 \pm 0.06	0.45 \pm 0.06	0.48 \pm 0.01	-	0.52 \pm 0.06	0.72 \pm 0.01	38.5

6 CONCLUSION

468 In this work, we introduced RELDIFF, a novel
 469 diffusion-based generative framework designed
 470 for synthesizing complete relational databases
 471 by explicitly modeling their inherent foreign
 472 key graph structure. Our approach uniquely
 473 combines a joint graph-conditioned diffusion
 474 process for attribute synthesis across all inter-
 475 connected tables with our *D2K+SBM* graph
 476 generator for structure creation. This principled
 477 decomposition ensures both high fidelity in the
 478 generated data and strict adherence to referential
 479 integrity, addressing key limitations of existing
 480 relational data synthesis methods that often flatten
 481 the relational structure or impose restrictive
 482 assumptions.

483 Through extensive experiments on 11 bench-
 484 mark datasets, RELDIFF consistently achieves
 485 state-of-the-art performance. Our framework
 486 effectively captures complex structural and sta-
 487 tistical dependencies, leading to synthetic data



488 **Figure 3: Comparing single and multi-table**
 489 **C2ST performance.** As opposed to previous meth-
 490 ods, our approach incurs only a slight degra-
 491 dation between average multi-table (C2ST-Agg) and
 492 single-table (C2ST) performance indicated by the
 493 relational $\Delta = \text{Acc}(\text{C2ST-Agg}) - \text{Acc}(\text{C2ST})$.

486 that better reflects the intricacies of real-world relational data. This advancement holds significant
 487 promise for various downstream applications, including privacy-preserving data sharing, data
 488 augmentation for relational learning tasks, and imputation of missing values in complex relational
 489 datasets.

490

491 REPRODUCIBILITY STATEMENT

492

493 We provide the full implementation of our method as supplementary material as well as pseudocode
 494 of the training procedure in Algorithm 1. Detailed descriptions of the datasets, preprocessing steps,
 495 model architecture, training procedure, and evaluation setup are included in Appendix B. Our goal is
 496 to ensure full transparency and to facilitate reproducibility in machine learning research.

497

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741 A RELATED WORK OVERVIEW

742 Here we present a more detailed overview of synthetic relational database generation approaches.
 743 The Synthetic Data Vault (SDV) (Patki et al., 2016) introduced the first learning-based method for
 744 generating relational databases. They introduce a hierarchical modeling approach using Gaussian
 745 Copulas, incorporating recursive conditional parameter aggregation to preserve relational structure.

746 Mami et al. (2022) proposed GraphVAE, a graph-based approach leveraging graph variational
 747 autoencoders. They represent relational databases as a single homogeneous graph where all rows
 748 become nodes. This contrasts with our approach, which uses a heterogeneous graph to better reflect

756 the distinct structure and relationships of the table. Foreign keys are implicitly handled by establishing
 757 edges between primary and secondary table rows within this homogeneous graph representation. The
 758 GraphVAE then uses message-passing layers during both encoding and decoding to model inter-table
 759 interactions and generate attribute values for synthetic rows.

760 Building upon the Conditional Tabular GAN (CT-GAN) method by Xu et al. (2019), two GAN-based
 761 methods have been proposed for relational data synthesis. Row Conditional-TGAN (RCTGAN) (Gu-
 762 eye et al., 2023) extends CT-GAN by integrating hierarchical dependencies, enabling the conditional
 763 synthesis of child tables based on their parent and grandparent rows. The Incremental Relational Gen-
 764 erator (IRG) (Li et al., 2024a) synthesizes relational databases through a sequential table generation
 765 process that follows a topological ordering. It constructs an extended table by integrating context
 766 from all relevant previously generated or related tables.

767 The transformer-based approach REaLTabFormer (Solorio & Dupriez, 2023) focuses on single-
 768 parent relational databases. It employs a GPT-2 encoder with a causal language model head to
 769 independently model the parent table. For dependent tables, a sequence-to-sequence (Seq2Seq)
 770 transformer is utilized, leveraging the frozen parent model as context. All attributes are transformed
 771 into a common vocabulary; however, this approach inherits the limitations of language models,
 772 particularly concerning the accurate modeling of numerical data.

773 Xu et al. (2023) introduced a framework for modeling many-to-many datasets using multipartite
 774 graphs under differential privacy. Their method utilizes a factorization of the joint data distribution,
 775 combining techniques from random graph generation to model structure with graph representation
 776 learning methods to conditionally generate tables based on node embeddings. Our approach builds
 777 upon their work by specifically capturing hierarchical characteristics during graph generation, mod-
 778 eling tables jointly rather than sequentially, and addressing general relational databases beyond
 779 many-to-many relationships.

780 Diffusion models have also been adapted for relational synthesis. ClavaDDPM (Pang et al., 2024)
 781 integrates clustering-guided diffusion models to preserve foreign-key dependencies, utilizing Gaussian
 782 mixture models to encode inter-table dependencies. Similarly, RGCLD (Hudovernik, 2024) uses
 783 conditional latent diffusion models, using a heterogeneous graph representation and GNNs to encode
 784 table relationships, which then guide the diffusion process within the latent space. A key limitation
 785 of both methods is their sequential modeling of tables, which introduces implicit assumptions on
 786 inter-table dependencies and may allow errors to propagate down the hierarchy during generation.

787 Finally, auto-regressive models have been explored for tabular and sequential relational synthesis.
 788 TabularARGN (Tiwald et al., 2025) employs an any-order auto-regressive network, trained on
 789 discretized attributes, to model conditional dependencies. While it specializes in single-table and
 790 sequential database modeling, TabularARGN also supports multi-parent schemas by preserving
 791 certain dependencies using context tables and maintaining referential integrity for the remaining
 792 relationships.

793 The marginal-based approaches for synthetic relational database generation primarily focus on
 794 preserving marginal queries, typically with differential privacy guarantees. PrivLava (Cai et al., 2023)
 795 synthesizes relational databases by modeling foreign key relationships as a directed acyclic graph with
 796 latent variables, generating tables incrementally. MARE (Kapenekakis et al., 2024), specializing in
 797 medical relational data, employs correlation partially directed acyclic graphs (CPDAGs) for selective
 798 correlation modeling and orchestrates two-phase data sampling. Alimohammadi et al. (2025) propose
 799 an approach to adapt single-table differentially private generators to relational data by learning
 800 a weighted bi-adjacency matrix to generate the relational structure. Finally, PrivPetal (Cai et al.,
 801 2025) synthesizes a flattened relational database using normalized permutation marginals and then
 802 iteratively decomposes it by sampling attributes of reference relations.

803 We acknowledge two concurrent works GCFM (Scassola et al., 2025) and GRDM (Ketata et al.,
 804 2025), which also approach relational data synthesis as a graph-based modeling task. All three
 805 methods move beyond sequential table generation to jointly model the database schema using graph-
 806 based generative models. While the overarching paradigms are similar, the methods differ in their
 807 generative backbones and structural assumptions. Regarding the generative mechanism, GRDM
 808 applies a Gaussian DDPM uniformly to both numerical and categorical features, whereas GCFM
 809 utilizes flow matching. In terms of structure generation, GRDM introduces a degree-preserving

random graph model, while GCFM utilizes the original graph or samples with replacement from the connected components of the graph.

We classify related work based on how it approaches attribute and structure generation in Table 4.

Table 4: **Classification of related work** based on the mechanisms for attribute and structure modeling as well as the general synthetic data generation (SDG) classification.

Method	Attribute Modeling	Structure Modeling	SDG Class
SDV (Patki et al., 2016)	conditional tabular	as attributes	statistical
GraphVAE (Mami et al., 2022)	homogeneous graph-based	retains original structure	neural
RCTGAN (Gueye et al., 2023)	conditional tabular	as attributes	neural
IRG (Li et al., 2024a)	conditional tabular	as attributes	neural
REaLTabFormer (Solatorio & Dupriez, 2023)	conditional tabular	sequential modeling	neural
ClavaDDPM (Pang et al., 2024)	conditional tabular	conditional modeling + matching	neural
RGCLD (Hudovernik, 2024)	conditional tabular	retains original structure	neural
TabularARGN (Tiwaldi et al., 2025)	conditional tabular	sequential modeling	neural
BayesM2M, NeuralM2M (Xu et al., 2023)	conditional tabular	random graphs (BJDD)	neural & marginal
PrivLava (Cai et al., 2023)	conditional tabular	as attributes	marginal
MARE (Kapenekakis et al., 2024)	conditional tabular	sequential modeling	marginal
DP-Relational (Alimohammadi et al., 2025)	independent tabular	weighted bi-adjacency matrix	marginal
PrivPetal (Cai et al., 2025)	flattened tabular	flattening connected tables	marginal
Ours	heterogeneous graph-based	<i>D2K</i> + SBM random graphs	neural

B DETAILED EXPERIMENT SETUP

This section provides a comprehensive overview of our experimental setup, detailing the datasets (Appendix B.1) utilized and the evaluation metrics (Appendix B.2) employed.

B.1 DATASETS

Here we describe the datasets used in our evaluation. Table 5 provides detailed statistics for each dataset. The MovieLens and Berka datasets are used in both benchmarks so we only describe them once.

Rossmann Store Sales: The Rossmann Store Sales dataset (FlorianKnauer, 2015) features historical sales data for 1115 stores, organized into two connected tables.

Airbnb: The Airbnb dataset (Montoya et al., 2015) contains anonymized user interactions and demographics for predicting travel destinations. It comprises multiple tables detailing user sessions and summary information.

Walmart: The Walmart dataset (Walmart, 2014) contains historical sales data for 45 stores across three connected tables, including store details, features, and department sales.

Cora: The Cora dataset (McCallum et al., 2000) is a graph benchmark of 2708 academic papers classified into seven categories, linked by a citation network of 5429 relationships. Unlike the graph representation learning version with one-hot encoded node features, this relational version stores paper content in a separate table connected via a foreign key, and citation links are represented in a dedicated foreign-key-only table.

Biodegradability: The Biodegradability dataset (Blockeel et al., 1999) is a collection of 328 chemical compounds with biodegradation half-life labels, intended for regression analysis based on chemical features.

IMDB MovieLens: The IMDB MovieLens dataset (Harper & Konstan, 2015) includes information on movies, actors, directors, user ratings, and related details across seven tables.

Berka: The Berka dataset (Berka et al., 2000) is a real-world financial dataset focused on loan outcomes, encompassing loan details and transaction histories across multiple tables. For the SyntheRela benchmark, this dataset is split temporally to facilitate the evaluation of RDL utility.

F1: The F1 dataset (F1, 2021) contains historical Formula 1 racing data and statistics from 1950 onwards, covering drivers, races, and results across numerous tables.

California: The California dataset is a real-world anonymized census database (M. Center, 2020) on household information. It consists of two tables in the form of a basic parent-child relationship.

Instacart 05: The Instacart 05 is created by downsampling 5-percent from the Kaggle competition dataset Instacart (Stanley et al., 2017), which is a real-world transaction dataset of Instacart orders. This dataset consists of 6 tables in total with a maximum depth of 3.

CCS: The CCS dataset (Motl & Schulte, 2025) is a real-world transactional dataset Czech debit card company. It consists of 5 tables with a maximum depth of 2.

Table 5: Summary of the 11 benchmark datasets. The number of columns represents the number of non-id columns (The MovieLens and Berka datasets appear in both benchmarks). The collection is diverse and covers all types of relational structures.

Dataset Name	# Tables	# Rows	# Columns	# Relationships	Max Depth	Hierarchy Type
Rossmann	2	59,085	16	1	2	Linear
AirBnB	2	57,217	20	1	2	Linear
Walmart	3	15,317	17	2	2	Multi Child
Cora	3	57,353	2	3	2	Multi Child
Biodegradability	5	21,895	6	5	4	Multi Child & Parent
IMDB MovieLens	7	1,249,411	14	6	2	Multi Child & Parent
Berka	8	757,722	37	8	4	Multi Child & Parent
F1	9	74,063	33	13	3	Multi Child & Parent
California	2	2,076,141	25	1	2	Linear
CCS	5	423,050	11	4	2	Multi Child & Parent
Instacart 05	6	1,906,353	12	6	3	Multi Child & Parent
Berka	8	1,079,680	41	8	4	Multi Child & Parent
MovieLens	7	1,249,411	14	6	2	Multi Child & Parent

B.2 METRICS

B.2.1 SHAPE AND TREND SCORES

Shape and Trend are proposed by the commonly used synthetic data evaluation library SDMetrics¹. They are used to measure the column-wise density estimation performance and pair-wise column correlation estimation performance, respectively. Shape uses Kolmogorov-Sirnov test (KST) for numerical columns and the total variation distance (TVD) for categorical columns to quantify column-wise density estimation. Trend uses Pearson correlation for numerical columns and contingency similarity for categorical columns to quantify pair-wise correlation.

Shape. KST: Given two (continuous) distributions $p_r(x)$ and $p_s(x)$ (r denotes real and s denotes synthetic), KST quantifies the distance between the two distributions using the upper bound of the discrepancy between two corresponding cumulative distribution functions (CDFs):

$$KST = \sup_x |F_r(x) - F_s(x)|, \quad (7)$$

where $F_r(x)$ and $F_s(x)$ are the CDFs of $p_r(x)$ and $p_s(x)$, respectively:

$$F(x) = \int_{-\infty}^x p(x)dx. \quad (8)$$

TVD is defined as half the sum of the absolute differences between the real and synthetic probabilities across all categories:

$$TVD = \frac{1}{2} \sum_{\omega \in \Omega} |R(\omega) - S(\omega)|, \quad (9)$$

where ω describes all possible categories in a column Ω . $R(\cdot)$ and $S(\cdot)$ denotes the real and synthetic frequencies of these categories. To comply with previous work on relational data synthesis, we report the complement of the KST and TVD distances ($D(P||Q)$) as $1 - D_{KST/TVD}(P||Q)$

¹<https://docs.sdv.dev/sdmetrics>

918 **Trend.** The *Pearson correlation coefficient* measures whether two continuous distributions are
 919 linearly correlated and is computed as:
 920

$$921 \quad \rho_{x,y} = \frac{\text{Cov}(x,y)}{\sigma_x \sigma_y}, \quad (10)$$

923 where x and y are two continuous columns. Cov is the covariance, and σ is the standard deviation.
 924

925 Then, the performance of correlation estimation is measured by the average differences between the
 926 real data correlations and the synthetic data correlations:
 927

$$928 \quad \text{Pearson Score} = 1 - \frac{1}{2} \mathbb{E}_{x,y} [|\rho^R(x,y) - \rho^S(x,y)|], \quad (11)$$

930 where $\rho^R(x,y)$ and $\rho^S(x,y)$ denotes the Pearson correlation coefficient between column x and
 931 column y of the real data and synthetic data, respectively. As $\rho \in [-1, 1]$, the average distance of
 932 Pearson coefficients is divided by 2, to ensure that it falls in the range of $[0, 1]$, and subtracted from 1
 933 such that the larger the score, the better the estimation.
 934

935 *Contingency similarity:* For a pair of categorical columns A and B , the contingency similarity score
 936 computes the difference between the contingency tables using the TVD. The process is summarized
 937 by the formula below:
 938

$$939 \quad \text{Contingency Score} = 1 - \frac{1}{2} \sum_{\alpha \in A} \sum_{\beta \in B} |R_{\alpha,\beta} - S_{\alpha,\beta}|, \quad (12)$$

940 where α and β describe all the possible categories in column A and column B , respectively. $R_{\alpha,\beta}$
 941 and $S_{\alpha,\beta}$ are the joint frequency of α and β in the real data and synthetic data, respectively. To obtain
 942 a score, the distance is again subtracted from 1.
 943

944 B.2.2 K-HOP TREND

945 The k-hop Trend metric, proposed by (Pang et al., 2024), extends the Trend metric to evaluate the
 946 preservation of correlations across multiple tables connected by foreign keys. The 0-hop Trend is
 947 equivalent to the standard Trend metric, measuring pairwise column correlations within a single table.
 948 For $k > 0$, the k-hop Trend assesses correlations across tables reachable within k foreign key hops.
 949 This is achieved through a series of join operations:
 950

- 951 • 1-hop: Refers to the correlation between two columns in tables that are directly linked by a
 952 foreign key. To calculate this, we join the two related tables based on the foreign key and
 953 then compute the Trend metric on the resulting joined table.
 954
- 955 • ($k > 1$)-hop: Extends the process iteratively. Based on a foreign key sequence of length
 956 k , the tables are recursively joined. The Trend metric is then computed on the final joined
 957 table.
 958

959 As in the Trend metric, the Pearson correlation coefficient and contingency similarity are used to
 960 quantify the differences in joint distributions depending on the data type. The final k-hop Trend score
 961 is the average of these correlation/similarity scores across all relevant k-hop relationships within
 962 the database schema. Similar to the 0-hop Trend, a higher score indicates a better preservation of
 963 inter-table correlations up to k hops.
 964

964 B.2.3 CARDINALITY SIMILARITY

965 Given a parent table P with a primary key pk and a child table C with a foreign key fk referencing
 966 pk , this metric evaluates the similarity of the cardinality distribution between the real and synthetic
 967 datasets. The cardinality of a parent row $p \in P$ is defined as the number of child rows $c \in C$ for
 968 which $c.fk = p.pk$.
 969

970 Let $card_R(p)$ and $card_S(p)$ denote the cardinality of a parent row p in the real and synthetic
 971 datasets, respectively. This yields two numerical distributions: $D_R = \{card_R(p) \mid p \in P_{\text{real}}\}$ and
 $D_S = \{card_S(p) \mid p \in P_{\text{synthetic}}\}$.
 972

972 The cardinality similarity score is computed as the complement of the Kolmogorov-Smirnov statistic,
 973 defined as $1 - \text{KST}(D_R, D_S)$, where KST refers to the Kolmogorov-Smirnov Test statistic as defined
 974 in Appendix B.2.1. Cardinality similarity ranges from 0 to 1, where a score of 1 indicates identical
 975 cardinality distributions in the real and synthetic data, and a score of 0 indicates maximally different
 976 distributions.

977 B.2.4 C2ST

978 The Classifier Two-Sample Test (C2ST) evaluates the fidelity of synthetic data by training a discriminator
 979 (in our case, an XGBoost model) to distinguish it from real data. This detection-based approach,
 980 rooted in two-sample testing, uses the classifier's performance as a proxy for distributional similarity.
 981 If the discriminator achieves better-than-random accuracy, it indicates discernible differences between
 982 the real and synthetic datasets, suggesting lower fidelity. C2ST offers a comprehensive assessment
 983 of single table fidelity that captures complex higher-order dependencies between features beyond
 984 simple correlations when using expressive-enough models, as highlighted in (Zein & Urvoy, 2022).
 985

986 B.2.5 C2ST-AGG

987 The Classifier Two-Sample Test with Aggregations (C2ST-Agg) (Jurkovič et al., 2025) extends
 988 detection-based fidelity evaluation to relational data by capturing inter-table relationships. C2ST-Agg
 989 functions by augmenting parent tables with aggregated features derived from their connected child
 990 tables. This propositionalization approach, drawing inspiration from relational reasoning techniques,
 991 enables the C2ST to evaluate the preservation of interactions between columns in connected tables,
 992 as well as relationship cardinalities. Conceptually, C2ST-Agg assesses how well fundamental SQL
 993 operations such as join and groupby are maintained alongside complex interactions both within
 994 and between tables. By summarizing child-table information using aggregation functions (e.g.,
 995 mean, count, max), C2ST-Agg effectively accounts for both relationship cardinality and high-level
 996 interactions across related tables, offering a comprehensive assessment of relational data fidelity.
 997

998 In our evaluation we use the aggregation functions used by the original authors *mean* - for nu-
 999 mercial attributes, *count* of connected rows and *count distinct* for categorical variables and use an
 1000 XGBoost (Chen & Guestrin, 2016) model with k=5 fold cross-validation as the discriminative model.
 1001

1002 B.2.6 RDL UTILITY

1003 The relational deep learning utility (RDL utility) (Jurkovič et al., 2025) metric evaluates the capacity
 1004 of synthetic relational data to support effective learning on downstream tasks. This metric adheres to
 1005 the RelBench (Robinson et al., 2024) framework, which transforms relational databases into temporal
 1006 heterogeneous graphs with predefined tasks specifically designed for GNN training.
 1007

1008 To assess utility, the RDL utility metric employs the RelBench GNN pipeline. A heterogeneous
 1009 variant of the GraphSage model (Hamilton et al., 2017) is trained on both the real and synthetic data.
 1010 These trained models are subsequently evaluated on a dedicated test set composed entirely of real
 1011 data.

1012 Data splitting is handled via a time-based splitting strategy. Our evaluation incorporates five datasets
 1013 from the SyntheRela benchmark that possess a temporal feature, with the following predictive tasks:

- 1014 • Rossmann: Prediction of daily number of customers for each store and date.
- 1015 • Walmart: Prediction of weekly sales for each department within each store.
- 1016 • Airbnb: Binary prediction of whether a user has previously made a booking.
- 1017 • Berka: Prediction of the binary loan status (successful or unsuccessful).
- 1018 • F1: Prediction of whether a driver will qualify in the top-3 for a race in the next month.

1019 All datasets and models utilize the default RelBench hyperparameters.
 1020

1021 B.2.7 DISTANCE TO CLOSEST RECORD

1022 The distance to closest record (DCR) (Zhao et al., 2021) evaluates privacy protection by measuring
 1023 how closely synthetic data resembles the training data. The DCR score (Zhang et al., 2024) quantifies

1026 the fraction of synthetic records whose nearest neighbor is in the training set; a value near 0.5 suggests
 1027 the model samples from the true distribution rather than overfitting.
 1028

1029 Nearest neighbor distances are calculated using the l^2 norm of synthetic, training, and holdout records.
 1030 For a synthetic record i , distances to its nearest neighbors in training (N_{trn}) and holdout (N_{hold})
 1031 datasets are:

$$d(i)_{\text{trn}} = \min_{j \in N_{\text{trn}}} \| \text{syn}_i - \text{trn}_j \|_2, \quad d(i)_{\text{hold}} = \min_{j \in N_{\text{hold}}} \| \text{syn}_i - \text{hold}_j \|_2.$$

1032 An indicator function, $I(i)_{\text{trn}}$, determines if the nearest neighbor of synthetic record i is in the training
 1033 set:

$$I(i)_{\text{trn}} = \begin{cases} 1 & \text{if } d(i)_{\text{trn}} < d(i)_{\text{hold}}, \\ 0 & \text{if } d(i)_{\text{trn}} > d(i)_{\text{hold}}, \\ 0.5 & \text{if } d(i)_{\text{trn}} = d(i)_{\text{hold}}. \end{cases}$$

1034 The DCR score is then computed as:

$$\text{DCR score} = \frac{1}{N_{\text{syn}}} \sum_{i=1}^{N_{\text{syn}}} I(i)_{\text{trn}}.$$

1044 B.3 MEMBERSHIP INFERENCE ATTACKS

1045 Membership inference attacks (MIA) are used to evaluate the privacy risk of a synthetic data generator.
 1046 An MIA measures the vulnerability of a model by assessing whether an adversary, with access to
 1047 both the synthetic data and real data from the same population, can infer if a specific record from the
 1048 real data was used in the training process of the synthetic data generator (El Emam et al., 2022). Our
 1049 evaluation uses the implementation provided by Lautrup et al. (2025), which assumes a scenario
 1050 where the adversary knows the population but has no prior knowledge of the synthesis algorithm.
 1051 We model the attacker as a LightGBM classifier (Shi et al., 2025b) trained to distinguish between
 1052 synthetic records and real records from an external holdout dataset. The classifier’s performance
 1053 in identifying whether a sample is real or synthetic serves as a proxy for the risk of membership
 1054 disclosure. We report both the *recall* and *precision* of the attacking model, where recall is analogous to
 1055 the membership inference risk. Similarly as with the DCR metric, we evaluate MIA at the individual
 1056 table level, as no multi-table implementation currently exists.

1058 C IMPLEMENTATION DETAILS

1059 We implemented RELDIFF in PyTorch. We performed our experiments on two Nvidia H100 GPUs
 1060 with 80G memory.

1061 **Data preprocessing.** Raw relational datasets often contain missing values. Our initial preprocessing
 1062 step involves imputing these, following approaches in (Pang et al., 2024; Shi et al., 2025a): numerical
 1063 missing values are replaced by the column average, and categorical missing values are treated as a
 1064 distinct new category. For the SyntheRela benchmark, we adopt a more nuanced approach for missing
 1065 values, similar to (Patki et al., 2016; Hudovernik, 2024). We introduce an additional binary indicator
 1066 variable for each attribute to explicitly denote whether a value was originally missing; this indicator
 1067 is modeled as a separate categorical variable, allowing us to recover the original missingness pattern
 1068 after sampling. To mitigate training instability caused by the diverse ranges of numerical columns,
 1069 we transform the numerical values with the QuantileTransformer² and recover the original values
 1070 after sampling.

1071 **Hyperparameters Setting.** RELDIFF employs a consistent hyperparameter setting across all datasets,
 1072 with the sole exception of the number of epochs and batch size, which is mainly dependent on
 1073 the graph structure. We train our models for 10000 epochs on most datasets. For larger network
 1074 datasets, specifically *Instacart 05* and *MovieLens*, we utilize a reduced number of epochs (400 and
 1075 4000 respectively) to manage computational load. We use the AdamW optimizer with learning rate
 1076 $\gamma = 6e - 4$ and weight decay $w = 1e - 5$ in all experiments.

1077
 1078 ²<https://scikit-learn.org/stable/modules/generated/sklearn.preprocessing.QuantileTransformer.html>

1080 Regarding the specific hyperparameters within RELDIFF, the values for σ_{\min} and σ_{\max} are set to
 1081 0.002 and 80.0, respectively, referencing the optimal setting in (Karras et al., 2022). The parameter
 1082 δ is set to $1e - 3$. For the loss weightings, we fix λ_{cat} to 1.0 and linearly decay λ_{num} from 1.0 to
 1083 0.0 as training proceeds. In all our experiments, the number of GNN layers is set to $k = 2$. During
 1084 inference, we select the checkpoint with the lowest training loss. And utilize 100 discretization steps
 1085 ($T = 100$) during sampling.

1086 **Model Architecture** Our model is parameterized by a heterogeneous graph neural network, featuring
 1087 transformer encoders and decoders and an MLP backbone. Each column is initially projected into a
 1088 d -dimensional vector using a linear layer, with $d = 4$, matching the size used in Zhang et al. (2024)
 1089 and Shi et al. (2025a). These tokenized columns are then processed by a two-layer transformer.
 1090 Subsequently, the concatenated columns are projected to a dimension of $\dim_h = 128$, to which noise
 1091 embeddings of the same dimensionality are added, consistent with the approach of Pang et al. (2024).

1092 The embeddings are then processed by a heterogeneous variant of the GraphSAGE network (Hamilton
 1093 et al., 2017), which is used as the RDL baseline in Fey et al. (2024). For databases containing records
 1094 at fixed time intervals, and given our use of a permutation-invariant GNN, positional encodings are
 1095 added to the embeddings before message passing to preserve record order. The GNN embeddings
 1096 for each table are then further processed by five-layer MLPs, conditioned on a time embedding. The
 1097 size of these MLPs for each table is comparable to those used in experiments by Kotelnikov et al.
 1098 (2023). Finally, the hidden representation is decoded back into the data space by another two-layer
 1099 transformer. It is worth noting that the MLP backbone accounts for the majority of the model's
 1100 parameters, and the memory consumed by these parameters is typically less than that used by the
 1101 intermediate data representations, especially since relational databases often contain larger tables
 1102 than typical tabular datasets.

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D ADDITIONAL EXPERIMENTS

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D.1 SINGLE TABLE FIDELITY RESULTS

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1108

1109 **Table 6: Single-table results.** For each dataset and metric we report the average detection accuracy
 1110 (C2ST - lower is better), column shapes and column pair trends (Shape, Trend - higher is better)
 1111 across all tables for three independent samples. DNC denotes *Did Not Converge* and "-" denotes
 1112 a method is unable to generate the dataset. The best result is **bolded**. We report the percentage
 1113 improvement of RELDIFF over the state-of-the-art in [blue](#).

1114

	Metric	TabARGN	ClavaDDPM	RCTGAN	REALTABF.	SDV	RelDiff	Improv.
Airbnb	C2ST (↓)	64.23 \pm 0.20	78.10 \pm 0.03	88.37 \pm 0.14	83.97 \pm 4.36	99.75 \pm 5e-3	54.11 \pm 0.34	15.76
	Shape (↑)	95.70 \pm 0.05	94.42 \pm 0.01	89.18 \pm 0.17	71.66 \pm 0.92	59.37 \pm 0.04	98.14 \pm 0.07	2.55
	Trend (↑)	93.48 \pm 0.33	87.78 \pm 0.12	79.37 \pm 0.29	53.90 \pm 1.26	49.03 \pm 0.08	95.76 \pm 0.25	2.43
Rossmann	C2ST (↓)	56.07 \pm 0.58	66.77 \pm 0.14	88.02 \pm 0.50	74.70 \pm 0.55	96.90 \pm 0.21	52.46 \pm 0.32	6.44
	Shape (↑)	96.96 \pm 0.19	94.05 \pm 0.07	91.31 \pm 0.04	90.65 \pm 0.38	81.05 \pm 0.19	98.04 \pm 0.07	1.12
	Trend (↑)	91.34 \pm 0.08	84.78 \pm 0.80	84.38 \pm 0.40	84.58 \pm 0.88	67.77 \pm 0.25	95.93 \pm 0.65	5.02
Walmart	C2ST (↓)	83.54 \pm 0.84	53.50 \pm 1.95	76.40 \pm 0.55	70.87 \pm 1.07	87.02 \pm 0.81	60.30 \pm 0.50	0.0
	Shape (↑)	89.09 \pm 0.33	92.21 \pm 0.52	82.31 \pm 0.51	81.71 \pm 0.40	81.80 \pm 0.10	94.04 \pm 0.53	1.99
	Trend (↑)	83.89 \pm 0.21	94.02 \pm 0.14	86.60 \pm 0.25	83.10 \pm 0.46	87.61 \pm 0.23	95.42 \pm 0.45	1.49
Berka	C2ST (↓)	72.31 \pm 0.17	54.48 \pm 0.11	68.12 \pm 0.44	-	82.40 \pm 0.33	50.23 \pm 0.05	7.80
	Shape (↑)	82.20 \pm 0.26	91.62 \pm 0.10	81.90 \pm 0.38	-	56.27 \pm 0.29	97.72 \pm 0.03	6.65
	Trend (↑)	70.43 \pm 0.25	88.54 \pm 1.19	74.22 \pm 0.28	-	64.01 \pm 0.11	98.81 \pm 0.02	11.59
F1	C2ST (↓)	81.93 \pm 0.49	71.42 \pm 0.46	80.67 \pm 0.31	-	89.84 \pm 0.22	63.50 \pm 0.03	11.09
	Shape (↑)	84.71 \pm 1.15	84.63 \pm 0.28	89.68 \pm 0.40	-	52.62 \pm 0.57	94.89 \pm 0.08	5.81
	Trend (↑)	81.31 \pm 0.45	84.65 \pm 0.05	90.17 \pm 0.03	-	73.05 \pm 0.19	95.10 \pm 0.12	5.46
IMDB	C2ST (↓)	50.92 \pm 0.22	49.83 \pm 0.07	55.38 \pm 0.11	-	-	52.07 \pm 0.36	0.0
	Shape (↑)	98.40 \pm 0.14	99.01 \pm 0.05	92.70 \pm 0.09	-	DNC	96.91 \pm 0.49	0.0
	Trend (↑)	97.80 \pm 0.13	98.66 \pm 0.10	81.65 \pm 0.03	-	-	93.88 \pm 0.87	0.0
Biodeg.	C2ST (↓)	58.79 \pm 0.19	-	58.26 \pm 0.15	-	68.59 \pm 0.14	48.28 \pm 0.20	17.13
	Shape (↑)	90.85 \pm 0.14	-	90.91 \pm 0.40	-	79.46 \pm 0.47	95.95 \pm 0.11	5.55
	Trend (↑)	74.82 \pm 0.35	-	85.44 \pm 2.19	-	97.58 \pm 0.50	99.38 \pm 0.13	1.85
Cora	C2ST (↓)	50.94 \pm 0.37	-	48.97 \pm 0.14	-	75.45 \pm 0.16	54.03 \pm 0.61	0.0
	Shape (↑)	92.65 \pm 0.12	-	96.38 \pm 0.15	-	50.24 \pm 0.17	87.85 \pm 1.22	0.0

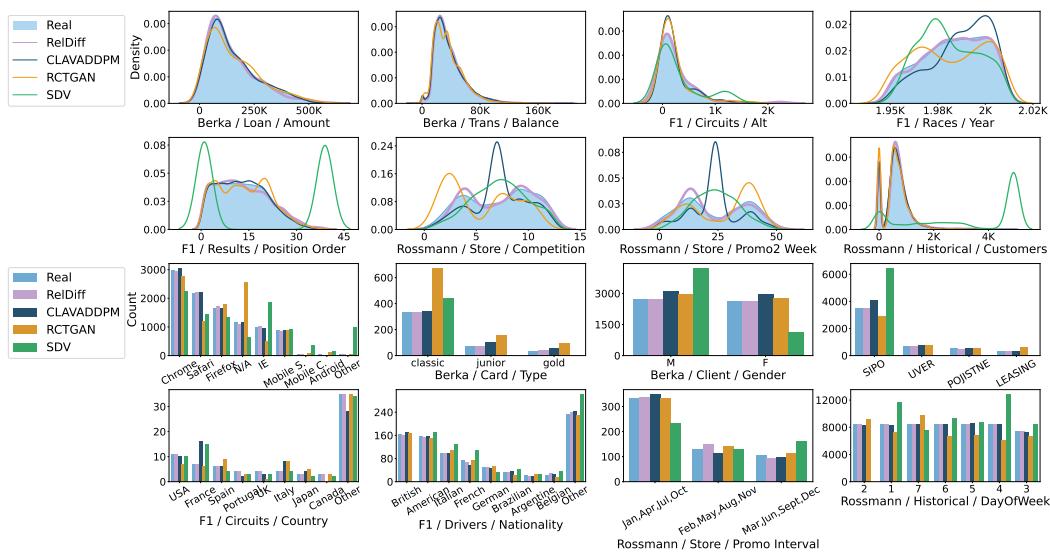
1133

1134
 1135 In this section, we present detailed single-table fidelity results on the SyntheRela benchmark. We
 1136 evaluate the Shape, Trend, and C2ST scores. As shown in Table 6, RELDIFF consistently demonstrates
 1137 the strongest performance. When employing the original structure, RELDIFF is only outperformed on
 1138 the IMDB and Cora datasets, indicating a trade-off where some single-table fidelity is exchanged for
 1139 superior multi-table fidelity. This behavior on the IMDB dataset can be attributed to structural motifs
 1140 that may cause bottlenecks in GNN message passing. Notably, our approach can achieve near-perfect
 1141 performance on Cora if its schema is normalized to third normal form.

1142 D.2 VISUALIZATION OF GENERATED DATA

1143 To complement our quantitative evaluation and offer a qualitative evaluation of generation quality, we
 1144 provide visual comparisons between the real and synthetic data distributions.

1145 Figure 4 presents the column-wise feature densities for selected tables. We utilize kernel density
 1146 estimation (KDE) curves for numerical features and histograms for categorical features. The plots
 1147 demonstrate that the marginal densities produced by RELDIFF consistently align closely with the real
 1148 data distributions, visually corroborating RELDIFF’s superior performance on the Shape metric.



1149
 1150 Figure 4: **Visualizations of column-wise densities.** The horizontal axis labels stand for “<Dataset> /
 1151 <Table> / <Feature>”.

1152 In Figure 5, we visualize the pairwise column correlations within single tables. As illustrated by the
 1153 plots, RELDIFF faithfully captures the correlations in the data, exhibiting a higher degree of fidelity
 1154 compared to the baseline methods.

1155 Finally, we visualize long-range dependencies in Figure 6. These plots highlight RELDIFF’s ability to
 1156 preserve statistical interactions between columns across connected tables, validating the effectiveness
 1157 of our joint graph-based diffusion process.

1158 D.3 PRIVACY EVALUATION

1159 **Distance to Closest Record (DCR).** We follow related work (Kotelnikov et al., 2023; Zhang et al.,
 1160 2024; Shi et al., 2025a; Pang et al., 2024) to perform a privacy sanity check against SMOTE (Chawla
 1161 et al., 2002), an interpolation-based method that generates new data through convex combinations
 1162 of real data points. To quantify the privacy level, we evaluate the distance to closest record (Zhao
 1163 et al., 2021). Specifically, we compare the DCR distributions of RELDIFF against SMOTE on two
 1164 datasets, adhering to the evaluation protocol of Pang et al. (2024): *California*, a real-world census
 1165 dataset containing anonymized household and individual information, and a subset of tables from the
 1166 *Berka* dataset, which holds anonymized financial information from a Czech bank. The results of the
 1167 DCR score (Zhang et al., 2024) are presented in Table 7.

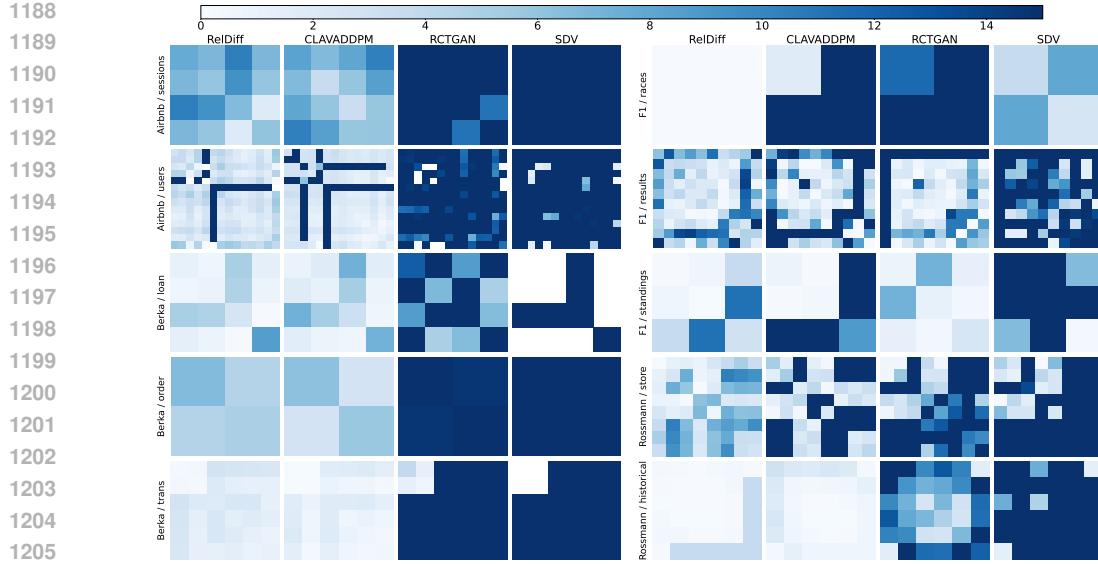


Figure 5: **Visualizations of single-table column-wise correlations.** The shades represents the error rate (i.e., difference in Trend metric), and the vertical axis labels stand for “<Dataset> / <Table>”.

Table 7: **DCR score** represents the probability that a synthetic example is closer to the training set rather than the holdout set (% , a score closer to 50% is better).

Method	Table	Household	Individual	Transaction	Order
SMOTE		77.22 \pm 0.0	76.25 \pm 0.0	99.94 \pm 0.0	99.40 \pm 0.0
ClavaDDPM		50.23 \pm 0.0	50.31 \pm 0.0	53.47 \pm 0.0	91.84 \pm 0.0
RelDiff		50.54 \pm 0.0	50.45 \pm 0.0	50.71 \pm 0.0	52.38 \pm 0.0

RELDIFF consistently achieves DCR scores around 50%. This outcome is indicative of the model’s ability to sample from the underlying data distribution rather than memorizing the training data.

Membership Inference Attack (MIA).

The results of the membership inference attack (MIA) are summarized in Table 8, which reports the *Macro F1 score* of an adversarial classifier. The Macro F1 aggregates precision and recall across both the member and non-member classes, providing a balanced measure of the attack’s effectiveness even under class imbalance (a score of 50% corresponds to random guessing).

Table 8: **MIA Macro F1** denotes the effectiveness of an adversarial classifier in distinguishing training members from non-members. We report means with standard errors; values near or below 50% indicate performance near random guessing and thus limited privacy risk.

Method	Table	Household	Individual	Transaction	Order
SMOTE		50.99 \pm 0.05	50.59 \pm 0.05	63.23 \pm 0.21	59.28 \pm 0.62
CLAVADDPM		49.59 \pm 0.07	50.08 \pm 0.04	51.70 \pm 0.20	55.89 \pm 0.35
RelDiff		48.81 \pm 0.14	48.7 \pm 0.07	49.01 \pm 0.05	51.55 \pm 0.35

In principle, if a generative model were perfectly private, the expected F1 score of an attacker would be close to 50%. In our experiments, the observed values cluster tightly around this baseline and are consistently lower than those of competing methods, indicating limited membership leakage. On three tables, the F1 scores are even slightly below 50%, which can be explained by the presence of a small fraction of outlier values generated by our model. Since such values never occur in the original data, the attacker learns to separate them from the holdout distribution, but this signal is not informative for true membership inference. Overall, our privacy evaluation shows strong empirical privacy protection.

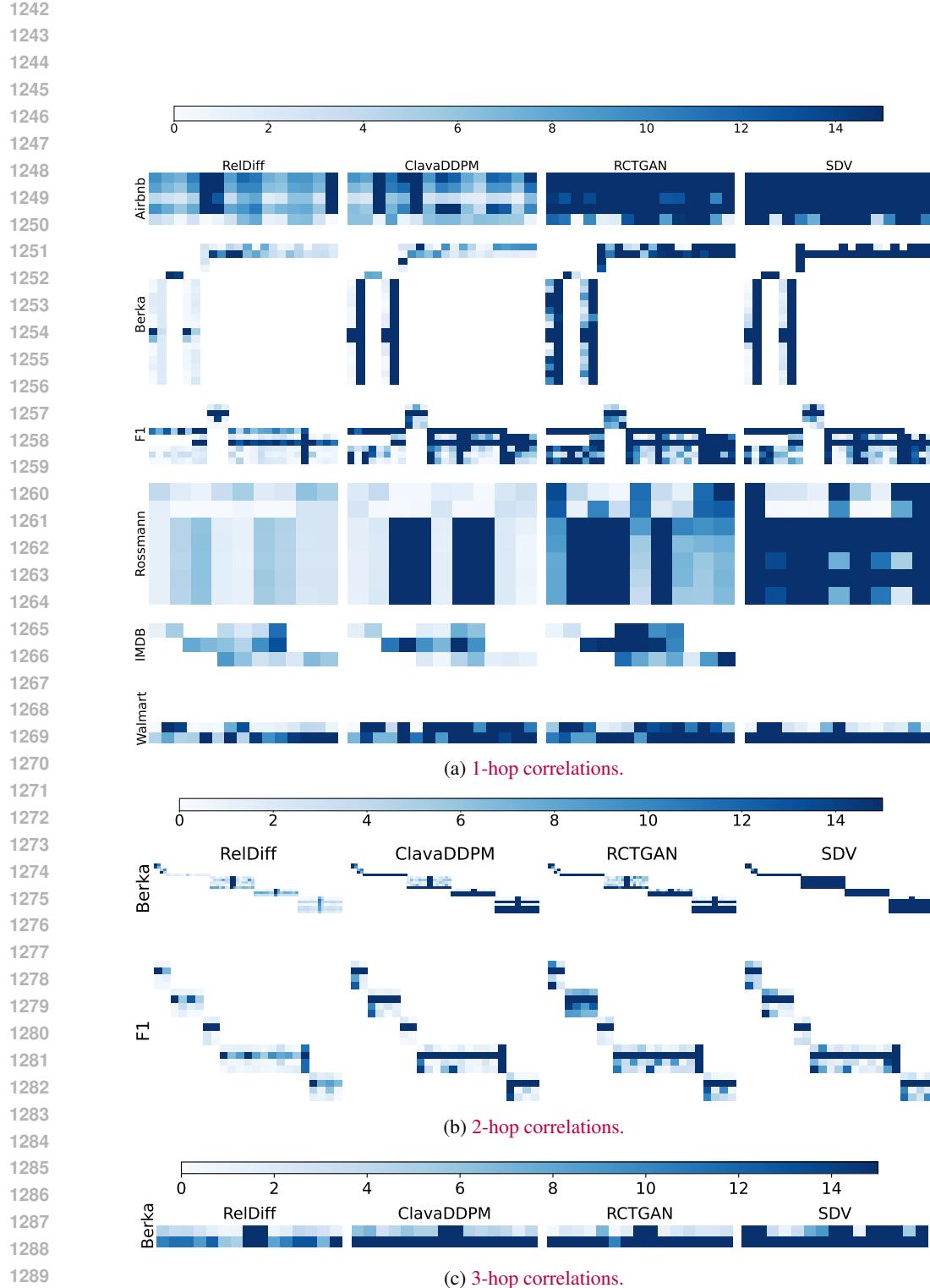


Figure 6: **Visualizations of k-hop correlations** for $k \in \{1, 2, 3\}$. The shades represent the error rate (i.e., difference in k-hop Trend metrics between columns in tables at distance k).

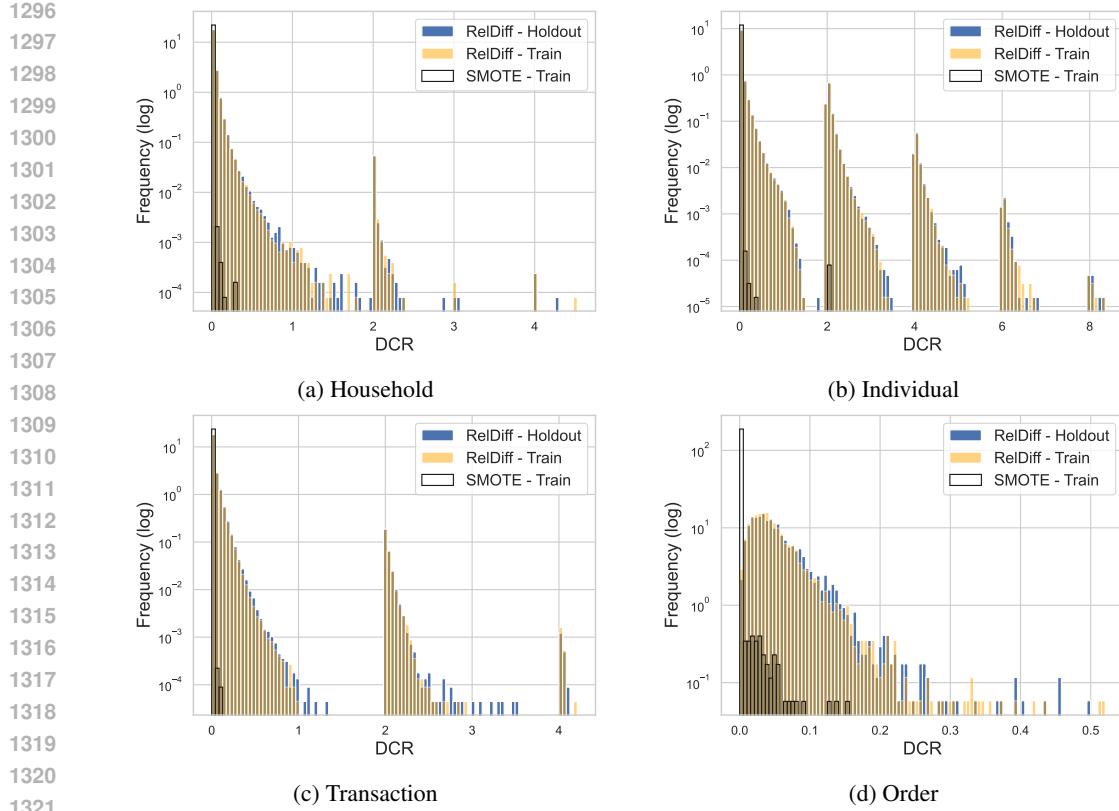


Figure 7: **DCR distributions** on the *California* and *Berka* datasets (log-transformed y-axis). RELDIFF exhibits DCR values for the training set that are significantly higher than SMOTE, indicating enhanced privacy protection. The distribution of DCR values for the held-out data remains consistent with that of the training data.

D.4 ABLATION STUDIES

We conducted an ablation study to evaluate the contribution of key components of our method. The results are summarized in Table 9.

Graph Generation Strategy. We conducted a series of ablations on our graph generation approach. We compared our full method, which uses the *D2K+SBM* generator, against two baselines: a simpler bipartite *2K* model and our method when provided with the original ground-truth graph (*original* column). For the simple two-table Airbnb dataset, both graph generators perfectly recover the original graph structure. However, as schema complexity increases for the Berka and F1 datasets, the effectiveness of our *D2K+SBM* generator becomes evident, as it significantly outperforms the bipartite *2K* variant. This highlights the importance of our approach for accurately modeling complex relational structures.

Model Architectures. Next, we ablate the GNN architectures to examine their importance in capturing inter-table relationships. When we remove the GNN entirely, the method reverts to a single-table generative model **with graph based sampling of rows and preserved relational structure**. While this variant preserves the distributions of individual columns, it fails to model long-range inter-table relationships, as evidenced by the significant performance drop on all HOP metrics. This confirms the GNN’s central role in our approach. We also evaluated a more complex, heterogeneous variant of the Graph Attention Network (GAT) as a replacement for our GNN. The results were mixed, with performance improving on some datasets while decreasing on others. Given that the GAT significantly increases runtime, we conclude that our basic GNN offers a sufficient and efficient trade-off between performance and efficiency for our joint diffusion approach. We also ablate the transformer layers. Contrary to previous work (Shi et al., 2025a), we observed that removing the

1350 Table 9: **Ablation Study** on three databases with increasingly complex schemas. We present results
 1351 for our method when trained with the ground-truth entity graph (RELDIFF original), our proposed
 1352 RELDIFF *D2K+SBM* variant, a version with a more complex GAT network (RELDIFF GAT), and
 1353 variants where the transformer layers or GNN are removed entirely. Additionally, we include a variant
 1354 that uses a simpler bipartite 2K entity graph generator, as well as how sampling parameters, such
 1355 as number of neighbors (S) and hops (K), affect performance. For each metric, we highlight the
 1356 best-performing variant in **bold** and underline the second-best.

Metric	RelDiff (original)	RelDiff Bipartite 2K	RelDiff D2K+SBM	RelDiff GAT	RelDiff w/o Transformer	RelDiff S=64	RelDiff K=1	RelDiff w/o GNN
Airbnb	Shape (\uparrow)	98.14 \pm 0.07	98.14 \pm 0.07	98.14 \pm 0.07	<u>98.44</u> \pm 0.03	98.40 \pm 0.05	98.40 \pm 0.08	98.02 \pm 0.04
	Trend (\uparrow)	95.76 \pm 0.25	95.76 \pm 0.25	95.76 \pm 0.25	<u>95.92</u> \pm 0.04	96.46 \pm 0.20	95.54 \pm 0.36	95.12 \pm 0.31
	1-HOP (\uparrow)	<u>89.37</u> \pm 0.38	<u>89.37</u> \pm 0.38	<u>89.37</u> \pm 0.38	86.63 \pm 0.69	<u>88.66</u> \pm 0.33	87.72 \pm 0.29	86.97 \pm 0.37
Berka	Shape (\uparrow)	98.75 \pm 8e-4	97.94 \pm 0.06	97.92 \pm 0.01	96.79 \pm 0.06	97.94 \pm 0.02	95.07 \pm 0.02	97.83 \pm 0.09
	Trend (\uparrow)	<u>99.35</u> \pm 5e-3	98.70 \pm 0.01	98.68 \pm 0.02	98.01 \pm 0.01	98.77 \pm 0.04	97.40 \pm 0.01	98.91 \pm 0.01
	1-HOP (\uparrow)	98.03 \pm 0.06	96.59 \pm 0.06	<u>96.70</u> \pm 0.06	94.83 \pm 0.01	96.53 \pm 0.05	94.01 \pm 0.06	92.48 \pm 0.08
F1	2-HOP (\uparrow)	97.96 \pm 0.03	95.33 \pm 0.03	<u>95.61</u> \pm 0.03	95.00 \pm 0.09	95.59 \pm 0.04	93.72 \pm 0.06	95.59 \pm 0.05
	3-HOP (\uparrow)	91.97 \pm 0.13	90.43 \pm 0.22	89.54 \pm 0.38	90.33 \pm 0.33	90.95 \pm 0.15	93.20 \pm 0.17	90.41 \pm 0.12
	Shape (\uparrow)	97.92 \pm 0.05	82.80 \pm 0.16	<u>96.80</u> \pm 0.03	92.95 \pm 0.36	96.69 \pm 0.14	96.66 \pm 0.07	93.94 \pm 0.17
F1	Trend (\uparrow)	98.04 \pm 0.01	89.35 \pm 0.13	<u>97.35</u> \pm 0.05	95.86 \pm 0.07	<u>97.51</u> \pm 0.07	96.50 \pm 0.01	94.88 \pm 0.06
	1-HOP (\uparrow)	95.10 \pm 0.14	84.48 \pm 0.28	<u>94.02</u> \pm 0.11	86.18 \pm 0.83	<u>93.55</u> \pm 0.06	93.84 \pm 0.20	90.02 \pm 0.46
	2-HOP (\uparrow)	97.92 \pm 0.05	91.26 \pm 0.11	96.19 \pm 0.01	96.07 \pm 0.04	96.22 \pm 0.03	<u>97.73</u> \pm 0.03	95.14 \pm 0.11

1368
 1369 transformer encoders and decoders can lead to slight performance improvements on certain metrics.
 1370 This suggests that users can opt for a simplified architecture without the transformer layers. On tables
 1371 with a high number of columns, this might even result in significant speedups due to the quadratic
 1372 complexity of attention.
 1373

1374 **Subgraph Sampling.** Finally, we investigate the sensitivity of our model to subgraph sampling
 1375 parameters, specifically the number of sampled neighbors (S) and the number of GNN layers (K).
 1376 By default, we sample all neighbors and use $K = 2$. When limiting the neighborhood size to $S = 64$,
 1377 we observe that the method retains the majority of its performance across metrics. Surprisingly,
 1378 this constraint even leads to improved preservation of long-range dependencies in certain cases,
 1379 such as the 3 and 2 HOP metrics on the Berka and F1 datasets respectively. Additionally, reducing
 1380 the receptive field to a single hop ($K = 1$) results in only a marginal degradation in fidelity while
 1381 offering substantial computational benefits. We attribute this effectiveness to the multi-step nature of
 1382 the diffusion process, where information propagates iteratively, effectively expanding the receptive
 1383 field to $\mathcal{O}(T \times K)$ hops over T denoising steps. Consequently, $K = 1$ successfully captures long-
 1384 range dependencies while offering significant computational gains, reducing training and generation
 1385 times by over 40% on the Berka dataset. These findings suggest that neighborhood subsampling
 1386 and shallower architectures can be effectively leveraged to further enhance scalability on massive
 1387 databases with minimal impact on generation quality.

E STUDY OF MODEL EFFICIENCY AND SCALABILITY

1390 In this section, we analyze the efficiency and scalability of RELDIFF. We provide a comparative
 1391 complexity analysis against alternative graph generation approaches and evaluate runtime performance
 1392 relative to the state-of-the-art baseline ClavaDDPM. Both training and sampling runtimes are reported.
 1393 Finally, we study RELDIFF’s scalability by generating (i) a database with an order of magnitude
 1394 more rows than those considered in prior work and (ii) a database whose schema contains an order of
 1395 magnitude more tables.

E.1 COMPLEXITY ANALYSIS

Graph Generation.

1400 We compare the computational complexity of our proposed *D2K+SBM* generator against common
 1401 deep learning-based graph generative approaches. Let N denote the number of nodes in the relational
 1402 entity graph. Transformer-based autoregressive models incur $\mathcal{O}(N^4)$ complexity, since each genera-
 1403 tion step requires full self-attention across all nodes and sequentially generates the adjacency matrix.
 1404 This quickly becomes intractable as N grows. Diffusion-based graph generators reduce this cost but

1404 still require $\mathcal{O}(TN^2)$ operations, where T is the number of diffusion steps. At each denoising step,
 1405 dense pairwise computations across nodes are performed, leading to quadratic cost per step.
 1406

1407 In contrast, our *D2K+SBM* generator achieves $\mathcal{O}\left(\ln^2(N) + \sum_{i=1}^{|R|}(N + 2e_i + B)\right)$ complexity,
 1408 where $|R| \ll N$ is the number of foreign key relationships, e_i the number of edges per relation,
 1409 $E = \sum_{i=1}^{|R|} e_i$ the total number of edges, and $B < N$ the number of blocks in the stochastic block
 1410 model. This formulation yields near-linear scaling with the number of nodes and edges in the graph,
 1411 in sharp contrast to the polynomial scaling of deep learning-based methods. **If we opt to transform**
 1412 ***two-parent-no-child* nodes into many-to-many edges we further improve efficiency as we decrease**
 1413 **the number of nodes and edges in the graph.**

1414 Moreover, while existing graph generators require an additional post-processing step to enforce refer-
 1415 ential integrity, our method guarantees it by construction. Taken together, this analysis demonstrates
 1416 the superior scalability of *D2K+SBM*: whereas autoregressive and diffusion-based approaches be-
 1417 come prohibitively expensive on large graphs, our generator remains efficient, making it particularly
 1418 well-suited for synthesizing large relational databases with many entities and complex schemas.
 1419

1420 **Data Generation.** We analyze the computational complexity of our joint diffusion model, comparing
 1421 its computational efficiency against its closest competitor, ClavaDDPM, during both training and
 1422 inference phases. The training complexity of both *RELDIFF* and ClavaDDPM is primarily dictated by
 1423 their diffusion-based backbones, which have a comparable number of parameters. The key difference
 1424 in efficiency emerges from their respective denoising conditioning mechanisms.
 1425

1426 ClavaDDPM conditions the sampling process with a separate MLP classifier for each relationship
 1427 within the database schema. Consequently, its complexity scales linearly with the number of
 1428 relationships. This approach becomes computationally expensive as the complexity of the schema
 1429 increases.

1430 In contrast, *RELDIFF* employs a single heterogeneous GNN. The GNN’s per-batch complexity is
 1431 $\mathcal{O}\left(\prod_{i=1}^K S_i\right)$, controlled by the structure of local neighborhoods with small, bounded depth $K = 2$,
 1432 and sparse node degrees or explicit sampling thresholds S_i (Hamilton et al., 2017). This allows
 1433 *RELDIFF* to maintain superior efficiency as the database schema becomes more complex.
 1434

1435 The inference complexity is proportional to the training complexity. Again, ClavaDDPM repeats
 1436 denoising steps for tables with multiple parents, whereas *RELDIFF* performs a single forward pass per
 1437 diffusion step across the entire relational graph. This significantly minimizes redundant computations
 1438 and enhances efficiency as the schema grows more complex. Additionally, *RELDIFF* employs more
 1439 efficient diffusion modeling, which allows it to generate high-quality samples with 1/20 of the
 1440 steps required by DDPM-based methods. This allows our method to achieve significant runtime
 1441 improvements compared to ClavaDDPM.
 1442

1443 E.2 RUNTIMES

1444 We now report the training and sampling runtimes of *RELDIFF* and ClavaDDPM, measured across
 1445 the benchmark datasets used in our experiments. **As shown in Table 10, *RELDIFF* achieves training**
 1446 **runtimes that are competitive with, and frequently faster than, ClavaDDPM. Notably, *RELDIFF* is**
 1447 **significantly more efficient on the complex F1 schema (2.7h vs. 12.3h).** While training times are
 1448 higher on Berka and IMDB due to our default full-neighbor sampling strategy on these denser graphs,
 1449 our ablation studies (Appendix D.4) demonstrate that users can alleviate this cost by subsampling
 1450 neighbors, achieving speedups of up to 30% with minimal impact on fidelity. Furthermore, we
 1451 emphasize that training represents a one-time cost. In practice, sampling will be the more commonly
 1452 executed operation, where *RELDIFF* achieves significantly lower runtimes.
 1453

1454 Table 11 highlights this key advantage, showing that *RELDIFF* is significantly faster during sampling.
 1455 For instance, on the California dataset, *RELDIFF* is more than 20 times faster, a direct result of our
 1456 method’s ability to jointly generate data, avoiding the need for sequential generation. This finding is
 1457 particularly important as sampling is the process that will be executed most frequently in a production
 1458 environment.

1459 **We report the maximum GPU memory usage during training and the sampling and training batch**
 1460 **sizes in Table 12.**

1458

Table 10: **Training runtimes** (hours).

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Dataset	RelDiff	ClavaDDPM
California	3.3	3.3
CCS	2.5	3.2
Instacart 05	4.8	6.6
Airbnb	0.4	0.4
Rossman	0.4	1.2
Walmart	0.6	0.9
Berka	11.8	6.5
IMDB	7.7	3.1
F1	2.7	12.3
Biodegradability	0.3	-
CORA	0.5	-

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Table 11: **Sampling runtimes** (seconds).

Dataset	RelDiff	ClavaDDPM
California	221.2	5,032
CCS	126.3	1,232
Instacart 05	5,107	9,968
Airbnb	8.6	178.5
Rossman	5.7	194.9
Walmart	17.8	40.2
Berka	780.6	1,662
IMDB	499.4	2,314
F1	32.3	643.6
Biodegradability	8.4	-
CORA	9.4	-

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Table 12: **Memory usage and batch size**.

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Dataset	Batch size	Training	Sampling
		Max Mem. Usage	
California	50k	42 GB	100k
CCS	100k	19 GB	200k
Instacart 05	40k	45 GB	80k
Airbnb	Full	6 GB	Full
Rossman	2048	7 GB	Full
Walmart	512	7 GB	Full
Berka	50k	21 GB	100k
IMDB	200k	34 GB	400k
F1	20k	5 GB	Full
Biodegradability	512	3 GB	Full
CORA	25k	7 GB	50k

E.3 LARGE-SCALE DATABASE GENERATION

We perform additional experiments on large-scale databases to further validate the scalability of our method. The loss curves in Figure 8 demonstrate that RELDIFF successfully trains on the large rel-hm dataset, proving its capability to handle millions of rows within a reasonable training time (under 48 hours). Similarly, the stable loss curve on the highly complex AdventureWorks schema confirms that our graph-based approach can effectively model databases with a large number of interconnected tables, a crucial test for its applicability to real-world, enterprise-level data. This suggests that future methods should now be evaluated not just on small-to-medium datasets but on large-scale databases with millions of rows and highly complex schemas (See Figure 9 for the AdventureWorks schema).

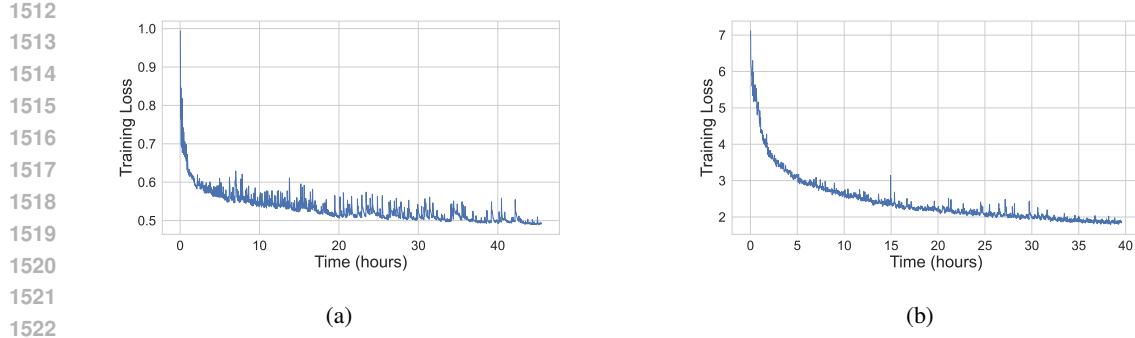
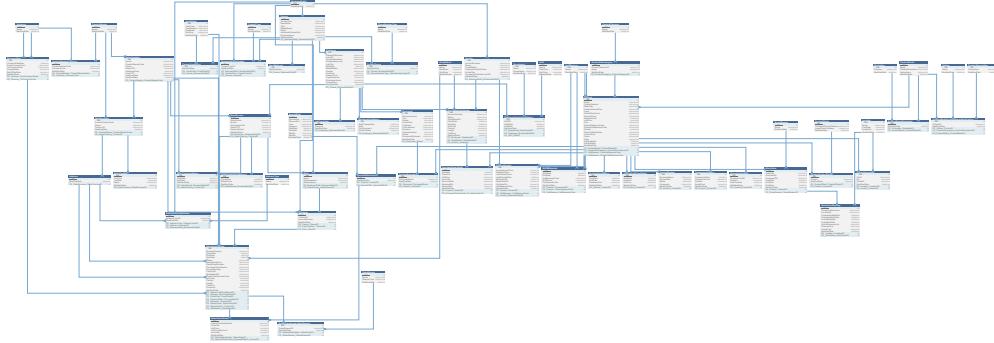
Figure 8: Loss curves for the *rel-hm* (a) and *AdventureWorks* (b) datasets.

Figure 9: Schema of the AdventureWorks database.

F MODELING THE JOINT DISTRIBUTION $p(\mathcal{V}, \mathcal{E})$

To construct a general, data-agnostic generative framework, we formulate the synthesis of relational databases as sampling from the joint distribution over entities and edges of the relational entity graph (Section 3), $p(\mathcal{V}, \mathcal{E})$. An ideal simulation of the data generating process might attempt to mimic the temporal evolution of a database, sequentially adding users, products, and transactions. However, such an approach requires extensive, dataset-specific domain knowledge. Consequently, we seek a factorization of the joint distribution that remains computationally tractable without relying on specific assumptions about the data’s origin. We identify three viable factorizations: (1) direct joint modeling $p(\mathcal{V}, \mathcal{E})$, (2) an attributes-first decomposition $p(\mathcal{V})p(\mathcal{E}|\mathcal{V})$, and (3) a structure-first decomposition $p(\mathcal{E})p(\mathcal{V}|\mathcal{E})$.

We posit that the first two approaches face a fundamental scalability bottleneck. In both direct joint modeling and features-first generation, the edges are generated alongside or subsequent to the entity attributes. This necessitates evaluating the probability of a link existing between every possible pair of entities. For a relational database containing tables of size N and M , this implies modeling interactions over a search space of size $N \times M$ (See Appendix E.1). This quadratic complexity renders factorizations (A) and (B) computationally prohibitive for large-scale real-world databases, where the number of potential interactions grows exponentially with the number of tables.

In contrast, the structure-first factorization $p(\mathcal{E})p(\mathcal{V}|\mathcal{E})$ effectively circumvents this combinatorial explosion. By generating the relational skeleton first, we can exploit the exchangeability of nodes and utilize efficient random graph models—such as the D2K+SBM proposed in this work that scales linearly with the number of edges rather than quadratically with the number of nodes. While the dependency between structure and attributes is essentially bidirectional, generating the lower-dimensional topology first provides a stable computational graph for the subsequent attribute generation. Crucially, this factorization does not assume that only the attributes can depend on structure. Our parameterization of $p(\mathcal{V}|\mathcal{E})$ employs an expressive GNN model that jointly models attributes across the entire schema. This enables the model to capture bidirectional dependencies between node attributes and structure, effectively filling in the graph with attributes that respect the underlying distribution. For

1566 example the model can learn to assign a subset of values that would result in the row having few
1567 foreign key connections, to a node with the corresponding structure in the graph. This is analogous to
1568 autoregressive language modeling, where an imposed sequential generation order does not preclude
1569 the model from capturing complex, non-causal semantic dependencies between tokens.
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1571 G BROADER IMPACTS 1572

1573 This research introduces a novel method for generating synthetic relational databases, which presents
1574 potential benefits in fields with privacy restrictions, such as healthcare, finance, and education, and in
1575 scenarios involving limited or biased data. However, there are potential negative impacts to consider.
1576 While our empirical privacy analysis does not raise immediate concerns, our method is not equipped
1577 with provable privacy guarantees like differential privacy. Additionally, due to the method’s ability
1578 to generate datasets that closely resemble the original data, it might inadvertently amplify biases
1579 already present in the original data. Furthermore, synthetic data that closely mirrors real data could
1580 be misused. Consequently, we believe that future work should prioritize research directions focused
1581 on enhancing privacy protection and developing effective bias reduction techniques for synthetic
1582 relational data.
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