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## **Do LSTMs Learn Compositionally?**

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#### Abstract

LSTM-based language models exhibit compositionality in their representations, but how this behavior emerges over the course of training has not been explored. Analyzing synthetic data experiments with contextual decomposition, we find that LSTMs learn long-range dependencies compositionally by building them from shorter constituents during training.

## 1 Introduction

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Consider the process of backpropagation through time for a language model. As an example, the language model should learn that an occurrence of "either" increases the later likelihood of "or". To do so, it must backpropagate information from the occurrence of "or" through some intervening constituent, which we will refer to as a conduit because the association of either/or is carried through it to affect the representation of "either". Perhaps it encounters a training example that uses a conduit that is predictable by being structured in familiar ways, here italicized: "Either Socrates is mortal or not all men are mortal." However, what if the conduit is unpredictable and the structure cannot be interpreted by the model, for example, if the conduit includes unknown tokens, as in: "Either slithy toves gyre or mome raths outgrabe"? Which conduit will carry the gradient from "or" to "either" easily?

Formally, as the gradient of the error  $e_t$  at timestep t is backpropagated k timesteps through the hidden state h:

$$\frac{\partial e_t}{\partial h_{t-k}} = \frac{\partial e_t}{\partial h_t} \prod_{i=1}^k \frac{\partial h_{t-i+1}}{\partial h_{t-i}}$$

The backpropagated message is multiplied repeatedly by the gradients associated with each item in the conduit. If the recurrence derivatives  $\frac{\partial h_{i+1}}{\partial h_i}$  are large at some parameter, the correspondingly larger backpropagated gradient  $\frac{\partial e_t}{\partial h_{t-k}}$  will accelerate descent in that direction.

When we ask which conduit will carry the gradient message to learn a long-range dependency faster, the answer will depend on the magnitude and distribution of the recurrence gradients. If the language model relies on linguistic structure in the conduit in order to pass the message effectively, then the more predictable conduit will facilitate learning a long-range pattern.

In order to investigate whether long-range dependencies are built from short constituents, we train models on synthetic data which varies the predictability of short sequences. We find that memorizing local patterns allows a language model to learn a long-range dependency faster but ultimately inhibits its ability to fully acquire longrange rules.

## 2 Related Work

How do neural language models learn? The key to answering this question is to understand the compositionality of LSTM training. To this end, we connect the hierarchical structure of language model representations with the incremental nature of neural learning dynamics.

We have extensive evidence that hierarchical structure is integral to the high performance of fully trained neural language models. LSTMs learn more effectively from natural language data than from similarly distributed random data, implying that they take advantage of linguistic structure (Liu et al., 2018). The representations they produce seem to be hierarchical in nature (Hewitt and Manning, 2019; Blevins et al., 2018; Hupkes et al., 2017). They implicitly exhibit a number of compositionality assumptions linguists tend to make by encoding information about part of speech (Belinkov et al., 2017), morphology (Vania and Lopez, 2017), and verb agreement (Lakretz et al., 2019). But the compositional nature of these representations tells us little about the process by which they are learned.

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Humans learn by memorizing short rote phrases and later mastering the ability to construct deep syntactic trees from them (Lieven and Tomasello, 2008). LSTM models, meanwhile, learn by backpropagation through time, leading to different inductive priors compared to a human. We may not therefore expect an LSTM to exhibit similarly compositional learning behavior. However, language models are known to encode hierarchical syntax, so we must consider whether they learn hierarchically as well, by building longer constituents out of shorter ones during training.

Recognizing the role of inductive priors in train-117 ing is critical. LSTMs have the theoretical capac-118 ity to encode a wide range of context-sensitive lan-119 guages, but in practice their ability to learn such 120 rules from data is limited, implicating the impor-121 tance of the training process (Weiss et al., 2018). 122 However, we may find that the hierarchical na-123 ture of the representation is entirely a result of the 124 data, rather than induced by the biases of the train-125 ing process. LSTMs by default learn associations 126 from the most recent items in a sequence, but they 127 are still capable of learning to encode grammatical 128 inflection from the first word in a sequence rather 129 than the most recent (Ravfogel et al., 2019). In-130 ductive priors play a critical role in the ability of 131 an LSTM to learn effectively, but they are neither 132 necessary nor sufficient in determining what the 133 model can learn.

We therefore investigate further into LSTM learning dynamics. In general, work in deep learning has supported the assumption that easy examples are learned before hard examples (Arpit et al., 2017). A controversial proposal by Shwartz-Ziv and Tishby (2017) held that learning begins with a memorization phase followed by a compression phase which makes the model more general, a claim that has been extensively debated with evidence for (Noshad and Hero III, 2018) and against (Saxe et al., 2018) it. If the hypothesis holds generally, the transition from memorized to compressed rules is another example of, or potential explanation for, easy-first learning.

148In the case of an LSTM, dependency range is149one aspect of difficulty that might affect the or-

der of learning. For example, an either/or matching over a short distance can be memorized, but over a long distance requires an encoding of concepts like constituency in order to be applied generally. The learning dynamics of an LSTM cause lower layers to converge faster than higher layers when there are many layers (Raghu et al., 2017), which combined with findings of hierarchy (Blevins et al., 2018; Belinkov et al., 2018) imply that the local connections encoded by lower layers are learned before the more distant connections encoded by higher layers. Even within a single layer, Saphra and Lopez (2019) found that local properties, such as syntactic tags, were learned earlier than the less local property of topic. The transition from local dependencies to more global dependencies is yet another example of how simple patterns are required before complex ones.

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However, even if simple local rules are learned first, they might not be used compositionally in constructing longer rules. In fact, simple rules learned early on might inhibit the learning of more complex rules through the phenomenon of gradient starvation (Combes et al., 2018), in which more frequent features reduce the gradient directed at rarer features. Simple local rules could slow down the training process by affecting the recurrence from timestep to timestep to degrade the gradient, or by trapping the model in a local minimum which makes the long-distance rule harder to reach. The compositional view of training is not a given and must be verified.

## 3 Methods

All experiments use a one layer LSTM, with inputs taken from an embedding layer and outputs processed by a softmax layer. All hidden dimensions are 200. We train with a learning rate set at 1 throughout and gradients clipped at 0.25. We found momentum and weight decay to slow rule learning in this setting, so they are not used.

#### 3.1 Contextual Decomposition

In our running example, we need to determine when our language model has learned that "either" implies an appearance of "or" later in the sequence. It is difficult to directly measure the influence of "either" on the later occurrence of "or", so in order to dissect the sequence and understand the impact of individual elements in the sequence, we employ contextual decomposition (CD).

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200 First introduced by Murdoch et al. (2018), CD 201 is a method of looking at the individual influences of words and phrases in a sequence on the out-202 put of a recurrent model. CD converts the output 203 vector from an LSTM layer into a sum of relevant 204 (contributed only by the input word or phrase of 205 interest  $x_{\gamma}$ ; represented as  $v_{\gamma}$ ) and irrelevant (con-206 tributed by, or involving interactions with, other 207 words in the sequence; represented as  $v_{\beta}$ ) parts, 208  $v = v_{\gamma} + v_{\beta}$ . Because the individual contribu-209 tions of the items in a sequence interact in nonlin-210 ear ways, it is difficult to disentangle the impact of 211 a specific word or phrase on the label distribution 212 predicted. However, the dynamics of LSTMs are 213 approximately linear in natural settings, as found 214 by Morcos et al. (2018), who used canonical corre-215 lation analysis to find close linear projections be-216 tween the activations at each timestep in a repeat-217 ing sequence and the activations at the end of the 218 sequence. It is therefore unsurprising that approx-219 imation error is low for the CD approach of lin-220 earizing the output of a LSTM layer so it can be 221 viewed as the sum of a relevant component and 222 the contributions of the rest of the sequence. 223

While Murdoch et al. (2018) were primarily interested in analyzing the importance and classification tendencies of the phrases and words that formed a sequence, we are interested in understanding whether a dependency between two words has been learned at all. Because the decomposed logits can be used as inputs for a softmax, we convert the decomposed elements into probability distributions by  $P(x|x_{\gamma}) = \operatorname{softmax}(v_{\gamma})$ . This allows us to analyze the effect of  $x_{\gamma}$  on a later element x while controlling for the influence of the rest of the sequence. We consider the running either-or example dependency to have been effectively learned when the contribution of the opening token ('either') places a high probability on its mate ('or') at the appropriate timestep when 'or' occurs in the data.

### 4 Experiments

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We use synthetic data to test the ability of an LSTM to learn a consistent rule with a longdistance dependency. This controls for the irregularity of natural language as well as for the confounding factor of rule frequency. While LSTMs in natural language model settings learn shortrange dependencies first, we must consider the possibility that this pattern is unrelated to any inductive prior. It could be that longer-range dependencies are simply rarer and therefore learned later. Our synthetic data sets instead have a fixed number of occurrences of the long distance relationship, regardless of the conduit length.

We generate data uniformly at random from a vocabulary  $\Sigma$ . However, we insert *n* instances of the long-distance rule  $\alpha \Sigma^k \omega$  (with conduit length *k*), where we consider an **open symbol**  $\alpha$  and a **close symbol**  $\omega$ , with  $\alpha, \omega \notin \Sigma$ . Relating to our running example,  $\alpha$  stands for "either" and  $\omega$  stands for "or". We use a corpus of 1m tokens with  $|\Sigma| = 1k$  types, which leaves a low probability that any conduit sequence longer than 1 token appears elsewhere by chance.

For all analyses, CD yielded an approximation error  $\frac{\|(v_{\gamma}+v_{\beta})-v\|}{\|v\|} < 10^{-5}$ , when running the true LSTM model to generate v for comparison.

**Limitations** While we hope to isolate the role of long range dependencies through synthetic data, we must consider the possibility that the natural predictability of language data differs in relevant ways from the synthetic data, in which the conduits are predictable only through pure memorization. Because LSTM models take advantage of linguistic structure, we cannot be confident that predictable natural language exhibits the same cell state dynamics that make a memorized uniformly sampled conduit promote or inhibit long-range rule learning.

## 4.1 The Effect of Rule Frequency

First, we investigate how the frequency of a rule affects the ability of the model to learn the rule. We vary the conduit length k while keeping nconstant. The results in Figure 1 illustrate how a longer conduit length requires more examples before the model can learn the corresponding rule. We consider the probability assigned to the close symbol according to the contributions of the open symbol, excluding interaction from any other token in the sequence. For contrast, we also show the extremely low probability assigned to the close symbol according to the contributions of the conduit taken as an entire phrase. In particular, note the pattern when the rule is extremely rare: The probability of the close symbol as determined by the open symbol is low but steady regardless of the conduit length, while the probability as determined by the conduit declines with conduit length due to the accumulated low probabilities from



Figure 1: The predicted probability  $P(x_t = \omega)$ , according to the contributions of open symbol  $x_{t-k} = \alpha$  and of the conduit sequence  $x_{t-k+1} \dots x_{t-1}$ , for various rule occurrence counts *n*. Shown at 40 epochs.



Figure 2: The predicted  $P(x_t = \omega | x_{t-k} \dots x_{t-k+i})$  according to CD, varying *i* as the x-axis and with  $x_{t-k} = \alpha$  and k = 8. Shown at 50 epochs.

each element in the sequence.

To understand the impact of the open symbol in context, see Figure 2, which illustrates that the conduit interacts with the open symbol to increase the probability slightly, a sign that the model is counting the intervening symbols rather than registering only the effect of the open symbol.

#### 4.2 The Effect of Conduit Predictability

To understand the effect of conduit predictability, we modify the synthetic data such that the sequence in the conduit appears frequently outside of the long-distance rule. In this experiment, the conduits are actually taken from a randomly generated vocabulary of 100, so that each unique conduit q appears in the training corpus 10 times in the context  $\alpha q \omega$ . This repetition is necessary in order to fit n = 1000 occurrences of the rule in all settings. In the unpredictable-conduit setting, q appears only in this context as a conduit, so the conduit remains random and unpredictable. In the **predictable-conduit setting**, we randomly distribute m = 1000 occurrences of each conduit q throughout the corpus outside of the rule patterns. In the predictable-conduit setting, each con-



Figure 3: The predicted  $P(x_t = \omega | x_{t-k} = \alpha)$ , according to CD. Solid lines are in the unpredictable conduit setting, while dashed lines are in the predictable conduit setting.

duit is seen often enough to be memorized.

As we see in Figure 3, copying each conduit 100 times throughout the corpus inhibits learning of the symbol-matching rule over the long run of training, but promotes early learning of the rule. This result implies that long-range dependencies are learned from the structure of their constituents. Therefore the model is delayed during training in representing longer dependencies in part because it depends on constituents being effectively learned first.

## 5 Conclusions

We confirm that the longer the span of a rule, the more examples are required for an LSTM model to effectively learn the rule. We then find1 that a more predictable conduit between the rule symbols promotes the early learning of the rule, implying that the process by which an LSTM learns long-range rules is compositional. However, the representation learned through the predictable conduit ultimately prevents the model from confidently learning these long-range connections.

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