Quantum gravity in superspace momentum

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General relativity is an excellent model of gravity, but the theory is classical, not described by a quantum theory. There are many models of quantum gravity, they can be divided into background and independent from the background of the theory.

In this paper, we consider the canonical approach to quantum gravity. But a completely different geometry and the concept of space is used.

Instead of real space, consider the geometry of the fields in the momentum space. For this, the coordinates are considered by operators who are set in the space of the vector field.

$$x_\beta \psi = i\, \delta\psi / \delta A_\beta$$

$$A = A(x)$$

Note that this vector field is similar to the gauge field. Thus, this definition is an independent about the background. Next, can use a differential geometry to analyze the space of this vector field.

Consider the metric of space the vector field

$$dA^2 = g_{\alpha\beta} dA_\alpha dA_\beta$$

Metric has spatial components, that is, it describes the three-dimensional geometry of this the space.

Consider the definition of volume the space of vector field

$$dV_A = dA_x dA_y dA_z$$

If there is time, we use a 4-dimensional volume for this geometry in the form of

$$d\Omega = dt\, dV_A$$

The concept of density the Hamiltonian is defined as the energy ratio to the volume of this vector field.

$$dE = H\, dV_A$$

These definitions need to form a quantum geometry.

We make certain steps, consider combinations of fundamental constants, we find the desired combination that corresponds this the density of Hamiltonian

$$k = Gh/c^2 = c\, L^2$$

Quantum geometry is described by a wave function, which is considered on the space metric.

We formulate a hypothesis. It is possible that there is a Schrödinger equation for this geometry, then a quantum evolution and description of the metric of the vector field will be considered in the form of

$$i\, \delta\Psi/\delta\Omega = -\, k\, \delta^2\Psi/\delta g^2$$

As can be seen, the evolution of the wave function occurs along the specified 4-dimensional volume and depends on the state of the metric of the field space.

$$\Psi = \Psi(\Omega, g)$$

In general, this equation has only an mathematum definition. It can describe the quantum dynamics of the field metric in terms of wave function and evolution in general form.

However, the optimism is exist, perhaps this structure has a geometric formulation of the quantum field theory in form the equation Wheeler-de Witt.

It is convenient to consider the canonical momentum in the form of an operator in the space of the field metric

$$\delta^2\Psi/\delta g^2 = -\pi^2\Psi$$

$$H = k\,\pi^2$$

Hamiltonian has yet has a free kinetic part in the form of square the momentum in geometry the vector field.

For further study of this model, it is necessary to consider the differential analysis in the form of Riemann geometry

$$\Delta = \delta^\beta\delta_\beta = \delta^2/\delta A^2$$

$$R_{\alpha\beta} \sim \Delta g_{\alpha\beta}$$

Here, differential operations occur in the field space.

Special attention should be done for the square of the curvature

$$R^2 = R_{\alpha\beta}R^{\alpha\beta} - \delta_\alpha\delta_\beta R^{\alpha\beta}$$

For such the field geometry, can find a general action. In determining and normalizing the constant, the action has a physical meaning only for a square of curvature

$$dS = \sigma\, R^2\, d\Omega$$

$$\sigma \sim c^4/G$$

Thus, a mathematum description of the metric and curvature of the space of the vector field in the form of a variational method is obtained.

This allows to find a complete Hamiltonian as a sum of the kinetic and potential part in this model

$$H = k\,\pi^2 + \sigma\, R^2$$

It is interesting to note that this definition is almost equivalent to the equation Wheelar-de Witt for ordinary real space.

In our model, a vector field geometry is used, which is independent of the background, in this sense the real space is a secondary concept.

It can be noted that the curvature of the field space is proportional to the surface area. Perhaps our model can describe the statistical behavior of the field geometry. For instance information for horizon the black hole

$$\Phi = F/4L^2$$

$$\Phi = \ln p$$

This indicates that the quantum field theory can be formulated by geometrical form, in particular on the surface.

In this case, only a small part of this model was considered. In the next paper, we will make a deeper review and consequences for geometry, gravity, and even problem the time. In general, further research will show which geometric shape of the world will be expressed in the general model.