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Training Fails to Promote
Translation Equivariance in Transformers

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Abstract

We present a result that would seem to have remarkable implications for the design of transformers. We observe that in a trained Gemma3 model, 27% of the variance in individual key-query attention coefficients can be attributed to the absolute position of the query in the context window, roughly the same as an untrained model. Training thus produces no move toward translation equivariance.

The elements of language—words, phrases, and sentences—have intrinsic meaning apart from their position in a text. The phrase "San Diego" can be understood whether it is at the beginning or end of a sentence. "The conference is in San Diego" states a fact that does not depend on whether it is presented in isolation or following "I am looking forward to the conference." Robust understanding by language models requires *translation equivariance*: shifting the position of a text string in the context window should not intrinsically alter how that text is interpreted. Of course, relevant preceding context can shade meaning, but merely moving a sentence in the window from, say, position 47 to position 98 should not alter how it is understood. Without translation equivariance, models would need to learn what "San Diego" means from scratch for every position within the context window and language understanding would be extremely brittle.

Transformers appear to have an inductive bias that favors translation equivariance. The query-key attention mechanism itself is agnostic to the order of preceding tokens. Position codes are incorporated to represent token ordering and distance. Commonly used position codes encode *relative* position—the number of positions between tokens—not *absolute* position—the index of a token within the context window (Ke et al., 2020; Huang et al., 2020; Shaw et al., 2018; Su et al., 2024).

We analyze translation equivariance by examining attention patterns in a transformer. Specifically, we characterize the *persistence* of a token on influencing subsequent processing in terms of the attention allocated to its key from queries that follow. To formalize, we use the notation $a_{t,\Delta t,l,h}$ to denote the attention coefficient associated with a key at position t and a query at position $t + \Delta t$ in layer l and head h of the transformer. If a model exhibits translation equivariance, $a_{t,\Delta t,l,h}$ will not depend on t.

We conducted analyses using a pretrained Gemma3-500m model (Gemma Team et al., 2025) run on C4 eval sequences with a 1024-token context window. Figure 1a plots the persistence of a key as a function of Δt, averaged across test sequences, t, l, and h. The curve is well fit by a power function: the coefficient of determination (R^2) is 0.933. There is of course variability across tokens, layers, and heads: Figure 1b shows typical curves for individual heads of a randomly selected key and layer. The decrease in attention with Δt is still evident even for single heads and single instances.

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Although the mean persistence curve is well characterized by a power function, when we condition the analysis on the absolute position of the key (t), Figure 1c indicates that the early positions in the context window are best fit by a power function (purple) and later positions by an exponential (red).

This mysterious pattern has a simple explanation. Suppose that a model distributes its causal attention *uniformly*. The query at position $t + \Delta t$ will attend to itself and the $t + \Delta t$ preceding tokens (assuming zero indexing), leading to an attention coefficient of $1/(t + \Delta t + 1)$ for each key. We refer to the function obtained by conditioning on t and varying Δt as the *uniform causal prior*. When we compare this parameter-free curve to our persistence function, we obtain a better fit for every key position t (cyan curve of Figure 1c) than either exponential or power functions, both having two free parameters.

This result indicates that even after training, the transformer still distributes its attention quite widely, at least for aggregated attention across layers, heads, and tokens. Figure 1b suggests that the same pattern is observed for individual instances (i.e., individual layers, heads, and tokens). To obtain direct evidence, we measure the fraction of variance explained in the attention coefficients by the query position ($t + \Delta t$) for individual instances. Figure 1d is a scatterplot of attention coefficients, $a_{t,\Delta t,l,h}$, sampled over all documents, t, Δt, l, and h, versus the causal attention prior, $1/(t + \Delta t + 1)$. To capture the dynamic range, we use a log-log plot and obtain a coefficient of determination, R^2, of 0.269 for the log values. (The non-log values produce $R^2 = 0.568$, but because this value is inflated by a few outliers, we adopt the more conservative log-based R^2.) To underscore this result, *if you pick a random token in a random layer and a random attention head, over one quarter of the variance in the attention coefficient is due to the absolute position of the query, independent of token content.*

Contrasting with an *untrained* model with zero weight initialization, we would obtain $R^2 = 1.0$; with actual Gemma initialization, we obtain $R^2 = 0.269$, the same as the trained model. Thus, the training process barely has an effect on narrowing the distribution of attention to a query. One might well have imagined that a model would learn to focus on the local context most of the time, which would make the absolute query position irrelevant—exact translation equivariance—and would yield an R^2 closer to 0.

Our findings have yet to be evaluated on other transformer architectures. We conjecture they will behave similarly because training must start from a roughly uniform attention distribution in order to initially learn to detect relevant context. And with a sufficiently overparameterized model, it appears that solutions are found without narrowing the focus of attention. It remains to be determined whether this property is detrimental to transformer generalization, and if so, what can be done to mitigate the problem. Solutions include local attention windows (currently in Gemma, though the locality spans the 1024 tokens context we studied here) and top-k attention mechanisms, possibly decreasing k over the course of training, and attention normalization schemes (e.g., Miller, 2023).
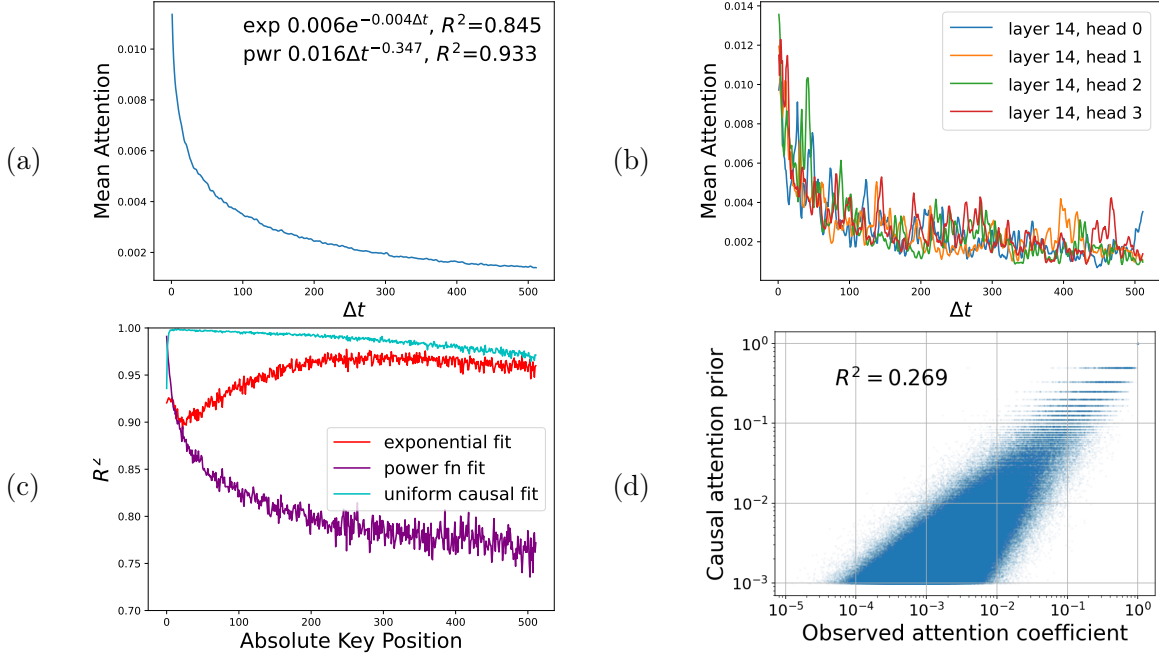
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Figure 1: (a) Attention coefficient as a function of key-query distance (Δt), averaged over documents, key position t, layers l, and heads h. (b) Attention coefficient as a function of Δt for a speciific document, layer l, and head h, averaged over key positions t. (c) Quality of fit (coefficient of determination, R^2), as a function of t, between persistence function and exponential, power function, and the uniform causal prior, $(t + \Delta t + 1)^{-1}$. (d) Scatterplot of observed attention coefficients versus uniform causal prior.

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