Proceedings of the 44th Chinese Control Conference
July 28-30, 2025, Chongqing, China

Adaptive Finite-Time Consensus Protocol for Multiple Mechanical Manipulator under Stochastic Vibration Conditions Based on Command Filtered Backstepping Method

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Abstract: As robotic arms are increasingly mounted on mobile platforms, traditional control methods, which often focus solely on either the robotic arm or the mobile platform, scarcely address the integrated control of the entire system. This often leads to inadequate handling of stochastic vibrations and environmental disturbances. This paper introduces a consensus protocol designed to enhance rapid and precise collaboration among robotic arms on mobile robots. A distributed neural network-based finite-time controller is proposed for a dynamic system model of robotic arms affected by random vibrations. The effectiveness of this protocol in achieving finite-time convergence is substantiated through a Lyapunov-like function. Simulations are presented to demonstrate the efficacy and advantage of the proposed approach, confirming its potential to significantly improve operational performance for complex robotic systems.

Key Words: Stochastic multi-manipulators, Command filtered backstepping, Finite-time Consensus, Stochastic Vibration

1 Introduction

In modern industry and services, fixed robotic arms play a vital role in boosting productivity and flexibility. However, their limitations in tasks requiring high adaptability have become evident. To address this, robotic arms are increasingly mounted on mobile platforms such as wheeled robots, legged robots, and drones, enhancing their manipulative capabilities. This integration, however, introduces significant complexities, transforming their dynamics into nonlinear systems influenced by disturbances and stochastic vibrations [1–3].

Addressing the nonlinearities inherent in these systems, numerous nonlinear control strategies have been developed. For instance, sliding mode control with observers has been widely studied [4, 5] , alongside other robust control approaches such as adaptive neural network (NN) control[6, 7]. Despite these advancements, the single robotic arm often falls short when tasked with complex operations due to mechanical and spatiotemporal constraints. This deficiency has shifted research focus towards multi-robot collaboration, where consensus problems become essential for coordinated task execution. Networked robots employing simple trajectory tracking under a consensus protocol can significantly enhance operational efficiency and reduce costs.

In this paper, backstepping methods are recognized for their effectiveness in constructing tracking protocols for higher-order nonlinear systems. However, they suffer from the ”explosion of differential” issue[8, 9], which increases computational burden, thus impeding real-time performance in systems with limited computational resources. On the other hand, recent advancements have been made with finite-time control techniques, known for their rapid convergence and precision, garnering attention across theoretical and applied domains. Finite-time command filtered (FTCF) control methods, addressing both the explosion of differential problem and achieving faster convergence rates, have proven effective in various studies[10–12].

Building upon these insights, this paper proposes an adaptive command filtering approach based on finite-time convergence for a consensus protocol in robotic arms subjected to stochastic vibrations. The main contributions of this study are twofold:

- This work represents an innovative application of adaptive FTCF to nonlinear models of robotic arms experiencing stochastic vibrations. It introduces a consensus protocol that achieves faster convergence compared to traditional backstepping controls. The effectiveness of this protocol is demonstrated through simulation results.
- Considering the impacts of random vibrations and external disturbances on networks of robotic arms, this study incorporates an adaptive approach to approximate dynamic disturbances. The effectiveness of the proposed method in achieving rapid consensus is validated through a finite-time convergence proof.

2 Preliminaries and System Descriptions

In this section, a definition of finite-time convergence for stochastic dynamic systems is first provided. Subsequently, a stochastic dynamic system model of robotic arms is presented, accompanied by necessary assumptions and expla-

This work is supported in part by the Major Key Project (PCL2023AS1-1 and PCL2024A04-1) of Pengcheng Laboratory, and in part by the National Natural Science Foundation of China under Grants 62273156. (Corresponding authors: Min Wang & Bowen Peng.)

nations.

2.1 Stochastic Dynamic Systems

The dynamic description of the stochastic system in continuous time is designed as follows

$$d\zeta(t) = F(\zeta(t))dt + \Phi(\zeta(t))dw(t), \quad \zeta(t_0) = \zeta_0, \tag{1}$$

where $\zeta(t)$ is n-dimensional stochastic state variable, ζ_0 is the stochastic initial condition, and $w(t)$ is a set of white gaussian noise process, which is independent of $\zeta(t)$.

The practical finite-time stable equilibrium in mean square is defined as:

Lemma 1 *[13] The equilibrium $\zeta(t) = 0$ of system (1) is practical finite-time stable in mean square, if there is a stochastic settling time $T(\epsilon, \zeta_0) < \infty$ and ϵ is a positive constant, for $\forall t > t_0 + T$, the system will satisfy $E[\|\zeta(t)\|^2] < \epsilon$.*

2.2 Stochastic Lagrangian System

Following the same line as in [14, 15], the kth stochastic manipulator can be modeled in the form of the stochastic Lagrangian as follows

$$M_k(q_k)\ddot{q}_k + C_k(q_k, \dot{q}_k)\dot{q}_k + h_k(q_k) = u_k + \Lambda_k(q_k)\xi_k, \tag{2}$$

where $q_k \in \mathbb{R}^n$ is a vector of generalized coordinates, $M_k(q_k) \in \mathbb{R}^{n\times n}$ is the inertia matrix (generalized mass) which is symmetric positive definite, $C_k(q_k, \dot{q_k}) \in \mathbb{R}^{n\times n}$ is the Coriolis/centrifugal matrix, $h_k(q_k) \in \mathbb{R}^n$ is the potential force, and $\Lambda_k(q_k)\xi$ is the random excitation force caused by the white noise $\xi_k \in \mathbb{R}^m$, $u_k \in \mathbb{R}^n$ is the control force acting on the system.

Based on the aforementioned description, several reasonable assumptions are subsequently outlined.

Assumption 1 *The Inertia matrix M_k as well as the Coriolis matrix C_k can be partitioned to the nominal part $\bar{M}_k$ and the unknown uncertain part ΔM_k.*

Assumption 2 *The nominal part $\bar{M}_k$ satisfies $\bar{M}_{k,\min}\|x\|^2 \le x^T\bar{M}x \le \bar{M}_{k,\max}\|x\|^2$, where $x \in \mathbb{R}^3$, $\bar{M}_{k,\max}$ and $\bar{M}_{k,\min}$ are known positive constants. For the unknown uncertain part ΔM_k, there exists an unknown $\Delta M_k^* > 0$ such that $\|\Delta M_k\| \le \Delta M_k^*$.*

Lemma 2 *[13] For matrix $M \in \mathbb{R}^{n\times n}$, if $M = \bar{M} + \Delta M$, there holds*

$$M^{-1} = \bar{M}^{-1} + \Delta\tilde{M}, \tag{3}$$

where $\Delta\tilde{M} = -\bar{M}^{-1}\Delta M(I_n + \bar{M}\Delta M)^{-1}\bar{M}^{-1}$.

According to Lemma 2 and Assumption 1, we have $M_k^{-1} = \bar{M}_k^{-1} + \Delta\tilde{M}_k$, $C_k^{-1} = \bar{C}_k^{-1} + \Delta\tilde{C}_k$, then (2) can be rewritten as the following equations:

$$\begin{aligned}\ddot{q}_k = &-C_k^\star\dot{q}_k - h_k^\star + \bar{M}_k^{-1}u_k + \Delta_k\tilde{M}_k u_k \\ &+ \Delta_k + (\Lambda_k^\star + \Delta\Lambda_k)\xi_k,\end{aligned} \tag{4}$$

where

$$\begin{aligned}C_k^\star &= \bar{M}_k^{-1}\bar{C}_k, h_k^\star = \bar{M}_k^{-1}h_k(q_k), \\ \Delta_k &= -\Delta\tilde{M}_k\bar{C}_k - M_k^{-1}\Delta\tilde{C}_k - \Delta\tilde{M}_k h_k(q_k), \\ \Lambda_k^\star &= M_k^{-1}\Lambda_k(q_k), \Delta\Lambda_k = \Delta\tilde{M}_k\Lambda_k(q_k), \\ \Delta\tilde{M}_k &= -\bar{M}_k^{-1}\Delta M_k(I_n + \bar{M}_k\Delta M_k)^{-1}\bar{M}_k^{-1}, \\ \Delta\tilde{C}_k &= -\bar{C}_k^{-1}\Delta C_k(I_n + \bar{C}_k\Delta C_k)^{-1}\bar{C}_k^{-1}.\end{aligned}$$

Let $v_k = \dot{q}_k$, the Itô stochastic differential equation of (4) can be obtained

$$\begin{cases} dq_k = v_k dt, \\ dv_k = (\bar{M}_k^{-1}u_k + \varphi_k)dt + \tilde{\varphi}_k\varpi_k dw_k, \\ y_k = q_k, \end{cases} \tag{5}$$

where $\varphi_k = -C_k^\star x_{k,1} - h_k^\star + \Delta\tilde{M}_k u_k + \Delta_k$, $\tilde{\varphi}_k = \Lambda_k^\star + \Delta\Lambda_k$. $\frac{1}{2\pi}\varpi$ is the power spectral density of the white noise ξ_k, $\varpi_k \in \mathbb{R}^{m\times m}$ is a positive matrix, and w_k is an m-dimensional independent standard Wiener process. The leader can be modeled as follows:

$$\dot{q}_d = v_d, \tag{6}$$

where $q_d, v_d \in \mathbb{R}^2$ are the desired position and the constant velocity, respectively.

3 Consensus Control Protocol

In this section, a new adaptive FTCF based control strategy is designed to ensure that the output q of the robot manipulator in a random vibration environment can track a given trajectory q_d, where an adaptive is used to approximate the unknown dynamics of the manipulator.

The tracking errors $e_{k,i}$ are defined as follows:

$$\begin{cases} e_{k,1} = \sum_{j=1}^N \rho_{k,j}(q_k - q_j) + o_k(q_k - q_d), \\ e_{k,2} = v_k - \bar{\alpha}_k, \end{cases} \tag{7}$$

where $\bar{\alpha}_k = [\psi_{k,1}, \psi_{k,2}, \cdots, \psi_{k,n}]^T \in \mathbb{R}^n$ is the output of FTCF with the virtual signal $\alpha_k = [\alpha_{k,1}, \alpha_{k,2}, \cdots, \alpha_{k,n}]^T$ as the input. The FTCF is designed as:

$$\begin{aligned}\dot{\psi}_{k,i} &= \zeta_{k,i}, \\ \zeta_{k,i} &= -\varrho_{k,1}\left|\psi_{k,i} - \alpha_{k,i}\right|^{\frac{1}{2}}\operatorname{sign}(\psi_{k,i} - \alpha_{k,i}) + \psi_{k,i}, \\ \dot{\psi}_{k,i} &= -\varrho_{k,2}\operatorname{sign}(\psi_{k,i} - \zeta_{k,i}).\end{aligned} \tag{8}$$

Lemma 3 *[16]For the case of absent input noise, if choosing $\varrho_{k,1}$ and $\varrho_{k,2}$ properly, the following equations are achieved in finite time,*

$$\psi_{i,1} = \alpha_{i,0}, \zeta_i = \dot{\alpha}_{i,0}. \tag{9}$$

Then, in the case of present input noise, it means that $\alpha_i = \alpha_{i,0}$. Assuming the input noise satisfies $|\alpha_i - \alpha_{i,0}| \le \Xi_i$, there the following inequalities hold in finite time

$$\begin{aligned}|\psi_{i,1} - \alpha_{i,0}| &\le \Theta_i\Xi_i = \pi_1, \\ |\zeta_i - \dot{\alpha}_{i,0}| &\le \Upsilon_i\Xi_i^{\frac{1}{2}} = \pi_2.\end{aligned} \tag{10}$$

where $\Theta_i > 0$ and $\Upsilon_i > 0$ are constants.

Remark 1 *Taking the virtual controller α as input, the $\bar{\alpha}$ and its first-order derivative $\dot{\bar{\alpha}}$ is obtained. Compared with [16–19], the command filter in this paper not only guarantees the filter effect of α, but also achieves stability in finite time.*

The virtual signal α_k and controller u_k are designed as follows

$$\begin{cases} \alpha_k = (o_k + \sum_{j=1}^{N})^{-1}(-t_{k,1}{v_{k,1}}^{\kappa} + \sum_{j=1}^{N} v_j + o_k\dot{q}_d, \\ \quad - \frac{3}{4}v_{k,1}(o_k + \sum_{j=1}^{N}\rho_{k,j} + s_{k,1})), \\ u_k = \bar{M}(-t_{k,2}{z_{k,2}}^{\kappa} - \frac{1}{4}(4 + 3s_{k,2} + a_{k,2})z_{k,2} \\ \quad - \frac{\hat{\theta}_k z_{k,2} B_k^T B_k}{4\tau_k}), \end{cases} \tag{11}$$

where $0 < \kappa < 1$, $k_i > 0$, and $c_i > 0, (i = 1, 2)$ are designed positive constants, $\hat{\theta}$ represents the estimate of θ and θ is a constant estimate of as $\theta = \max\{\|H\|^2\}$, where $\|\cdot\|$ denotes the 2-norm of the vector. And the updating process of $\hat{\theta}$ is designed as

$$\dot{\hat{\theta}}_k = -\iota_k\hat{\theta}_k + \frac{\lambda_k}{4\tau_k}(v_{k,2}^T v_{k,2})^2 B_k^{\mathrm{T}} B_k, \tag{12}$$

where $\iota_k > 0$ is a constant.

$$z_{k,1} = e_{k,1} - \varsigma_{k,1}, \quad z_{k,2} = e_{k,2} - \varsigma_{k,2} - \rho_k. \tag{13}$$

The ς_i is the error compensation system defined by

$$\begin{aligned} \dot{\varsigma}_{k,1} &= -s_{k,1}\varsigma_{k,1} - t_{k,1}\varsigma_{k,1}^{\kappa} \\ &\quad + (o_k + \sum_{j=1}^{N}\rho_{k,j})[(\bar{\alpha}_k - \alpha_k) + \varsigma_{k,2}], \\ \dot{\varsigma}_{k,2} &= -s_{k,1}\varsigma_{k,2} - t_{k,2}\varsigma_{k,2}^{\kappa}, \end{aligned} \tag{14}$$

with $\varsigma_{k,i}(0) = 0 (i = 1, 2)$.

Remark 2 *The finite-time control of the closed-loop system is achieved when $0 < \kappa < 1$, which means that fractional power functions are included in the virtual control signal α and the error compensation system ς_i. When $\kappa = 1$, the control algorithm no longer achieves finite-time convergence and the closed-loop system can only converge asymptotically over time, which is a special case of finite-time control.*

Theorem 1 *Consider the manipulator systems (2). It satisfies Assumption 1 and 2. Choose FTCF as in (8). Design the error compensation system as in (14). Construct the virtual signal α and controller u as in (11) with the adaptive updating law (12). Then, the tracking error z_1 is practical finite-time stable in mean square. Also, all signals in the closed-loop system are bounded in mean square in finite time.*

To prove Theorem 1, we have the following definition and Lemmas:

Definition 1 *[13] For $V(\zeta) \in C^2$, associated with stochastic system (1), we define a differential operator L as follows:*

$$LV(\zeta) = \frac{\partial V}{\partial \zeta}F(\zeta(t)) + \frac{1}{2}Tr\{\Phi(\zeta(t))\frac{\partial^2 V}{\partial \zeta^2}\Phi(\zeta(t))\}, \tag{15}$$

where Tr represents a matrix trace.

Lemma 4 *[18] For $z_i \in \mathbb{R}, i = 1, \cdots, K, 0 < \kappa \leq 1$, there holds*

$$(\sum_{i=1}^{K}|z_i|)^{\kappa} \leq \sum_{i=1}^{K}|z_i|^{\kappa} \leq K^{1-\kappa}(\sum_{i=1}^{K}|z_i|)^{\kappa}. \tag{16}$$

Lemma 5 *[13] For $x, z \in \mathbb{R}$, $p > 0, q > 0$ and $\alpha(x, z) > 0$,*

$$|x|^p|z|^q \leq \frac{p\alpha(x,z)|x|^{p+q}}{p+q} + \frac{q\alpha(x,z)^{-\frac{p}{q}}|z|^{p+q}}{p+q}. \tag{17}$$

Lemma 6 *[13] If there are three positive constants $\Delta, \Gamma, \kappa \in (0, 1)$ and two κ_∞−functions β_1 and β_2 ,which make a C^2 function $V(\zeta(t))$ such that*

$$\begin{cases} \beta_1(\|\zeta(t)\|) \leq V(\zeta(t)) \leq \beta_2(\|\zeta(t)\|), 0 \leq s \leq t, \\ W(\zeta(t)) - W(\zeta(t)) \leq -\Delta\int_s^t W^{\kappa}(\zeta(v))dv + \Gamma(t-s). \end{cases}$$

Then, for $\forall t \geq T$, there holds $\|\zeta(t)\| < \epsilon$, where

$$T = \frac{1}{(1-\kappa)\sigma\Delta}\left[V^{1-\kappa}(\zeta(0)) - \left(\frac{\Gamma}{(1-\sigma)\Delta}\right)^{(1-\kappa)/\kappa}\right],$$

$$\epsilon = \beta_1^{-1}\left[([\Gamma/((1-\sigma)\Delta)])^{1/\kappa}\right], 0 < \sigma < 1.$$

Now, the design and proof of Theorem 1 are presented as follow.

Proof 1 ***Step** 1: Consider the stochastic system (5), according to the Itô formula, the following equation is obtained:*

$$dz_{k,1} = ((o_k + \sum_{j=1}^{N}\rho_{k,j})v_k - \sum_{j=1}^{N}\rho_{k,j}v_j - o_k v_d - \dot{\varsigma}_{k,1})dt. \tag{18}$$

The following stochastic Lyapunov function is chosen:

$$V_{k,1} = \frac{1}{4}({z_{k,1}}^T z_{k,1})^2. \tag{19}$$

Based on Definition 1 and (14), the following is yielded

$$\begin{aligned} LV_{k,1} &= {z_{k,1}}^T z_{k,1}{z_{k,1}}^T[(o_k + \sum_{j=1}^{N}\rho_{k,j})(\alpha_k + z_{k,2}) \\ &\quad - \sum_{j=1}^{N}\rho_{k,j}x_{j,2} - o_k\dot{q}_d + s_{k,1}\varsigma_{k,1} + t_{k,1}\varsigma_{k,1}^{\kappa}]. \end{aligned} \tag{20}$$

By applying Young's inequality, Lemma 4, and Lemma 5, the following inequality is obtained

$$\begin{aligned} {z_{k,1}}^T z_{k,1}{z_{k,1}}^T\varsigma_{k,1}^{\kappa} &\leq \frac{3}{\kappa+3}({z_{k,1}}^T z_{k,1})^{\frac{3+\kappa}{2}}, \\ &\quad + \frac{\kappa}{\kappa+3}({\varsigma_{k,1}}^T\varsigma_{k,1})^{\frac{3+\kappa}{2}}. \end{aligned} \tag{21}$$

When (11) is substituted, the following is obtained

$$\begin{aligned} LV_{k,1} &\leq -\frac{t_{k,1}\kappa}{\kappa+3}({v_{k,1}}^T v_{k,1})^{\frac{3+\kappa}{2}} + \frac{t_{k,1}\kappa}{\kappa+3}({\varsigma_{k,1}}^T\varsigma_{k,1})^{\frac{3+\kappa}{2}} \\ &\quad + \frac{s_{k,1}}{4}({\varsigma_{k,1}}^T\varsigma_{k,1})^2 + \frac{o_k + \sum_{j=1}^{N}\rho_{k,j}}{4}({v_{k,2}}^T v_{k,2})^2. \end{aligned} \tag{22}$$

***Step* 2:** *Since $z_{k,2}=e_{k,2}-\varsigma_{k,2}$, according to Itô formula, the following equation is obtained*

$$dz_{k,2}=(\bar{M}_k^{-1}u_k+\varphi_k-L\bar{\alpha}_k-\dot{\varsigma_{k,2}})dt+\bar{\varphi}_k\varpi dw, \tag{23}$$

where $\bar{\varphi}_k=\tilde{\varphi}_k-\frac{\partial\bar{\alpha}_k}{\partial q_k}$. The candidate Lyapunov function is chosen as

$$V_{k,2}=V_{k,1}+\frac{1}{4}({z_{k,2}}^{\mathrm{T}}z_{k,2})^2+\frac{\tilde{\theta}_k^2}{2\lambda_k}. \tag{24}$$

where $\lambda_k>0$ is a constant. Form the formula (18) and (14), the following is obtained

$$\begin{aligned}LV_{k,2}=&LV_{k,1}+{z_{k,2}}^{\mathrm{T}}z_{k,2}{z_{k,2}}^{\mathrm{T}}(\bar{M}_k^{-1}u_k+\varphi_k-L\bar{\alpha}_k\\&+s_{k,1}\varsigma_{k,2}+t_{k,2}\varsigma_{k,2}^{\kappa})-\frac{\tilde{\theta}_k\dot{\hat{\theta}}_k}{\lambda_k}\\&+\frac{1}{2}\{{\bar{\varphi}_k}^{\mathrm{T}}(2z_{k,2}{z_{k,2}}^{\mathrm{T}}+{z_{k,2}}^{\mathrm{T}}z_{k,2}I)\bar{\varphi}_k\}.\end{aligned} \tag{25}$$

According to Young's inequality, the following inequality is obtained

$${z_{k,2}}^{\mathrm{T}}z_{k,2}{z_{k,2}}^{\mathrm{T}}\varsigma_{k,2}\le\frac{3}{4}({z_{k,2}}^{\mathrm{T}}z_{k,2})^2+\frac{1}{4}({\varsigma_{k,2}}^{\mathrm{T}}\varsigma_{k,2})^2. \tag{26}$$

And according to the property of norm, the following inequality is obtained

$$\begin{aligned}&\frac{1}{2}\{{\bar{\varphi}_k}^{\mathrm{T}}(2z_{k,2}{z_{k,2}}^{\mathrm{T}}+{z_{k,2}}^{\mathrm{T}}z_{k,2}I)\bar{\varphi}_k\}\\&\le\frac{3m\sqrt{m}}{2}{z_{k,2}}^{\mathrm{T}}z_{k,2}\|\bar{\varphi}_k^{\mathrm{T}}\bar{\varphi}_k\|.\end{aligned} \tag{27}$$

Substituting (26) and (27) into (25), the following inequality is obtained

$$\begin{aligned}LV_{k,2}\le&LV_{k,1}+{z_{k,2}}^{\mathrm{T}}z_{k,2}{z_{k,2}}^{\mathrm{T}}(\bar{M}_k^{-1}u_k+\varphi_k\\&-L\bar{\alpha}_k+\frac{3}{4}s_{k,2}z_{k,2}+t_{k,2}{\varsigma_{k,2}}^{\kappa})-\frac{\tilde{\theta}_k\dot{\hat{\theta}}_k}{\lambda_k}\\&+\frac{s_{k,2}}{4}({\varsigma_{k,2}}^{\mathrm{T}}\varsigma_{k,2})^2+\frac{3m\sqrt{m}}{2}{z_{k,2}}^{\mathrm{T}}z_{k,2}\|\bar{\varphi}_k^{\mathrm{T}}\bar{\varphi}_k\|.\end{aligned} \tag{28}$$

Then, letting $\hat{\varphi}_k={z_{k,2}}^{\mathrm{T}}(\varphi_k-L\bar{\alpha}_k)+\frac{3m\sqrt{m}}{2}\|\bar{\varphi}_k^{\mathrm{T}}\bar{\varphi}_k\|$, we adopt a FLS $H^{\mathrm{T}}B(X)$ to approximate it. For $\forall\varepsilon_k>0$, the following equality is obtained:

$$\hat{\varphi}_k=H_k^{\mathrm{T}}B_k(X)+\delta_k(X),|\delta(X)|\le\varepsilon_k. \tag{29}$$

By using Young's inequality, the following inequality is obtained

$$\begin{aligned}({z_{k,2}}^{\mathrm{T}}z_{k,2})\hat{\varphi}_k\le&\frac{({z_{k,2}}^{\mathrm{T}}z_{k,2})^2\|H_k\|^2B_k^{\mathrm{T}}B_k}{4\tau_k}\\&+\tau_k+\frac{1}{4}({z_{k,2}}^{\mathrm{T}}z_{k,2})^2+\varepsilon_k^2,\end{aligned} \tag{30}$$

where $\tau_k>0$ is a constant. Substituting (12), (30) and (11) into (28) yields

$$\begin{aligned}LV_{k,2}\le&-\sum_{i=1}^{2}\frac{t_{k,i}\kappa}{\kappa+3}({z_{k,i}}^{\mathrm{T}}z_{k,i})^{\frac{3+\kappa}{2}}\\&+\sum_{i=1}^{2}\frac{t_{k,i}\kappa}{\kappa+3}({\varsigma_{k,i}}^{\mathrm{T}}\varsigma_{k,i})^{\frac{3+\kappa}{2}}\\&+\sum_{i=1}^{2}\frac{s_{k,i}}{4}({\varsigma_{k,i}}^{\mathrm{T}}\varsigma_{k,i})^2+\frac{\iota_k}{\lambda_k}\tilde{\theta}_k\dot{\hat{\theta}}_k+(\tau_k+\varepsilon_k^2).\end{aligned} \tag{31}$$

***Step* 3:** *Based on Lemma 2 that the inequality $\|\bar{\alpha}_k-\alpha_k\|\le\pi_{k,1}$ holds at a finite time. By employing the Young's inequality, one can obtain*

$${\varsigma_{k,1}}^{\mathrm{T}}\varsigma_{k,1}{\varsigma_{k,1}}^{\mathrm{T}}(\bar{\alpha}_k-\alpha_k)\le\frac{3}{4}({\varsigma_{k,1}}^{\mathrm{T}}\varsigma_{k,1})^2+\frac{1}{4}{\pi_{k,1}}^4, \tag{32}$$

$${\varsigma_{k,1}}^{\mathrm{T}}\varsigma_{k,1}{\varsigma_{k,1}}^{\mathrm{T}}\varsigma_{k,2}\le\frac{3}{4}({\varsigma_{k,1}}^{\mathrm{T}}\varsigma_{k,1})^2+\frac{1}{4}({\varsigma_{k,2}}^{\mathrm{T}}\varsigma_{k,2})^2. \tag{33}$$

For the error compensation system, choose the following Lyapunov function

$$\bar{V}_k=\sum_{i=1}^{2}\frac{({\varsigma_{k,i}}^{\mathrm{T}}\varsigma_{k,i})^2}{4}. \tag{34}$$

Differentiating it, one can get

$$\begin{aligned}L\bar{V}_k=&{\varsigma_{k,1}}^{\mathrm{T}}\varsigma_{k,1}{\varsigma_{k,1}}^{\mathrm{T}}\dot{\varsigma}_{k,1}+{\varsigma_{k,2}}^{\mathrm{T}}\varsigma_{k,2}{\varsigma_{k,2}}^{\mathrm{T}}\dot{\varsigma}_{k,2}\\=&-\sum_{i=1}^{2}s_{k,i}({\varsigma_{k,i}}^{\mathrm{T}}\varsigma_{k,i})^2-\sum_{i=1}^{2}t_{k,i}{\varsigma_{k,i}}^{\mathrm{T}}\varsigma_{k,i}{\varsigma_{k,i}}^{\mathrm{T}}\varsigma_{k,i}^{\kappa}\\&+(o_k+\sum_{j=1}^{N}\rho_{k,j}){\varsigma_{k,1}}^{\mathrm{T}}\varsigma_{k,1}{\varsigma_{k,1}}^{\mathrm{T}}[(\bar{\alpha}_k-\alpha_k)+\varsigma_{k,2}].\end{aligned} \tag{35}$$

Then, let $V=V_2+\bar{V}$ and it can be obtained that

$$\begin{aligned}LV_k\le&-\sum_{i=1}^{2}\frac{t_{k,i}\kappa}{\kappa+3}({z_{k,i}}^{\mathrm{T}}z_{k,i})^{\frac{3+\kappa}{2}}\\&-\sum_{i=1}^{2}\frac{3t_{k,i}}{\kappa+3}({\varsigma_{k,i}}^{\mathrm{T}}\varsigma_{k,i})^{\frac{3+\kappa}{2}}-\frac{\iota_k}{2\lambda_k}\tilde{\theta}_k^2\\&-(\frac{3}{4}s_{k,1}-\frac{3}{2}(o_k+\sum_{j=1}^{N}\rho_{k,j}))({\varsigma_{k,1}}^{\mathrm{T}}\varsigma_{k,1})^2\\&-(\frac{3}{4}s_{k,2}-\frac{1}{4}(o_k+\sum_{j=1}^{N}\rho_{k,j}))({\varsigma_{k,2}}^{\mathrm{T}}\varsigma_{k,2})^2+\Gamma_k',\end{aligned} \tag{36}$$

where $\Gamma_k'=\tau_k+\varepsilon_k^2+\frac{1}{4}{\pi_{k,1}}^4(o_k+\sum_{j=1}^{N}\rho_{k,j})+\frac{\iota_k}{2\lambda_k}\theta_k^2$.

Moreover, there is

$$\frac{\iota_k}{\lambda_k}\tilde{\theta}_k\dot{\hat{\theta}}_k\le-\frac{\iota_k}{2\lambda_k}\tilde{\theta}_k^2+\frac{\iota_k}{2\lambda_k}\theta_k^2. \tag{37}$$

Based on Lemma 5, choosing $x=\frac{\tilde{\theta}_k^2}{2\lambda_k}$, $z=1$, $p=\frac{\kappa+3}{4}$, $q=1-p=\frac{1-\kappa}{4}$,$\alpha(x,z)=\frac{4}{\kappa+3}$, it follows that

$$(\frac{\tilde{\theta}_k^2}{2\lambda_k})^{\frac{\kappa+3}{4}}\le\frac{\tilde{\theta}_k^2}{2\lambda_k}+\frac{1-\kappa}{4}(\frac{4}{\kappa+3})^{-\frac{\kappa+3}{1-\kappa}}. \tag{38}$$

Let $\frac{3}{4}s_{k,1} - \frac{3}{2}(o_k + \sum_{j=1}^{N}\rho_{k,j}) = 0$*, it obtains* $s_{k,1} = 2(o_k + \sum_{j=1}^{N}\rho_{k,j})$*, similarly,* $s_{k,2} = \frac{1}{3}(o_k + \sum_{j=1}^{N}\rho_{k,j})$*. Substituting that into (36), it is straightforward to show that*

$$LV_k \leq -\sum_{i=1}^{2}\frac{t_{k,i}\kappa}{\kappa+3}(z_{k,i}{}^{\mathrm{T}}z_{k,i})^{\frac{3+\kappa}{2}} - \sum_{i=1}^{2}\frac{3t_{k,i}}{\kappa+3}(\varsigma_{k,i}{}^{\mathrm{T}}\varsigma_{k,i})^{\frac{3+\kappa}{2}} - \iota_k(\frac{\tilde{\theta}_k^2}{2\lambda_k})^{\frac{\kappa+3}{4}} + \Gamma_k. \tag{39}$$

where $\Gamma_k = \Gamma'_k + \frac{1-\kappa}{4}(\frac{4}{\kappa+3})^{-\frac{\kappa+3}{1-\kappa}}$*. Letting* $\Delta_k = \min\{4^{\frac{\kappa+3}{4}}\frac{t_{k,i}\kappa}{\kappa+3}, 4^{\frac{\kappa+3}{4}}\frac{3t_{k,i}}{\kappa+3}, \iota_k\}$*, then we have*

$$LV_k \leq -\Delta_k V_k^{\frac{\kappa+3}{4}} + \Gamma_k. \tag{40}$$

Step 4***:*** *Based on the consensus error (7), the stability analysis for all agents is discussed below. Consider a Lyapunov function as:* $V = \sum_{k=1}^{N}V_k$*, Just like before, it is clear that*

$$LV = -\sum_{k=1}^{N}\Delta_k V_k^{\frac{\kappa+3}{4}} + \sum_{k=1}^{N}\Gamma_k. \tag{41}$$

Let $\Delta = \min\{\Delta_k\}$*,* $\Gamma = \sum_{k=1}^{N}\Gamma_k$*, and based on Lemma 4 it can obtain*

$$LV \leq -\Delta(\sum_{k=1}^{N}V_k)^{\frac{\kappa+3}{4}} + \Gamma = -\Delta V^{\frac{\kappa+3}{4}} + \Gamma. \tag{42}$$

Denoting $V(x(t)) = V$*, the Itô formula allows it to obtain the following result for* $0 \leq s_1 \leq s_2$*,*

$$E[V(x(s_2))] = EV(x(s_1)) + \int_{s_1}^{s_2}E[LV(x(t))]dt. \tag{43}$$

Taking equation (40) into (43) and applying the Jensen's inequality, it yields

$$E[LV(x(t))] \leq -\Delta[E[V(x(t))]]^{\frac{\kappa+3}{4}} + \Gamma. \tag{44}$$

Substituting (44) into (43), it yields

$$\begin{aligned} &E[V(x(s_2))] - E[V(x(s_1))] \\ &\leq -\Delta\int_{s_1}^{s_2}[E[V(x(t))]]^{\frac{\kappa+3}{4}}dt + \Gamma(s_2 - s_1). \end{aligned} \tag{45}$$

Using $\zeta(x(t)) = EV(x(t))$*, and based on Lemma 6, it can deduce that there is a setting time* $T = \frac{4}{(1-\kappa)\sigma\Delta}[[EV(x)]^{\frac{1-\kappa}{4}} - (\frac{\Gamma}{(1-\sigma)\Delta})^{\frac{1-\kappa}{\kappa+3}}]$*, such that* $E[V(x(t))] \leq \epsilon$ *for* $\forall t \geq T$*,* $\epsilon = 4(\frac{\Gamma}{(1-\sigma)\Delta})^{\frac{1}{\kappa+3}}$*. Because of this, the following inequality is satisfied*

$$E\left(\sum_{k=1}^{N}\sum_{i=1}^{2}(z_{k,i}{}^{T}z_{k,i})^2\right) \leq 4E[V(x(t))] \leq 4\epsilon, t \geq T. \tag{46}$$

Using the mathematical expectation property, it can be inferred that

$$\begin{aligned} &[E\|z_{k,i}\|^2]^2 \leq E(\|z_{k,i}\|^4) \\ &\leq E\left(\sum_{k=1}^{N}\sum_{i=1}^{2}(z_{k,i}{}^{T}z_{k,i})^2\right) \leq 4\epsilon, t \geq T. \end{aligned} \tag{47}$$

Thus

$$E\|z_{k,i}\|^2 \leq 2\sqrt{\epsilon}, t \geq T. \tag{48}$$

Similarly, it is concluded that

$$E\|\varsigma_{k,i}\|^2 \leq 2\sqrt{\epsilon}, E\|\tilde{\theta}_{k,i}\|^2 \leq 2\sqrt{\epsilon}, t \geq T. \tag{49}$$

Since $z_1 = e_1 - \varsigma_1$*, one has*

$$E\|e_{k,1}\|^2 \leq 2E\|z_{k,1}\|^2 + 2E\|\varsigma_{k,1}\|^2 \leq 4\sqrt{\epsilon}, t \geq T. \tag{50}$$

Q.E.D

4 Simulation Results

We consider a two-link multi-manipulator system suspended from a randomly vibrating ceiling to validate the proposed approach. Figure 1 illustrates the communication topology of the system, consisting of one leader and four followers, represented under a directed graph.

The system is modeled as a double pendulum on a vertical plane, neglecting air resistance. Let $\xi_{k,1}$ and $\xi_{k,2}$ represent the horizontal and vertical accelerations of the suspension point O, respectively. These accelerations are modeled as independent white noise. The parameters $m_{k,i}$ and $l_{k,i}$ $(k \in N, i = 1, 2)$ denote the masses and lengths of the "upper" and "lower" pendulums, with their specific values listed in TABLE I. The gravitational acceleration is denoted by g (unit: m/s^2).

The generalized coordinates $q_{k,i}$ $(i = 1, 2)$ represent the angles (in radians) between the pendulums and the vertical axis, while the control inputs $u_{k,i}$ $(i = 1, 2)$ are the torques applied at the joints (unit: $\mathrm{N} \cdot \mathrm{m}$).

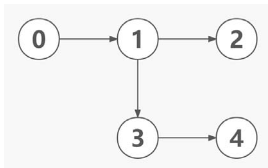

Fig. 1: Communication topology.

Table 1: Parameters in all two-link manipulator systems.

Parameters	**Description**	**Values**	**Unit**
m	the mass of the link	$m = [0.5, 0.6]^{\mathrm{T}}$	kg
l	the distance of the link	$l = [0.8, 1.2]^{\mathrm{T}}$	m
g	acceleration of gravity	$g = 9.8$	m/s^2

Refer to the example given in [15], the system functions is of the form of Equation (2),

in which $q_k = [q_{k,1}, q_{k,2}]$, $M_k(q_k) = [M_{mn}] \in \mathbb{R}^{2\times2}$ and $C_k(q_k, \dot{q}_k) \in \mathbb{R}^{2\times2}$ are given by

$$M_k = \begin{bmatrix} (m_1+m_2)l_1^2 & m_2 l_1 l_2\cos(q_{k,2} - q_{k,1}) \\ m_2 l_1 l_2\cos(q_{k,2} - q_{k,1}) & m_2 l_2^2 \end{bmatrix},$$

$$C_k = \begin{bmatrix} 0 & C_{12} \\ C_{21} & 1 \end{bmatrix},$$

where

$$C_{12} = -m_2 l_1 l_2\sin(q_{k,2} - q_{k,1})\ddot{q}_{k,2},$$

$$C_{21} = m_2 l_1 l_2 \cos(q_{k,2} - q_{k,1})\dot{q}_{k,1}.$$

$h_k(q) \in \mathbb{R}^2$ and $\Lambda_k(q) \in \mathbb{R}^{2\times 2}$ are given by

$$h_k = \begin{bmatrix} (m_1 + m_2) g l_1 \sin(q_{k,1}) \\ m_2 g l_2 \sin(\dot{q}_{k,2}) \end{bmatrix};$$

$$\Lambda_{11} = -m_1 l_1 \cos(q_{k,1}) + 0.5 m_2 l_1 \sin(q_{k,1}) \sin(2(q_{k,2} - q_{k,1})),$$
$$\Lambda_{12} = -m_1 l_1 \sin(q_{k,1}) - 0.5 m_2 l_1 \cos(q_{k,1}) \sin(2(q_{k,2} - q_{k,1})),$$
$$\Lambda_{21} = m_2 l_2 \sin(q_{k,1}) \sin(2(q_{k,2} - q_{k,1})),$$
$$\Lambda_{22} = -m_2 l_2 \cos(q_{k,1}) \sin(2(q_{k,2} - q_{k,1})).$$

The control parameters are chosen as $t_{k,1} = 2, t_{k,2} = 1/3$, $s_{k,1} = 2$, $s_{k,2} = 45$, $\varrho_{k,1} = 750$, $\varrho_{k,2} = 180$, $\iota_k = 100$, $b_{k,1} = b_{k,2} = 1$, and $\kappa = 3/5$.

Choosing the reference signal $q_d = [1.5\sin(t), \sin(2t)]^{\mathrm{T}}$ and initial conditions $q_k(0) = [-0.1, 0.25]^{\mathrm{T}}$, $\dot{q}_k(0) = [-0.3, 0.45]^{\mathrm{T}}$, Fig. 2 shows the trajectories of q and desired signal q_d under the proposed control scheme.

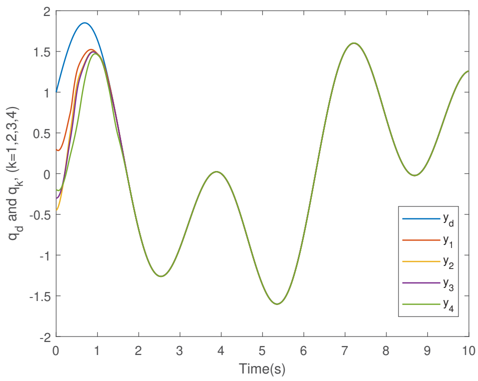

Fig. 2: Schematic diagram of trajectory tracking for q_d and q_k

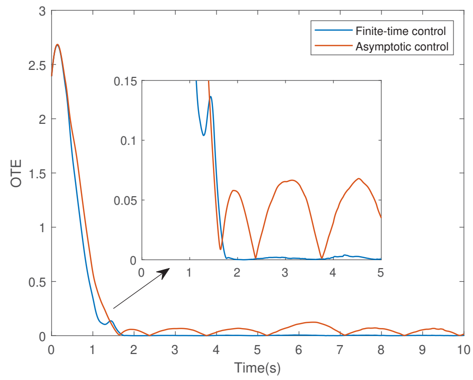

Fig. 3: The overall tracking error comparison by Finite-time and Asymptotic control.

To evaluate the effectiveness of the finite-time control algorithm proposed in this paper compared to the asymptotic control method in [19], we define the overall tracking error as ERR $= |q - q_d|$.

Figure 3 illustrates the the overall tracking error under varying values of κ, while keeping all other parameters constant. Clearly, the proposed finite-time CFAB algorithm demonstrates a faster convergence rate and higher tracking accuracy compared to the asymptotic control method.

5 Conclusions

The finite-time tracking control problem of an n-linked manipulator system with parameter uncertainty in a random vibration environment is investigated in this paper. After establishing a model based on a stochastic Lagrangian control system, an adaptive neural network control strategy combining the command filtered backstepping method is given. Even with inertial uncertain parameters, random vibrations and input saturation, the proposed scheme is able to overcome the complexity explosion problem of conventional backstepping and guarantee that tracking error be practically finite-time stable in mean square.

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