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Hyperbolic Embedding of Multilayer Networks

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Extended Abstract

Multilayer networks provide a powerful framework to describe systems where multiple types of interactions coexist. Most embedding methods collapse these layers into a single representation, obscuring key structural properties. We introduce a hyperbolic embedding framework that preserves both intra-layer structures and inter-layer dependencies, extending coalescent embedding to the multilayer setting with a tunable coupling parameter. The method is validated on synthetic benchmarks and brain connectivity data, where it reveals meaningful community organization and disease-related patterns. Our work highlights the advantages of hyperbolic geometry for comparative analysis of multilayer networks [1].

Complex systems often involve multiple types of relationships, which can be naturally described using *multilayer networks* [2, 3]. For instance, social networks combine friendship, professional, and online interactions, while brain networks are studied across different modalities or temporal epochs. A major challenge is to embed these multilayer systems into low-dimensional geometric spaces for visualization, clustering, and inference. Hyperbolic geometry has been shown to capture the scale-free and hierarchical nature of single-layer networks [4, 5], but existing embeddings largely ignore multilayer structure, leading to information loss.

We propose a method for *multi-layer hyperbolic embeddings* that: *i)* represents each layer in the Poincaré disk while preserving intra-layer latent geometry, *ii)* controls inter-layer coupling with a parameter μ, *iii)* accommodates heterogeneous node sets and inter-layer edges, and *iv)* supports comparative analysis across layers and integrated visualization.

Our approach extends the coalescent embedding framework [6] to multilayer contexts. Nodes are first placed in the hyperboloid model and projected onto the Poincaré disk, where radial coordinates reflect node popularity and angular coordinates capture similarity. Two key innovations are introduced: 1) coupling parameter μ: Controls the degree of similarity between layer embeddings, from independent layouts ($\mu = 0$) to maximally coupled ones ($\mu = 1$) and, 2) joint optimization: Balances intra-layer likelihood maximization with inter-layer consistency, enabling the method to exploit both local and global information.

Synthetic data: We tested the method on multilayer stochastic block models (SBM). Compared to independent embeddings, our framework preserves community structures more effectively and achieves lower distortion between latent and reconstructed distances. *Brain networks:* We applied the method to multilayer anatomical brain networks from epileptic patients. Each layer corresponds to a subject, with edges representing functional connectivity. Our embeddings consistently clustered disease-associated regions, demonstrating potential for comparative network neuroscience. In contrast, independent embeddings produced inconsistent results across layers.

The results confirm that hyperbolic geometry is particularly suitable for multilayer networks. By preserving both intra-layer and cross-layer organization, our method enables: *ii) Comparability:* layers can be analyzed individually or compared within the same geometric framework, and *iii) Generalization:* beyond neuroscience, applications include social multiplexes, temporal interaction modeling, and multilayer infrastructures.

To conclude, we present a multilayer hyperbolic embedding framework that extends coalescent embedding and introduces tunable layer coupling. Validated on synthetic and empirical data, the method preserves structural patterns and enhances interpretability. Future work will focus on higher-dimensional hyperbolic embeddings, integration with machine learning pipelines, and scaling to massive multilayer networks.

References

[1] M. Guillemaud, V. Dinkelacker, and M. Chavez. "Hyperbolic embedding of multilayer networks". In: *arXiv preprint arXiv:2505.20378* (2025).

[2] M. Kivelä et al. "Multilayer networks". In: *Journal of Complex Networks* 2 (2014), pp. 203–271.

[3] S. Boccaletti et al. "The structure and dynamics of multilayer networks". In: *Physics Reports* 544 (1 2014), pp. 1–122.

[4] M. Boguñá, F. Papadopoulos, and D. Krioukov. "Sustaining the internet with hyperbolic mapping". In: *Nature communications* 1 (2010), p. 62.

[5] P. Fragkiskos et al. "Popularity versus similarity in growing networks". In: *Nature* 489 (2012), pp. 537–540.

[6] A. Muscoloni et al. "Machine learning meets complex networks via coalescent embedding in the hyperbolic space". In: *Nature Communications 2017 8:1* 8 (2017), pp. 1–19.

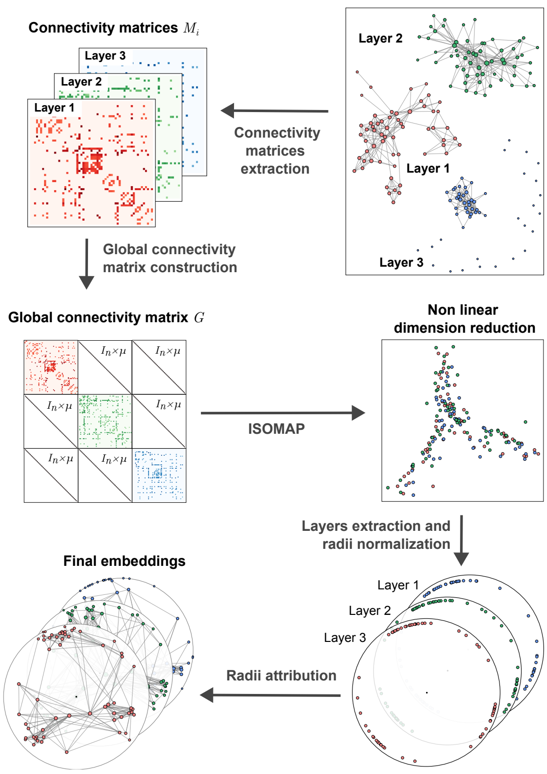

Figure 1: Schematic representation of multilayer hyperbolic embedding. The connectivity matrices of each layer are combined to form a global connectivity matrix G. The dimension reduction algorithm is applied to G to obtain a two-dimensional representation of the dataset. From this embedding, the angular coordinates of the nodes in each layer are extracted, and their radii are initially normalized to one. In a final step, the radial coordinates are reassigned based on the centrality of each node.