

000 001 002 003 004 005 xKV: CROSS-LAYER KV-CACHE COMPRESSION VIA 006 ALIGNED SINGULAR VECTOR EXTRACTION 007 008 009

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ABSTRACT

032 Large Language Models (LLMs) with long context windows enable powerful
033 applications but come at the cost of high memory consumption to store the key
034 and value states (KV-Cache). Recent studies attempted to merge KV-Caches from
035 multiple layers into shared representations, yet these approaches either require
036 expensive pretraining or rely on per-token cosine similarity across layers, which
037 may not always be observed in practice. We find that the dominant singular vectors
038 are remarkably well-aligned across multiple layers of the KV-Cache. Exploiting
039 this insight, we propose \times KV, a post-training compression method that applies
040 Singular Value Decomposition (SVD) on the KV-Cache of grouped layers. \times KV
041 consolidates the KV-Cache of multiple layers into a shared low-rank subspace,
042 significantly reducing KV-Cache sizes. Through extensive evaluations on the
043 RULER long-context benchmark with widely-used LLMs (e.g., Llama-3.1 and
044 Qwen2.5), \times KV achieves up to $8\times$ KV-Cache compression rate while keeping the
045 accuracy gap within 2–3 percentage points of the non-compressed baseline over a
046 set of representative long-context tasks, and remains robust in multi-turn settings.
047 Coupled with the designed *Selective Reconstruction* (SR) at decode time, \times KV–SR
048 (keys only, values offloaded to CPU memory) yields 2.53% higher accuracy than
049 the state-of-the-art system that combined token selection with single-layer SVD and
050 delivers up to **3.23** \times end-to-end generation speedups over full attention on an A100
051 GPU. At a similar accuracy level, \times KV–SR (keys and values on GPU) achieves up
052 to **4.23** \times faster speedups. These results highlight \times KV as a versatile, plug-and-play
053 solution to alleviate both memory and latency bottlenecks in long-context LLM
054 inference.

055 1 INTRODUCTION

056 Large language models (LLMs) (Touvron et al., 2023; OpenAI et al., 2024; Team et al., 2024; Ila,
057 2024; Jiang et al., 2023; Anthropic, 2023) have revolutionized numerous artificial intelligence (AI)
058 applications with advanced cognitive capabilities that were previously unattainable with conventional
059 machine learning (ML) models. Recent efforts to extend the context lengths of LLMs have further
060 expanded their potential: open-sourced models now support up to 1M tokens (Pekelis et al., 2024;
061 Yang et al., 2025), and proprietary ones like Gemini push this limit even further to 10M tokens
062 (Team et al., 2024). These extended context windows unlock a wide range of previously impractical
063 applications, such as large-scale information retrieval and debugging or extending a large-scale
064 codebase (DeepSeek-AI et al., 2025; Dubey et al., 2024; Yang et al., 2025; OpenAI et al., 2024).

065 However, this expanded capability on long-context introduces significant challenges, particularly in
066 the management of key-value (KV) caches during inference (Fu, 2024; Li et al., 2024a). Typically,
067 KV states are cached to avoid redundant computations; yet, under extended context lengths, the
068 memory consumption of KV-Cache rapidly becomes prohibitive. This inflated memory footprint
069 severely limits the number of concurrent inference requests, causing substantial throughput reduction.
070 To address this, researchers have proposed various approaches to mitigate the large memory footprint
071 of KV-Caches. These include quantization (Hooper et al., 2024; Liu et al., 2024c; Chen et al., 2025;
072 Zhao et al., 2023), token eviction (Adnan et al., 2024; Ge et al., 2024; Xiao et al., 2024; Zhang et al.,
073 2024b; Li et al., 2024b; Cai et al., 2024), and low-rank decomposition (Sun et al., 2024a; Chang
074 et al., 2025; Zhang et al., 2024a; Yuan et al., 2023). These approaches have primarily focused on
075 intra-layer redundancies that compress the KV-Cache of each layer separately. While this often yields
076

054 respectable per-layer compression, these methods do not utilize potential redundancy across layers
 055 (Gromov et al., 2024).
 056

057 To exploit cross-layer redundancy, two main lines of work have emerged. The first, represented
 058 by Cross-Layer Attention (CLA) (Brandon et al., 2024) and YOCO (Sun et al., 2024b), introduces
 059 new architectures that share a single set of KV-Cache across groups of adjacent layers. While
 060 effective, these methods require architectural modifications and thus expensive pretraining from
 061 scratch, limiting their applicability to existing pretrained models. A second direction, exemplified
 062 by MiniCache (Liu et al., 2024b), operates in a post-hoc manner by merging adjacent layers’ KV-
 063 Cache under the assumption of high cosine similarity, implemented via spherical linear interpolation
 064 (SLERP) (Shoemake, 1985). Our analysis, however, shows that such similarity, though present to
 065 some extent, is not consistently strong enough across layers to support robust compression, leading to
 066 nontrivial accuracy degradation in practice (see §3.1). Together, prior methods are limited either by
 067 costly pretraining or by fragile similarity assumptions, motivating the need for a new approach.
 068

069 We revisit inter-layer similarity using Centered
 070 Kernel Alignment (CKA) (Kornblith et al.,
 071 2019). Our analysis reveals that, although the
 072 KV-Cache of adjacent layers exhibit low cosine
 073 similarity, their dominant singular vectors re-
 074 main highly aligned (see §3.2). This observation
 075 enables us to share basis vectors across mul-
 076 tiple adjacent layers’ KV-Cache, yielding a more
 077 compact representation.
 078

079 Building on this insight, we propose \times KV, a
 080 fully *plug-and-play* compression method that re-
 081 quires no additional fine-tuning or architectural
 082 modifications. \times KV simultaneously compresses
 083 the KV-Cache of multiple layers by extracting
 084 a *shared* set of singular vectors through cross-
 085 layer SVD, producing a compact token basis
 086 reused across adjacent layers as illustrate in Fig-
 087 ure 1. To further reduce overhead at inference,
 088 we introduce *Selective Reconstruction (SR)*: instead of reconstructing all tokens, we selectively
 089 reconstruct only those relevant to the query (§ 4.3). The pairing of cross-layer compression with SR
 090 substantially lowers reconstruction cost while preserving model accuracy, making \times KV practical for
 091 real-world deployment.
 092

093 To adapt \times KV to diverse deployment requirements, we further design two decoding modes (§ 4.4).
 094 When the target application is latency-sensitive, we use \times KV-SR, which compresses *both* keys and
 095 values and keeps them fully in GPU memory, yielding fastest decoding. When accuracy must be
 096 preserved, we use \times K-SR, which compresses *keys only* while offloading values to CPU memory,
 097 delivering near-lossless accuracy with reduced GPU memory usage.
 098

099 Extensive experiments on RULER with Llama (Ila, 2024) and Qwen (Yang et al., 2024; 2025)
 100 models show that \times KV achieves up to $8\times$ compression rate with minimal accuracy degradation (<3%),
 101 significantly outperforming representative token eviction and quantization baselines. With SR enabled,
 102 \times K-SR yields **>2.5 percentage points** higher accuracy than state-of-the-art single-layer SVD systems.
 103 Most importantly, by keeping the compressed cache entirely on-device, \times KV-SR eliminates PCIe
 104 bottlenecks, translating these efficiency gains into **3.6 \times** faster attention operation and up to **4.23 \times**
 105 higher end-to-end generation throughput over Full KV-Cache baseline with FlashAttention-2 CUDA
 106 kernel on Llama-3.1-8B.
 107

108 2 RELATED WORK

109 **Low-Rank KV-Cache Compression.** A broad line of research exploits the *low-rank nature* of the
 110 KV-Cache to reduce its memory footprint. For instance, Multi-Head Latent Attention (MLA) (Liu
 111 et al., 2024a; DeepSeek-AI et al., 2025) projects tokens onto a low-rank subspace and caches those
 112 latent representations instead of the original key and value states, however, MLA requires training the
 113 model from scratch. In contrast, several *post-training* techniques decompose the key/value parameter

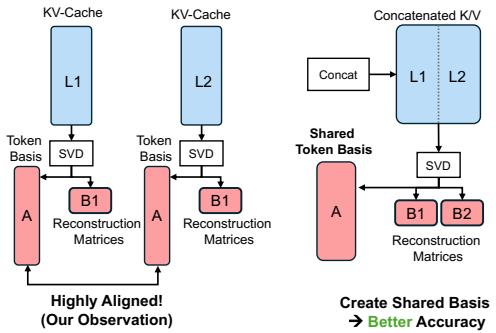
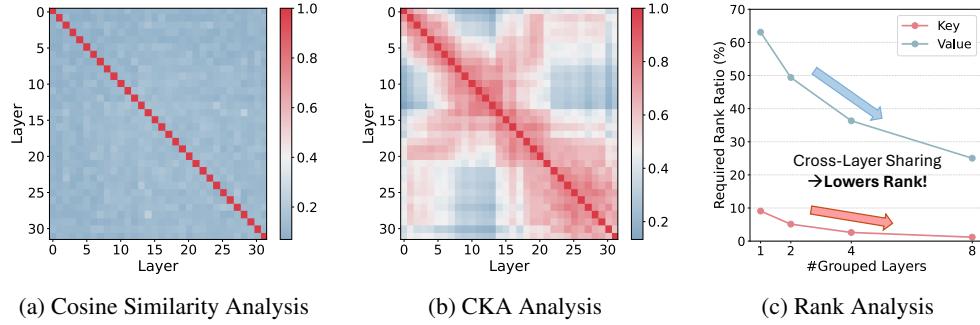


Figure 1: The token basis (singular vectors) of two different layers’ KV-Cache are highly aligned. \times KV concatenates adjacent layers and performs one SVD to obtain a shared basis **A** with layer-specific **B₁** and **B₂**, improving accuracy at a fixed rank and reducing memory.

108 matrices to obtain low-rank projection modules similar to MLA, such as ASVD (Yuan et al., 2023),
 109 Palu (Chang et al., 2025), and LoRC (Zhang et al., 2024a). Other methods decompose the KV-Cache
 110 directly: EigenAttention (Saxena et al., 2024) applies SVD to a calibration dataset to derive projection
 111 matrices, whereas ShadowKV (Sun et al., 2024a) performs online SVD to capture the dynamics of
 112 different contexts. In xKV, we also exploit the low-rank nature of KV-Cache. However, unlike prior
 113 methods focusing on per-layer compression, xKV further considers the shared information among
 114 multiple layers and extends the usage of low-rank projections to a new cross-layer dimension.

115 **Cross-Layer KV-Cache Optimization.** Going beyond the intra-layer perspective, another stream
 116 of research explores inter-layer redundancy of KV-Cache (Brandon et al., 2024; Sun et al., 2024b;
 117 Wu & Tu, 2024; Liu et al., 2024b; Dong et al., 2025). CLA (Brandon et al., 2024) and YOCO(Sun
 118 et al., 2024b) both modify the Transformer model architecture so that later layers can directly reuse or
 119 reference KV states from earlier layers. LCKV (Wu & Tu, 2024) restricts full KV storage to a small
 120 subset of layers, foregoing caches in other layers. However, these methods rely on retraining or model
 121 fine-tuning, which makes them less flexible. Minicache (Liu et al., 2024b), in contrast, provides a
 122 flexible post-training alternative by merging the key and value tokens from adjacent similar layers
 123 using spherical linear interpolation. Our approach goes further by extracting shared singular vectors
 124 of multiple layers’ KV-Caches, thereby enabling higher compression.

125 **Dynamic Token Selection and KV Offloading.** A complementary line of work accelerates decoding
 126 by selecting a small subset of context tokens per step (dynamic sparse attention). Quest (Tang
 127 et al., 2024) proposes query-aware page selection to reduce attention cost without compressing the
 128 KV-Cache. ShadowKV (Sun et al., 2024a) stores a low-rank key cache on GPU, offloads values to
 129 CPU, and employs an accurate landmark-guided selector with a small static outlier set to reconstruct
 130 minimal sparse KV pairs on-the-fly, improving throughput under long contexts. In contrast, xKV
 131 targets cross-layer KV compression: we extract a shared low-rank token basis across adjacent layers
 132 and pair it with selective reconstruction. This lets us (i) match ShadowKV’s “keys-only + offloaded
 133 values” regime via xK-SR, and (ii) run xKV-SR with both keys and values compressed on GPU,
 134 avoiding host-device transfer. Empirically, at matched token budgets, xK-SR/xKV-SR achieve
 135 higher accuracy than Quest and ShadowKV while offering stronger speedups when values remain
 136 on-device.



147 Figure 2: (a) Average Token-wise Cosine Similarity for value-caches across different layers. For each
 148 pair of layers, we compute the token-level cosine similarities between their embeddings and average
 149 these values into a single similarity score. (b) CKA Matrix for the value-cache. The higher (warmer)
 150 values indicate stronger singular vector alignment across layers. (c) Required rank ratio (percentage
 151 of total dimension) for capturing 95% of the cumulative eigenvalues in the key (red) and value (blue)
 152 matrices, plotted against the number of grouped layers. For each group, we horizontally concatenate
 153 the key/value caches and compute the rank needed to achieve 95% of the cumulative eigenvalues.
 154 As the grouping increases, a smaller rank (relative to total dimension) is required, implying a higher
 155 compression rate for the same level of information preservation. We perform these analyses on
 156 the KV-Cache obtained from Llama-3.1-8B-Instruct, using the multi-valued NIAH dataset from the
 157 RULER (Hsieh et al., 2024) benchmark.

3 ANALYSIS AND MOTIVATION

161 In this section, we examine the cross-layer similarity of KV-Caches with different metrics to reveal
 162 the motivation behind the design of xKV.

162 3.1 CROSS-LAYER COSINE SIMILARITY (PRIOR WORK)
163

164 To understand the assumption used in the previous work (Liu et al., 2024b), we first measure token-
165 wise cosine similarity across various layer-pairs. The measurement on the cosine similarity is
166 presented in Figure 2a. Notably, the adjacent layers exhibit low token-wise similarity. This modest
167 similarity fundamentally limits the compression rate achieved by prior representative methods (Liu
168 et al., 2024b).

169 3.2 REVISIT CROSS-LAYER SIMILARITY WITH CKA
170

171 While token-wise (cosine) similarity offers a local perspective, a more holistic view can reveal deeper
172 patterns in how an entire KV-Cache is aligned across layers. Specifically, we adopt Centered Kernel
173 Alignment (CKA) (Kornblith et al., 2019) to measure the similarity in the overall structure of two
174 layers’ KV-Caches. Concretely, for a layer ℓ with KV-Cache $\mathbf{X}_\ell \in \mathbb{R}^{n \times d}$, we first define the centered
175 Gram matrix

$$176 \mathbf{G}_\ell = \mathbf{H} \mathbf{X}_\ell \mathbf{X}_\ell^\top \mathbf{H}, \quad \text{where } \mathbf{H} = \mathbf{I}_n - \frac{1}{n} \mathbf{1} \mathbf{1}^\top.$$

177 Then, the CKA between two layers ℓ_1 and ℓ_2 is

$$178 \text{CKA}(\mathbf{X}_{\ell_1}, \mathbf{X}_{\ell_2}) = \frac{\text{trace}(\mathbf{G}_{\ell_1} \mathbf{G}_{\ell_2})}{\sqrt{\text{trace}(\mathbf{G}_{\ell_1}^2) \text{trace}(\mathbf{G}_{\ell_2}^2)}}.$$

182 Unlike the token-wise cosine similarity metric, which simply compares corresponding token embed-
183 dings, CKA reflects the similarity of *the entire distribution* of token embeddings in each layer. If
184 CKA($\mathbf{X}_{\ell_1}, \mathbf{X}_{\ell_2}$) is high, then the dominant left singular vectors of \mathbf{X}_{ℓ_1} are highly aligned to those
185 of layer ℓ_2 (ref. Appendix A). In other words, the basis vectors that define how the token varies in
186 these two layers might be similar.

187 **Observation 1: Highly Aligned Basis.** In Figure 2b, we show the CKA value between each layers’
188 KV-Cache of Llama-3.1-8B-Instruct. As shown in Figure 2b, many pairs of layers exhibit remarkably
189 high CKA (red blocks) even though their token-wise cosine similarities are quite modest. This
190 observation suggests that, although individual token embeddings differ across layers, the dominant
191 singular vectors (*i.e.*, *basis*) that span the KV-Cache are, in fact, *well-aligned*. Thus, focusing solely
192 on the cosine similarity between pairs of token embeddings can underestimate the potential for
193 *cross-layer* merging and compression.

194 3.3 EIGENVALUE ANALYSIS OF KV-CACHE
195

197 **Observation 2: Horizontally Concatenated KV-Caches Exhibit Lower Rank.** Motivated by the
198 observation that different layers’ basis are well aligned, we examine the rank to achieve a certain
199 level of information preservation after horizontally concatenating the KV-Caches across multiple
200 layers. Because each layer shows substantial cross-layer overlap (§3.2), a *single* set of low-rank
201 basis vectors can effectively approximate the KV-Caches of all layers in the group. As illustrated
202 in Figure 2c, a larger group size reduces the fraction of total rank needed to preserve the same
203 cumulative eigenvalues. Compared with creating separate low-rank subspaces for each layer, this
204 shared approach avoids storing nearly identical basis vectors multiple times, yielding a more compact
205 yet expressive representation. In §4, we leverage these observations to propose our xKV method that
206 achieves significantly higher compression ratios while preserving model accuracy.

207 4 METHODOLOGY: xKV
208

209 4.1 NOTATION

210 We consider a Transformer with N decoder blocks and a long prompt of length L . Let d denote the
211 KV hidden size. Under GQA, $d = H_{\text{kv}} \cdot d_h$ with H_{kv} KV heads and per-head width d_h . Because the
212 same decomposition/reconstruction pipeline applies to both keys and values, we use a *unified* symbol

$$213 \mathbf{X}_\ell^\tau \in \mathbb{R}^{L \times d}, \quad \tau \in \{K^{\text{pre}}, V\},$$

214 to denote the cache of type τ at layer ℓ . For RoPE models, we always decompose *pre-RoPE keys*
($\tau = K^{\text{pre}}$) and re-apply RoPE after reconstruction.

216 **Decode-time head mapping and row selection.** Let H_q be the number of query heads and
 217 $\rho : [H_q] \rightarrow [H_{kv}]$ the GQA mapping from query heads to KV heads. At decode step t , for each layer
 218 ℓ and KV head g , we will use an index set $\mathcal{S}_{t,\ell,g} \subseteq [L]$ of selected prompt rows with $M_{t,\ell,g} = |\mathcal{S}_{t,\ell,g}|$
 219 (§. 4.3).

220 4.2 CORE METHOD: CROSS-LAYER SVD

222 Motivated by our empirical finding that the dominant left singular vectors of KV-Caches are
 223 well-aligned across adjacent layers (§ 3), we group layers into contiguous strides of size G :

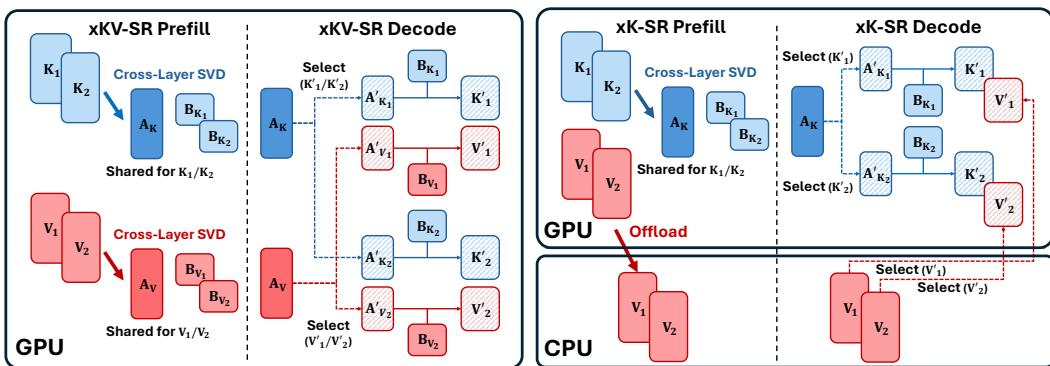
$$224 \quad \mathcal{G}_k = \{kG, \dots, kG + G - 1\}, \quad k = 0, 1, \dots, \frac{N}{G} - 1.$$

225 For a group $\mathcal{G}_k = \{\ell_1, \dots, \ell_G\}$ and type $\tau \in \{K^{\text{pre}}, V\}$, we horizontally concatenate the group's
 226 caches and compute a single low-rank factorization:

$$228 \quad [\mathbf{X}_{\ell_1}^\tau, \dots, \mathbf{X}_{\ell_G}^\tau] \approx \mathbf{U}_{k,r_\tau}^\tau \mathbf{S}_{k,r_\tau}^\tau (\mathbf{V}_{k,r_\tau}^\tau)^\top = \underbrace{\mathbf{A}_k^\tau}_{\in \mathbb{R}^{L \times r_\tau}} \left[\underbrace{\mathbf{B}_{\ell_1}^\tau}_{\in \mathbb{R}^{r_\tau \times d}} \cdots \underbrace{\mathbf{B}_{\ell_G}^\tau}_{\in \mathbb{R}^{r_\tau \times d}} \right], \quad (1)$$

229 where $\mathbf{A}_k^\tau = \mathbf{U}_{k,r_\tau}^\tau \mathbf{S}_{k,r_\tau}^\tau$ is the *shared token basis* for the group, and \mathbf{B}_ℓ^τ are layer-specific reconstruc-
 230 tion matrices. Compared to single-layer SVD, this *cross-layer* factorization learns a shared subspace
 231 across adjacent layers and is effective for *both keys and values*. Each layer-specific reconstruction
 232 matrix can also be viewed as the concatenation of KV-head specific reconstruction matrices:

$$233 \quad \mathbf{B}_\ell^\tau = [\mathbf{B}_{\ell,1}^\tau \cdots \mathbf{B}_{\ell,H_{kv}}^\tau], \quad \mathbf{B}_{\ell,g}^\tau \in \mathbb{R}^{r_\tau \times d_h}$$



249 Figure 3: Illustration of different optimized operation modes. xKV-SR design (**Left**) keeps both
 250 low-rank key and value caches on the GPU. xK-SR design (**Right**) keeps the low-rank key cache on
 251 the GPU and offloads the full value cache to the CPU.

252 4.3 PROCESS DURING INFERENCE

254 **Prefill Compression.** During prefill, we compute (1) *separately* for $\tau = K^{\text{pre}}$ and $\tau = V$ for every
 255 group k :

$$256 \quad \{\mathbf{A}_k^K, \{\mathbf{B}_\ell^K\}_{\ell \in \mathcal{G}_k}\}, \quad \{\mathbf{A}_k^V, \{\mathbf{B}_\ell^V\}_{\ell \in \mathcal{G}_k}\}.$$

257 We perform the decomposition online during prefill to capture prompt dynamics (the added cost is a
 258 small fraction of prefill and diminishes as L grows). Empirically, the online cross-layer SVD accounts
 259 for only 3.9% of prefill time at sequence length of 128K (See Appendix C.1). Newly generated
 260 tokens are left uncompressed by default (their length is typically $\ll L$ in long-context use); for very
 261 long generations, we may reapply cross-layer SVD to those tokens.

263 **Dense reconstruction (baseline cost).** A direct use of the factors would, for each $\ell \in \mathcal{G}_k$ and head
 264 g , reconstruct all L rows:

$$265 \quad \hat{\mathbf{X}}_{\ell,g}^\tau = \mathbf{A}_k^\tau \mathbf{B}_{\ell,g}^\tau.$$

266 For keys in RoPE models, we then set $\hat{\mathbf{K}}_{\ell,g}^{\text{rope}} = \text{RoPE}(\hat{\mathbf{X}}_{\ell,g}^{K^{\text{pre}}})$ by applying RoPE per row using its
 267 original position index. This dense strategy reconstruction FLOPs \mathbf{AB} cost that scales with sequence
 268 L at every step (Appendix D.5). Despite the memory saving that the decomposition can offer, this
 269 additional computation cost can pose an extra latency overhead during decoding.

270 **Selective reconstruction.** Prior work shows that LLMs exhibit strong *attention sparsity* during
 271 decoding, with most queries attending only to a small subset of context tokens (Sun et al., 2024a;
 272 Tang et al., 2024; Cai et al., 2024). Inspired by this characteristic, we leverage this inherent sparsity
 273 nature and reconstruct only the tokens that are likely to matter at that step. Specifically, at step t we
 274 reconstruct only rows in $\mathcal{S}_{t,\ell,g} \subseteq [L]$:

$$\hat{\mathbf{X}}_{\ell,g}^\tau[\mathcal{S}_{t,\ell,g}, :] = \mathbf{A}_k^\tau[\mathcal{S}_{t,\ell,g}, :] \mathbf{B}_{\ell,g}^\tau. \quad (2)$$

277 For any query head h with $\rho(h) = g$, attention is then computed using $\hat{\mathbf{X}}_{\ell,g}^\tau$ restricted to $\mathcal{S}_{t,\ell,g}$.
 278 (For RoPE models, we decompose pre-RoPE keys and apply RoPE after reconstruction.) In our
 279 implementation, the sets $\mathcal{S}_{t,\ell,g}$ are produced by a landmark-guided Top- k chunk selector with a small
 280 static outlier set (Sun et al., 2024a). We provide the detailed workflow on how the indices $\mathcal{S}_{t,\ell,g}$ in
 281 Appendix B.1 and further analysis on the reconstruction FLOPs in Appendix D.5.

283 4.4 OPERATION MODES

285 We design two operation modes, **xKV-SR** and **xK-SR**, optimized for different scenarios. The
 286 overview is presented in Figure 3.

288 **Joint key-value compression with selective reconstruction (xKV-SR).** Leveraging cross-layer
 289 SVD, xKV can effectively compress *both* keys and values while maintaining strong accuracy, reducing
 290 the total KV footprint in device memory. With effective compression, we can fit the entire compressed
 291 KV on GPU’s memory and avoid the necessity of KV-Cache offloading that induces host-device
 292 transfer, which is crucial when host-device bandwidth is limited (e.g., PCIe-only servers) or on
 293 unified-memory/embedded platforms (e.g., Jetson-class devices), allowing more requests per GPU
 294 and lower end-to-end latency.

296 **Key-only compression with selective reconstruction and value offloading (xK-SR).** When
 297 host-to-device bandwidth is sufficient (e.g., 900GB/s on GB200 Goldwasser et al. (2024)), we
 298 adopt a key-only compression strategy that offloads the value cache to CPU memory, similar to
 299 ShadowKV (Sun et al., 2024a). Our analysis (Figure 2c) shows that values are relatively high-rank
 300 and more sensitive to compression, so leaving them uncompressed preserves accuracy. To mitigate
 301 the added memory cost of this design, we overlap key reconstruction (Eq. 2) with host-device
 302 value transfers, effectively hiding reconstruction latency behind data movement. Unlike ShadowKV,
 303 however, xK-SR leverages xKV’s *cross-layer* key factorization, yielding higher accuracy under the
 304 same memory budget.

305 5 ACCURACY EVALUATIONS

307 **Models.** We evaluate xKV on three widely used language models using Grouped-Query Attention
 308 (GQA): Llama-3.1-8B-Instruct (Dubey et al., 2024) (8 KV heads) and Qwen2.5-7B-Instruct-1M
 309 (Yang et al., 2025) (4 KV heads). In Appendix E, we also evaluate xKV on DeepSeek-Coder-V2-Lite-
 310 Instruct (Dai et al., 2024) with Multi-head Latent Attention (MLA) and Mixture-of-Experts (MoE) to
 311 demonstrate xKV’s high compatibility with emerging efficient Transformer architectures.

313 **Datasets.** We select RULER (Hsieh et al., 2024) as our major benchmark, which features complex
 314 tasks such as retrieval, multi-hop tracking, and question-answering. We also evaluate our approach
 315 using Needle In A Haystack (NIAH) (Kamradt, 2023) under multi-turn setups. We also provide the
 316 LongBench evaluation in the Appendix D.2.

317 **Baselines.** We compare xKV with the baselines in two scenarios. Firstly, the pure KV-Compression
 318 without selective reconstruction for reducing KV-Cache memory footprint. In this scenario, we
 319 compare against six baselines: (1) MiniCache (Liu et al., 2024b), the inter-layer compression method
 320 based on cosine similarity cross-layer. (2) Single SVD (Sun et al., 2024a), which compresses
 321 KV-Cache by factorizing each layer’s key and value caches independently without exploiting the
 322 cross-layer similarity. (3) Token eviction baselines PyramidKV (Cai et al., 2024) and SnapKV (Li
 323 et al., 2024b). (4) A 2-bit quantization method KIVI (Liu et al., 2024c). (5) A token selection
 324 methodology, StreamingLLM (Xiao et al., 2024), Quest (Tang et al., 2024), that entails dynamic

324 Table 1: KV-Cache Compression Results: Performance of different methods on the RULER bench-
 325 mark evaluated at a context length of 64K. \times KV consistently achieves a higher accuracy than the Full
 326 Attns at the same compression rate or even at a significantly higher compression rate.

| Method | Comp. | N-S1 | N-S2 | N-MK1 | N-MK2 | N-MQ | N-MV | QA-1 | QA-2 | VT | FWE | Avg. |
|-------------------------------|-------|--------|--------|--------|--------|--------|-------|-------|-------|-------|-------|-------|
| Llama-3.1-8B-Instruct | | | | | | | | | | | | |
| Full Attn | 1.00 | 100.00 | 100.00 | 98.96 | 97.92 | 98.96 | 97.66 | 83.33 | 59.38 | 97.29 | 85.42 | 91.89 |
| MiniCache | 1.30 | 89.58 | 66.67 | 43.75 | 10.42 | 14.06 | 21.35 | 61.46 | 35.42 | 49.38 | 58.33 | 45.04 |
| KIVI-2 | 7.10 | 100.00 | 96.88 | 98.96 | 90.63 | 91.41 | 89.58 | 80.21 | 55.21 | 81.46 | 84.38 | 86.87 |
| StreamingLLM | 8.00 | 15.63 | 12.50 | 13.54 | 13.54 | 14.58 | 17.97 | 56.25 | 45.83 | 9.58 | 94.10 | 29.35 |
| PyramidKV | 8.00 | 100.00 | 100.00 | 100.00 | 96.88 | 100.00 | 98.44 | 83.33 | 57.29 | 95.42 | 68.06 | 89.94 |
| SnapKV | 8.00 | 100.00 | 100.00 | 98.96 | 94.79 | 100.00 | 97.66 | 83.33 | 58.33 | 95.00 | 68.75 | 89.68 |
| Single SVD | 8.40 | 25.00 | 51.04 | 61.46 | 96.88 | 28.91 | 44.79 | 47.92 | 36.46 | 3.54 | 61.11 | 45.71 |
| \times KV (Ours) | 8.03 | 100.00 | 96.88 | 97.92 | 97.92 | 96.09 | 96.62 | 78.13 | 56.25 | 86.67 | 78.47 | 88.50 |
| Qwen2.5-7B-Instruct-1M | | | | | | | | | | | | |
| Full Attn | 1.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 95.83 | 84.38 | 60.42 | 90.63 | 86.81 | 91.81 |
| MiniCache | 1.30 | 26.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 12.50 | 14.58 | 0.42 | 3.47 | 5.70 |
| KIVI-2 | 7.10 | 0.00 | 2.08 | 3.13 | 13.54 | 0.00 | 0.78 | 48.96 | 43.75 | 36.46 | 40.63 | 18.93 |
| StreamingLLM | 8.00 | 15.63 | 12.50 | 12.50 | 9.38 | 14.84 | 17.71 | 46.88 | 43.75 | 13.13 | 89.24 | 27.56 |
| PyramidKV | 8.00 | 100.00 | 93.75 | 96.88 | 16.67 | 90.37 | 80.73 | 84.38 | 59.38 | 89.17 | 76.39 | 78.77 |
| SnapKV | 8.00 | 100.00 | 96.88 | 97.92 | 31.25 | 95.31 | 83.07 | 84.38 | 59.38 | 91.25 | 80.56 | 82.00 |
| Single SVD | 8.40 | 100.00 | 97.92 | 96.88 | 98.96 | 97.40 | 91.15 | 64.58 | 56.25 | 73.75 | 61.46 | 83.84 |
| \times KV (Ours) | 8.03 | 100.00 | 100.00 | 100.00 | 98.96 | 100.00 | 90.63 | 80.21 | 58.33 | 82.08 | 81.94 | 89.22 |

344 token selection. (6) A state-of-the-art baseline, ShadowKV (Sun et al., 2024a), that applies single-
 345 layer SVD compression on keys, offloads values to CPU memory, and performs token selection.

346 **Setup.** For \times KV variants, we set the rank for key $r_{K^{pre}} = 384$ and $r_V = 576$ if value compression
 347 is applied. [We use `torch.svd_lowrank` API from PyTorch for performing decomposition](#). We
 348 set the cross-layer group size to be 4 as the default setting. For baseline, we align MiniCache’s official
 349 settings to merge half of the layers, from the middle to the end of the LLM, and vary the compression
 350 rate by adjusting the layer index at which merging begins. For the token eviction (e.g., SnapKV,
 351 PyramidKV) and quantization baseline (KIVI), we adopt the implementation from MInference (Jiang
 352 et al., 2024; Li et al., 2025) library. We keep the newly generated tokens uncompressed for all
 353 comparison targets to ensure fair comparison. Unless specified, we calculate the compression rate by
 354 assuming a context length of 64k.

355 5.1 RESULTS ON RULER DATASETS

357 **KV-Cache Compression Results.** Table 1 reports the performance of \times KV and several representa-
 358 tive compression methods on the RULER benchmark at a 64K context length. As shown in Table 1,
 359 MiniCache suffers dramatic accuracy loss even at a modest 1.3 \times compression rate. This degradation
 360 echos our finding in §3.1), the token-wise cosine similarity in KV-Cache across adjacent layers is
 361 generally low. Compared to single-layer SVD compression, \times KV yields substantial accuracy gains:
 362 at an 8 \times compression rate, \times KV improves average accuracy by 43% on Llama-3.1-8B-Instruct and by
 363 8% on Qwen2.5-7B-Instruct-1M, demonstrating its superior information preservation by exploiting
 364 the inherent alignment of KV-Cache representations across layers.

365 In comparison with token-eviction methods, \times KV achieves 88.50% accuracy on Llama-3.1-8B-Instruct
 366 at 8.03 \times compression, closely matching SnapKV. On Qwen2.5-7B-Instruct-1M, however, both
 367 SnapKV and Pyramid incur noticeable accuracy degradation. We attribute this to Qwen2.5’s inherently
 368 more compact KV cache—due to its smaller number of KV heads—which makes information
 369 preservation more challenging. Despite this, \times KV attains 89.22% average accuracy, narrowing the
 370 gap to the non-compressed baseline to just 2.6%. Moreover, \times KV surpasses the quantization baseline
 371 KIVI-2 by 1.7% on Llama-3.1-8B while maintaining accuracy on Qwen2.5, where KIVI-2 suffers
 372 significant drops. Finally, as shown in Appendix D.4, our approach can be combined with quantization
 373 to further increase compression without sacrificing accuracy.

374 **Results on Multi-turn Conversation Datasets.** We test our method using a multi-turn Needle-In-
 375 A-Haystack (NIAH) benchmark and compare its efficacy against token eviction-based approaches
 376 (e.g., SnapKV and PyramidKV). We conduct the evaluation at context length of 64K. Figure 4 shows
 377 results on Llama-3.1-8B-Instruct. SnapKV and PyramidKV both suffer steep declines after the first
 378 turn because they evict tokens using the initial attention patterns of the first query and cannot recover

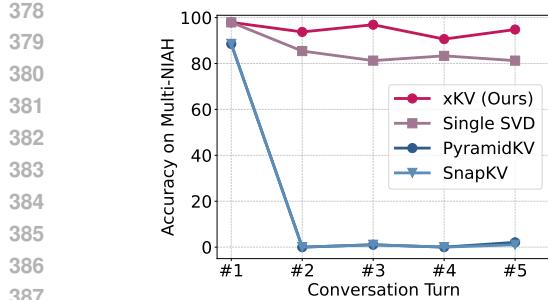


Figure 4: Accuracy of each conversation turn on Multi-turn NIAH. PyramidKV, SnapKV, and xKV are all at a compression rate of 8 \times .

Table 2: Accuracy across different group sizes on RULER with Llama-3.1-8B-Instruct. We align the rank setting with Table 1 and Table 3 for group size 4. For group sizes 1, 2, and 8, we scaled the rank linearly to maintain the same compression rate, with $(r_{K^{pre}}, r_V) = (96, 144)$ and $(192, 288)$, respectively.

| Group Size | xKV | xK-SR | xKV-SR |
|------------|-------|-------|--------|
| 1 | 45.71 | 87.17 | 72.27 |
| 2 | 75.15 | 88.43 | 86.06 |
| 4 | 88.50 | 89.70 | 89.69 |
| 8 | 88.91 | 89.74 | 89.72 |

Table 3: KV-Cache Compression with Selective Reconstruction Results: Accuracy of different methods on the RULER benchmark at a context length of 64K. Here, "Comp." indicates the total KV-Cache reduction, while the number in parentheses shows the effective GPU memory reduction considering KV-Cache offloading. ShadowKV* refers to a variant of ShadowKV that additionally compresses the value cache.

| Method | Comp. | N-S1 | N-S2 | N-MK1 | N-MK2 | N-MQ | N-MV | QA-1 | QA-2 | VT | FWE | Avg. |
|------------------------------|---------------------|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Llama-3.1-8B-Instruct | | | | | | | | | | | | |
| Full Attn Quest | 1.00 1.00 (8.00) | 100.00 93.75 | 100.00 90.63 | 98.96 96.88 | 97.92 87.50 | 98.96 94.27 | 97.66 85.42 | 83.33 83.33 | 59.38 57.29 | 97.29 77.71 | 85.42 81.94 | 91.89 84.87 |
| ShadowKV | 1.64 (9.08) | 100.00 | 100.00 | 98.96 | 97.92 | 96.88 | 94.53 | 82.29 | 60.42 | 66.04 | 74.65 | 87.17 |
| xKV-SR (Ours) | 1.63 (8.90) | 100.00 | 100.00 | 98.96 | 97.92 | 98.44 | 95.31 | 83.33 | 60.42 | 87.92 | 74.65 | 89.70 |
| ShadowKV* | 5.51 | 100.00 | 76.04 | 75.00 | 97.92 | 54.43 | 45.83 | 81.25 | 57.29 | 47.29 | 74.31 | 70.94 |
| xKV-SR (Ours) | 5.35 ¹ | 100.00 | 100.00 | 98.96 | 97.92 | 98.44 | 95.57 | 82.29 | 60.42 | 87.29 | 76.04 | 89.69 |

context for later queries (Li et al., 2025). In contrast, our xKV maintains stable performance across all turns and consistently preserves critical information.

KV-Cache Compression with Selective Reconstruction Results In Table 3, we compare xK-SR, xKV-SR, and two representative token selection baselines, Quest and ShadowKV, using the RULER benchmark at a 64K context length for Llama-3.1-8B-Instruct. For a fair comparison, we fix the token budget (*i.e.*, the number of tokens selected for each decoding step) to be 2k for evaluation targets. Compared with Quest, both xK-SR and xKV-SR showcase superior accuracy with around 4% higher in average. As Quest does not entail KV-Cache compression but only dynamic loading, it does not reduce the size of the KV-Cache and necessitates KV-Cache offloading to avoid out-of-memory (OOM). Compared against ShadowKV, xK-SR extends its by replacing the single-layer SVD compression key cache with a cross-layer alternative. At a 1.64 \times KV-compression rate (8.9 \times GPU memory reduction considering value offloading), xKV-SR closes the accuracy gaps from 4.7% to around 2.1%, demonstrating xKV’s better capability in preserving information. Leveraging the cross-layer alignment that we observed, xKV-SR is able to compress and reduce the KV-Cache to a significant 5.35 \times while maintaining 89.69% accuracy, roughly 19% higher than ShadowKV*. This enables retaining all tensors on GPUs and unlocking the faster inference that avoids the host-device transfer, which improves decoding efficiency over offloading scenarios (See Section 6).

Impact of xKV on Compressing Value and Key Only. To understand how xKV affects key and value compression, we conduct ablation experiments on four subtasks from RULER (Hsieh et al., 2024) to evaluate how xKV (cross-layer low-rank SVD) affects key and value compression. We show the results in Figure 5. Overall, xKV consistently boosts accuracy under varying compression rates. Also, keys exhibit higher compressibility than values, matching the eigenvalue analysis in Figure 2c. A closer inspection of the results reveals that the achievable compression ratio appears

¹This set up have the 8 \times compressed KV-Cache using cross-layer SVD. The final compression rate is calculated, including the memory cost of the landmark for computing selective indices. See Appendix D.5 for more details.

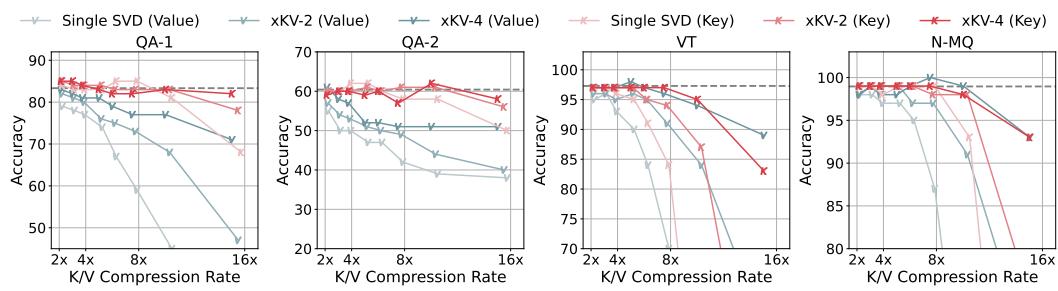


Figure 5: Accuracy comparison of applying different methods to key and value separately on Llama-3.1-8B-Instruct using RULER benchmark. The number after xKV denotes the cross-layer group size.

to be task-dependent. On the questions-answering subtasks (QA-1 and QA-2) xKV can push the compression rate to $16\times$ while still preserving performance. In Variable Tracking (VT) and NIAH multi-queries (N-MQ) (Kamradt, 2023), accuracy begins to decline beyond $8\times$ compression; however, in these same tasks, values tolerate compression more easily than in QA subtasks. These observations underscore how different tasks may demand different “sweet spots” for key versus value compression. In xKV, we employ a fixed compression ratio for all different tasks. Exploring task-specific or context-aware (Liu et al., 2023b; Akhauri et al., 2025; 2024) rank allocation is a promising avenue for future work.

Impact of Cross-layer Group Size to Accuracy. To quantify the impact of cross-layer compression, we conduct a group size ablation on the RULER benchmark at a fixed compression rate (Table 2). For example, xKV improves from 45.71% with group size 1 to 75.15% at size 2, and further to 88.50% at size 4. Similar trends are observed for xK-SR and xKV-SR, where performance likewise climbs steadily as group size increases. These results confirm that sharing across more layers consistently enhances reconstruction fidelity under an identical compression rate. However, at a group size of 8, the accuracy of xKV, xK-SR, and xKV-SR all saturates, with accuracy nearly identical to that at a size of 4. Therefore, we use a group size of 4 in all main experiments.

6 EFFICIENCY STUDIES

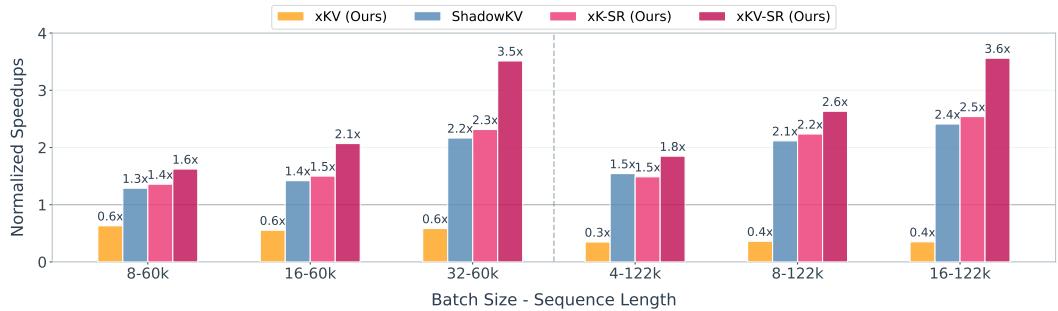


Figure 6: Attention latency evaluation. Normalized speedup relative to FlashAttention-2.

Setup. We evaluate performance on Llama-3.1-8B (GQA) using an A100 (80GB). Figure 7 reports end-to-end generation throughput, while Figure 6 isolates the normalized attention latency relative to FlashAttention-2.

Dense Reconstruction (xKV). xKV reduces memory usage and enables larger batch sizes than Full Attention, but its runtime is limited by the cost of reconstructing dense KV-Cache tensors. As Figure 6 shows, dense reconstruction cost grows with sequence length, which increases attention latency ($0.6\times$ speed at 64k and $0.3\times$ at 128k). This transition from memory-bound to compute-bound

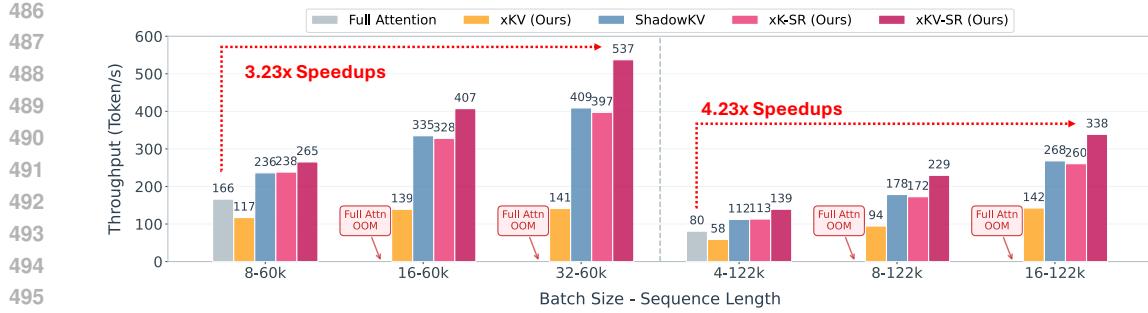


Figure 7: Generation throughput (tokens/s) on an A100.

execution is further reflected in the end-to-end throughput (Figure 7). When memory is no longer a bottleneck, xKV performs similarly to or slightly worse than the baseline.

Selective Reconstruction (xKV-SR). xKV-SR addresses the compute bottleneck via selective reconstruction, delivering the highest performance across all metrics. By keeping compressed KV-Cache entirely on the GPU, it achieves consistent attention latency gains, reaching up to $3.6\times$ speedup in Figure 6. This translates directly to generation throughput (Figure 7), where xKV-SR attains up to $3.23\times$ and $4.23\times$ speedups at 60k and 122k tokens, respectively.

Selective reconstruction with offloaded values (xK-SR and ShadowKV). Both xK-SR and ShadowKV operate in a “keys-only compression + offloaded values” regime. This regime is the setups with most memory saving. However, their performance tracks closely and is strictly bounded by PCIe bandwidth rather than compute. Comparisons in Figure 6 confirm that reconstructing compressed values on-chip (xKV-SR) is significantly faster than fetching uncompressed values over PCIe (xK-SR and ShadowKV), making it the superior choice for high-throughput, long-context decoding.

7 LIMITATIONS AND FUTURE WORK

Long Generation Scenario. Our study focuses on the long-prefill setting, where only the initial context is compressed while tokens generated during decoding remain uncompressed. This regime covers many long-context applications (e.g., information retrieval (Perplexity, 2025) and database QA), but it does not address test-time scaling under extended generation, which the cumulative KV-Cache can also become the bottleneck. We leave to future work how to leverage the observed cross-layer alignment of the KV-cache’s dominant singular vectors and proposed cross-layer SVD to tackle long-generation scenarios.

8 CONCLUSION

We introduce xKV, a plug-and-play compression method for key-value (KV) caches that exploits inter-layer redundancy. Our approach reveals that KV-Caches across different layers share highly aligned basis vectors. Leveraging this property, we apply a cross-layer SVD to compress multiple KV-Caches into a shared low-rank subspace. Experiments demonstrate that xKV outperforms accuracy on all other compression methods, including representative inter-layer approaches and intra-layer methods such as quantization, token eviction, and single-layer SVD. At roughly $8\times$ compression, xKV keeps average accuracy within 2–3 percentage points of the non-compressed baseline, and it remains robust in multi-turn settings. With *Selective Reconstruction* (SR), our fastest alternative xKV-SR reaches up to $4.23\times$ faster generation speed on A100 GPU, highlighting xKV as a practical approach to reduce both memory footprint and latency for long-context LLM inference.

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756 A CKA AND INDICATION OF ALIGNED LEFT SINGULAR VECTORS
757758 A.1 NOTATION AND DEFINITIONS
759760 For each layer ℓ , let

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$$\mathbf{X}_\ell \in \mathbb{R}^{n \times d},$$

762 where each of the n rows corresponds to a token (data point). Define the centering matrix

763
$$\mathbf{H} = \mathbf{I}_n - \frac{1}{n} \mathbf{1} \mathbf{1}^\top,$$

764 which subtracts the (row) mean from each token embedding. We define the *centered* embeddings
765

766
$$\tilde{\mathbf{X}}_\ell = \mathbf{H} \mathbf{X}_\ell,$$

767 and the *centered Gram matrix*

768
$$\mathbf{G}_\ell = \tilde{\mathbf{X}}_\ell \tilde{\mathbf{X}}_\ell^\top \in \mathbb{R}^{n \times n}.$$

769 Because \mathbf{G}_ℓ is symmetric and positive semidefinite, its largest-eigenvalue directions capture the most
770 “energetic” dimensions of $\tilde{\mathbf{X}}_\ell$.
771772 Given two layers ℓ_1 and ℓ_2 , the *Centered Kernel Alignment (CKA)* between their token embeddings is
773

774
$$\text{CKA}(\mathbf{X}_{\ell_1}, \mathbf{X}_{\ell_2}) = \frac{\text{trace}(\mathbf{G}_{\ell_1} \mathbf{G}_{\ell_2})}{\sqrt{\text{trace}(\mathbf{G}_{\ell_1}^2) \text{trace}(\mathbf{G}_{\ell_2}^2)}},$$

775

776 which measures how similarly \mathbf{G}_{ℓ_1} and \mathbf{G}_{ℓ_2} encode pairwise relationships (dot products) among the
777 n token embeddings.
778779 A.2 SVD PERSPECTIVE AND OVERLAP
780781 **SVD of centered embeddings.** Consider the (compact) SVD of $\tilde{\mathbf{X}}_\ell$:

782
$$\tilde{\mathbf{X}}_\ell = \mathbf{U}_\ell \Sigma_\ell \mathbf{V}_\ell^\top,$$

783

784 where:

785 $\mathbf{U}_\ell \in \mathbb{R}^{n \times r}$ (orthonormal columns), $\Sigma_\ell = \text{diag}(\sigma_1, \dots, \sigma_r)$, $\mathbf{V}_\ell \in \mathbb{R}^{d \times r}$ (orthonormal columns),
786 and $r \leq \min(n, d)$ is the rank. Then the centered Gram matrix factors as

787
$$\mathbf{G}_\ell = \tilde{\mathbf{X}}_\ell \tilde{\mathbf{X}}_\ell^\top = \mathbf{U}_\ell \Sigma_\ell^2 \mathbf{U}_\ell^\top,$$

788

789 so the columns of \mathbf{U}_ℓ are exactly the eigenvectors of \mathbf{G}_ℓ , and σ_i^2 are the corresponding eigenvalues.
790791 **CKA in terms of left singular vectors.** Let $\tilde{\mathbf{X}}_{\ell_1} = \mathbf{U}_{\ell_1} \Sigma_{\ell_1} \mathbf{V}_{\ell_1}^\top$ and $\tilde{\mathbf{X}}_{\ell_2} = \mathbf{U}_{\ell_2} \Sigma_{\ell_2} \mathbf{V}_{\ell_2}^\top$. Then
792

793
$$\mathbf{G}_{\ell_1} = \mathbf{U}_{\ell_1} \Sigma_{\ell_1}^2 \mathbf{U}_{\ell_1}^\top, \quad \mathbf{G}_{\ell_2} = \mathbf{U}_{\ell_2} \Sigma_{\ell_2}^2 \mathbf{U}_{\ell_2}^\top.$$

794

We compute

795
$$\text{trace}(\mathbf{G}_{\ell_1} \mathbf{G}_{\ell_2}) = \text{trace}(\mathbf{U}_{\ell_1} \Sigma_{\ell_1}^2 \mathbf{U}_{\ell_1}^\top \mathbf{U}_{\ell_2} \Sigma_{\ell_2}^2 \mathbf{U}_{\ell_2}^\top) = \sum_{i=1}^{r_1} \sum_{j=1}^{r_2} \sigma_{\ell_1, i}^2 \sigma_{\ell_2, j}^2 (\mathbf{u}_{\ell_1}^{(i)\top} \mathbf{u}_{\ell_2}^{(j)})^2,$$

796

797 where $\mathbf{u}_{\ell_1}^{(i)}$ and $\mathbf{u}_{\ell_2}^{(j)}$ are the i -th and j -th columns of \mathbf{U}_{ℓ_1} and \mathbf{U}_{ℓ_2} , respectively. Meanwhile,
798

799
$$\text{trace}(\mathbf{G}_{\ell_1}^2) = \sum_{i=1}^{r_1} \sigma_{\ell_1, i}^4, \quad \text{trace}(\mathbf{G}_{\ell_2}^2) = \sum_{j=1}^{r_2} \sigma_{\ell_2, j}^4.$$

800

801 Hence,

802
$$\text{CKA}(\mathbf{X}_{\ell_1}, \mathbf{X}_{\ell_2}) = \frac{\sum_{i,j} \sigma_{\ell_1, i}^2 \sigma_{\ell_2, j}^2 (\mathbf{u}_{\ell_1}^{(i)\top} \mathbf{u}_{\ell_2}^{(j)})^2}{\sqrt{(\sum_i \sigma_{\ell_1, i}^4)(\sum_j \sigma_{\ell_2, j}^4)}}.$$

803

804 Because the eigenvalues $\sigma_{\ell, i}^2$ reflect how “dominant” each left singular vector is, a **large CKA** value
805 requires significant overlap $(\mathbf{u}_{\ell_1}^{(i)\top} \mathbf{u}_{\ell_2}^{(j)})^2$ for the most important (largest- σ^2) directions, implying the
806 principal subspaces of \mathbf{G}_{ℓ_1} and \mathbf{G}_{ℓ_2} align closely.
807

810 A.3 CONCLUSION
811

812 In summary, when $\text{CKA}(\mathbf{X}_{\ell_1}, \mathbf{X}_{\ell_2})$ is high, the dominant *left singular vectors* of $\tilde{\mathbf{X}}_{\ell_1}$ and $\tilde{\mathbf{X}}_{\ell_2}$ are
813 well aligned. Since these vectors also serve as the *largest-eigenvalue* directions of the centered
814 Gram matrices, high CKA implies that the *principal subspace* geometry of the token embeddings
815 in layers ℓ_1 and ℓ_2 is *structurally* very similar—even if token-by-token (cosine) matches are small.
816 Thus, CKA goes beyond individual token similarities, capturing **how** tokens vary collectively in a
817 shared subspace.

818 B IMPLEMENTATION DETAILS
819820 B.1 LANDMARK-GUIDED CHUNK SELECTOR FOR SELECTIVE RECONSTRUCTION
821822 **Algorithm 1** Landmark Construction (Prefill)

823 **Require:** Post-RoPE keys $K_{\ell}^{\text{rope}} \in \mathbb{R}^{H_{\text{kv}} \times N \times d_h}$, chunk size c , optional #outliers o
824
825 **Ensure:** Landmarks $L_{\ell} \in \mathbb{R}^{H_{\text{kv}} \times n_c \times d_h}$, optional outlier indices $\{\mathcal{O}_{\ell,g} \subseteq [n_c]\}_{g=1}^{H_{\text{kv}}}$
826
827 1: $n_c \leftarrow \lceil N/c \rceil$;
828 2: **Chunking the sequence:** $\tilde{K} \leftarrow \text{View}(K_{\ell}^{\text{rope}}) \in \mathbb{R}^{H_{\text{kv}} \times n_c \times c \times d_h}$
829 3: **Chunk means (landmarks):** $L_{\ell} \leftarrow \text{mean}(\tilde{K}, \text{axis} = 2) \in \mathbb{R}^{H_{\text{kv}} \times n_c \times d_h}$
830 4: **(Optional) Static outliers, per head:** $S_{\text{cos}} \leftarrow \cos(\tilde{K}, L_{\ell} \text{ broadcast along } c) \in \mathbb{R}^{H_{\text{kv}} \times n_c \times c}$
831 5: $m \leftarrow \min(S_{\text{cos}}, \text{axis} = 2) \in \mathbb{R}^{H_{\text{kv}} \times n_c}$; $I^{\text{out}} \leftarrow \text{ArgTopK}(-m, o)$; $\mathcal{O}_{\ell,g} \leftarrow I^{\text{out}}[g, :]$
832 6: **return** L_{ℓ} and (optionally) $\{\mathcal{O}_{\ell,g}\}$

834
835 **Algorithm 2** Landmark-Guided Top- k Chunk Selection (Decode)

836 **Require:** Landmarks $L_{\ell} \in \mathbb{R}^{H_{\text{kv}} \times n_c \times d_h}$, queries $Q_{t,\ell} \in \mathbb{R}^{H_q \times d_h}$, GQA map $\rho : [H_q] \rightarrow [H_{\text{kv}}]$, token
837 budget k , chunk size c , optional outliers $\{\mathcal{O}_{\ell,g}\}$
838 **Ensure:** Per-KV head selected chunk indices $\{S_{t,\ell,g} \subseteq [n_c]\}_{g=1}^{H_{\text{kv}}}$
839 1: $k_{\text{ch}} \leftarrow \lceil k/c \rceil$ ▷ convert token budget to chunk budget
840 2: **Scores to landmarks (batched MatMul):**
841
$$P \in \mathbb{R}^{H_q \times H_{\text{kv}} \times n_c} \leftarrow \langle Q_{t,\ell}[:, \cdot], L_{\ell}[:, :, \cdot] \rangle_{d_h} / \sqrt{d_h}$$

842 3: **Pool from query heads to KV heads (GQA):**
843
$$S[g, j] \leftarrow \max_{h: \rho(h)=g} P[h, g, j] \quad \text{for all } g \in [H_{\text{kv}}], j \in [n_c]$$

844 4: **Top- k_{ch} per KV head:** $I \in \mathbb{R}^{H_{\text{kv}} \times k_{\text{ch}}} \leftarrow \text{ArgTopK}(S, k_{\text{ch}})$
845 5: **(Optional) add static outliers:** $S_{t,\ell,g} \leftarrow \text{Union}(I[g, :], \mathcal{O}_{\ell,g}) \quad \text{for each } g$
846 6: **return** $\{S_{t,\ell,g}\}_{g=1}^{H_{\text{kv}}}$

852 We adopted the landmark-guided selection techniques from ShadowKV (Sun et al., 2024a) to decide
853 the token indices for selective reconstruction. We detail the workflow in the paragraphs below.
854

855 **Landmark construction (prefill).** At layer ℓ , we split the post-RoPE key sequence into $n_c =$
856 $\lceil N/c \rceil$ contiguous chunks of size c . For each KV head g and chunk j , we define the *landmark* as the
857 mean key of that chunk:

$$\ell_{j,g} = \frac{1}{|C_j|} \sum_{x \in C_j} K_{\ell,g}^{\text{rope}}(x).$$

858 Optionally, we keep a tiny per-head *static outlier* set to guard against heterogeneous chunks whose
859 mean is a weak representative. We identify these by computing the minimum within-chunk cosine
860 similarity to the landmark,
861

$$r_{g,j} = \min_{x \in C_j} \cos(K_{\ell,g}^{\text{rope}}(x), \ell_{j,g}),$$

864 and marking the o chunks with the smallest $r_{g,j}$ as outliers for each head g . This metric indicates
 865 how well the landmark summarizes its chunk: lower values signal that at least one token is poorly
 866 captured by the mean, so those chunks are always considered during decoding. We summarize the
 867 procedure in Algorithm. 1.

868
 869 **Landmark-guided selection (decode).** At each decode step t , given queries $Q_{t,\ell} \in \mathbb{R}^{H_q \times d_h}$,
 870 we score every chunk via a batched scaled dot-product between $Q_{t,\ell}$ and the landmarks. With
 871 grouped-query attention, scores are pooled from query heads to KV heads using the GQA map
 872 $\rho : [H_q] \rightarrow [H_{\text{kv}}]$ by taking a max over the query heads mapped to each KV head. Given a token
 873 budget k and chunk size c , we convert to a chunk budget $k_{\text{ch}} = \lceil k/c \rceil$ and keep the top k_{ch} chunks
 874 per KV head. Optionally, we union these with the static outliers $\mathcal{O}_{\ell,g}$. The selected chunk indices
 875 are then expanded to row indices $\mathcal{S}_{t,\ell,g}$ and used to reconstruct only the corresponding tokens. We
 876 summarize the procedure in Algorithm. 2.

877 C MORE LATENCY STUDIES

880 C.1 ON-THE-FLY SVD OVERHEAD

882 Table 4 reports the latency of the prefilling phase as well as the cross-layer SVD on A6000 GPU.
 883 On a sequence length of $L = 64k$ tokens, the SVD accounts for 6.92% of the forward-pass time.
 884 This fraction steadily decreases as L increases, reaching only 2.05% at $L = 256k$, where L denotes
 885 the sequence length. The reduction can be attributed to the fact that the cost of attention grows
 886 quadratically with L , whereas the low-rank decomposition scales only linearly (Li et al., 2019). As a
 887 result, for very long contexts, the one-time decomposition performed during the prefill phase becomes
 888 practically negligible, contributing minimally to the overall computation time. Similar trends also
 889 hold on A100 GPU as demonstrated in Table 5.

890 Table 4: The latency data of on-the-fly SVD under different context lengths. Measured on an A6000
 891 GPU with Qwen2.5-14B-Instruct. (Unit: seconds)

| Seqlen | 64k | 128k | 160k | 256k |
|--------------------|--------------|--------------|--------------|--------------|
| Prefill Time | 39.02 | 122.30 | 182.54 | 425.42 |
| SVD time ($G=2$) | 1.98 (5.04%) | 3.48 (2.85%) | 4.37 (2.39%) | 6.36 (1.49%) |
| SVD time ($G=4$) | 2.70 (6.92%) | 4.76 (3.90%) | 5.89 (3.23%) | 8.74 (2.05%) |

900 Table 5: The latency data of on-the-fly SVD under different context lengths. Measured on an A100
 901 GPU with Llama-3.1-8B. (Unit: seconds)

| Seqlen | 64k | 128k | 160k | 256k |
|--------------------|---------------|--------------|--------------|--------------|
| Prefill Time | 18.98 | 47.87 | 71.55 | 159.27 |
| SVD time ($G=2$) | 1.90 (10.03%) | 3.48 (7.26%) | 4.38 (6.12%) | 6.35 (3.99%) |
| SVD time ($G=4$) | 2.56 (13.46%) | 4.75 (9.93%) | 5.89 (8.23%) | 8.74 (5.48%) |

910 D MORE EXPERIMENTAL RESULTS

912 D.1 MORE RESULTS ON RULER

914 **KV-Cache More Compression with Selective Reconstruction Results.** Table 6 reports results
 915 at different compression rates on the RULER benchmark. At the high compression setting (around
 916 $11.5 \times$ effective GPU memory reduction), $\times\text{K-SR}$ outperforms ShadowKV by a striking 36%. This
 917 demonstrates that $\times\text{KV-SR}$ is significantly more effective at preserving performance under extreme
 compression.

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Table 6: More KV-Cache Compression with Selective Reconstruction Results: Accuracy of different methods on the RULER benchmark at a context length of 64K. Here, "Comp." indicates the total KV-Cache reduction, while the number in parentheses shows the effective GPU memory reduction considering KV-Cache offloading. ShadowKV* refers to a variant of ShadowKV that additionally compresses the value cache.

| Method | Comp. | N-S1 | N-S2 | N-MK1 | N-MK2 | N-MQ | N-MV | QA-1 | QA-2 | VT | FWE | Avg. |
|------------------------------|--------------|--------|--------|--------|-------|-------|-------|-------|-------|-------|-------|-------|
| Llama-3.1-8B-Instruct | | | | | | | | | | | | |
| Full Attn | 1.00 | 100.00 | 100.00 | 98.96 | 97.92 | 98.96 | 97.66 | 83.33 | 59.38 | 97.29 | 85.42 | 91.89 |
| Quest | 1.00 (8.00) | 93.75 | 90.63 | 96.88 | 87.50 | 94.27 | 85.42 | 83.33 | 57.29 | 77.71 | 81.94 | 84.87 |
| ShadowKV | 1.60 (7.94) | 100.00 | 100.00 | 100.00 | 97.92 | 99.22 | 95.83 | 83.33 | 59.38 | 78.33 | 73.96 | 88.80 |
| xK-SR (Ours) | 1.59 (7.76) | 100.00 | 100.00 | 98.96 | 97.92 | 98.70 | 96.35 | 82.29 | 61.46 | 88.33 | 75.69 | 89.97 |
| ShadowKV | 1.64 (9.08) | 100.00 | 100.00 | 98.96 | 97.92 | 96.88 | 94.53 | 82.29 | 60.42 | 66.04 | 74.65 | 87.17 |
| xK-SR (Ours) | 1.63 (8.90) | 100.00 | 100.00 | 98.96 | 97.92 | 98.44 | 95.31 | 83.33 | 60.42 | 87.92 | 74.65 | 89.70 |
| ShadowKV | 1.68 (10.61) | 100.00 | 71.88 | 73.96 | 97.92 | 27.34 | 24.22 | 68.75 | 58.33 | 52.71 | 73.96 | 64.91 |
| xK-SR (Ours) | 1.68 (10.45) | 100.00 | 98.96 | 98.96 | 97.92 | 94.53 | 93.49 | 82.29 | 60.42 | 80.83 | 76.04 | 88.34 |
| ShadowKV | 1.71 (11.59) | 96.88 | 6.25 | 5.21 | 80.21 | 0.78 | 2.34 | 65.62 | 56.25 | 49.79 | 72.57 | 43.59 |
| xK-SR (Ours) | 1.70 (11.44) | 100.00 | 96.88 | 92.71 | 97.92 | 62.50 | 56.25 | 80.21 | 59.38 | 69.58 | 76.39 | 79.18 |
| ShadowKV* | 4.52 | 100.00 | 98.96 | 96.88 | 97.92 | 93.49 | 91.67 | 82.29 | 58.33 | 67.92 | 75.69 | 86.32 |
| xK-SR (Ours) | 4.37 | 100.00 | 100.00 | 98.96 | 96.88 | 99.48 | 96.61 | 82.29 | 60.42 | 87.92 | 75.69 | 89.83 |
| ShadowKV* | 5.51 | 100.00 | 76.04 | 75.00 | 97.92 | 54.43 | 45.83 | 81.25 | 57.29 | 47.29 | 74.31 | 70.94 |
| xK-SR (Ours) | 5.35 | 100.00 | 100.00 | 98.96 | 97.92 | 98.44 | 95.57 | 82.29 | 60.42 | 87.29 | 76.04 | 89.69 |

D.2 RESULTS ON LONGBENCH

KV-Cache Compression Results. Table 7 presents the comprehensive evaluation of xKV against representative compression methods on the LongBench dataset, demonstrating consistent performance across diverse long-context tasks, including single-document QA, multi-document QA, summarization, few-shot learning, synthetic tasks, and code completion. Experiments were conducted on Llama-3.1-8B-Instruct and Qwen2.5-7B-Instruct-1M models.

MiniCache exhibits severe performance degradation, with accuracy dropping by 12.57% on Llama-3.1-8B-Instruct and a catastrophic 26.91% on Qwen2.5-7B-Instruct-1M compared to the baseline, reinforcing our earlier observation that cross-layer compression methods fail when token-wise cosine similarity assumptions are violated across different model architectures and task types.

At 8.03 \times compression, xKV achieves 42.27% average accuracy on Llama-3.1-8B-Instruct, closely matching PyramidKV and SnapKV. On Qwen2.5-7B-Instruct-1M, xKV reaches 40.37% accuracy, demonstrating competitive performance against PyramidKV and SnapKV, with a slight accuracy degradation.

These LongBench results validate xKV’s robustness across heterogeneous task domains, confirming that our shared low-rank subspace approach effectively preserves critical information for diverse long-context reasoning scenarios while achieving aggressive compression rates comparable to leading token eviction methods.

KV-Cache Compression with Selective Reconstruction Results. In Table 8, we evaluate xK-SR and xK-SR against Quest and ShadowKV baselines on the LongBench dataset using Llama-3.1-8B-Instruct. Quest achieves 42.63% accuracy through dynamic token loading with 8 \times GPU memory reduction via offloading, demonstrating minimal performance degradation while requiring host-device transfers.

At comparable compression ratios, xK-SR consistently outperforms ShadowKV across different settings. With 1.68 \times compression and 10.45 \times GPU memory reduction, xK-SR achieves 42.50% accuracy, surpassing ShadowKV by 1.99%. This improvement demonstrates the effectiveness of our cross-layer key compression approach over single-layer SVD methods.

Most notably, xK-SR enables aggressive 5.35 \times compression while achieving 42.40% accuracy, outperforming ShadowKV* by 0.89%. These consistent improvements across both RULER and LongBench benchmarks validate that our cross-layer alignment approach effectively adapts to diverse evaluation frameworks, preserving critical information across heterogeneous long-context tasks ranging from retrieval and reasoning to code completion and summarization. Moreover, the significant gains observed on LongBench further corroborate the robustness and generality of our method beyond the RULER benchmark.

972 Table 7: KV-Cache Compression Results: Accuracy of different methods on LongBench. xKV
 973 consistently achieves a higher accuracy than the Full Attns at the same compression rate or even at a
 974 significantly higher compression rate.

| 975 | Method | 976 Comp. | Single-doc QA | Multi-doc QA | Summarization | Few-shot | Synthetic | Code | Avg. |
|-------------------------------|--------------|-----------|---------------|--------------|---------------|----------|-----------|-------|-------|
| Llama-3.1-8B-Instruct | | | | | | | | | |
| 978 | Full Attn | 1.00 | 44.23 | 44.72 | 28.52 | 25.88 | 53.44 | 62.41 | 43.20 |
| 979 | MiniCache | 1.30 | 22.01 | 26.79 | 20.51 | 25.05 | 52.29 | 37.11 | 30.63 |
| 980 | KIVI | 7.10 | 40.87 | 42.45 | 27.40 | 26.96 | 51.70 | 59.42 | 41.47 |
| 981 | StreamingLLM | 8.00 | 30.04 | 37.79 | 23.61 | 25.49 | 49.75 | 61.15 | 37.97 |
| 982 | PyramidKV | 8.00 | 42.92 | 43.99 | 25.73 | 27.62 | 53.02 | 61.54 | 42.47 |
| 983 | SnapKV | 8.00 | 43.17 | 44.13 | 26.09 | 27.75 | 53.27 | 62.56 | 42.83 |
| | Single SVD | 8.40 | 30.34 | 23.93 | 20.26 | 27.41 | 44.75 | 52.63 | 33.22 |
| | xKV (Ours) | 8.03 | 44.39 | 38.82 | 26.14 | 27.34 | 55.50 | 61.44 | 42.27 |
| Qwen2.5-7B-Instruct-1M | | | | | | | | | |
| 985 | Full Attn | 1.00 | 40.52 | 49.30 | 26.67 | 37.07 | 54.00 | 45.11 | 42.11 |
| 986 | MiniCache | 1.30 | 11.29 | 15.93 | 6.87 | 28.51 | 5.38 | 23.21 | 15.20 |
| 987 | KIVI | 7.10 | 33.54 | 38.35 | 21.34 | 35.55 | 34.32 | 39.73 | 33.80 |
| 988 | StreamingLLM | 8.00 | 28.86 | 38.04 | 22.42 | 47.58 | 17.50 | 42.75 | 32.86 |
| 989 | PyramidKV | 8.00 | 39.48 | 48.31 | 23.32 | 44.13 | 54.00 | 43.33 | 42.09 |
| 990 | SnapKV | 8.00 | 40.21 | 48.32 | 24.93 | 43.73 | 54.00 | 44.19 | 42.56 |
| | Single SVD | 8.40 | 39.13 | 43.32 | 24.04 | 32.00 | 36.25 | 38.42 | 35.53 |
| | xKV (Ours) | 8.03 | 39.73 | 47.97 | 26.62 | 34.42 | 53.00 | 40.48 | 40.37 |

991
 992 Table 8: KV-Cache Compression with Selective Reconstruction Results: Accuracy of different methods
 993 on the LongBench. Here, "Comp." indicates the total memory reduction, while the number
 994 in parentheses shows the effective GPU memory reduction considering KV-Cache offloading. Shad-
 995 owKV* refers to a variant of ShadowKV that additionally compresses the value cache.

| 996 | Method | 997 Comp. | Single-doc QA | Multi-doc QA | Summarization | Few-shot | Synthetic | Code | Avg. |
|------------------------------|-----------|--------------|---------------|--------------|---------------|----------|-----------|-------|-------|
| Llama-3.1-8B-Instruct | | | | | | | | | |
| 999 | Full Attn | 1.00 | 44.23 | 44.72 | 28.52 | 25.88 | 53.44 | 62.41 | 43.20 |
| 1000 | Quest | 1.00 (8.00) | 43.18 | 44.40 | 28.20 | 26.57 | 52.88 | 60.55 | 42.63 |
| 1001 | ShadowKV | 1.68 (10.61) | 37.98 | 44.11 | 25.26 | 24.43 | 53.35 | 57.92 | 40.51 |
| 1002 | xK-SR | 1.68 (10.45) | 43.64 | 44.47 | 27.62 | 25.31 | 52.63 | 61.32 | 42.50 |
| 1003 | ShadowKV | 1.64 (9.08) | 43.35 | 44.87 | 27.15 | 25.76 | 52.63 | 59.53 | 42.21 |
| 1004 | xKV-SR | 1.63 (8.90) | 44.38 | 44.63 | 27.98 | 25.55 | 52.13 | 61.50 | 42.69 |
| 1005 | ShadowKV* | 5.51 | 41.76 | 44.89 | 26.02 | 24.74 | 52.73 | 58.91 | 41.51 |
| | xKV-SR | 5.35 | 44.58 | 45.20 | 27.76 | 25.32 | 52.63 | 58.94 | 42.40 |

1007 D.3 RESULTS ON REASONING-HEAVY TASK

1009 To demonstrate the broad applicability of our method on
 1010 reasoning-intensive benchmarks, we evaluated xKV on
 1011 GSM8K and the BIG-Bench Hard (BBH) suite, where
 1012 retention of all intermediate states is critical. As shown
 1013 in Table 9, token eviction approaches suffer catastrophic
 1014 declines, with accuracy plunging from approximately 78%
 1015 to just over 10% on BBH and into the mid-50s on GSM8K
 1016 at a 7 \times compression rate. Even the quantization baseline
 1017 KIVI experiences significant degradation on BBH. In stark contrast, xKV preserves strong per-
 1018 formance across both benchmarks, underscoring that our shared low-rank subspace compression
 1019 achieves a consistently superior accuracy-compression trade-off, even under the most demanding
 1020 reasoning conditions.

1021 D.4 INTEGRATE WITH QUANTIZATION

1023 xKV can be combined with other cache management techniques. To illustrate this capability, we
 1024 conducted preliminary experiments integrating xKV with Quantization. Specifically, we applied a
 1025 simple round-to-nearest (RTN) quantization method to the compressed cache. With 4-bit quantization,
 the cache achieves a substantial 25.6 \times compression while maintaining model accuracy.

1006
 1007 Table 9: Accuracy of different methods
 1008 on GSM8K and BBH with Llama-3.1-
 1009 8B-Instruct.

| 1010 | Method | 1011 Comp. | 1012 GSM8K | 1013 BBH |
|------|-----------|------------|------------|----------|
| | Full Attn | 1.00 | 78.47 | 69.70 |
| | PyramidKV | 7.00 | 54.66 | 10.89 |
| | SnapKV | 7.00 | 59.06 | 10.59 |
| | KIVI | 7.10 | 67.55 | 52.96 |
| | xKV | 7.00 | 71.42 | 69.19 |

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Table 10: Accuracy of xKV integrated with naive quantization on RULER benchmark.

| Method | Comp. | N-S1 | N-S2 | N-MK1 | N-MK2 | N-MQ | N-MV | QA-1 | QA-2 | VT | FWE | Avg. |
|------------------------------|-------|--------|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Llama-3.1-8B-Instruct | | | | | | | | | | | | |
| Full Attn | 1.00 | 100.00 | 100.00 | 98.96 | 97.92 | 98.96 | 98.18 | 83.33 | 60.42 | 97.71 | 85.42 | 92.09 |
| xKV | 8.03 | 100.00 | 98.96 | 97.92 | 97.92 | 96.35 | 97.14 | 78.13 | 57.29 | 86.67 | 78.13 | 88.85 |
| xKV-4bit | 25.7 | 100.00 | 96.88 | 97.92 | 97.92 | 96.35 | 93.23 | 77.08 | 55.21 | 83.33 | 78.47 | 87.64 |
| xKV-3bit | 32.12 | 93.75 | 94.79 | 95.83 | 96.88 | 95.05 | 90.89 | 77.08 | 52.08 | 73.33 | 76.74 | 84.64 |

Table 10 presents the performance of xKV with naive quantization on the RULER benchmark, evaluated using Llama-3.1-8B-Instruct. We observe that xKV alone provides an $8\times$ compression with minimal accuracy loss. Further applying 4-bit quantization yields a total compression of $25.6\times$, with only a slight drop in the average score from 88.85% to 87.64%. Even more aggressive 3-bit quantization achieves $32\times$ compression, with a moderate decrease in performance (average 84.64%), demonstrating that xKV can be effectively combined with other cache reduction techniques without severely impacting accuracy.

D.5 FLOPs & MEMORY COST

Dense Reconstruction (no selection). For a given type τ , if one reconstructs all L rows per layer at decode, the per-layer cost of $\mathbf{A}^\tau \mathbf{B}^\tau$ is

$$\text{FLOPs}_{\text{dense}}^{(\tau)} = L \cdot r_\tau \cdot d. \quad (3)$$

If both keys and values are reconstructed densely, costs add: $\text{FLOPs}_{\text{dense}} = \sum_\tau L r_\tau d$.

Selective Reconstruction (per step). With index sets $\mathcal{S}_{t,\ell,g}$ and $M_{t,\ell,g} = |\mathcal{S}_{t,\ell,g}|$, the per-step cost for a given τ becomes

$$\text{FLOPs}_{\text{sparse}}^{(\tau)} = \sum_{g=1}^{H_{\text{kv}}} M_{t,\ell,g} r_\tau d_h, \quad (4)$$

and $\text{FLOPs}_{\text{sparse}} = \sum_\tau \text{FLOPs}_{\text{sparse}}^{(\tau)}$ when compressing both types. When $M_{t,\ell,g} \ll L$, selective reconstruction is a small fraction of the dense cost. Computing $\mathcal{S}_{t,\ell,g}$ itself involves light matrix–vector operations and is independent of the cross-layer factors.

Compressed-cache memory. xKV stores, per group k , the shared token bases $\mathbf{A}_k^\tau \in \mathbb{R}^{L \times r_\tau}$ (one per type) and, per layer, the reconstructions $\mathbf{B}_\ell^\tau \in \mathbb{R}^{r_\tau \times d}$. Summed over all groups/layers, the total memory is

$$\sum_{\tau \in \{K^{\text{pre}}, V\}} \left(\underbrace{\frac{N}{G} L r_\tau}_{\text{shared bases}} + \underbrace{N r_\tau d}_{\text{layer reconstructions}} \right) + \underbrace{\frac{N}{c} d}_{\substack{\text{landmark (optional)}}}, \quad (5)$$

compared to $2N L d$ for the full KV-Cache (keys and values). In Mode (A) (key-only compression), only the K terms in (5) apply and values are fetched (or lightly quantized) from host memory. In Mode (B), both types are compressed and resident on GPU. When activating SR, we have to store the landmark L_l of size $\frac{N}{c} d$ for computing the indices for selective reconstruction.

How we compute the KV-Cache compression ratio. Let C denote the compression ratio achieved by the xKV cache (i.e., the ratio of the original KV-Cache size to the compressed size). The landmark requires storing $\frac{L}{8} \times d$ elements, which is exactly one-eighth the size of the full K-Cache. The outlier set is a constant and can be ignored when the context length is long. The numerator 2 in each formula represents the combined original size of keys and values; the denominator represents the post-compression storage of the KV-Cache plus the landmark set (and, in the total memory case, the full value cache).

For xK-SR (key-only compression, value offloading):

Effective GPU memory compression ratio:

$$R_{\text{xK-SR, GPU}} = \frac{2}{\frac{1}{C} + \frac{1}{8}} \quad (6)$$

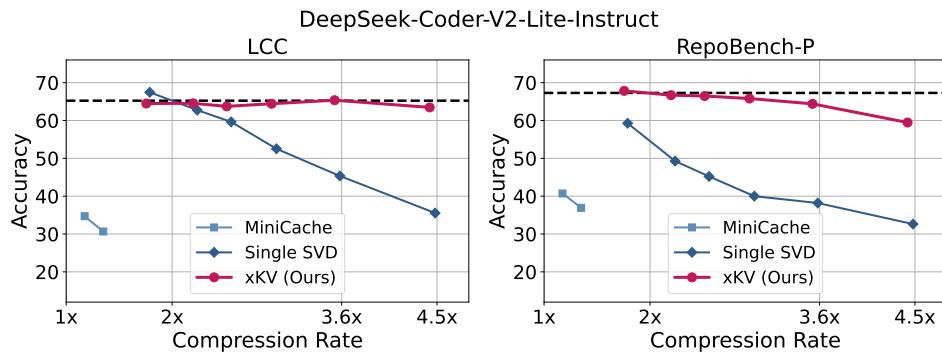
1080 Total memory compression ratio (counting values at original size):
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$$R_{\text{xKV-SR, total}} = \frac{2}{\frac{1}{C} + \frac{1}{8} + 1} \quad (7)$$

1084 For xKV-SR (both keys and values compressed):
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$$R_{\text{xKV-SR}} = \frac{2}{\frac{2}{C} + \frac{1}{8}} \quad (8)$$

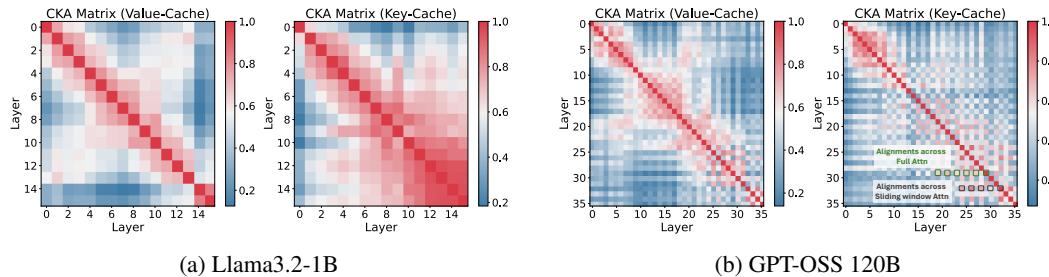
1090 E EXTENDING xKV ON MULTI-HEAD LATENT ATTENTION (MLA)



1104 Figure 8: Evaluation results of different KV-Cache methods on DeepSeek-Coder-V2-Lite-Instruct
 1105 model using RepoBench-P (Liu et al., 2023a) and LCC(Guo et al., 2023). The accuracy denotes
 1106 the edit similarity (Svyatkovskiy et al., 2020), and the dotted line represents the baseline score with
 1107 uncompressed KV-Cache.
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1109 To demonstrate the effectiveness of xKV on emerging attention variants, we evaluate xKV on
 1110 DeepSeek-V2-Coder-Lite (Liu et al., 2024a), which employs the efficient Multi-head Latent Attention
 1111 (MLA) architecture (Liu et al., 2024a). MLA is proposed to reduce the KV-Cache size per layer
 1112 through low-rank projections. As shown in Figure 8, we can further compress the compact latent
 1113 cache by exploiting the cross-layer redundancy by using our xKV. With a group size of 4, xKV
 1114 achieves a 3× compression rate on RepoBench (Liu et al., 2023a) and 3.5× on LCC (Guo et al.,
 1115 2023) without compromising accuracy. In contrast, other methods, such as MiniCache (Liu et al.,
 1116 2024b) and Single SVD, fail to preserve accuracy on the MLA architecture even at substantially
 1117 lower compression rates. These results underscore xKV’s versatility and compatibility with emerging
 1118 memory-efficient attention architectures (Liu et al., 2024a).
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1120 F BROADER CKA ANALYSIS



1132 Figure 9: Extended CKA analysis of different models. GPT-OSS is a hybrid architecture that
 1133 interleaves window attention and full attention layers in a 1:1 ratio.
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