

WirelessMathLM: Teaching Mathematical Reasoning for LLMs in Wireless Communications with Reinforcement Learning

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Abstract

Large Language Models (LLMs) struggle with specialized mathematical reasoning in domains governed by strict physical constraints, such as wireless communications. Progress is currently stifled by a lack of training-scale resources; existing datasets are either purely evaluative or insufficient in volume for robust adaptation. To bridge this gap, we present **WirelessMathBench-XL**, the first training-scale benchmark for wireless mathematics, comprising 4,027 problems derived from 970 state-of-the-art papers. We employ a rigorous construction pipeline combining automated extraction with human expert verification to ensure robust generalization assessment. The dataset features a hierarchical taxonomy consisting of multiple-choice, progressive fill-in-the-blank, and full equation completion tasks, explicitly designed to test comprehension depth. To validate the corpus, we train baseline models (**WirelessMathLM**) using reinforcement learning with verification rewards. Our 7B model achieves 39.5% accuracy, rivaling GPT-4o (40.4%) and demonstrating that the dataset provides sufficient signal for small models to master complex domain logic. Further analysis reveals that training on this specialized corpus improves performance on general mathematical benchmarks without catastrophic forgetting, while also showing consistent improvements across broad knowledge, science, and coding tasks.

1 Introduction

Large language models (LLMs) demonstrate remarkable general reasoning capabilities (Achiam et al., 2023; Dubey et al., 2024; Google DeepMind, 2024; Guo et al., 2025), yet they frequently fail when confronted with specialized technical mathematics constrained by

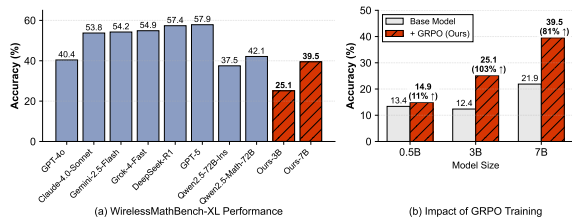


Figure 1: **WirelessMathBench-XL enables small models to achieve competitive performance.** (a) WirelessMathLM-7B trained on our dataset achieves 39.5%, approaching GPT-4o (40.4%) while using far fewer parameters than DeepSeek-R1 (57.4%). (b) Training improvements validate dataset quality: WirelessMathLM-3B (+103%) and WirelessMathLM-7B (+81%) models nearly double their performance.

physical laws (Frieder et al., 2024; Li et al., 2025; Lu et al., 2023; He et al., 2024). This failure is particularly acute in wireless communications, where mathematical reasoning is not merely symbolic but physically bound. Problems in this field require precise application of information-theoretic bounds and complex-valued matrix algebra, and are significantly bound by unstated physical constraints such as the nonnegativity of transmit power ($P \geq 0$), causal signal processing, phase shift bounds ($[0, 2\pi)$), and noncommutative operations on high-dimensional matrices ($\mathbb{C}^{N \times M}$).

Developing specialized models to bridge this gap has been hindered by a fundamental data scarcity. While the recent WirelessMathBench (Li et al., 2025) introduced an evaluation benchmark of 587 problems, the field lacks a *training-scale* dataset necessary for robust model adaptation. Evaluation-only datasets are insufficient for teaching models the breadth of domain-specific reasoning patterns and physical constraints required for graduate-level problem solving.

To address this, we present **WirelessMathBench-XL**, the first training-scale benchmark for wireless mathematics. This dataset is 24 times larger than the prior benchmark (Li et al., 2025) and contains 4,027 expert-validated problems derived from 970 papers, covering 20 technical subfields, ranging from traditional Shannon capacity derivations to the latest 5G and 6G technologies. Our dataset employs a three-tiered question design: multiple-choice questions for concept recognition, progressive fill-in-the-blank questions (with masking rates ranging from 25% to 75%) for structured reasoning, and full equation completion questions for comprehensive knowledge mastery. This design provides both training signals and refined evaluation. Each question includes complete variable definitions and context, allowing for automatic validation of student responses. Furthermore, our dataset construction itself utilizes a rigorous two-tiered quality assurance mechanism, combining automatic selection and expert validation.

To validate the dataset’s utility, we train **WirelessMathLM**, a family of small models (0.5B, 3B, 7B) optimized using Group Relative Policy Optimization (GRPO) (Shao et al., 2024) with binary verification rewards. As Figure 1 demonstrates, WirelessMathLM-7B achieves 39.5% accuracy, approaching GPT-4o (40.4%) and significantly outperforming general open-source baselines, all while using approximately 100 times fewer parameters than models like DeepSeek-R1 (671B).

Beyond benchmarking, our training experiments yield three critical scientific findings regarding domain specialization:

RL significantly outperforms SFT in verifiable domains: Verification-based reinforcement learning outperforms supervised fine-tuning by +23% relative (25.1% vs 20.4%) on identical data. This challenges the assumption that SFT is sufficient for technical domain adaptation, demonstrating that exploration against a deterministic verifier is crucial for mastering complex reasoning paths.

Specialization strengthens foundational capabilities: Contrary to widespread concerns about catastrophic forgetting, WirelessMathLM models gain an average of +8.4 points across five general mathematical bench-

marks and show improvements on knowledge-intensive (MMLU) and reasoning (GPQA) tasks. Learning to navigate tight physical constraints appears to sharpen general reasoning precision rather than degrade it.

Generalizable principle learning: Performance gains are distributed uniformly across all 20 subdomains regardless of training data prevalence. Notably, small categories (e.g., Semantic Communications) see massive gains (+300%), indicating the model acquires transferable mathematical principles (e.g., optimization logic) rather than merely memorizing frequent patterns.

2 WirelessMathBench-XL: Dataset Construction

Creating a high-quality benchmark for wireless communication mathematics requires addressing three key challenges: (1) extracting structured mathematical content from dense technical papers, (2) ensuring problem correctness and solvability, and (3) maintaining consistency across diverse mathematical formulations. We present a systematic pipeline that constructs WirelessMathBench-XL from 970 papers, yielding 4,027 problems. Figure 2 illustrates our three-stage pipeline for constructing WirelessMathBench-XL from raw arXiv papers to validated mathematical questions.

2.1 Data Collection Pipeline

We developed an automated pipeline that comprehensively collects and processes wireless communication papers from arXiv. Our approach prioritizes broad coverage with sophisticated filtering rather than narrow targeting.

Paper Collection and Filtering. We query 24 arXiv categories spanning core wireless domains (cs.NI, eess.SP, cs.IT), AI/ML (cs.LG, stat.ML), and interdisciplinary areas. Our crawler initially retrieves 47,000 papers from 2005-2025 using broad keyword queries across communication, signal processing, and networking terms. Each paper receives a deterministic relevance score computed as a weighted sum of keyword hits in title (weight 0.6) and abstract (weight 0.3), plus category bonuses (e.g., eess.SP: +0.4, cs.NI: +0.35, cs.IT: +0.3), yielding a reproducible

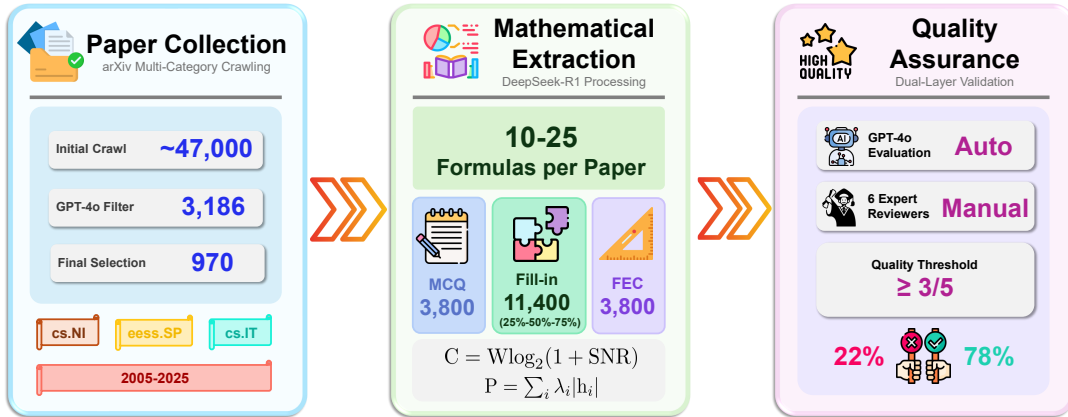


Figure 2: **The WirelessMathBench-XL construction pipeline.** A multi-stage filter reduces $\sim 47,000$ papers to 970 high-value sources via deterministic scoring and GPT-4o verification. DeepSeek-R1 extracts mathematical models to generate three task types (MCQ, Fill-in, Equation Completion), followed by a dual-layer QA process (Automated + Expert) that yields a 78% acceptance rate.

score in $[0,1]$ without any LLM involvement. We then apply GPT-4o-based filtering as a second-stage verification to identify 3,186 papers containing substantial mathematical content, from which we select the top $\sim 1,000$ based on mathematical rigor, citation impact, and topical diversity. Full implementation details are provided in Appendix E.

2.2 Mathematical Content Extraction and Problem Generation

Structured Model Extraction. We employ DeepSeek-R1 (Guo et al., 2025) to extract mathematical models from each paper’s LaTeX source. Our extraction preserves complete context including system equations, variable definitions with units and domain restrictions, underlying assumptions, and boundary conditions. Each paper yields a structured summary with properly formatted mathematical notation (e.g., \mathbf{v} for vectors, $\mathbf{H} \in \mathbb{C}^{N \times M}$ for complex matrices). Appendix I presents three representative examples of extracted system models, demonstrating the comprehensiveness of our approach across different wireless domains, including Stacked Intelligent Metasurfaces (SIM)-based air-ground communication, UAV-assisted mobile edge computing (UAV-MEC) systems, and Reconfigurable Intelligent Surfaces (RIS)-aided random access.

Automated Problem Generation. From extracted models, we generate three types of exam-style questions using carefully designed prompt templates (see Appendix H for com-

plete specifications):

- **Multiple Choice Questions (MCQ):** Equations are presented with masked right-hand sides, accompanied by four carefully designed options. Distractors reflect common errors such as matrix dimension mismatches or incorrect operator sequences.
- **Progressive Fill-in-the-Blank (Fill-in):** Three difficulty levels with 25%, 50%, and 75% of equation components masked, testing incremental understanding.
- **Full Equation Completion (FEC):** Complete 100% masking requiring full equation recall.

2.3 Quality Assurance Framework

Automated Evaluation. Each generated question undergoes systematic evaluation by GPT-4o across four critical dimensions: mathematical correctness, variable completeness, answer verifiability, and pedagogical value. The evaluation employs a comprehensive 5-point quality rubric, which categorizes problems as invalid (score 1), poor (score 2), acceptable (score 3), good (score 4), or excellent (score 5). This automated screening utilizes specialized prompt templates described in Appendix H to ensure consistent evaluation criteria across all question types.

Expert Validation. Questions passing automated evaluation proceed to human expert

235	review conducted by a team of six domain spe-	3 Validation Methodology:	283
236	cialists comprising four PhD students and two	Verification-Based RL	284
237	postdoctoral researchers with expertise span-	To validate the utility of WirelessMathBench-	285
238	ning optimization theory, information theory,	XL as a training resource, we employ Group	286
239	signal processing, and network analysis. Each	Relative Policy Optimization (GRPO) (Shao	287
240	question undergoes independent evaluation by	et al., 2024). Unlike standard Supervised Fine-	288
241	at least two experts who assess mathematical	Tuning (SFT) which mimics static solutions,	289
242	rigor, notational consistency, problem clar-	GRPO leverages the verifiable nature of our	290
243	ity, and relevance to wireless communications.	dataset to reward correct final answers, al-	291
244	Questions must achieve a minimum consensus	lowing the model to explore diverse reasoning	292
245	score of 3/5 to qualify for dataset inclusion.	paths.	293
246	The final acceptance rate of 78% reflects our	3.1 Training Objective and Reward	294
247	stringent quality standards. Detailed scoring	Formulation	295
248	criteria and representative examples across all	We optimize the policy π_θ to maximize a	296
249	quality levels are provided in Appendices F	group-wise advantage objective. For each	297
250	and J.	problem x , we sample a group of outputs	298
		$\{y_1, \dots, y_G\}$ and compute the advantage of each	299
251	2.4 Dataset Statistics and Analysis	output based on its relative reward within the	300
		group. This approach eliminates the need for	301
252	The WirelessMathBench-XL dataset com-	a separate value function critic, significantly	302
253	prises 4,027 problems derived from 970 papers,	reducing memory overhead. The full mathe-	303
254	providing comprehensive coverage across wire-	matical formulation of the objective function	304
255	less communications mathematics.	is detailed in Appendix A.	305
256	Technical Coverage. Deep learning	The core of our validation lies in the reward	306
257	dominates (259 papers, 14.0%), followed	function, which utilizes the ground truth pro-	307
258	by convex optimization (206, 11.2%) and	vided in WirelessMathBench-XL. We define a	308
259	MIMO/Massive MIMO (192, 10.4%). The pa-	composite reward $r(x, y)$:	309
260	pers in our dataset are distributed across es-		
261	tablished topics and emerging paradigms, with	$r(x, y) = \alpha \cdot r_{\text{format}}(y) + (1 - \alpha) \cdot r_{\text{accuracy}}(x, y)$	
262	strong representation of beamforming (185),	(1)	310
263	RIS/IRS (156), channel coding (115), feder-	where $\alpha = 0.1$. The reward components are:	311
264	ated learning (110), semantic communications		
265	(75), and NOMA (54).	<ul style="list-style-type: none">• Format Reward (r_{format}): A binary	312
266	Temporal Distribution. The dataset spans	check ensuring the model adheres to the	313
267	three technological generations: 3G/4G (2005-	requested L ^A T _E X format and encloses the	314
268	2018: 28 papers, 2.9%), 5G deployment	final result in <code>\boxed{\}</code> .	315
269	(2019-2023: 317 papers, 32.7%), and 5G-	<ul style="list-style-type: none">• Accuracy Reward (r_{accuracy}): A hi-	316
270	Advanced/6G research (2024-2025: 625 pa-	erarchical verification mechanism. We	317
271	pers, 64.4%).	first attempt a deterministic string match	318
272	Quality Distribution. Expert evaluation re-	against the normalized ground truth. If	319
273	veals that 35.53% of questions achieve accept-	that fails (e.g., due to commutative dif-	320
274	able quality (score 3), 30.89% are rated good	ferences like $A + B$ vs $B + A$), we employ	321
275	(score 4), and 11.08% reach excellence (score	a lightweight semantic verifier (GPT-4o-	322
276	5). Questions scoring below threshold (scores	mini) to check mathematical equivalence.	323
277	1-2: 22.50%) undergo revision or exclusion.	4 Experiments	324
278	Dataset Split. The 4,027 problems partition	4.1 Experimental Setup	325
279	into training (3,227, 80%) and test (800, 20%)	Baselines. We benchmark WirelessMathLM	326
280	sets with balanced representation. Training	against comprehensive baselines spanning	327
281	set: Fill-in-75% (900), FEC (751), Fill-in-50%		
282	(680), MCQ (551), Fill-in-25% (345).		

proprietary and open-source models. Proprietary models include GPT-5 (OpenAI, 2025), GPT-4o (Hurst et al., 2024), Claude-4.0-Sonnet (Anthropic, 2025), Gemini-2.5-Flash, and Gemini-2.5-Pro (Google DeepMind, 2025), representing state-of-the-art commercial systems. For open-source comparisons, we evaluate against general-purpose models including DeepSeek-R1 (671B) (Guo et al., 2025), DeepSeek-V3.1 (671B) (DeepSeek-AI, 2025), Llama-3.3-70B-Instruct (Grattafiori et al., 2024), and Qwen2.5-72B-Instruct (Yang et al., 2024a), as well as math-specialized models such as Qwen2.5-Math-72B-Instruct (Yang et al., 2024b) and DeepSeekMath-7B-RL (Shao et al., 2024). To isolate the specific value of verification-based RL, we include two critical ablations: (1) Qwen2.5 base models (0.5B, 3B, 7B) without any post-training, and (2) supervised fine-tuning (SFT) baseline trained on identical WirelessMathBench-XL data for 6 epochs to capture peak SFT performance.

Training Configuration. We train the **WirelessMathLM** family using Qwen2.5-Base checkpoints (0.5B, 3B, 7B) and Qwen3-4B-Base. This allows us to assess how dataset effectiveness scales with model size. All models are trained for 40 epochs with a learning rate of 10^{-6} and a KL-divergence penalty of $\beta = 0.01$ to prevent reward hacking. Complete hyperparameter settings and computational infrastructure details are provided in Appendix A.

Standardized Evaluation Protocol. All models are evaluated using uniform prompt templates containing explicit context and formatting instructions (see Appendix H). MCQs require `\boxed` answers, while fill-in tasks demand complete correctness of all masked slots. We employ a hierarchical evaluation metric: exact string matching serves as the primary filter, with GPT-4o-mini providing semantic equivalence checks for algebraic variations. A reliability analysis of this judge is provided in Appendix C.

4.2 Main Results on WirelessMathBench-XL

Table 1 presents comprehensive evaluation results on the WirelessMathBench-XL test set.

GRPO enables competitive performance

with dramatic parameter reduction. Our 7B WirelessMathLM trained with GRPO achieves 39.5% overall accuracy, approaching the performance of GPT-4o (40.4%) while using orders of magnitude fewer parameters. This result is particularly striking when compared against open-source math-specialized models: our approach outperforms both Qwen2.5-Math-7B-Instruct (21.6%) and DeepSeekMath-7B-RL (21.5%) by nearly 2 \times , despite these models being explicitly trained for mathematical reasoning. The performance gain stems from our domain-specific training strategy. While general math models struggle with the specialized notation and problem structures in wireless communications, our targeted approach with verifiable rewards enables efficient learning of domain-specific patterns.

Verification-based RL outperforms supervised fine-tuning. Our controlled ablation on Qwen2.5-3B demonstrates that GRPO (25.12%) substantially exceeds the peak SFT performance (20.37% at Epoch 4), yielding a +4.75% absolute gain (+23% relative). While SFT rapidly improves the base model (12.37% \rightarrow 20.37%), it hits a performance ceiling by Epoch 4 and shows instability in subsequent epochs (Fill-in accuracy drops from 17.44% to 13.24% by Epoch 6). This suggests that the performance gap between GRPO and the base model is not solely due to exposure to domain data (as does SFT), but rather because the reinforcement signal enables the model to explore solution paths beyond those captured by the true labels.

GRPO training yields consistent improvements across all model scales and architectures. The impact of GRPO training is substantial and scale-dependent. The 7B model nearly doubles its performance, improving from 21.9% to 39.5% (+81% relative), reaching within 0.9 percentage points of GPT-4o (40.4%). The 3B model demonstrates the most dramatic gains, more than doubling its accuracy from 12.4% to 25.1% (+103% relative). Improvements persist even at the minimal 0.5B scale (+11%). This effectiveness extends beyond the Qwen2.5 series models. When applied to the newer Qwen3-4B model, GRPO still achieves a significant increase (a relative increase of 31%,

Table 1: Performance on WirelessMathBench-XL test set (800 problems). MCQ: Multiple Choice Questions, Fill-in: Fill-in-the-blank, FEC: Full Equation Completion. Best result per category in **bold**.

Model	Size	MCQ (%)	Fill-in (%)	FEC (%)	Overall (%)
<i>Proprietary Models</i>					
GPT-5	-	63.91	63.20	41.36	57.87
GPT-5-mini	-	67.67	53.99	40.31	53.00
GPT-5-nano	-	57.14	37.82	30.37	39.25
GPT-4o	-	54.14	43.62	24.61	40.37
o4-mini	-	67.67	49.56	40.31	50.38
Claude-4.0-Sonnet	-	60.15	56.30	42.93	53.75
Gemini-2.5-Flash	-	63.16	56.09	43.46	54.25
Gemini-2.5-Pro	-	66.17	50.42	36.65	49.75
Grok-4-Fast	-	70.31	56.33	40.33	54.89
<i>Open-Source General Models</i>					
DeepSeek-R1	671B	65.41	60.50	43.98	57.37
DeepSeek-V3.1	671B	66.17	58.85	45.03	56.87
Llama-3.3-70B-Instruct	70B	54.14	38.03	28.27	38.37
Qwen2.5-72B-Instruct	72B	51.88	35.50	32.46	37.50
Qwen2.5-7B-Instruct	7B	39.1	21.85	26.18	25.75
Gemma 3 27B	27B	42.11	30.04	27.75	31.50
Gemma 3 12B	12B	36.84	21.43	21.99	24.12
<i>Open-Source Math-Specialized Models</i>					
Qwen2.5-Math-72B-Instruct	72B	60.15	40.55	33.51	42.13
Qwen2.5-Math-7B-Instruct	7B	42.11	14.71	24.61	21.62
DeepSeekMath-7B-RL	7B	43.61	13.66	25.65	21.50
<i>WirelessMathLM (Ours)</i>					
Qwen2.5-7B-Base	7B	44.36	14.29	25.13	21.88
+ GRPO	7B	53.38	36.97	36.13	39.50
Qwen3-4B-Base	4B	41.35	5.25	15.18	13.63
+ GRPO	4B	47.37	8.82	19.90	17.87
Qwen2.5-3B-Base	3B	26.32	7.14	15.71	12.37
+ GRPO	3B	48.87	17.02	28.80	25.12
Qwen2.5-0.5B-Base	0.5B	27.07	5.25	24.08	13.38
+ GRPO	0.5B	30.08	6.09	26.18	14.87
<i>Ablation: Supervised Fine-Tuning</i>					
Qwen2.5-3B-Base + SFT (Epoch 1)	3B	39.85	11.97	20.42	18.62
Qwen2.5-3B-Base + SFT (Epoch 4, Peak)	3B	40.60	17.44	13.61	20.37
Qwen2.5-3B-Base + SFT (Epoch 6)	3B	44.26	13.24	16.23	19.13

from 13.63% to 17.87%), despite its better performance on basic multiple-choice questions (41.35%), demonstrating that our dataset and reinforcement strategy enable effective learning independent of specific model capacity or pre-training distributions.

Performance patterns reveal task-specific strengths. Analyzing performance across question types reveals interesting patterns. All models perform best on multiple-choice questions (MCQ), where our 7B model achieves 53.4% accuracy, within striking distance of proprietary models like GPT-4o (54.1%) and approaching DeepSeek-R1 (65.4%). Performance on fill-in-the-blank questions shows the largest improvement from GRPO training (14.3% \rightarrow 37.0% for 7B), suggesting that the reinforcement learning

particularly helps with partial equation completion. Full equation completion (FEC) remains challenging across all models, though our 7B model’s 36.1% accuracy is competitive with GPT-5-mini (40.3%) and exceeds many larger open models.

Comparison with state-of-the-art reveals efficiency-performance trade-offs. While DeepSeek-R1 (671B) achieves the highest open-source performance at 57.4%, it requires $\approx 100\times$ more parameters than our 7B model. The performance gap of 17.9 percentage points represents a favorable trade-off for deployment scenarios. Our model achieves 69% of DeepSeek-R1’s performance with just 1% of its parameters. Among proprietary models, only GPT-5 (57.9%) significantly outperforms our approach, while models like

Claude-4.0-Sonnet (53.8%) and Gemini-2.5-Flash (54.3%) show more modest advantages despite their substantially larger scale and computational requirements.

4.3 Generalization to General Mathematics

Surprisingly, training on wireless-specific mathematics enhances general mathematical reasoning (Table 2).

Domain-specific training strengthens fundamental mathematical capabilities.

Our GRPO-trained models show substantial improvements on general mathematics benchmarks without any explicit training on these tasks. The 7B model improves from 52.0% to 67.0% on MATH 500 (Hendrycks et al., 2021b) (+28.8% relative), while the 3B model gains even more dramatically (41.6% \rightarrow 58.2%, +39.9% relative). These improvements extend across diverse mathematical domains: Minerva-Math (Lewkowycz et al., 2022) sees modest but consistent gains (7B: 12.1% \rightarrow 14.3%), OlympiadBench (He et al., 2024) improves substantially (7B: 25.3% \rightarrow 30.2%), and AMC (Li et al., 2024) performance increases significantly (7B: 27.7% \rightarrow 41.0%). Even on the challenging AIME24 (Li et al., 2024), the 7B model doubles its performance (6.7% \rightarrow 13.3%).

4.4 No Regression on Foundational Capabilities

To rigorously test whether domain specialization causes catastrophic forgetting, we evaluate pre- and post-GRPO models on four diverse general-purpose benchmarks spanning knowledge (MMLU) (Hendrycks et al., 2021a), science reasoning (GPQA) (Rein et al., 2024), instruction-following (IFEval) (Zhou et al., 2023), and programming (HumanEval) (Chen et al., 2021).

Key Findings: (1) **No catastrophic forgetting:** Both 3B and 7B models show improvements across all benchmarks except a negligible -0.39 regression on IFEval for 7B, well within measurement noise. (2) **Reasoning-intensive tasks benefit most:** The 7B model gains +11.90 on MMLU and +10.10 on GPQA, suggesting that learning strict physical constraints in wireless mathematics strengthens general reasoning capa-

bilities. (3) **Transfer to unrelated domains:** Even HumanEval (programming) improves (+3.65 for 3B, +1.22 for 7B) despite zero code in training, indicating that domain-specific RL develops generalizable problem-decomposition skills.

4.5 Qualitative Analysis of Reasoning Capabilities

We analyzed 800 solutions generated by WirelessMathLM-7B to characterize the reasoning behaviors emerging from specialized reinforcement learning. Detailed case studies are provided in Appendix D.

Structured Decomposition and Coherence.

The model exhibits a systematic decomposition strategy that transcends simple pattern matching. Across evaluated solutions, 99.1% demonstrate explicit multi-step reasoning connected by logical transitions (e.g., “therefore”, “substituting Eq. 1”). Crucially, the model effectively bridges the gap between physical principles and mathematical formulation. In complex optimization scenarios, such as beamforming design under limited power, the model consistently defines the feasibility region (e.g., total power constraints) before invoking Lagrangian duality, preventing the generation of mathematically valid but physically impossible solutions.

Domain-Specific Operational Competency.

A key differentiator of WirelessMathLM is its handling of domain-specific operations that defy standard arithmetic rules. For instance, in Cell-Free Massive MIMO tasks, the model correctly applies conjugate beamforming logic, specifically the complex conjugation of channel estimates (\hat{g}^*), citing the physical necessity to “cancel phase shifts introduced by the channel.” This integration of procedural knowledge (matrix algebra) with conceptual understanding (signal propagation) allows the model to solve problems where general-purpose models frequently fail due to dimension mismatches or incorrect operator application.

Emergent Constraint Awareness. Perhaps the most significant finding is the model’s internalization of unstated physical laws. Analysis reveals that WirelessMathLM-7B automatically incorporates implicit constraints—such as the non-negativity of trans-

Table 2: Transfer learning effects on general mathematical reasoning benchmarks.

WirelessMathLM	MATH 500	Minerva-Math	OlympiadBench	AMC	AIME24	Average
Qwen2.5-7B-Base	52.00	12.13	25.33	27.71	6.67	24.77
+ GRPO	67.00	14.34	30.22	40.96	13.33	33.17
Δ (GRPO vs Base)	+15.00	+2.21	+4.89	+13.25	+6.66	+8.40
Qwen2.5-3B-Base	41.60	5.88	14.67	18.07	0.00	16.04
+ GRPO	58.20	9.93	22.96	21.69	0.00	22.56
Δ (GRPO vs Base)	+16.60	+4.05	+8.29	+3.62	0.00	+6.52

Table 3: Performance on general-purpose benchmarks before and after GRPO training.

WirelessMathLM	MMLU	GPQA	IFEval	HumanEval
Qwen2.5-3B-Base	42.49	23.74	26.99	61.59
+ GRPO	45.30	28.28	32.90	65.24
Δ (GRPO vs Base)	+2.81	+4.54	+5.91	+3.65
Qwen2.5-7B-Base	38.84	28.79	36.06	63.41
+ GRPO	50.74	38.89	35.67	64.63
Δ (GRPO vs Base)	+11.90	+10.10	-0.39	+1.22

mit power ($P \geq 0$) or the causality of signal processing filters—into its derivations without explicit prompting. This is frequently accompanied by "self-verification" behaviors; for example, when deriving matrix all-pass filters, the model was observed to autonomously check the unitary property $\mathbf{G}(z)\mathbf{G}^{-1}(z) = \mathbf{I}$ after factorization, confirming its result against the physical definition of lossless systems.

5 Related Work

Mathematical Reasoning in LLMs. Chain-of-thought prompting (Wei et al., 2022) demonstrated that eliciting step-by-step reasoning significantly improves mathematical problem-solving in large language models. This was extended through process supervision (Lightman et al., 2024), where models receive feedback on intermediate steps rather than just final answers, and tool-augmented approaches like ToRA (Gou et al., 2023) that integrate external computation for complex calculations. While these advances have been evaluated on benchmarks ranging from elementary word problems (GSM8K (Cobbe et al., 2021)) to competition mathematics (MATH (Hendrycks et al., 2021b)) and formal theorem proving (MiniF2F (Zheng et al., 2021)), such benchmarks do not capture the symbolic manipulation and domain knowledge required in technical fields.

Domain Adaptation. Continued pre-training on domain-specific corpora (Gururangan et al., 2020) and instruction tuning (Chung et al., 2022) have proven effective for adapting language models to specialized fields. Scientific models like Galactica (Taylor et al., 2022) attempted broad scientific reasoning, while BioBERT (Lee et al., 2019) and MedPaLM (Singhal et al., 2022) achieved strong performance in biomedicine.

Reinforcement Learning from Verifiable Rewards. While RLHF (Ouyang et al., 2022) successfully aligns language models with human preferences, it requires expensive annotation that limits scalability. Recent alternatives include Constitutional AI (Bai et al., 2022) using principle-based self-critique, RLAIIF (Lee et al., 2023) leveraging model-generated feedback, and GRPO (Shao et al., 2024) using outcome-based rewards for mathematics.

6 Conclusion

This work introduces **WirelessMathBench-XL**, filling a critical resource gap for reasoning in specialized engineering domains. By curating a rigorously validated, document-split dataset of 4,027 problems, we provide the community with the first training-scale benchmark for physically constrained mathematics. Our extensive benchmarking validates the resource’s utility: training on this corpus enables small models (7B) to achieve performance competitive with closed-source frontier models. Crucially, we find that domain specialization does not degrade general capabilities; rather, the rigor required to satisfy physical constraints transfers positively to general mathematical benchmarks. We envision this resource serving as a testbed for future research in automated verification, neuro-symbolic solvers, and reliable generation in technical fields.

640 Limitations

641 While our work demonstrates the viability of
642 verification-based RL for wireless mathemat-
643 ics, several limitations warrant discussion: Ab-
644 solute Performance Ceiling: Despite substan-
645 tial improvements, our best 7B model achieves
646 39.5% accuracy approaching GPT-4o (40.4%),
647 but it is still far from being deployed for
648 high-risk engineering applications. This re-
649 flects the extreme difficulty of graduate-level
650 wireless problems extracted from state-of-the-
651 art research papers. Future work should ex-
652 plore whether scaling to 70B+ parameters, in-
653 corporating tool-augmentation (e.g., symbolic
654 math engines), or leveraging multi-modal in-
655 puts (circuit diagrams, simulation results) can
656 bridge this gap. Evaluation Methodology:
657 Our hierarchical verification minimizes but
658 does not eliminate LLM dependence. While
659 cross-evaluator analysis quantifies potential
660 bias at 3-4% absolute impact developing a
661 fully deterministic symbolic verifier trained
662 on WirelessMathBench-XL remains critical fu-
663 ture work for removing all LLM-based evalu-
664 ation and enabling truly reproducible assess-
665 ment. Paradoxically, achieving this end-to-end
666 automation requires exactly the type of high-
667 quality human-verified datasets we provide
668 here. A robust Auto-Judge model trained on
669 the verified solutions in WirelessMathBench-
670 XL could eventually replace human verifica-
671 tion entirely, creating a closed-loop system for
672 continuous dataset expansion and model im-
673 provement without manual intervention. This
674 represents a key direction for transforming our
675 current semi-automated pipeline into a fully
676 autonomous framework.

677 Ethics Statement

678 This work focuses on advancing the math-
679 ematical reasoning capabilities of language
680 models in the specialized domain of wireless
681 communications. The WirelessMathBench-
682 XL dataset was constructed from publicly ac-
683 cessible academic papers on arXiv, respect-
684 ing the norms of scientific dissemination. Our
685 data collection process did not involve hu-
686 man subjects or personally identifiable infor-
687 mation. The expert validation phase was con-
688 ducted by graduate students and postdoctoral
689 researchers as part of their standard research

690 activities. While any powerful AI technol-
691 ogy carries potential for misuse, our work is
692 foundational and does not present immediate
693 dual-use concerns. We acknowledge that our
694 dataset, being derived from existing literature,
695 may reflect the inherent biases present in the
696 field. We encourage responsible use of our
697 models and dataset, and we are committed
698 to addressing any ethical concerns that may
699 arise.

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A Training Implementation Details

A.1 Mathematical Formulation of GRPO

We utilize the Group Relative Policy Optimization (GRPO) objective. Given a query $x \sim P(X)$ and a policy π_θ , we sample a group of G outputs $\{y_i\}_{i=1}^G$ from the old policy $\pi_{\theta_{\text{old}}}$. The objective is defined as:

$$\mathcal{J}_{\text{GRPO}}(\theta) = \mathbb{E}_{x,y} \left[\frac{1}{G} \sum_{i=1}^G \min \left(r_i(\theta) A_i, \text{clip}(r_i(\theta), 1 - \epsilon, 1 + \epsilon) A_i \right) \right] \quad (2)$$

where $r_i = \frac{\pi_\theta(y_i|x)}{\pi_{\theta_{\text{old}}}(y_i|x)}$ is the probability ratio, ϵ is the clipping parameter (set to 0.2), and A_i is the standardized advantage computed within the group:

$$A_i = \frac{R_i - \text{mean}(\{R_j\}_{j=1}^G)}{\text{std}(\{R_j\}_{j=1}^G)} \quad (3)$$

Here, R_i is the raw reward obtained from our verification system. This standardization is crucial for stabilizing training when absolute reward scales vary across different problem types.

A.2 Hyperparameters and Infrastructure

Table 4 lists the detailed hyperparameters used for training WirelessMathLM. We utilized a cluster of 4 NVIDIA A6000 GPUs. Training duration varied by model size: 14 hours for 0.5B, 40 hours for 3B, and 61 hours for 7B.

Table 4: Hyperparameters for GRPO Training

Hyperparameter	Value
Base Model	Qwen2.5 (0.5B/3B/7B)
Optimizer	AdamW
Learning Rate	1×10^{-6}
Weight Decay	0.1
LR Scheduler	Cosine Annealing
Global Batch Size	64
Group Size (G)	8
KL Coefficient (β)	0.01
Clip Range (ϵ)	0.2
Max Context Length	2048
Epochs	40

B Per-Subdomain Performance Analysis

We analyze performance at the problem level across 20 wireless subdomains in the 800-problem test set.

Key Findings: (1) **Uniform improvements across all subdomain sizes:** GRPO enhances 19/20 categories regardless of training data prevalence. Notably, the smallest categories show the largest gains (Semantic Communications: +300%, Edge Computing: +450%), while heavily-represented topics show moderate gains (Deep Learning: +58%, Convex Optimization: +39%). (2) **No correlation between problem count and performance:** Large categories like Channel Estimation (160 problems, +61%) and small categories like Cell-Free (9 problems, +200%) both benefit substantially. (3) **Models learn generalizable principles:** The lack of correlation between subdomain size and improvement magnitude suggests models acquire transferable wireless mathematics reasoning rather than memorizing frequent patterns.

C Evaluation Robustness and Potential Bias

This appendix analyzes the robustness of our evaluation and bounds the potential bias introduced by LLM-based semantic checking. Our verifier is hierarchical: it first applies deterministic matching and only falls back to an LLM judge when the answer is mathematically correct but not syntactically identical. Because different models produce different surface forms, the fallback rate is *model-dependent*.

Evaluation Architecture. For each of the 800 test problems, we perform:

- **Stage 1 (Deterministic).** MCQ answers are graded by direct letter extraction. Fill-in / FEC answers are normalized by removing whitespace and LaTeX styling tokens (e.g., `\mathbf`, `\boldsymbol`) and then checked via exact string match.
- **Stage 2 (LLM Semantic Fallback).** Only when Stage 1 fails, we invoke an

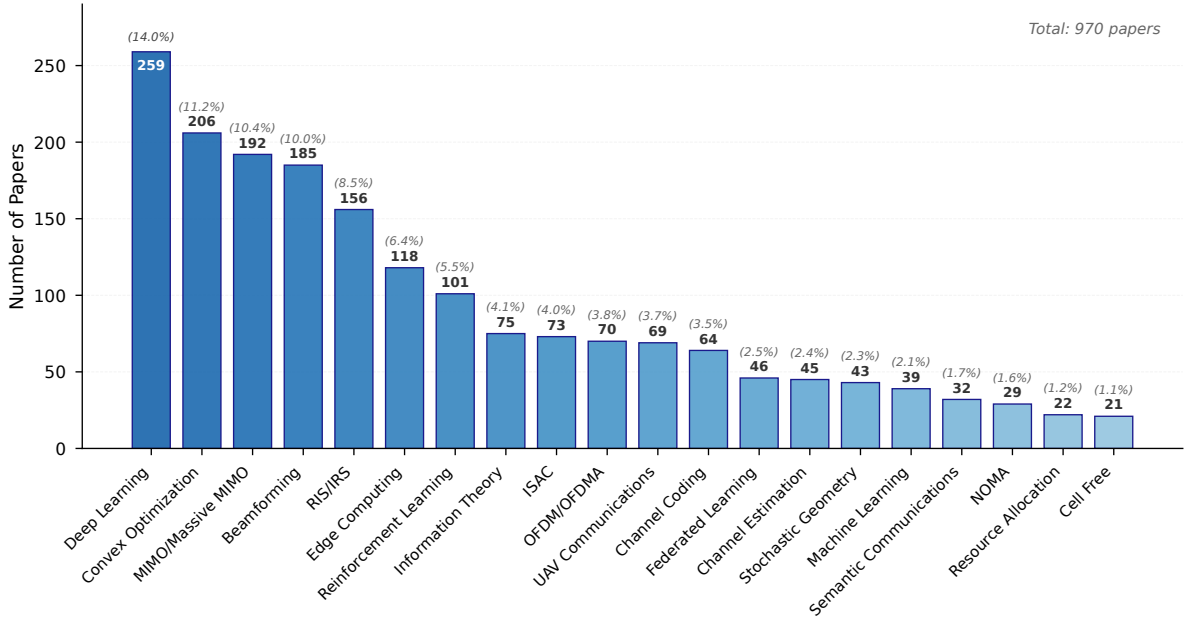


Figure 3: Distribution of the top 20 key techniques across the 970 source papers in WirelessMathBench-XL. Deep learning leads with 259 papers (14.0%), followed by convex optimization (206, 11.2%) and MIMO/Massive MIMO (192, 10.4%). The distribution spans from foundational techniques (beamforming, channel coding) to emerging paradigms (RIS/IRS, semantic communications, NOMA).

LLM judge to verify semantic equivalence for cases such as scalar commutativity (AB vs. BA) or equivalent conjugate-transpose notation (H^H vs. H^*).

Thus, even under pessimistic assumptions, semantic-judge variability produces an overall uncertainty of **approximately 4.6–7.3% absolute**, depending on the model.

On the 800-problem test set, the number of cases requiring Stage 2 varies across models: **GPT-5: 270/800 (33.75%)**, **Gemini-2.5 Pro: 183/800 (22.88%)**, and **Claude-Sonnet-4: 293/800 (36.63%)** (Table 6).

Cross-Evaluator Bias Quantification. To bound evaluator-induced variance, we regraded the Stage-2 subset for the above three top-performing models using three independent judges (GPT-4.1-mini, Claude-3-Haiku, Gemini-2.0-Flash). Agreement rates on the Stage-2 subset differ by **10–20 percentage points** across judges (Table 6). Let ρ denote the Stage-2 fraction and δ the maximum judge disagreement on that subset; then a conservative upper bound on overall-score variation is $\rho \cdot \delta$. With $\delta \leq 0.20$, this yields worst-case absolute impacts of:

$$\begin{aligned} \text{Gemini-2.5 Pro: } & 0.2288 \times 0.20 \approx 4.6\% \\ \text{GPT-5: } & 0.3375 \times 0.20 \approx 6.8\% \\ \text{Claude-Sonnet-4: } & 0.3663 \times 0.20 \approx 7.3\%. \end{aligned}$$

Key Findings. (1) **Most grading is bias-free.** The majority of evaluations are handled deterministically (about 63–77% across top models), substantially limiting exposure to judge bias. (2) **Overall impact is bounded and smaller than our main effects.** The worst-case 4.6–7.3% absolute uncertainty is far below GRPO gains over base/SFT and well below the large gaps between WirelessMathLM-7B (39.5%) and open-source baselines (21–25%). (3) **Rankings are robust.** Across all three judges, relative ordering among models remains unchanged, indicating that our conclusions do not depend on a particular evaluator.

D Detailed Qualitative Analysis

To supplement the summary in the main text, we provide specific case studies illustrating the three key reasoning capabilities observed in WirelessMathLM-7B.

Table 5: Problem-level subdomain performance on 800-problem test set. GRPO improves 19/20 categories, with gains independent of subdomain size.

Category	Count	%	Base (7B)	+GRPO	Δ Abs	Δ Rel
Channel Estimation	160	20.0%	22.50%	36.25%	+13.75%	+61.1%
Beamforming	113	14.1%	18.58%	43.36%	+24.78%	+133.4%
Resource Allocation	110	13.8%	21.82%	39.09%	+17.27%	+79.2%
MIMO/Massive MIMO	104	13.0%	23.08%	43.27%	+20.19%	+87.5%
Information Theory	103	12.9%	21.36%	46.60%	+25.24%	+118.2%
RIS/IRS	98	12.3%	17.35%	33.67%	+16.32%	+94.1%
Machine Learning	86	10.8%	25.58%	50.00%	+24.42%	+95.5%
OFDM/OFDMA	50	6.3%	18.00%	28.00%	+10.00%	+55.6%
ISAC	49	6.1%	24.49%	36.73%	+12.24%	+50.0%
Convex Optimization	49	6.1%	26.53%	36.73%	+10.20%	+38.5%
Deep Learning	44	5.5%	27.27%	43.18%	+15.91%	+58.4%
Stochastic Geometry	40	5.0%	35.00%	32.50%	-2.50%	-7.1%
UAV Communications	32	4.0%	12.50%	31.25%	+18.75%	+150.0%
Edge Computing	28	3.5%	7.14%	39.29%	+32.15%	+450.0%
Reinforcement Learning	28	3.5%	32.14%	35.71%	+3.57%	+11.1%
Channel Coding	20	2.5%	25.00%	45.00%	+20.00%	+80.0%
Federated Learning	19	2.4%	26.32%	36.84%	+10.52%	+40.0%
NOMA	19	2.4%	15.79%	26.32%	+10.53%	+66.7%
Semantic Communications	13	1.6%	15.38%	61.54%	+46.16%	+300.0%
Cell-Free	9	1.1%	11.11%	33.33%	+22.22%	+200.0%
Average across all 20 categories	—	—	—	—	+17.7%	+81.0%

Table 6: Cross-evaluator agreement on the LLM-judged subset.

Model Evaluated	LLM Cases	GPT-4.1-mini	Claude-3-Haiku	Gemini-2.0-Flash
GPT-5	270	41.11%	56.67%	42.22%
Gemini-2.5 Pro	183	53.01%	55.74%	56.28%
Claude-Sonnet-4	293	41.64%	60.75%	45.05%

D.1 Case Study 1: Domain-Specific Operations

The model demonstrates strong competency in handling complex-valued operations unique to wireless communications. For instance, in a Cell-Free Massive MIMO conjugate beamforming problem (Question ID 18369), the model correctly identified that conjugate beamforming requires the complex conjugation of estimated channel coefficients. It provided the physical rationale (“to cancel out phase shifts introduced by the channel”) and derived the correct signal expression:

$$s_m = \sqrt{P_m} \sum_{k=1}^K \sqrt{\eta_{mk}} \hat{g}_{mk}^* u_k \quad (4)$$

This response demonstrates an understanding of power scaling ($\sqrt{P_m}$), summation logic, and the specific syntax of complex conjugation (\hat{g}^*), which are distinct from general algebraic rules.

D.2 Case Study 2: Method Justification

Correct solutions routinely include explicit rationales for chosen approaches. In a matrix all-pass filter factorization problem (Ques-

tion ID 11325), the model explained: “A matrix all-pass filter is a filter whose frequency response has a magnitude of 1 for all frequencies...” before deriving the factorization $\mathbf{G}(z) = \mathbf{N}(z)\mathbf{D}^{-1}(z)$. It further verified the result by checking the property $\mathbf{G}(z)\mathbf{G}^{-1}(z) = \mathbf{I}_m$, demonstrating a “self-verification” behavior ingrained during training.

D.3 Case Study 3: Physical Intuition

Solutions frequently connect mathematical properties to physical hardware constraints. When deriving XOR operations for backscattered data processing, the model explicitly cited the “commutative and associative” properties of XOR as the mathematical basis for recovering data from superimposed wireless tag signals.

E Dataset Construction Details

E.1 Detailed Paper Collection Methodology

Multi-Category Coverage. We query across 24 arXiv categories to capture interdisciplinary research:

- Core categories: cs.NI (Networking),

1098	eess.SP (Signal Processing), cs.IT (Information Theory)		1143
1099			1144
1100	• AI/ML categories: cs.LG, stat.ML, cs.AI for learning-based approaches		1145
1101			
1102	• Systems categories: cs.SY, cs.DC, cs.MA for distributed and multi-agent systems		1146
1103			1147
1104	• Physics categories: physics.optics, quantum for emerging physical layer techniques		1148
1105			1149
1106	• Mathematical categories: math.OC, math.IT for optimization and theory		1150
1107			1151
1108	Query Construction Strategy. We implement four complementary query strategies:		1152
1109			1153
1110	queries = [1154
1111	{name: 'basic_communication_terms',		1155
1112	keywords: [communication, network,		1156
1113	wireless, radio, signal, antenna,		1157
1114	frequency, spectrum, transmission]},		1158
1115	{name: 'system_algorithm_terms',		1159
1116	keywords: [system, algorithm,		1160
1117	optimization, performance, model,		1161
1118	framework, architecture]},		1162
1119	{name: 'application_computing_terms',		1163
1120	keywords: [computing, sensing, iot,		1164
1121	edge, cloud, distributed, energy,		1165
1122	security]},		1166
1123	{name: 'data_intelligence_terms',		1167
1124	keywords: [learning, intelligence,		1168
1125	neural, prediction, detection,		1169
1126	processing, estimation]}		1170
1127]		1171
1128	Relevance Scoring and Annotation.		1172
1129	For each paper, we calculate:		1173
1130	• Relevance Score (0-1): Weighted sum		1174
1131	of keyword presence in title (0.6 weight)		1175
1132	and abstract (0.3 weight), plus category		1176
1133	bonuses (eess.SP: 0.4, cs.NI: 0.35, cs.IT:		1177
1134	0.3)		1178
1135	• Technology Focus: Detected across		1179
1136	8 categories (wireless_basic, advanced_wireless,		1180
1137	next_gen, emerging_tech,		1181
1138	signal_processing, network_protocol,		1182
1139	ai_ml, iot_apps)		1183
1140	• Quality Tier: Based on relevance score		1184
1141	(high: ≥ 0.7 , medium: 0.4-0.7, low: 0.1-		1185
1142	0.4)		1186
			1187
			1188
			1189
			1190
			1191
			1192
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			1195
			1196
			1197
			1198
			1199
			1200

Table 7: Detailed expert question quality assessment rubric

Score	Criteria
1 - Invalid	Problem statement or solution is clearly wrong or contradictory; Not related to wireless communications domain; Cannot be used as a valid question
2 - Poor	Statement correct but problem too trivial (answerable instantly); Problem too vague or nearly impossible to answer correctly; Very little learning or evaluation value
3 - Acceptable	Statement and solution reasonable with no major errors; Difficulty and relevance are average; Can be kept but adds limited value (baseline quality)
4 - Good	Clear and well-structured problem; Relevant to domain and moderately challenging; Provides meaningful assessment of understanding; Worth keeping and recommending
5 - Excellent	Highly relevant to the domain; Strong depth, creativity, or insight required; Excellent for differentiating levels of understanding; Strongly recommended for inclusion

G.1 Quality Assessment Framework

Our quality assessment framework employs a systematic approach to evaluate technical questions across six dimensions:

- Question Clarity** (1-5): Measures the clarity and unambiguousness of the question statement
- Background Relevance** (1-5): Evaluates the completeness and relevance of provided context
- Answer Accuracy** (1-5): Assesses the correctness and formatting of the provided answer
- Technical Appropriateness** (1-5): Determines if the difficulty level matches the target audience
- Mathematical Rigor** (1-5): Evaluates mathematical notation and conventions
- Wireless Communication Relevance** (1-5): Measures domain relevance to wireless communications

G.2 Real LLM Annotation Examples

To demonstrate the practical effectiveness of our LLM-assisted quality assessment system, we present three representative examples from our evaluation dataset, showcasing different quality levels and the corresponding LLM assessments.

H Prompt Construction for Dataset Generation and Evaluation

We employ specialized prompt templates for dataset construction, quality assessment, and standardized evaluation to ensure consistency and fairness across all stages of our methodology.

H.1 System Model Extraction Prompt

The following prompt template guides the extraction of mathematical models from research papers:

System Model Extraction Template

Task: Act as an expert in wireless communications and mathematical modeling. Extract and summarize the mathematical system modeling from the paper.

STRUCTURE REQUIREMENTS

1. Model Extraction:

- Identify ALL system equations with context
- For each equation:
 - List ALL variables with units/dimensions
 - Specify underlying assumptions
 - Note domain restrictions

2. Summary Organization:

- \paragraph{Background} (2-3 sentences contextualizing the model)

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Table 8: LLM Annotation Examples Across Quality Levels

Quality Level	Question Content	LLM Assessment
High Quality	<p>Background: Federated fine-tuning system with low-rank adaptation matrices $\mathbf{A}_k \in \mathbb{R}^{d \times r}$, $\mathbf{B}_k \in \mathbb{R}^{r \times d}$</p> <p>Question: Which term completes: $\mathbf{W} + [\text{MASK}]$?</p> <p>Options: A) $\mathbf{A}_k \mathbf{B}_k$ B) $\mathbf{B}_k \mathbf{A}_k$</p> <p>Human: 4/5</p>	<p>LLM Score: 4/5</p> <p>Strengths: Clear structure, complete context, accurate answer, rigorous notation</p> <p>Weaknesses: Could benefit from brief explanation of low-rank adaptation significance</p> <p>Agreement: ✓</p>
Medium Quality	<p>Background: H-NOMA system with variable definitions partially provided</p> <p>Question: Fill in [MASK] for the equation</p> <p>Human: 2/5</p>	<p>LLM Score: 3/5</p> <p>Strengths: Wireless relevance, accurate answer</p> <p>Weaknesses: Ambiguous [MASK] usage, lacks clarity in instructions</p> <p>Technical Issues: Missing variable definitions</p> <p>Disagreement: LLM too optimistic</p>
Low Quality	<p>Background: Transformer model context with incomplete variable definitions</p> <p>Question: What replaces the "full key matrix"?</p> <p>Human: 1/5</p>	<p>LLM Score: 3/5</p> <p>Strengths: Clear structure, accurate answer</p> <p>Weaknesses: Limited wireless relevance, focuses more on tensor parallelism</p> <p>Technical Issues: "Full key matrix" not defined</p> <p>Bias: LLM shows optimistic scoring pattern</p>

- \paragraph{Key Assumptions} (bullet points with •)
- \paragraph{Parameter Definitions} (table-like structure)
- \paragraph{Core Equations} (numbered with original labels)

3. Equation Formatting:

- Vectors: \mathbf{v}
- Matrices: \mathbf{M}
- Operators: diag, tr
- Complex numbers: j for imaginary unit

CONTENT GUIDELINES

- Variable Explanations:
 - For each symbol: θ (Type: Phase shift; Domain: $[0, 2\pi)$; Unit: rad)
 - Matrix dimensions: $\mathbf{H} \in \mathbb{C}^{N \times M}$
 - Distinguish similar symbols: h_{ij} vs $h_i^{(j)}$
- Model Validation:
 - Verify dimensional consistency
 - Check boundary conditions
 - Confirm parameter unit homogeneity

H.2 Question Generation Prompt

The following template generates exam-style questions from extracted models:

Question Generation Template

Task: Generate exam-style questions from research paper summaries.

STRUCTURE REQUIREMENTS

- Per Equation Processing:**
 - Identify ALL system model equations
 - For EACH equation:
 - Mask the RHS with [MASK]
 - Generate 1 MCQ with 4 plausible options
 - Create 4 progressive fill-in-the-blank subquestions:
 - 25%, 50%, 75%, and 100% key symbols masked
- Question Components:**
 - For MCQs:
 - Background: MUST include detailed variable definitions
 - Format: "where \mathbf{x} is the transmitted signal vector, $\mathbf{H} \in \mathbb{C}^{N \times M}$ represents the channel matrix..."
 - Equation: Masked equation in display math mode
 - Question: Explicitly ask to replace [MASK]
 - Options: 4 LaTeX-formatted choices (A)-(D)
 - Answer: Detailed derivation walkthrough

ENHANCED BACKGROUND REQUIREMENTS

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- Variable Definition Format:
 - Start with system context: "In this [type of system]..."
 - List EVERY symbol that appears in the equation
 - Include matrix/vector dimensions
 - Specify units where applicable: "(in watts)", "(in Hz)"
 - Explain subscripts and superscripts
- Distractor Design:
 1. Matrix dimension mismatches
 2. Incorrect operator sequences
 3. Missing diag() operators
 4. Channel matrix transposition errors
 5. Incorrect matrix multiplication order
- Masking Strategy:
 - 25%: Single critical variable
 - 50%: Two interdependent terms
 - 75%: Multiple components
 - 100%: Full equation recall

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To ensure consistent quality evaluation across the dataset, we employ a comprehensive assessment framework with few-shot learning enhancement. This framework guides both automated LLM evaluation and human expert review.

Quality Assessment Prompt with Few-Shot Learning

You are an expert evaluator specializing in wireless communication and mathematics education. Your task is to assess the quality of technical questions designed for advanced undergraduate and graduate students in wireless communications.

EVALUATION METHODOLOGY

Follow this systematic approach:

STEP 1: Initial Question Analysis

- Read the question, background, equation, and answer carefully
- Identify the technical domain and complexity level
- Check for obvious errors or inconsistencies

STEP 2: Multi-Dimensional Quality Assessment

Evaluate each dimension on a 1-5 scale:

1. **Question Clarity (1-5):** Crystal clear vs confusing/incomprehensible
2. **Background Relevance (1-5):** Comprehensive context vs inadequate background
3. **Answer Accuracy (1-5):** Completely correct vs incorrect/flawed
4. **Technical Appropriateness (1-5):** Perfect difficulty vs inappropriate level
5. **Mathematical Rigor (1-5):** Excellent notation vs poor rigor
6. **Wireless Relevance (1-5):** Highly relevant vs not relevant

HUMAN EXPERT EXAMPLES

Learn from these actual expert evaluations:

Example 1 - Score: 1 (Very Poor)

Question: "Which expression correctly calculates the sensitivity metric?"

Human Feedback: "The definition of TN is not given"

→ Missing variable definitions make question unsolvable

Example 2 - Score: 3 (Acceptable)

Question: "Which performance metric should replace [MASK]?"

Human Feedback: "Some variables in choices are not given"

→ Minor gaps but workable with assumptions

Example 3 - Score: 5 (Excellent)

Question: Complete differential privacy equation with full context

Human Feedback: "Well-structured with complete information"

→ Ready for immediate use

CRITICAL EVALUATION GUIDELINES

Be especially strict about:

- **Missing Variable Definitions:** Any undefined variables → Score ≤ 2
- **Incomplete Context:** Key background missing → Score ≤ 2
- **Vague Problem Statements:** Ambiguous questions → Score ≤ 3
- **Technical Accuracy:** Mathematical/technical errors → Score ≤ 2

OUTPUT FORMAT

Provide assessment in JSON:

```
{
  "overall_score": [1-5 integer],
  "dimension_scores": {
    "question_clarity": [1-5],
```

```

"background_relevance": [1-5],
"answer_accuracy": [1-5],
"technical_appropriateness": [1-5],
"mathematical_rigor": [1-5],
"wireless_relevance": [1-5]
},
"binary_flags": {
  "is_correct": [true/false],
  "is_wireless_related": [true/false]
},
"quality_analysis": {
  "strengths": ["Key strengths"],
  "weaknesses":
    ["Areas for improvement"],
  "specific_improvements":
    ["Detailed suggestions"]
}
}
}
Question Type: {question_type}
Question Text: {question_text}
Background: {background}
Equation: {equation}
Options: {options}
Correct Answer: {correct_answer}

```

Please solve this problem step by step. Fill in the [MASK] placeholder(s) with the correct mathematical expression(s).

For single mask: Your final answer should be given at the end in the format: `\boxed{your answer}`

For multiple masks: Your final answers should be given at the end in the format: `\boxed{answer1}, \boxed{answer2}, ...` (for the N blanks in order)

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H.4 Standardized Evaluation Prompts

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To ensure reproducible evaluation, all models receive identical prompts constructed from the following templates:

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MCQ Evaluation Template

Background

[Complete variable definitions and system context]

Question

[Question text]

Equation

[Equation with [MASK] placeholder]

Options

A: [Option A]

B: [Option B]

C: [Option C]

D: [Option D]

Please analyze this problem step by step. Show your reasoning and calculations. Your final answer should be given at the end in the format: `\boxed{X}` where X is the letter of the correct option.

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Fill-in-the-Blank Evaluation Template

Background

[Complete variable definitions and system context]

Question

[Question text]

Equation

[Equation with [MASK] placeholder(s)]

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I Representative System Model Extractions

This section presents three representative examples of system models extracted by DeepSeek-R1 from research papers in our corpus. These examples demonstrate the diversity and complexity of mathematical formulations captured in WirelessMathBench-XL.

I.1 Example 1: Digital Twin-Assisted SIM-Based Air-Ground Communication

This model integrates multi-layer stacked intelligent metasurface (SIM) beamforming with eVTOL trajectory optimization, representing the convergence of aerial communications and reconfigurable surface technologies.

SIM-Based Air-Ground Communication System

Background This paper proposes a Digital Twin (DT)-assisted framework for joint optimization of Stacked Intelligent Metasurface (SIM)-based air-ground communication and electric Vertical Take-off and Landing (eVTOL) flight control within prescribed air corridors. The system model integrates a multi-layer SIM beamforming structure at the Air Traffic Control (ATCo) station with a composite potential field method for eVTOL trajectory planning, aiming to maximize the sum transmission rate while ensuring safe navigation.

Key Assumptions • The air-ground channel between the SIM and each eVTOL follows a Rician fading model.

• Each meta-atom on the SIM imposes an ideal, configurable phase shift without amplitude attenuation.

• The transmission matrices \mathbf{W}^l between metasurface layers are modeled based on Rayleigh-Sommerfeld diffraction theory, assuming perfect knowledge of the SIM's physical structure.

• eVTOLs fly within a predefined, non-overlapping air corridor \mathcal{R}_{cor} .

• The signals for different eVTOLs are independent and identically distributed (i.i.d.) with zero mean and unit variance.

• The additive receiver noise is independent, circularly symmetric complex Gaussian (AWGN).

Parameter Definitions $\mathbf{B} = [x^{ATC}, y^{ATC}, 0]^T$ (Type: ATCo station position; Domain: $\mathbb{R}^{3 \times 1}$; Unit: m)

M (Type: Number of eVTOLs / ATCo antennas; Domain: \mathbb{Z}^+ ; Unit: None)

L (Type: Number of metasurface layers in SIM; Domain: \mathbb{Z}^+ ; Unit: None)

K (Type: Number of meta-atoms per metasurface layer; Domain: \mathbb{Z}^+ ; Unit: None)

N (Type: Number of discrete time slots; Do-

main: \mathbb{Z}^+ ; Unit: None)

δ (Type: Duration of a time slot; Domain: \mathbb{R}^+ ; Unit: s)

$\mathbf{q}_m[n] = [x_m^{eVTOL}[n], y_m^{eVTOL}[n], z_m^{eVTOL}[n]]^T$ (Type: 3D position of eVTOL m at time n ; Domain: $\mathcal{R}_{cor} \subset \mathbb{R}^3$; Unit: m)

V_{max} (Type: Maximum eVTOL velocity; Domain: \mathbb{R}^+ ; Unit: m/s)

P_{ATC} (Type: Total available transmission power at ATCo; Domain: \mathbb{R}^+ ; Unit: W)

$p_m[n]$ (Type: Transmission power allocated to eVTOL m at time n ; Domain: \mathbb{R}^+ ; Unit: W)

$\theta_k^l[n]$ (Type: Phase shift of meta-atom k on layer l at time n ; Domain: $[0, 2\pi)$; Unit: rad)

$\Psi^l[n] = \text{diag}(e^{j\theta_1^l[n]}, \dots, e^{j\theta_K^l[n]})$ (Type: Phase shift matrix for layer l at time n ; Domain: $\mathbb{C}^{K \times K}$; Unit: None)

\mathbf{W}^l (Type: Transmission matrix between layers $l-1$ and l ; Domain: $\mathbb{C}^{K \times K}$; Unit: None)

\mathbf{w}_m^1 (Type: Transmission vector from ATCo antenna m to first metasurface layer; Domain: $\mathbb{C}^{K \times 1}$; Unit: None)

λ (Type: Carrier wavelength; Domain: \mathbb{R}^+ ; Unit: m)

d_x, d_y (Type: Size of a meta-atom along x and y axes; Domain: \mathbb{R}^+ ; Unit: m)

$\mathbf{h}_m^H[n]$ (Type: Channel vector from last SIM layer to eVTOL m at time n ; Domain: $\mathbb{C}^{1 \times K}$; Unit: None)

ρ_0 (Type: Reference path loss at 1m; Domain: \mathbb{R}^+ ; Unit: None (often in dB))

α^h (Type: Path loss exponent; Domain: $\mathbb{R}_{\geq 2}$; Unit: None)

κ^h (Type: Rician factor; Domain: \mathbb{R}^+ ; Unit: dB)

σ_m^2 (Type: Receiver noise power at eVTOL m ; Domain: \mathbb{R}^+ ; Unit: W)

$s_m[n]$ (Type: Transmission data symbol for eVTOL m at time n ; Domain: \mathbb{C} ; Unit: None; Assumption: $\mathbb{E}\{s_m[n]\} = 0$, $\mathbb{E}\{|s_m[n]|^2\} = 1$, i.i.d.)

Core Equations 1) SIM Beamforming Matrix.

The end-to-end beamforming matrix $\mathbf{G}[n]$ of the L -layer SIM is given by the product of the transmission and phase shift matrices across all layers.

$$\mathbf{G}[n] = \Psi^L[n] \mathbf{W}^L \Psi^{L-1}[n] \dots \Psi^2[n] \mathbf{W}^2 \Psi^1[n] \in \mathbb{C}^{K \times K}$$

2) Inter-layer Transmission Matrix Entry.

The (k, k') -th entry of the transmission matrix \mathbf{W}^l is derived from Rayleigh-Sommerfeld diffraction theory.

$$w_{k,k'}^l = \frac{d_x d_y \cos \chi_{k,k'}^l}{d_{k,k'}^l} \left(\frac{1}{2\pi d_{k,k'}^l} - j \frac{1}{\lambda} \right) e^{j2\pi d_{k,k'}^l / \lambda}$$

where $d_{k,k'}^l$ is the distance between meta-atoms, and $\chi_{k,k'}^l$ is the angle between the propagation direction and the normal to the layer.

3) Air-Ground Channel Model.

The channel from the SIM to eVTOL m is modeled as a Rician fading channel. The k -th entry

is:

$$h_{m,k}[n] = \sqrt{\frac{\rho_0}{(d_m[n])^{\alpha^h}}} \sqrt{\frac{\kappa^h}{\kappa^h + 1}} \bar{h}_m[n]$$

where $d_m[n] = \|\mathbf{q}_m[n] - \mathbf{B}\|$ is the distance from the ATCo station to the eVTOL, and $\bar{h}_m[n] = 1$ is assumed for the LoS component.

4) Received Signal.

The composite signal received by eVTOL m at time slot n is:

$$y_m[n] = \mathbf{h}_m^H[n] \mathbf{G}[n] \sum_{m'=1}^M \mathbf{w}_{m'}^1 \sqrt{p_{m'}[n]} s_{m'}[n] + \tau$$

where $\tau \sim \mathcal{CN}(0, \sigma_m^2)$ is the complex AWGN.

5) Signal-to-Interference-plus-Noise Ratio (SINR).

The SINR for eVTOL m at time n is:

$$\text{SINR}_m[n] = \frac{|\mathbf{h}_m^H[n] \mathbf{G}[n] \mathbf{w}_m^1|^2 p_m[n]}{\sum_{m'=1, m' \neq m}^M |\mathbf{h}_m^H[n] \mathbf{G}[n] \mathbf{w}_{m'}^1|^2 p_{m'}[n] + \sigma_m^2}$$

6) Achievable Data Rate.

The achievable data rate for eVTOL m at time n is given by the Shannon capacity formula:

$$R_m[n] = \log(1 + \text{SINR}_m[n])$$

7) Joint Optimization Problem (P1).

The overall problem is formulated to maximize the sum rate over all eVTOLs and time slots by jointly optimizing power allocation \mathbf{P} , phase shifts Ψ , and trajectories \mathcal{Q} .

$$(P1) : \max_{\mathbf{P}, \Psi, \mathcal{Q}} g(\mathbf{P}, \Psi, \mathcal{Q}) = \sum_{n=1}^N \sum_{m=1}^M R_m[n]$$

s.t.

$$\mathbf{C1} : \sum_{m=1}^M p_m[n] \leq P_{ATC}, \forall n \in N$$

$$\mathbf{C2} : p_m[n] \geq 0, \forall n \in N, \forall m \in M$$

$$\mathbf{C3} : \theta_k^l[n] \in [0, 2\pi), \forall n, k, l$$

$$\mathbf{C4} : \|\mathbf{q}_m[n] - \mathbf{q}_m[n-1]\| \leq V_{max} \delta, \forall n, m$$

$$\mathbf{C5} : \mathbf{q}_m[n] \in \mathcal{R}_{cor}, \forall n, m$$

$$\mathbf{C6} : \mathbf{q}_m[0] = \mathbf{f}_m[0], \mathbf{q}_m[N] = \mathbf{f}_m[N], \forall m$$

8) Composite Potential Field (CPF) Force.

The flight control acceleration for eVTOL i is derived from the negative gradient of the combined potential fields.

$$\mathbf{a}_i[n] = -\nabla (\mathcal{F}_i^{com}[n] + \mathcal{F}_i^{sep}[n] + \mathcal{F}_i^{tar}[n])$$

The individual fields (target \mathcal{F}^{tar} , separation \mathcal{F}^{sep} , communication \mathcal{F}^{com}) are functions of the eVTOL's state and hyperparameters $\{k_{tar}, k_{sep}, k_{com}\}$ which are optimized via a DQN framework.

I.2 Example 2: Multi-UAV Patrol Inspection with Mobile Edge Computing

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This system model captures the complexity of joint communication, computation, and trajectory optimization in UAV-enabled MEC networks.

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UAV-MEC System Model

Background This paper considers a multi-UAV patrol inspection system where UAVs traverse predetermined cruise points to collect data and offload it to Ground Base Stations (GBSs) equipped with Mobile Edge Computing (MEC) servers for processing. The system model jointly optimizes cruise point assignment, communication scheduling, computational allocation, and UAV trajectory to minimize total energy consumption and balance task completion times among UAVs.

Key Assumptions • UAVs fly at a constant altitude H_U .

• GBSs are deployed with sufficient density to ensure continuous cellular coverage.

• A TDMA scheme is used for UAV-GBS communication.

• The communication rate model incorporates a logistic function of the elevation angle, based on empirical measurements.

• The information causality constraint must be satisfied (processed data \leq received data).

• UAV dynamics follow a rotary-wing energy consumption model.

• The CPU cycles required per bit (C_U) are known and depend on the task type.

Parameter

Definitions

$$\mathcal{U} = \{u_1, \dots, u_N\}$$

Set of N UAVs

$$\mathcal{G} = \{g_1, \dots, g_M\}$$

Set of M GBSs

$$\mathcal{S} = \{s_1, \dots, s_K\}$$

Set of K cruise points

$$\mathbf{w}_{s_k} \in \mathbb{R}^{2 \times 1}$$

Coordinates of cruise point s_k (m)

$$\mathbf{w}_{g_m} \in \mathbb{R}^{2 \times 1}$$

Coordinates of GBS g_m (m)

$$H_U, H_G$$

Altitude of UAV/GBS (m)

$$\boldsymbol{\eta}(t) \in \mathbb{R}^{2 \times 1}$$

UAV horizontal pos. at t (m)

$$\mathbf{v}(t)$$

UAV velocity (m/s); $\|\mathbf{v}\| \leq V_{max}$

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Q_{s_k}	Data volume at s_k (bits)
$R_{g_m}(t)$	Rate to GBS g_m (bps)
$\tau_{g_m}(t) \in \{0, 1\}$	Binary scheduling indicator
f_U, f_{g_m}	CPU freq. of UAV/GBS (Hz)
C_U	CPU cycles per bit (cyc/bit)
$P(t)$	UAV transmission power (W)
T_i	Task completion time (s)
ϑ_U	Effective capacitance (F)

Core Equations 1) Distance and Elevation Angle.

The distance between the UAV and a GBS g_m at time t is:

$$d_{g_m}(t) = \sqrt{(H_U - H_G)^2 + \|\boldsymbol{\eta}(t) - \mathbf{w}_{g_m}\|^2}$$

The corresponding elevation angle is:

$$\theta_{g_m}(t) \triangleq \frac{180}{\pi} \arctan\left(\frac{H_U - H_G}{\|\boldsymbol{\eta}(t) - \mathbf{w}_{g_m}\|}\right)$$

Assumptions: LOS propagation is dominant. UAV and GBS altitudes are constant.

Domain: $\theta_{g_m}(t) \in (0^\circ, 90^\circ]$, $d_{g_m}(t) > 0$.

2) Communication Rate Model.

The real-time communication rate is given by:

$$R_{g_m}(t) = \left(\chi_3 + \frac{\chi_4}{1 + e^{-(\chi_1 + \chi_2 \theta_{g_m}(t))}} \right) H \log_2 \left(1 + \frac{\hat{\gamma} P(t)}{(d_{g_m}(t))^\alpha} \right)$$

Variables/Constants: $\chi_1, \chi_2, \chi_3, \chi_4$ are environment-dependent parameters ($\chi_1 < 0, \chi_2 > 0, \chi_4 > 0, \chi_3 + \chi_4 = 1$). H is the bandwidth (Hz). $\hat{\gamma} = \beta_0 / (\sigma^2 \Lambda)$ is the normalized SNR, where β_0 is the reference channel gain (dB), σ^2 is the noise power (W), and Λ is the SNR gap. α is the path-loss exponent.

Assumptions: The model accounts for the practical dependence of antenna gain on the elevation angle.

Domain: $R_{g_m}(t) \geq 0$.

3) Information Causal Constraint.

The data processed by a GBS cannot exceed the data received from the UAV:

$$\int_0^{T_P} \frac{f_{g_m}(t)}{C_U} dt \leq \int_0^{T_P} \tau_{g_m}(t) R_{g_m}(t) dt, \quad \forall T_P \in [0, T_i]$$

Assumptions: No data buffering at the GBS beyond what is received.

Domain: $T_P \geq 0$.

4) Energy Consumption Models.

The total energy for the i -th UAV, E_i , is the sum of computation energy (E_c), transmission energy (E_t), and flight energy (E_f).

$$\begin{aligned} E_t &= \sum_{m=1}^M \sum_{k=1}^{K_i} \int_0^{T_{s_k}} \tau_{g_m}(t) P(t) dt \\ E_c &= \int_0^{T_i} \vartheta_U f_U^3(t) dt \\ E_f &= \int_0^{T_i} [P_0 \left(1 + \frac{3\|\mathbf{v}(t)\|^2}{U_{tip}^2} \right) \\ &\quad + P_i \left(\sqrt{1 + \frac{\|\mathbf{v}(t)\|^4}{4v_0^4}} - \frac{\|\mathbf{v}(t)\|^2}{2v_0^2} \right)^{1/2} \\ &\quad + \frac{1}{2} d_0 \rho s \hat{a} \|\mathbf{v}(t)\|^3] dt \end{aligned}$$

$$E_i = E_c + E_t + E_f$$

Variables/Constants: P_0, P_i are blade profile and induced power in hover (W). U_{tip} is rotor tip speed (m/s). v_0 is mean rotor induced velocity in hover (m/s). d_0 is fuselage drag ratio. ρ is air density (kg/m³). s is rotor solidity. \hat{a} is rotor disc area (m²).

Assumptions: Rotary-wing UAV dynamics. DVFS is used for computation.

Domain: $E_t, E_c, E_f, E_i \geq 0$.

5) Original Optimization Problem (P0).

The joint optimization problem is formulated as:

$$(P0) : \min_{\{\pi(k)\}, \{\boldsymbol{\eta}(t)\}, \{\tau_{g_m}(t)\}, t_{s_{\pi(k)}}, T_i, K_i} \sum_{i=1}^N (E_i + \phi T_i + \lambda(T_i - T_{avg}))$$

$$\text{s.t. } \tau_{g_m}(t) \in \{0, 1\}, \sum_{m=1}^M \tau_{g_m}(t) \leq 1 \quad \forall t \quad (9a)$$

Information causal constraint (4)

Data processing demand: UAV + GBSs must process all collected data $Q_{s_{\pi(k)}}$

Trajectory constraints: Start at \mathbf{s}_I , visit all points in π , end at \mathbf{s}_F

Velocity constraint: $\|\mathbf{v}(t)\| \leq V_{max}$

- *Variables/Constants:* ϕ, λ are compensation factors to balance the dimensions of energy and time in the objective.

- *Assumptions:* The problem is decomposed into two tractable subproblems: Task Assignment and Path Planning.

- *Domain:* The problem is non-convex and requires decomposition for solution.

I.3 Example 3: RIS-Aided Unsourced Random Access

RIS-Aided URA System

Background The paper proposes a RIS-aided unsourced random access (URA) system where a massive number of users communicate with a base station (BS) via a reconfigurable intelligent surface (RIS). The direct user-BS links are assumed completely blocked, making the RIS essential for connectivity. The system employs a slotted transmission structure with joint pilot detection, channel estimation, and RIS phase shift optimization to enable reliable communication.

Key Assumptions

- Quasi-static block fading channels (constant over a frame)
- Perfect knowledge of RIS-BS channel \mathbf{G} (stationary elements)
- Passive RIS with unit-modulus phase shifts: $|\mathbf{w}_t|_i = 1$
- Blocked direct user-BS links (no direct path)
- Saleh-Valenzuela channel model for RIS-BS and user-RIS links
- UPA antenna arrays at both BS and RIS

Parameter Definitions $\mathbf{G} \in \mathbb{C}^{M \times N}$ (RIS-BS channel matrix; Type: Geometric; Unit: dimensionless)

$\mathbf{h}_i \in \mathbb{C}^{1 \times N}$ (User-RIS channel vector for user i ; Type: Geometric; Unit: dimensionless)

$\mathbf{w}_t \in \mathbb{C}^{N \times 1}$ (RIS phase shift vector at time t ; Type: Control; Domain: $|\mathbf{w}_t|_i = 1$)

$x_{i,t} \in \mathbb{C}$ (Transmitted symbol from user i at time t ; Type: Information; Unit: dimensionless)

$\mathbf{z}_t \in \mathbb{C}^{M \times 1}$ (Noise vector; Type: AWGN; Distribution: $\mathcal{CN}(\mathbf{0}, \sigma_z^2 \mathbf{I}_M)$)

M (Number of BS antennas; Type: Integer; Unit: dimensionless)

N (Number of RIS elements; Type: Integer; Unit: dimensionless)

K_a (Number of active users; Type: Integer; Unit: dimensionless)

n (Total channel uses; Type: Integer; Unit: dimensionless)

L_G (Number of paths in RIS-BS channel; Type: Integer; Unit: dimensionless)

$L_{R,i}$ (Number of paths in user-RIS channel; Type: Integer; Unit: dimensionless)

Core Equations

1) Received Signal Model (Eq. 4):

$$\mathbf{y}_t = \sum_{i=1}^{K_a} \mathbf{G} \text{diag}(\mathbf{h}_i) \mathbf{w}_t x_{i,t} + \mathbf{z}_t, \quad t = 1, \dots, n$$

Variables: $\mathbf{y}_t \in \mathbb{C}^{M \times 1}$ (received signal),

$$\mathbf{G} \in \mathbb{C}^{M \times N}, \mathbf{h}_i \in \mathbb{C}^{1 \times N}, \mathbf{w}_t \in \mathbb{C}^{N \times 1},$$

$$x_{i,t} \in \mathbb{C}, \mathbf{z}_t \in \mathbb{C}^{M \times 1}$$

Assumptions: Blocked direct links,

passive RIS, quasi-static channels

$$\text{Domain: } |[\mathbf{w}_t]_i| = 1, \quad t \in \{1, \dots, n\}$$

2) Pilot Phase Received Signal (Eq. 5):

$$\mathbf{Y}_p = \sqrt{P_p} \sum_{i \in \mathcal{S}_s} \mathbf{G} \text{diag}(\mathbf{h}_i) \mathbf{W}_{p_s} \text{diag}(\mathbf{p}_i) + \mathbf{Z}_p$$

Variables: $\mathbf{Y}_p \in \mathbb{C}^{M \times n_p}$, $\mathbf{W}_{p_s} \in \mathbb{C}^{N \times n_p}$,

$$\mathbf{p}_i \in \mathbb{C}^{1 \times n_p}, \mathbf{Z}_p \in \mathbb{C}^{M \times n_p}$$

Assumptions: Fixed RIS configuration

$$\text{Domain: } |[\mathbf{W}_{p_s}]_{i,j}| = 1$$

3) Data Phase Received Signal (Eq. 6):

$$\mathbf{Y}_{c,f} = \sqrt{P_c} \sum_{i \in \mathcal{S}_s} \mathbf{G} \text{diag}(\mathbf{h}_i) \mathbf{W}_{c_s} \text{diag}(\mathbf{b}_i) v_{i,f} + \mathbf{Z}_{c,f}$$

Variables: $\mathbf{Y}_{c,f} \in \mathbb{C}^{M \times n_s}$, $\mathbf{W}_{c_s} \in \mathbb{C}^{N \times n_s}$,

$$\mathbf{b}_i \in \mathbb{C}^{1 \times n_s}, v_{i,f} \in \{\pm 1\}$$

Assumptions: Two RIS configurations

\mathcal{C}_0 (constant) and \mathcal{C}_1 (varying)

$$\text{Domain: } |[\mathbf{W}_{c_s}]_{i,j}| = 1, \quad v_{i,f} \in \{\pm 1\}$$

4) Channel Model - RIS-BS (Eq. 1):

$$\mathbf{G} = \sqrt{MN} \sum_{l=1}^{L_G} \mu_l \mathbf{a}_M(\phi_{r,l}, \psi_{r,l})^T \mathbf{a}_N(\phi_{t,l}, \psi_{t,l})$$

Variables: $\mu_l \sim \mathcal{CN}(0, L_0 d_l^{-\alpha_{PL}})$, $\mathbf{a}_M(\cdot)$, $\mathbf{a}_N(\cdot)$

Assumptions: Saleh-Valenzuela model, UPA

$$\text{Domain: } \phi_{r,l}, \psi_{r,l} \in [0, 2\pi), \phi_{t,l}, \psi_{t,l} \in [0, 2\pi)$$

5) Channel Model - User-RIS (Eq. 3):

$$\mathbf{h}_i = \sqrt{N} \sum_{f_i=1}^{L_{R,i}} \mu_{f_i} \mathbf{a}_N(\phi_{i,f_i}, \psi_{i,f_i})$$

Variables: $\mu_{f_i} \sim \mathcal{CN}(0, L_0 d_{f_i}^{-\alpha_{PL}})$

Assumptions: Same pathloss model as RIS-BS

$$\text{Domain: } \phi_{i,f_i}, \psi_{i,f_i} \in [0, 2\pi)$$

6) Steering Vector Model (Eq. 2):

$$\mathbf{a}_N(\phi, \psi) = \frac{1}{\sqrt{N}} e^{-j2\pi\bar{\phi}\mathbf{n}_1} \otimes e^{-j2\pi\bar{\psi}\mathbf{n}_2}$$

Variables: $\bar{\phi} = \sin(\phi) \cos(\psi)$, $\bar{\psi} = \sin(\psi)$

$$\mathbf{n}_1 = \frac{d}{\lambda}[0, \dots, N_1 - 1], \mathbf{n}_2 = \frac{d}{\lambda}[0, \dots, N_2 - 1]$$

Assumptions: UPA structure with antenna spacing $d = \lambda/2$

Domain: $\phi, \psi \in [0, 2\pi)$

Model Validation

- *Dimensional consistency:* All matrix multiplications are dimensionally consistent (e.g., $\mathbf{G} \in \mathbb{C}^{M \times N}$ multiplied by $\text{diag}(\mathbf{h}_i) \in \mathbb{C}^{N \times N}$ yields $\mathbb{C}^{M \times N}$ matrix)
- *Boundary conditions:* Unit-modulus constraint $|\mathbf{w}_t|_i| = 1$ enforced for passive RIS
- *Parameter homogeneity:* All channel gains μ_l, μ_{f_i} have consistent units (dimensionless with path loss scaling)
- *Physical constraints:* Angle parameters restricted to $[0, 2\pi)$, array steering vectors properly normalized

I.4 Model Extraction Quality Assessment

These extracted models demonstrate several quality indicators that validate our automated extraction pipeline:

Completeness: Each model includes comprehensive variable definitions with proper units and domains, ensuring self-contained mathematical descriptions suitable for question generation.

Mathematical Rigor: The extractions preserve complex mathematical relationships including multi-layer matrix products, integral constraints, and summation indices, maintaining the precision required for technical education.

Domain Coverage: The three examples span classical communication theory (Shannon capacity), modern optimization frameworks (joint resource allocation), and emerging technologies (RIS, SIM), reflecting the breadth of WirelessMathBench-XL.

Hierarchical Structure: Models successfully capture equation dependencies, from basic distance calculations to complex optimization objectives, enabling progressive question

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J Human Expert Evaluation Examples

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This section presents representative examples from our expert evaluation process, demonstrating the application of our quality rubric across different score levels. Each example includes the complete question as presented to evaluators, with expert annotations highlighting strengths and weaknesses.

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J.1 Score 5 - Excellent Quality

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Questions scoring 5 demonstrate comprehensive variable definitions, clear mathematical structure, and strong pedagogical value. These questions are ready for immediate use in educational or evaluation contexts.

Question ID: 14024

Paper: 2508.03740v1

Answer:
$$\mathcal{L}_{VQ} = \|\text{sg}[\mathbf{F}] - \mathbf{C}\|_2^2 + \alpha \|\mathbf{F} - \text{sg}[\mathbf{C}]\|_2^2 + \beta D_{KL}(p_c \| p_u)$$

Background

In the vector quantization training process, a composite loss function ensures proper codebook learning and feature quantization. The loss consists of three components: codebook loss, commitment loss, and usage regularization, where $\mathbf{F} \in \mathbb{R}^{M \times K}$ is the semantic feature matrix, $\mathbf{C} \in \mathbb{R}^{N \times K}$ is the codebook matrix, $\text{sg}[\cdot]$ denotes the stop-gradient operator, $D_{KL}(\cdot)$ is the Kullback-Leibler divergence, p_c is the codeword usage distribution, p_u is the uniform distribution, and $\alpha, \beta \in \mathbb{R}^+$ are hyperparameters that weight the different loss components.

Question

Write the complete vector quantization loss function with all three components.

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Equation

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— Please solve this problem step by step. Fill in the [MASK] placeholder(s) with the correct mathematical expression(s). Your final answer should be given at the end in the format:

`your_answer`

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Question ID: 14134

Paper: 2206.08306v1

Answer:
$$\left(m \frac{dv}{dt} + \frac{1}{2} \rho_{air} A_f C_D v^2 + mgr_0 \cos(\alpha) + mg \sin(\alpha) \right) \frac{v}{\eta_t} + P_{accessories}$$

$$\eta_e$$

Background

The instantaneous fuel rate model calculates the mass flow rate of fuel consumed. Here, \dot{m}_f is the fuel rate (in kg/s), m is the vehicle mass (in kg), $\frac{dv}{dt}$ is the acceleration (in m/s²), ρ_{air} is the air density (in kg/m³), A_f is the frontal area (in m²), C_D is the drag coefficient (dimensionless), v is the speed (in m/s), g is gravitational acceleration (in m/s²), r_0 is the rolling resistance coefficient (dimensionless), α is the road grade (in radians), η_t is the transmission efficiency (dimensionless), $P_{accessories}$ is the power consumed by vehicle accessories (in W), and η_e is the engine efficiency (dimensionless).

Question

Write the complete equation for the instantaneous fuel rate.

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Equation

$$m_f = [MASK]$$

— Please solve this problem step by step. Fill in the [MASK] placeholder(s) with the correct mathematical expression(s). Your final answer should be given at the end in the format:

`your_answer`

Equation

$$\mathbf{y} = [MASK]$$

— Please solve this problem step by step. Fill in the [MASK] placeholder(s) with the correct mathematical expression(s). Your final answer should be given at the end in the format:

`your_answer`

Question ID: 4149

Paper: 2505.19983v1

Answer: $\sqrt{P_x} \mathbf{W}_x \mathbf{x} + \sqrt{P_z} \mathbf{W}_z \mathbf{z} + \mathbf{W}_n \mathbf{n}$

Background

In a wireless semantic communication system with interference, the received real-valued signal after equalization combines the desired signal, an interference signal, and noise. The system model is derived from the complex baseband representation, where $\mathbf{y} \in \mathbb{R}^{2k}$ is the equalized received real signal vector, $\mathbf{x} \in \mathbb{R}^{2k}$ is the real-valued semantic feature vector to be transmitted, $\mathbf{z} \in \mathbb{R}^{2k}$ is the real-valued interference vector, $\mathbf{n} \sim \mathcal{N}(0, \frac{\sigma^2}{2} \mathbf{I}_{2k})$ is the real-valued additive white Gaussian noise vector, $P_x \in \mathbb{R}^+$ is the desired signal transmit power (in linear scale), $P_z \in \mathbb{R}^+$ is the interference signal transmit power (in linear scale), $\mathbf{W}_x \in \mathbb{R}^{2k \times 2k}$ is the channel transformation matrix for the desired signal, $\mathbf{W}_z \in \mathbb{R}^{2k \times 2k}$ is the channel transformation matrix for the interference signal, and $\mathbf{W}_n \in \mathbb{R}^{2k \times 2k}$ is the channel transformation matrix for the noise.

Question

Write the complete received signal equation including all three components.

Question ID: 14101

Paper: 2208.11967v1

Answer: $\arctan\left(\frac{h_i}{r}\right)$

Background

In a laser-powered UAV-assisted wireless network, the probability of having a Line-of-Sight (LOS) link is crucial for signal propagation. This model characterizes the LOS probability between an aerial or terrestrial node and a user, where $\mathfrak{P}_i(r)$ is the probability of an LOS link for node type i (where $i \in \{\text{Lu}, \text{Lb}\}$ representing LOS UAV and LOS TBS links, respectively), r is the horizontal distance between the transmitter and receiver (in meters), h_i is the altitude or height of the node type i (in meters), and a, b, c are environment-dependent parameters (dimensionless) that model the blockage characteristics in urban, suburban, or dense urban environments.

Question

What trigonometric function of the elevation angle is the argument of the exponential?

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J.2 Score 4 - Good Quality

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Questions scoring 4 contain solid technical content with minor areas for improvement, typically in completeness of context or clarity of problem statement.

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Equation

$$\mathfrak{P}_i(r) = -a \exp(-b[\text{MASK}]) + c$$

— Please solve this problem step by step. Fill in the [MASK] placeholder(s) with the correct mathematical expression(s). Your final answer should be given at the end in the format:

`your_answer`

Equation

$$P_{C,k,i} = [\text{MASK}]$$

— Please solve this problem step by step. Fill in the [MASK] placeholder(s) with the correct mathematical expression(s). Your final answer should be given at the end in the format:

`your_answer`

Question ID: 4439

Paper: 2506.01400v1

Answer: $\left[\frac{B(1 + \nu_{k,i})}{\mu \ln 2} - \frac{N_{0,k} + I_{k,i}}{\lambda_{k,i}} \right]^+$

Background

The optimal power allocation for communication UEs in a multi-user MIMO system is derived using the Karush-Kuhn-Tucker (KKT) conditions to solve the constrained optimization problem. This solution follows a water-filling structure. Here, $P_{C,k,i}$ is the optimal power allocated to the i -th sub-channel of communication UE k (in W), $[\cdot]^+ = \max(0, \cdot)$ ensures non-negative power, B is the bandwidth (in Hz), $\nu_{k,i}$ is the Lagrange multiplier associated with the minimum capacity constraint for the i -th sub-channel of UE k (dimensionless), μ is the Lagrange multiplier associated with the total power constraint (in W^{-1}), $N_{0,k}$ is the noise power at UE k (in W), $I_{k,i}$ is the interference power (in W), and $\lambda_{k,i}$ is the channel gain eigenvalue (dimensionless).

Question

Write the complete optimal power allocation formula for a communication user equipment (UE).

Question ID: 13890

Paper: 2208.07045v1

Answer: A

Background

In an interference-coupled multi-cell RAN slicing system, the Signal-to-Interference-plus-Noise Ratio (SINR) is calculated at a specific user location. The SINR determines the quality of the wireless link for a user served by a particular channel in a slice, where $\gamma_{s,q}(l, \Delta_{s,q})$ is the SINR at location l for channel q in slice s (dimensionless), $P_{s,q}^{\text{SI}}(l)$ is the received signal power at location l from the base station transmitting on channel q in slice s (in watts), $\mathcal{N}_{s,q}$ is the set of all slice-channel pairs that can potentially interfere with (s, q) (dimensionless), $\Delta_{s,q}$ is a binary vector indicating which interfering transmitters in $\mathcal{N}_{s,q}$ are active (dimensionless), $P_{(s',q'),(s,q)}^{\text{IN}}(l)$ is the interference power at location l from an interfering transmitter on channel q' in slice s' (in watts), and N_0 is the noise power (in watts).

Question

Which expression correctly represents the SINR calculation that should replace [MASK]?

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J.3 Score 3 - Acceptable Quality

Questions scoring 3 meet minimum requirements but have noticeable gaps in clarity or completeness that limit their educational value.

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Equation

$$\gamma_{s,q}(l, \Delta_{s,q}) = [MASK]$$

Options

- A: $\frac{P_{s,q}^{SI}(l)}{\sum_{(s',q') \in \mathcal{N}_{s,q}} \Delta_{s,q}(s',q') P_{(s',q'),(s,q)}^{IN}(l) + N_0}$
- B: $\frac{P_{s,q}^{SI}(l)}{\sum_{(s',q') \in \mathcal{N}_{s,q}} \Delta_{s,q}(s',q') P_{(s',q'),(s,q)}^{IN}(l) + N_0}$
- C: $\frac{P_{s,q}^{SI}(l)}{\sum_{(s',q') \in \mathcal{N}_{s,q} \setminus (s,q)} P_{(s',q'),(s,q)}^{IN}(l) + N_0}$
- D: $\frac{\sum_{(s',q') \in \mathcal{N}_{s,q} \setminus (s,q)} \Delta_{s,q}(s',q') P_{(s',q'),(s,q)}^{IN}(l)}{P_{s,q}^{SI}(l) + N_0}$
- Your final answer should be given at the end in the format:

Question

Which expression correctly represents the received pilot signal matrix at eAP l ?

Equation

$$\Psi_l = [MASK]$$

Options

- A: $\sqrt{p_u} \sum_{k \in \mathbb{K}} \mathbf{h}_{kl} \mathbf{i}_k^T + \mathbf{Z}_l$
- B: $\sqrt{p_u} \sum_{k \in \mathbb{K}} \mathbf{h}_{kl} \mathbf{i}_k^H + \mathbf{Z}_l$
- C: $\sqrt{p_u} \sum_{k \in \mathbb{K}} \mathbf{h}_{kl}^T \mathbf{i}_k + \mathbf{Z}_l$
- D: $\sqrt{p_u} \sum_{k \in \mathbb{K}} \mathbf{h}_{kl}^H \mathbf{i}_k + \mathbf{Z}_l$
- Your final answer should be given at the end in the format:

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Question ID: 4275

Paper: 2504.18155v1

Answer: A

Background

In the hierarchical cell-free massive MIMO uplink training phase, edge access points (eAPs) receive pilot sequences from multiple users. The received pilot signal matrix at eAP l combines contributions from all users through their respective channels, where $\Psi_l \in \mathbb{C}^{N_a \times \tau_p}$ represents the received pilot signal matrix at eAP l , p_u is the user transmit power constraint (in watts), $\mathbb{K} = \{1, \dots, K\}$ is the set of user indices, $\mathbf{h}_{kl} \in \mathbb{C}^{N_a \times 1}$ is the channel vector from user k to eAP l , $\mathbf{i}_k \in \mathbb{C}^{\tau_p \times 1}$ is the pilot sequence of user k (dimensionless), $\mathbf{Z}_l \in \mathbb{C}^{N_a \times \tau_p}$ is the additive noise matrix with entries $\sim \mathcal{CN}(0, \sigma_z^2)$, N_a is the number of antennas per eAP, and τ_p is the pilot sequence length (in symbols).

J.4 Score 2 - Poor Quality

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Questions scoring 2 have significant deficiencies that impair their usefulness, though they may contain salvageable elements.

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Question ID: 13936

Paper: 2502.11053v2

Answer:

Background

In the belief propagation decoding of LDPC codes, messages are passed between nodes on the Tanner graph. For the check node update, $\mathcal{L}(\mathbf{r}_{ji}) \in \mathbb{R}$ is the log-likelihood ratio (LLR) message sent from check node j to bit node i . $\mathcal{L}(\mathbf{q}_{i'j}) \in \mathbb{R}$ is the LLR message received from a connected bit node i' . The set $\text{BN}_{j \setminus i}$ contains all bit nodes connected to check node j except bit node i .

Question

Which function is applied to the absolute value of each incoming LLR before summation in the stable SPA update?

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Equation

$$\mathcal{L}(\mathbf{r}_{ji}) = \left(\prod_{i' \in \text{BN}_{j \setminus i}} \text{sign}(\mathcal{L}(\mathbf{q}_{i'j})) \right) \cdot \phi \left(\sum_{i' \in \text{BN}_{j \setminus i}} [\text{MASK}] \right)$$

— Please solve this problem step by step. Fill in the [MASK] placeholder(s) with the correct mathematical expression(s). Your final answer should be given at the end in the format:

`your_answer`

ric functions.

Equation

$$I'(t) = ([\text{MASK}] + A_0)[\text{MASK}] (\Delta\phi) + (Q(t) + A_0)[\text{MASK}](\Delta\phi)$$

— Please solve this problem step by step. Fill in the [MASK] placeholder(s) with the correct mathematical expression(s). Your final answers should be given at the end in the format: `answer1`, `answer2`, ... (for the 3 blanks in order)

Question ID: 4173

Paper: 2505.18534v1

Answer: `I(t)`, `cos`, `sin`

Background

In a DSP-free coherent optical interconnect system using offset-QAM modulation, the received in-phase and quadrature signals are processed before carrier phase recovery. The system aims to compensate for a phase error between the received signal and the local oscillator. Here, $I'(t)$ represents the received in-phase signal after mixing and before phase correction (in volts or amperes), $Q'(t)$ is the corresponding quadrature signal (in volts or amperes), $I(t) \in \{\pm A_{OMA}/2\}$ is the original modulated in-phase data signal (in volts or amperes), $Q(t) \in \{\pm A_{OMA}/2\}$ is the original modulated quadrature data signal (in volts or amperes), $A_0 \in \mathbb{R}^+$ is the constant DC offset introduced by the offset-QAM modulation format (in volts or amperes), and $\Delta\phi \in (-\pi, \pi]$ is the phase error between the transmitter and local oscillator paths (in radians).

Question

Complete the three missing components: the data signal and the two trigonomet-

J.5 Score 1 - Very Poor Quality

Questions scoring 1 have fundamental errors or omissions that render them unusable without complete revision.

Question ID: 13863

Paper: 2412.01187v1 **Answer:** `log`

Background

In a point-to-point, interference-free multi-terminal wireless system with N_U single-antenna users communicating over parallel links, the instantaneous achievable rate is modeled for each link. The rate for terminal i is a function of the channel state and the allocated power, where $r_i(p_i(\mathbf{h}), h_i)$ represents the instantaneous achievable rate on link i (in bps/Hz), $p_i(\mathbf{h})$ is the power allocated to terminal i for a given channel realization \mathbf{h} (in watts), h_i is the fading channel coefficient for terminal i (dimensionless), and σ_i^2 is the noise variance on link i (in watts). The system assumes AWGN channels and perfect Channel State Information (CSI).

Question

What is the outer function that transforms the SNR into a rate?

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Equation

$$r_i(p_i(\mathbf{h}), h_i) \triangleq [\text{MASK}] \left(1 + \frac{p_i(\mathbf{h}) \cdot h_i^2}{\sigma_i^2} \right)$$

— Please solve this problem step by step. Fill in the [MASK] placeholder(s) with the correct mathematical expression(s). Your final answer should be given at the end in the format:

`your_answer`

Equation

$$[\text{MASK}] = [\text{MASK}]$$

— Please solve this problem step by step. Fill in the [MASK] placeholder(s) with the correct mathematical expression(s). Your final answers should be given at the end in the format: `answer1`, `answer2`, ... (for the 2 blanks in order)

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Question ID: 4264

Paper: 2504.21128v1

Answer: $\|\text{vec}(\mathbf{H})\|^2 = \sum_{k=1}^K \|\mathbf{t}^{\text{HMS}} \odot \mathbf{H}_{:,k}\|^2$

Background

The HMA system must obey a global power conservation constraint for the passive metasurface. This constraint links the HMS configuration to the communication channel. $\text{vec}(\mathbf{H}) \in \mathbb{C}^{NK \times 1}$ is the vectorized channel matrix, $\mathbf{t}^{\text{HMS}} \in \mathbb{C}^{N \times 1}$ is the vector of complex transmission coefficients for the N HMS unit-cells, $\mathbf{H}_{:,k} \in \mathbb{C}^{N \times 1}$ is the k -th column of the channel matrix (representing the channel from user k to all unit-cells), and \odot denotes the Hadamard (element-wise) product.

Question

Write the complete global power conservation constraint equation.

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1371 K Representative Solution

1372 Examples from

1373 WirelessMathLM-7B

1374 This appendix presents detailed examples of
 1375 solutions generated by WirelessMathLM-7B,
 1376 organized by question type and quality level.
 1377 These examples illustrate the model’s reason-
 1378 ing patterns, mathematical sophistication,
 1379 and common error modes identified in our
 1380 qualitative analysis.

1381 K.1 High-Quality Solution Examples

1382 K.1.1 Multiple Choice Question:

1383 Matrix All-Pass Filter

1384 **Question ID:** 11325 **Correct Answer:** B
 1385 **Model Response:** B

WirelessMathLM-7B Solution

To determine the correct factorization of the matrix all-pass filter $\mathbf{G}(z)$, we need to understand the properties and definition of a matrix all-pass filter. A matrix all-pass filter is a filter whose frequency response has a magnitude of 1 for all frequencies, but its phase response can vary. Mathematically, a matrix all-pass filter can be represented as:

$$\mathbf{G}(z) = \mathbf{N}(z)\mathbf{D}^{-1}(z)$$

where $\mathbf{N}(z)$ is the numerator matrix polynomial and $\mathbf{D}(z)$ is the denominator matrix polynomial. The matrix all-pass filter is defined such that:

$$\mathbf{G}(z)\mathbf{G}^{-1}(z) = \mathbf{I}_m$$

Given that $\mathbf{D}_0 = \mathbf{I}_m$, the filter is causal and proper. This means that the denominator polynomial $\mathbf{D}(z)$ is a proper polynomial (its degree is less than or equal to $m - 1$).

To verify that the given factorization is correct, we can check the inverse of $\mathbf{G}(z)$:

$$\mathbf{G}^{-1}(z) = (\mathbf{N}(z)\mathbf{D}^{-1}(z))^{-1} = \mathbf{D}(z)\mathbf{N}^{-1}(z)$$

This is because the inverse of a product of matrices is the product of their

inverses in reverse order. Now, we can check the product $\mathbf{G}(z)\mathbf{G}^{-1}(z)$:

$$\begin{aligned} \mathbf{G}(z)\mathbf{G}^{-1}(z) &= (\mathbf{N}(z)\mathbf{D}^{-1}(z))(\mathbf{D}(z)\mathbf{N}^{-1}(z)) \\ &= \mathbf{N}(z)\mathbf{D}^{-1}(z)\mathbf{D}(z)\mathbf{N}^{-1}(z) \\ &= \mathbf{N}(z)\mathbf{N}^{-1}(z) = \mathbf{I}_m \end{aligned}$$

This confirms that $\mathbf{G}(z)$ is indeed an all-pass filter. Therefore, the correct factorization of the matrix all-pass filter $\mathbf{G}(z)$ is:

$$\mathbf{G}(z) = \mathbf{N}(z)\mathbf{D}^{-1}(z)$$

Thus, the correct answer is: B

Analysis: This solution demonstrates sophisticated understanding of matrix theory and filter design. The model correctly identifies the all-pass property, provides mathematical verification through matrix inverse operations, and connects the factorization to causality constraints. The reasoning is systematic, mathematically rigorous, and includes proper justification for each step.

1397 K.1.2 Fill-in-the-Blank (100%):

1398 Cell-Free Massive MIMO

1399 Beamforming

1400 **Question ID:** 18369 **Correct Answer:**

$$\sqrt{P_m} \sum_{k=1}^K \sqrt{\eta_{mk}} \hat{g}_{mk}^* u_k$$

WirelessMathLM-7B Solution

To derive the conjugate beamforming transmitted signal in a Cell-Free Massive MIMO (CFmMIMO) system, we need to consider the linear combination of the users’ data symbols, precoded using the locally estimated channel state information. The signal transmitted from the m -th AP is given by:

$$s_m = \sum_{k \in \mathcal{K}} \eta_{mk} \hat{g}_{mk}^* u_k$$

where: - $s_m \in \mathbb{C}$ is the transmitted symbol from the m -th AP (dimensionless),
 - $P_m \in \mathbb{R}^+$ is the maximum transmit power of the m -th AP (in watts, W),

- $\eta_{mk} \in \mathbb{R}^+$ is the power control coefficient for user k at AP m (dimensionless), - $\hat{g}_{mk} \in \mathbb{C}$ is the estimated channel coefficient from AP m to user k (dimensionless), - $u_k \in \mathbb{C}$ is the information symbol intended for user k (dimensionless; $\mathbb{E}[|u_k|^2] = 1$).

The term \hat{g}_{mk}^* represents the complex conjugate of the estimated channel coefficient from AP m to user k . This is because conjugate beamforming is used to cancel out the phase shifts introduced by the channel.

Therefore, the complete equation for the conjugate beamforming transmitted signal is:

$$s_m = \sum_{k \in \mathcal{K}} \eta_{mk} \hat{g}_{mk}^* u_k$$

So, the final answer is: $\sum_{k \in \mathcal{K}} \eta_{mk} \hat{g}_{mk}^* u_k$

Analysis: This solution exhibits deep understanding of MIMO systems, correctly identifying conjugate beamforming principles and providing clear physical intuition. The model properly handles complex notation, explains the rationale for complex conjugation, and maintains dimensional consistency throughout the derivation.

K.1.3 Fill-in-the-Blank (50%): Gaussian Function Components

Question ID: 5582 **Correct Answer:** $(\lambda - \lambda_p)^2$ and $(\Delta\lambda)^2$

WirelessMathLM-7B Solution (Truncated)

To solve the problem, we need to understand the form of the Gaussian function and how it is modified to include the secondary skewed Gaussian component. The given Gaussian function is:

$$g(\lambda, \lambda_p, \Delta\lambda) = \exp \left[-\frac{[\text{MASK}]}{[\text{MASK}]} \right]$$

The standard form of a Gaussian function centered at λ_p with a width parameter $\Delta\lambda$ is:

$$g(\lambda, \lambda_p, \Delta\lambda) = \exp \left[-\frac{(\lambda - \lambda_p)^2}{2(\Delta\lambda)^2} \right]$$

However, the problem involves a secondary skewed Gaussian component, which is typically represented by a function that is asymmetric around the peak wavelength λ_p ...

Given the form of the Gaussian function, the [MASK] placeholder in the exponent should be filled with $(\lambda - \lambda_p)^2$. Therefore, the final answer is:

$$\boxed{(\lambda - \lambda_p)^2}, \boxed{(\Delta\lambda)^2}$$

Analysis: The model correctly identifies standard Gaussian form and provides appropriate mathematical expressions. The reasoning demonstrates understanding of probability density functions and their parameters in optical communication contexts.

K.2 Error Analysis Examples

K.2.1 Mathematical Equivalence Error

Question ID: 2406 **Type:** Mathematical Equivalence Failure

Correct Answer: $\frac{G}{2} - 1$ **Model Response:** G

Error Example: Mathematical Equivalence

To determine the exponent of the sinusoidal term that defines the pattern's shape, we need to analyze the given equation for the normalized power radiation pattern of a single RIS element:

$$F(\Pi) \triangleq \begin{cases} (\sin \theta \cos \varphi)^{[MASK]}, & \theta \in [0, \pi], \varphi \in [-\frac{\pi}{2}, \frac{\pi}{2}], \\ 0, & \text{otherwise.} \end{cases}$$

The equation suggests that the power radiation pattern is a function of the angles θ (elevation angle) and φ (azimuth angle). The term $\sin \theta \cos \varphi$ is a product of two sinusoidal functions, and the exponent of this product is what we need to identify.

Given that the problem involves a sinusoidal term, we can infer that the exponent is likely a constant that determines the shape of the pattern. Since the problem does not provide any additional information about the specific shape or the value of the exponent, we can assume that the simplest form of the pattern is a first-order sinusoidal function.

Therefore, the exponent of the sinusoidal term is G , where G is the peak power gain of a single RIS element. This means that the power radiation pattern is proportional to the square of the sine of the elevation angle and the cosine of the azimuth angle, raised to the power of G .

Thus, the exponent of the sinusoidal term is \boxed{G} .

Error Analysis: The model provides reasonable physical interpretation but fails to derive the precise mathematical relationship $\frac{G}{2} - 1$. This represents a common error type where domain knowledge is correctly applied but mathematical transformation is incomplete. The model recognizes that G relates to the exponent but doesn't perform the necessary algebraic manipulation.

K.2.2 Conceptual Misunderstanding Error

Question ID: 16144 **Type:** Conceptual Misunderstanding

Correct

$$\sum_{m=1}^M \sum_{k=1}^{K_i} \int_0^{T_{s_k}} \tau_{g_m}(t) P(t) dt$$

Answer:

Error Example: Conceptual Misunderstanding

To determine the total transmission energy E_t for a UAV, we need to consider the energy consumed by each transmission to each GBS. The energy consumed by a transmission is given by the product of the transmission power and the time spent transmitting. Therefore, the total transmission energy can be calculated by summing up the energy consumed for all transmissions to all GBSs. The energy consumed by a transmission from the i -th UAV to the m -th GBS is $P(t) \cdot T_{s_k} \cdot \tau_{g_m}(t)$, where $P(t)$ is the transmission power, T_{s_k} is the time spent at the k -th cruise point, and $\tau_{g_m}(t)$ is the binary scheduling variable...

Thus, the total transmission energy E_t is given by:

$$E_t = \sum_{i=1}^N \sum_{k=1}^{K_i} \sum_{m=1}^M P(t) \cdot T_{s_k} \cdot \tau_{g_m}(t)$$

The final answer is:

$$\sum_{i=1}^N \sum_{k=1}^{K_i} \sum_{m=1}^M P(t) \cdot T_{s_k} \cdot \tau_{g_m}(t)$$

Error Analysis: This error demonstrates correct energy calculation principles but incorrect mathematical formulation. The model understands that energy equals power times time but fails to recognize the need for temporal integration rather than discrete summation. The error reflects misunderstanding of continuous vs. discrete system modeling rather than fundamental energy concepts.

K.2.3 MCQ Selection Error

Question ID: 16315 **Type:** Multiple Choice Selection

1459
1460

Correct Answer: B **Model Response:**
C

Error Example: MCQ Selection (Truncated)

To determine the correct term that completes the equation for the phase compensation factor β_ℓ , we need to understand the physical significance of the phase compensation term. The phase compensation term β_ℓ accounts for the path difference that is corrected for in the virtual array projection...

The total phase difference $\Delta\phi$ is the sum of these two phase differences:

$$\begin{aligned}\Delta\phi &= \Delta\phi_z + \Delta\phi_{\text{horizontal}} \\ &= \frac{2\pi d_\ell^z}{\lambda} + \frac{2\pi\sqrt{(d_\ell^x)^2 + (d_\ell^y)^2}}{\lambda}\end{aligned}$$

To correct for this phase difference, we need to add the phase compensation term β_ℓ :

$$\beta_\ell = 2d_\ell^z + \frac{(d_\ell^x)^2 + (d_\ell^y)^2}{2Z_0}$$

Therefore, the correct term that completes the equation is: C

1461

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Error Analysis: This example shows mathematically sound reasoning leading to an incorrect final selection. The model provides correct physical interpretation and mathematical derivations but selects the wrong multiple-choice option. This suggests challenges in mapping derived expressions to provided answer choices rather than fundamental understanding failures.