A Peek into Token Bias: Large Language Models Are Not Yet Genuine Reasoners

Anonymous ACL submission

Abstract

This study introduces a hypothesis-testing framework to assess whether large language models (LLMs) possess genuine reasoning abilities or primarily depend on token bias. We go beyond evaluating LLMs on accuracy; rather, we aim to investigate their token bias in solving logical reasoning tasks. Specifically, we develop carefully controlled synthetic datasets, featuring conjunction fallacy and syllogistic problems. Our framework outlines a list of hypotheses where token biases are readily identifiable, with all null hypotheses assuming genuine reasoning capabilities of LLMs. The findings in this study suggest, with statistical guarantee, that most LLMs still struggle with logical reasoning. While they may perform well on classic problems, their success largely depends on recognizing superficial patterns with strong token bias, thereby raising concerns about their actual reasoning and generalization abilities.

1 Introduction

Large language models (LLMs) have achieved remarkable progress in understanding and generating human-like text, triggering growing interest in the LLMs' theory of minds (Kosinski, 2023; Jamali et al., 2023; Bubeck et al., 2023) and decision-making abilities (Lyu et al., 2023; Prasad et al., 2023). However, there is ongoing debate about whether LLMs possess genuine reasoning capabilities, as evidence suggests that the performance of LLMs on reasoning tasks is correlated with how much the input's semantic content supports a correct logical inference (Dasgupta et al., 2022; Li et al., 2023). Should valid reasoning be applied, such a correlation would not exist, since a genuine reasoner should be able to derive the correct inference regardless of the context.

In this paper, we formalize this observation and say that an LLM is subject to **token bias** in a reasoning task if systematic changes to some or all tokens in the task descriptions - while keeping the underlying logic intact - allow us to predict the direction of the shift in the model's output. A strong token bias suggests that the model is relying on superficial patterns in the input rather than truly understanding the underlying reasoning task. This could lead to brittle performance that fails to generalize well to novel examples and phrasings encountered in the wild that differ from the spurious patterns that the model has learned from the training data.

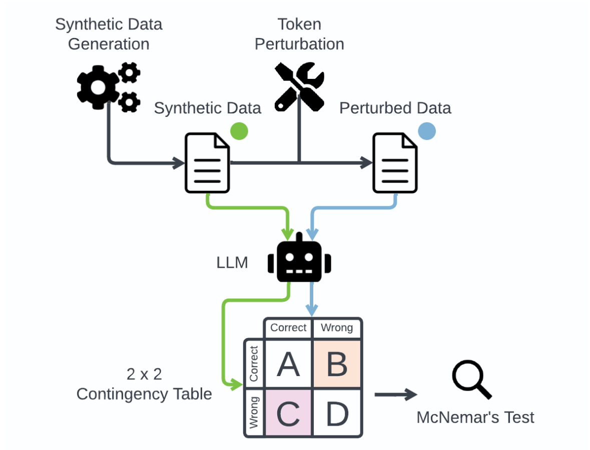


Figure 1: An illustration of the overall framework. We generate synthetic data, perform systematic token perturbations, and evaluate an LLM for comparative studies. The resulting contingency table, where A-D are integer values of counts, allows for subsequent statistical tests.

We explore several well-known logical fallacy problems from the cognitive science literature (Tversky and Kahneman, 1983; Kahneman, 2011), which provide a clear playground for assessing the reasoning capabilities of LLMs. Figure 2 depicts one kind of token bias found in our testing framework, where the model may be overfitting to specific tokens commonly found in classic problem statements. Since we observe many cases where state-of-the-art LLMs like GPT-4 (Achiam et al., 2023) successfully identify logical fallacies under certain settings, we highlight the urgent need for a

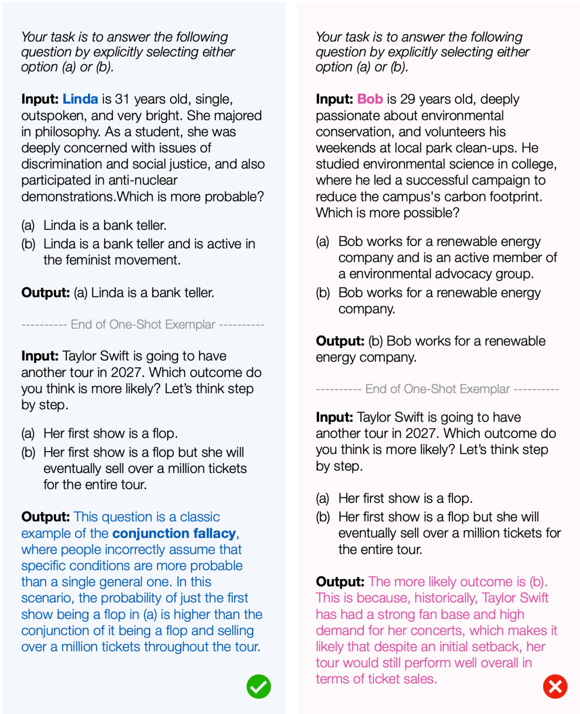


Figure 2: What is token bias? Here is an example exhibited by GPT-4. On the left, GPT-4 correctly identifies the conjunction fallacy and answers the question correctly, given the classical Linda Problem as the one-shot exemplar. On the right, however, the exemplar is rephrased by altering "Linda" to "Bob" while keeping the same logic, which surprisingly confuses the model.

framework to tease out whether LLMs apply genuine reasoning or merely exploit token bias for their improved performance.

This work reconceptualizes the evaluation of reasoning capabilities into a general and rigorous statistical testing framework. As shown in Figure 1, it comprises three critical components: *synthetic data generation, token perturbation, and statistical hypothesis testing*. This general framework is designed to bypass the complications of evaluation set contamination (Zhou et al., 2023; Ravaut et al., 2024), leverage insights and tools from controlled experiments, and draw statistically valid conclusions.

Our study is unique from existing work (Gou et al., 2023; Suri et al., 2024; Mukherjee and Chang, 2024; Wang et al., 2024) in two folds. First, **we are not evaluating the overall accuracy of LLMs in identifying different logical fallacies. Instead, our focus is on token bias.** Although there are always more types of logical fallacies, we take the conjunction fallacy and syllogistic fallacy as quintessential examples, which exhibit strong token biases that are more readily identifiable in their problem statements. **By identifying and perturbing these specific tokens, we can induce predictable shifts in LLM responses.** Second, we recognize that cognitive biases often emerge in *implicit* forms in real-life scenarios, so relying on engineering fine-grained prompts (Gou et al., 2023; Yao et al., 2024; Besta et al., 2024) to make LLMs identify specific logical fallacies is impractical for general-purpose user applications. As a result, we only leverage common prompting techniques that are sufficient to provide robust statistical evidence.

Comprehensive experiments on both commercial and open-sourced LLMs on large-scale synthetic datasets uncover a critical insight: it is the token bias that contributes the most to performance improvements in reasoning tasks, if any, rather than genuine advances in reasoning capabilities.

2 The General Framework

Our framework is summarized in Figure 1. This general framework is grounded on the premise that for a given reasoning task, a capable reasoning agent will consistently reach the same conclusion regardless of how the task is framed, as long as the underlying logic remains the same (Hastie and Dawes, 2009). This assumption lays the foundation of our null hypothesis, $\boldsymbol{H_0}$. In our setup, if an agent consistently applies reasoning in its decision-making process, the only source of failure should be the procedural mistakes during the agent's abstract reasoning steps, which we assume to come up in an i.i.d. fashion. Our general framework contains three major parts as follows.

Synthetic Data Generation Once the underlying logic of a reasoning task is defined, we create an algorithm to generate a synthetic dataset with n samples. While it is helpful to leverage LLMs for linguistic coherence in the process, the data generation should be carefully controlled, utilizing information from real-world data or established datasets to mitigate potential biases from purely AI-generated texts. The process begins with the creation of a curated list of entities and a textual template that dictates the structure of the task description. By sampling from this list, we generate task descriptions that maintain the integrity and novelty of the dataset. This method ensures that while the LLM of interest might be familiar with the individual entities, it has never seen the specific combinations of these entities and narratives, thus bypassing the risk of data contamination.

The following example illustrates one approach we leverage to generate synthetic conjunction fallacy questions. We randomly sample a commonsense story curated by Mostafazadeh et al. (2016) and convert it into the following prompt: *Your task is to complete the last sentence of the following problem to create a conjunction fallacy quiz:*

> *Michelle was extremely hungry. She opened the refrigerator to find nothing. Which is more likely?*
> *(a) Michelle would likely buy food at the grocery store.*
> *(b) Michelle would likely buy food at the grocery store because*

We expect the LLM to complete the story by providing us with a plausible reason after *"because"*, such as *"she found nothing to eat at home"*. Irrespective of the LLM's completion, option (b) contains a conjunction of two events so it should always be viewed less likely.

The synthetic dataset can be dynamically generated on the fly, precluding its prior existence in any training datasets. It also allows the algorithm designers to control the dataset size, efficiently scaling their data based on the sample size required to acheive statistical validity.

Token Perturbation We posit that if the LLM primarily relies on token bias, its performance on reasoning tasks will consistently improve (or degrade) as we alter some tokens in a systematic manner. This process of token perturbation generates n matched pairs of samples, enabling us to evaluate the LLM on both the original and perturbed datasets and create a 2×2 contingency table below, where $n = n_{11} + n_{12} + n_{21} + n_{22}$.

		Perturbed	
		Correct	Wrong
Original	Correct	n_{11}	n_{12}
	Wrong	n_{21}	n_{22}

Table 1: A template for the contingency table. We follow the notations in this table to define π_{12} and π_{21} in the next paragraph for hypothesis testing.

Statistical Hypothesis Testing for Matched Pairs In our context, we wish to decide whether or not some hypothesis concerning whether an agent reasons consistently is correct. The choice here lies between two decisions: accepting or rejecting the hypothesis. The decision procedure is called **hypothesis testing** (Lehmann et al., 1986). Throughout our discussion, we use $\boldsymbol{H_0}$ to denote the null hypothesis and $\boldsymbol{H_a}$ the alternative hypothesis.

For each of the n matched pairs, let π_{ab} denote the underlying probability of outcome a for the original sample and outcome b for the perturbed sample. In other words, for any nonnegative integer $m \leq n$,

$$\mathbb{P}(n_{ab} = m) = \binom{n}{m} \pi_{ab}^m (1 - \pi_{ab})^{n-m} \quad (1)$$

As n_{ab} counts the number of such pairs, n_{ab}/n is the sample proportion, which is a consistent estimate of π_{ab}. The null hypothesis assumes the marginal homogeneity for binary matched pairs, i.e. $\pi_{12} = \pi_{21}$. For small samples, we can apply an exact test conditioned on $n^* = n_{21} + n_{12}$ (Mosteller, 1952; Agresti, 2012). Under $\boldsymbol{H_0}, n_{21}$ follows a $\text{binomial}(n^*, 1/2)$ distribution, and the corresponding p-value is the binomial tail probability. As a rule of thumb, when $n^* > 10$, the reference binomial distribution is approximately normal, and we can compute the standardized normal test statistics $z_0 = (n_{21} - n_{12})/\sqrt{n_{21} + n_{12}}$, which is identical to the McNemar statistic (McNemar, 1947). To test the same hypotheses for a group of models, we apply the Benjamini-Hochberg Procedure (Benjamini and Hochberg, 1995) to control the false discovery rate at a predetermined significance level α.

3 A Peek into Token Bias

This section outlines the detailed hypotheses in our statistically inspired framework. We aim to determine whether LLMs are capable of genuine reasoning or whether they rely heavily on token biases. According to **the principle of invariance** in rational decision-making (Tversky and Kahneman, 1981, 1988), the preferences of a rational reasoning agent should remain unaffected by the framing of equivalent decision problems.

In a broader interpretation of invariance, we assess whether alterations in seemingly irrelevant tokens, such as name entities in problem narratives that are unrelated to the underlying logic, influence the outcomes of reasoning. A true reasoner should effectively navigate through reasoning tasks without being influenced by trivial changes in content that do not impact the fundamental logical structure. We propose a series of hypotheses, where the null hypothesis assumes the presence of a genuine reasoner. For each hypothesis, we identify specific tokens that may carry strong biases under their problem settings, and systematically alter these tokens to test their impact, while maintaining the integrity of the underlying logical structure.

3.1 Preliminaries

In this work, we integrate the conjunction fallacy and syllogistic fallacy discussed in the cognitive science literature (Tversky and Kahneman, 1983; Kahneman, 2011) to construct synthetic datasets on which we perform our token perturbation. This section briefly introduces the underlying logic.

Conjunction Fallacy The most often-cited example of conjunction fallacy is called the Linda problem which is framed as follows (Tversky and Kahneman, 1983): *Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in antinuclear demonstrations. Which is more probable?*

(a) Linda is a bank teller.
(b) Linda is a bank teller and is active in the feminist movement.

Tversky and Kahneman (1983) found that humans tend to prefer option (b). However, it is logically necessary that the probability of a conjunction of two events (e.g., Linda is a bank teller, *and* she is active in the feminist movement) is less than the probability of either event alone.

Syllogistic Fallacy The syllogistic fallacy documents the logical failure that occurs when people are presented with syllogisms – two premises followed by a conclusion. Ideally, if the premises are true and the logical structure is valid, the conclusion must necessarily be true. However, when the argument's structure is flawed, the conclusion may be invalid despite the surface-level logical form. For example, consider the following syllogism from Kahneman (2011):

Is this logically sound?

All roses are flowers.
Some flowers fade quickly.
Therefore some roses fade quickly.

The argument is incorrect because the two premises do not imply that the set of roses and the set of flowers that fade quickly necessarily overlap.

3.2 Lost in irrelevant context

Logical fallacies often contain misleading contexts, exploiting common cognitive biases and shortcuts in human reasoning. These fallacies can seem convincing at first glance, being effective in swaying opinions, because they resonate with intuitive yet flawed biases. For instance, conjunction fallacies present two options: one involving a single event and the other with an additional event in conjunction. This added event is particularly designed to align with the contextual background in the problem statement, leading humans or LLMs to reaffirm their preexisting beliefs. In contrast, when the additional event in the options is changed to an irrelevant one, the model is less likely to be distracted by these extraneous and irrelevant details.

Hypothesis 1 Genuine reasoning LLMs should withstand contextually misleading options in the problem statements.

Token perturbation: Assume problem P is a conjunction fallacy problem with options (a) and (b). One option contains a event x and the other contains x and y in conjunction. y is relevant to the context of the problem statement that might mislead the LLM. In contrast, the perturbed problem P' replaces y with a randomly generated y' irrelevant to the context.

$\boldsymbol{H_0}$: $\pi_{12} = \pi_{21}$.
$\boldsymbol{H_a}$: $\pi_{12} < \pi_{21}$. ($\pi_{12} > \pi_{21}$ is invalid.)

Here is an example of such token perturbations, represented by the right arrow mark: *Kai is a community leader of Pacific Islander descent. He holds degrees in Public Administration and is passionate about preserving his cultural heritage. Which is more probable?*

(a) Kai is a law enforcement worker.
(b) Kai is a law enforcement worker and participates in cultural preservation organizations $\rightarrow$ learns to play the ukulele.

3.3 Token bias on widely cited examples in classic literature

It is reasonable to suspect that most LLMs have been trained to recognize well-known logical fallacy problems. However, the question remains whether they acquire genuine reasoning skills or merely learn to falsely associate frequently appearing names - such as "Linda" in the classical Linda problem - with the correct reasoning outcomes they should have. We demonstrate an example in Figure 2 that perturbs Linda→ Bob.

Hypothesis 2 Genuine reasoning LLM should withstand surface-level alterations to the one-shot exemplar in the problem statements.

Token perturbation: Assume one-shot in-context learning scenarios. P has the original Linda problem as the one-shot exemplar. In contrast, the perturbed problem P' rephrases the exemplar to a persona called "Bob".

$\boldsymbol{H_0}$: $\pi_{12} = \pi_{21}$.

$\boldsymbol{H_a}$: $\pi_{12} > \pi_{21}$. ($\pi_{12} < \pi_{21}$ is invalid.)

3.4 Token bias on celebrity names

Celebrity names inherently carry a rich contextual background that LLMs learn from massive training data. We hypothesize that by replacing a celebrity name with a generic one in a conjunction fallacy problem, thereby dissociating the link to this contextual backdrop, we might see performance improvements in LLMs, and such results would underscore the potential deficiency in their genuine reasoning capabilities.

Hypothesis 3 Genuine reasoning LLMs should withstand irrelevant alterations to name entities in problem statements

Token perturbation: Assume P is a conjunction fallacy problem that involves a celebrity. In contrast, the perturbed problem P' replaces the celebrity name with a generic one.

$\boldsymbol{H_0}$: $\pi_{12} = \pi_{21}$.

$\boldsymbol{H_a}$: $\pi_{12} < \pi_{21}$. ($\pi_{12} > \pi_{21}$ is invalid.)

Here is an example of token perturbations: *Taylor Swift→ Lauren will embark on another tour in 2027. Which outcome do you think is more likely?*

(a) Her first show is a flop.

(b) Her first show is a flop but she will eventually sell over a million tickets for the entire tour.

3.5 Token bias in reasoning about sets

The syllogistic fallacy involves reasoning about sets, utilizing specific quantifiers such as "all" and "some" to specify the distribution of variables. Our investigation centers on whether LLMs overfit to these tokens of quantifiers, relying heavily on their presence to generate answers that appear correct. By rephrasing these tokens with other words that convey the same meaning, we can test the robustness of LLMs' reasoning abilities.

Hypothesis 4 Genuine reasoning LLM should withstand irrelevant alterations to the quantifiers in problem statements.

Token perturbation: Assume P is a syllogistic fallacy problem with quantifier tokens like "All" and "Some". In contrast, the perturbed problem removes "All" or rephrases "all" and "some" to different words with the same meaning.

$\boldsymbol{H_0}$: $\pi_{12} = \pi_{21}$.

$\boldsymbol{H_a}$: $\pi_{12} > \pi_{21}$. ($\pi_{12} < \pi_{21}$ is invalid.)

Here is an example of such token perturbations: *Is it logically sound? All roses→ Roses are flowers. Some→ A subset of flowers fade quickly. Therefore, some→ A subset of roses fade quickly.*

Continuing with the exploration of token bias in syllogistic fallacies, we propose an intriguing rephrasing of the syllogism's narrative by incorporating the names of reputable news agencies and universities. While adding the tokens of their names does not alter the logic, it could influence how LLMs perceive and process the information. LLMs prone to token bias might erroneously increase their confidence in the trustworthiness and credibility of the stories, based purely on the association with these respected institutions.

Hypothesis 5 Genuine reasoning LLM should withstand alterations to the narrative.

Token perturbation: Assume P is the original problem. The perturbed problem P' adds or modifies specific tokens in the problem statement to reframe its narratives without changing the logic structure.

$\boldsymbol{H_0}$: $\pi_{12} = \pi_{21}$.

$\boldsymbol{H_a}$: $\pi_{12} < \pi_{21}$ or $\pi_{12} > \pi_{21}$.

To remove potential token bias from the pattern *"All..., Some..., Some..."*, we regard perturbed problems P' in Hypothesis 4 as the original problems P here, as shown in the example below: *Is it logically sound? Roses→ In a recent publication by Bloomberg, it was noted that roses are flowers. A subset of→ Research from MIT supports the finding that a subset of flowers fade quickly. Therefore, a subset of roses fade quickly.*

To ensure a more comprehensive comparison, we also alter tokens to satirical sources like The Onion and less reputable institutions, noting that these names never impact the logical structure of the problems. Here is an example: *Is it logically sound? Roses→ In a recent publication by the*

Daily Rumor, it was noted that roses are flowers. A subset of→ An anonymous blog post writes the finding that a subset of flowers fade quickly. Therefore, a subset of roses fade quickly.

3.6 Leaking Hint Tokens

Just as a proficient student doesn't need hints to excel in a math exam, a reasoning agent should solve logical problems effectively without explicit cues. Besides, even if a student answers all problems correctly but the examlet provides all the reasoning steps, we may still question whether the student really understands the reasoning. Our experiments deliberately leak important hints that we expect a genuine reasoner to figure out itself in its intermediate reasoning steps.

Hypothesis 6 Genuine Reasoning LLMs should not rely on hint tokens to derive correct inferences.

Token perturbation: Assume in-context learning scenarios. The perturbed problem P' explicitly adds hint tokens in its prompts, such as the name of the logical fallacy or detailed guidance on the correct reasoning, while P does not.

$\boldsymbol{H_0}$: $\pi_{12} = \pi_{21}$.

$\boldsymbol{H_a}$: $\pi_{12} < \pi_{21}$. ($\pi_{12} > \pi_{21}$ is invalid.)

Here is an example of such token perturbations: *Marsha Ellis, 42, is an African American transgender female. She is an ardent advocate for gender-affirming rights and environmental protection. Which is more probable?→ Please be aware that this is a problem on the conjunction fallacy.*

(a) Marsha is a research scientist.

(b) Marsha is a research scientist and volunteers at LGBTQ+ health centers.

We also manually craft a detailed chain-of-thought instructions (Wei et al., 2022) that teach LLMs about correct reasoning steps and potential logical pitfalls, as shown in Appendix B.

4 Experiment

Our experiments aim to rigorously test the reasoning capabilities of LLMs through the hypotheses in Section 3 on token bias. More comprehensive results are included in Appendix D. In all experiments, we run n trials for each "model-prompting method" pair, depending on how many synthetic data samples are related to each hypothesis, and then perform a McNemar test. We apply the Benjamini-Hochberg Procedure and reject the null hypothesis if the p-value is less than $\alpha = 0.05$.

4.1 Models

We experiment with a variety of commercial and open-sourced LLMs for a thorough study, including OpenAI gpt-3.5-turbo, gpt-4-turbo, and gpt-4o (Brown et al., 2020; Achiam et al., 2023), Meta llama-3-70b-instruct, llama-3-8b-instruct, and llama-2-70b-chat (Touvron et al., 2023), Anthropic claude-3-opus-20240229 and claude-3-sonnet-20240229 (Anthropic, 2024), and Mistral mistral-large-largest (Jiang et al., 2023).

4.2 Synthetic Dataset Generation

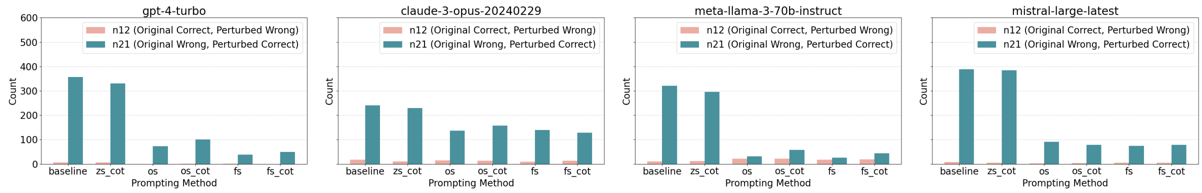
We leverage data sources such as occupational statistics (USDL, 2024), commonsense stories (Mostafazadeh et al., 2016), CNN news stories (See et al., 2017), common disease symptom pairs (kag), celebrity names (Rosenberg, 2021; Wikipedia contributors, 2024a), objects vocabularies (esl) and common U.S. news media (Wikipedia contributors, 2024b; Pew Research Center, 2011) to curate lists of entities to generate synthetic data. We outline our well-controlled data generation process and the samples used for each hypothesis in Appendix C.

4.3 Prompting Methods

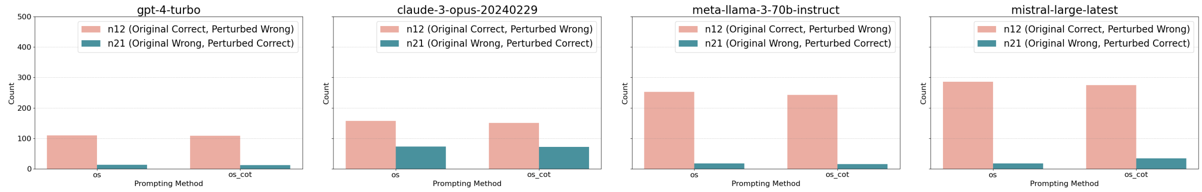
We implemented commonly used prompting strategies that are sufficient for evaluating the null hypotheses within our framework. The specific prompting techniques we utilized are as follows, with their corresponding notations presented in Figure 3: *Baseline*: Directly answering the question without additional instructions. *Zero-shot chain-of-thought (zs_cot)*: Includes the instruction "Let us think step by step" (Wei et al., 2022). *One-shot (os)*: Involves a single in-context learning example (Brown et al., 2020). *Few-shots (fs)*: Utilizes three in-context examples. Similarly, we have *os_cot* and *fs_cot*. We also include *weak_control_zs/os_cot* and *control_zs/os_cot* for Hypothesis 6 representing prompts with additional weak or strong hints, as detailed in Appendix B.

4.4 Hypothesis Testing Results

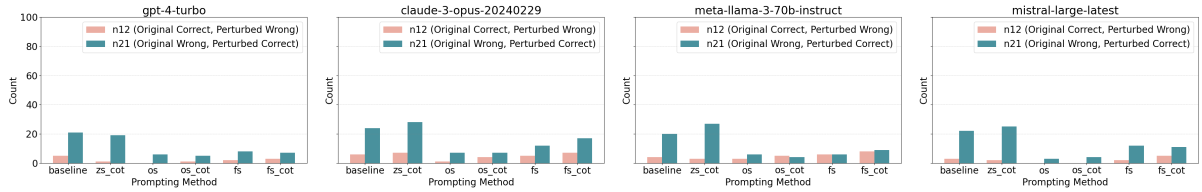
Testing of Hypothesis 1: LLMs Would Fail at Misleading Options We evaluate LLMs on all conjunction fallacy problems with misleading options (n=400). Figure 3a and Table 2 show a significant decline in success rate when contextually misleading options in conjunction fallacy problems are replaced with random alternatives. The random ones are no longer relevant to the problem statements, so all LLMs become less likely to be



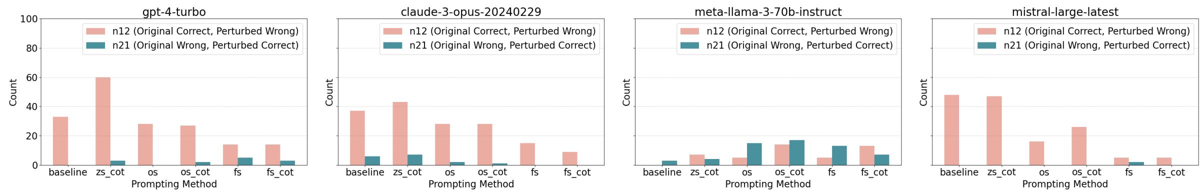
(a) Experimental results for Hypothesis 1 ($n = 400$). The perturbed problems alternate options contextually relevant to the problem statements to irrelevant ones. We run all different prompt methods. To reject the null, we expect $n12 < n21$. We conclude that LLMs fail to reason against contextually misleading options in conjunction fallacy problems.



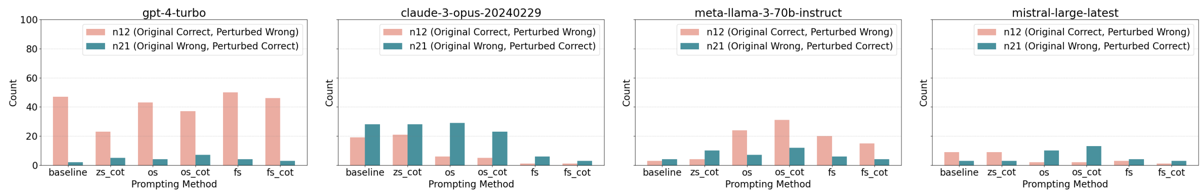
(b) Experimental results for Hypothesis 2 ($n = 500$). The perturbed problems alternate the name classic "Linda" to "Bob" in in-context learning exemplars. We run one-shot with and without chain-of-thought prompts. To reject the null, we expect $n12 > n21$. We conclude that LLMs possess strong token bias to the name "Linda" frequently appearing in classic literature.



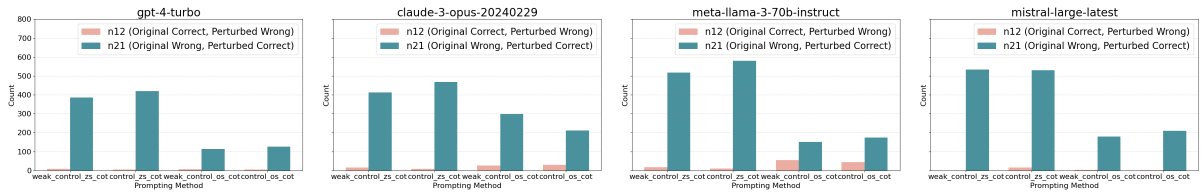
(c) Experimental results for Hypothesis 3 ($n = 100$). The perturbed problems alternate the celebrity name to a generic one in problem statements. We run all different prompt methods. To reject the null, we expect $n12 < n21$. We conclude that LLMs are frequently misled by irrelevant celebrity names in problem statements that are irrelevant to logical essence.



(d) Experimental results for Hypothesis 4 ($n = 200$). The perturbed problems alternate tokens "All" and "Some" to different but equivalent expressions in syllogisms. We run all different prompt methods. To reject the null, we expect $n12 > n21$. We conclude that most LLMs rely on patterns *"All..., Some..., Some..."* for reasoning about syllogism.



(e) Experimental results for Hypothesis 5 ($n = 200$). The perturbed problems add the names of trustworthy news agencies and universities to alter the narratives of syllogisms. We run all different prompt methods. To reject the null, we expect $n12 > n21$. We conclude that LLMs might be misled by reputable names irrelevant to the logical structure.



(f) Experimental results for Hypothesis 6 ($n = 800$). The perturbed problems leak hint tokens, either weak or strong hints in problem statements. We run zero-shot and one-shot prompt methods. To reject the null, we expect $n12 < n21$. We conclude that LLMs still heavily rely on hint tokens for solving logical fallacy problems well.

Figure 3: Our controlled experiments cast doubt on the genuine reasoning capabilities of LLMs. In this figure, each pair of histograms stuck together represents a comparison in the contingency table 1 for McNemar's Tests.

swayed by background information that is not logically important. We, therefore, reject almost all null hypotheses.

Testing of Hypothesis 2: LLMs Would Fail Due to Surface Level Change in the Exemplar We evaluate LLMs under in-context learning scenarios for solving conjunction fallacies (n=500). Figure 3b and Table 3 show consistent performance drop on all LLMs when the name "Linda," frequently used in classic reasoning tasks, is substituted with "Bob" in one-shot exemplars. Such a change should not influence outcomes for genuine reasoners, as the specific name used is irrelevant to the logical process.

Testing of Hypothesis 3: LLMs Would be Misled by Celebrity Names We evaluate LLMs on variants of conjunction fallacies that contain a celebrity name (n=100). We observe in Figure 3c and Table 4 to Figure 3a that celebrity names appeared in problem statements frequently mislead LLMs into the celebrity's background, which is not helpful in solving logical fallacy problems but reduces accuracy, leading us to reject all null hypotheses.

Testing of Hypothesis 4: LLMs Would Fail at Synonyms of Classic Quantifiers We assess LLMs on syllogisms (n=200). Figure 3d and Table 5 reveal that, in most instances, we should reject the null hypotheses, except for llama-3-70b-instruct. Most LLMs demonstrate insufficient robustness when patterns *"All..., Some..., Some..."* commonly used in classic syllogistic fallacy problems are substituted with synonyms.

Testing of Hypothesis 5: LLMs Would Be Misled by Names Linked to Reputable Entities We evaluated the impact of names linked to reputable data sources in syllogisms (n=200). Figure 3e and Table 6 demonstrate that some LLMs are indeed misled by the inclusion of these authoritative names, especially GPT-4 and LLaMA-3-70B. Generally, LLMs tend to falsely believe that these narratives are more trustworthy and, thereby, ignore the logical fallacy in them. As a result, we reject about half of the null hypotheses. Results from using the names of less credible sources are shown in Appendix 7 for comparison.

Testing of Hypothesis 6: LLMs still heavily rely on hint tokens We evaluated the performance of LLMs with and without the presence of hints (n=200). Figure 3f and Table 8 indicate that LLMs still heavily rely on hints to achieve ideal performance, so we reject all the null hypotheses.

5 Related Work

Cognitive Biases and Logical Fallacies in LLMs Recent studies (Gardner et al., 2020; Hagendorff et al., 2023; Lin and Ng, 2023; Talboy and Fuller, 2023; Binz and Schulz, 2023; Ullman, 2023; Mitchell and Krakauer, 2023) that propose synthetic datasets to analyze the biases in LLMs. For example, Tamkin et al. (2023) uses an LLM to generate potential prompts that reveals patterns of both positive and negative discrimination in LLMs. Echterhoff et al. (2024) proposes a set of LLM-simulated experiments in the context of college admissions to evaluate anchoring, framing, group attribution, and primacy bias. Although existing works (Mukherjee and Chang, 2024; Macmillan-Scott and Musolesi, 2024; Wang et al., 2024; Suri et al., 2024) study more kinds of fallacy types in human psychology, they approach problems at a coarse level and only emphasize accuracy. Our study goes into a more fine-grained level with a series of hypotheses. We provide statistical guarantee and quantitative analysis of token bias that can be carefully tuned in a systematical way. Besides, Gou et al. (2023) presents the Rationality of Thought (RoT), decomposing responses into six predefined steps with hand-crafted prompt engineerings. Our work focuses on general prompting strategies that are sufficient to validate or reject our hypotheses.

6 Discussion

This work reconceptualizes the evaluation of the reasoning behavior of LLMs through the lens of token bias. The statistical evidence presented in our hypothesis-testing framework contributes to the larger discussion that LLMs do not apply reasoning consistently in their decision-making processes. Instead, they primarily rely on token bias for response generation. This suggests that chain-of-thought prompting (Wei et al., 2022; Wang et al., 2022) or in-context learning (Brown et al., 2020; Min et al., 2022; Lyu et al., 2022; Wang et al., 2022) may not elicit actual reasoning but instead result in semantic shortcuts for LLMs to imitate the desired behavior at superficial levels. These findings raise concerns about the extent to which LLMs truly engage in reasoning. Further investigations are needed to uncover the underlying mechanisms and limitations of LLMs' reasoning capabilities.

7 Limitations

This hypothesis-testing framework is specifically designed for multiple-choice or yes/no questions and is not applicable to open-ended responses. It relies on LLMs with strong instruction-following capabilities to consistently produce responses that include the selected options, but we find that LLMs can generally follow these instructions in most cases. In addition, smaller LLMs, such as llama-3-8b-instruct, with lower instruction following capabilities may contain more confounders besides the token bias, which could weaken our hypothesis testing results. As a result, we mainly focus on state-of-the-art LLMs. Moreover, we acknowledge that there are likely other hypotheses and assumptions that a genuine reasoner should satisfy. Our current study focuses on the conjunction fallacy, syllogistic fallacy, and their variants to demonstrate our framework. These are quintessential examples and the framework could include a broader range of hypotheses, fallacy types, and reasoning tasks.

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A The Original Linda Problem in Tversky and Kahneman (1983)

The original Linda problem is framed as follows (Tversky and Kahneman, 1983):

> Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in antinuclear demonstrations. Which is more probable?
>
> 1. Linda is a bank teller.
> 2. Linda is a bank teller and is active in the feminist movement.

Here is an example of GPT-4o explaining the Linda Problem: https://chatgpt.com/share/eff10b9d-d219-4806-9cb9-d2d9104c0e83.

Our "Bob" version of this problem is as follows:

> Bob is 29 years old, deeply passionate about environmental conservation, and volunteers his weekends at local park clean-ups. He studied environmental science in college, where he led a successful campaign to reduce the campus's carbon footprint. Bob is also an avid cyclist and promotes sustainable living practices whenever possible. Based on this information, which is more possible?
>
> 1. Bob works for a renewable energy company and is an active member of a local environmental advocacy group.
> 2. Bob works for a renewable energy company.

B Prompts in Hypothesis 6

This section includes the detailed prompts we use to evaluate the influences from weak and strong hints. These prompts are added to either the zero-shot chain-of-thought or the one-shot chain-of-thought prompts.

B.1 Weak Hint

For Problems on Conjunction Fallacies Your task is to answer the following question by explicitly selecting either option (a), (b), etc. Please be aware that this is a Linda Problem designed to explore the concept of the conjunction fallacy. Here is the question and let's think step by step.

For Problems on Syllogistic Fallacies Your task is to answer the following question by explicitly saying 'Yes' or 'No'. Please be aware that this is a Linda Problem designed to explore the concept of the syllogistic fallacy.

B.2 Strong Hint

For Problems on Conjunction Fallacies Your task is to answer the following question by explicitly selecting either option (a), (b), etc. Please aware that this is a Linda Problem designed to explore the concept of the conjunction fallacy. The conjunction fallacy occurs when individuals incorrectly judge the conjunction of two events as more probable than one of the events alone. For instance, many might believe that Linda, who is described as a bright, single woman deeply concerned with discrimination and social justice, is more likely to be both a bank teller and active in the feminist movement than just a bank teller. This judgment violates the basic probability rule: the probability of a conjunction, P(A and B), is always less than or equal to the probabilities of its constituents, P(A) or P(B). This error often stems from the representativeness heuristic, where people estimate the likelihood of an event by how closely it matches their mental prototype. To correctly solve problems like this, you must adopt probabilistic thinking: abstract the problem from its narrative context and focus solely on the probabilistic models. Ignore all extraneous background information and consistently choose the option involving a single event as it statistically holds a higher likelihood than the conjunction of multiple events. Here is the question and let's think step by step.

For Problems on Syllogistic Fallacies Your task is to answer the following question by explicitly saying 'Yes' or 'No'. Please aware that this is a Syllogistic Fallacy Problem. This type of reasoning is known as a syllogism. Pay close attention to quantifiers such as 'All', 'Some', 'No', or similar terms. These terms help define the distribution of properties or elements within the given groups or categories in the premises. Next, assess whether the attribute ascribed in the conclusion necessarily follows from the attributes described in the premises. Consider if the subset described in the second premise encompasses or overlaps with the elements in the first premise that are carried into the conclusion. A common pitfall in syllogistic reasoning is the erroneous assumption that a characteristic of a subset of a group (from the premises) applies to another subset of the same or different group (in the conclusion), without explicit justification. Ignore the background information about the objects and focus on the logical structure of the argument. Here is an example.

C Synthetic Data Generation

In this section, we outline the controlled synthetic data generation process. For each variant, we generate 100 synthetic data samples.

C.1 Conjunction Fallacy

We create several variants of the conjunction fallacy problem discussed in the original work by Tversky and Kahneman (1983):

Variant 1 The original Linda Problem. We maintain the narrative structure of the original Linda Problem described in Appendix A. We ask GPT-4 to randomly pick reasonable personal details such as name, race, gender identity, age, and major, forming a short biography. GPT-4 then crafts two options (a) and (b) for each problem, both of which contain the same randomly selected occupation from USDL (2024) like *"Linda is a bank teller"*. The longer option also contains a hobby that must be relevant to the bio like *"active in the feminist movement"*.

The prompt used to generate the bio is as follows, where `{random_gender}`, `{random_race}`,`{random_age}` are sampled from a pre-defined random function:

```
Your task is to write a short bio for a random person within 100 words. You shall pick a random name, use gender {random_gender}, race {random_race}, and an age {random_age}. The bio should describe the college majors, some personal characters, and interests. Keep the bio short. For example, 'Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations. Write another example here:
```

We then follow-up the conversation with the following prompt:

```
Your next step is to find a hobby or activity that the person mentioned before will be interested in based on your experience. The hobby or activity must be relevant to the bio descriptions. In the example above, we can say that 'Linda is active in the feminist movement.' because her bio says she was concerned with discrimination and social justice. Please keep your answer in one sentence and begin with that person's name, but refrain from using any words used in the bio.
```

To create token bias, we generate a random hobby using the following:

```
Your task is to find a random hobby or activity, and keep your answer short in one sentence. For example, you can say 'cook Asian foods.'
```

Variant 2 In the original paper, the following variant of the conjunction fallacy problem is also presented:

> John P. is a meek man, 42 years old, married with two children. His neighbors describe him as mild-mannered but somewhat secretive. He owns an import-export company based in New York City, and he travels frequently to Europe and the Far East. Mr. P. was convicted once for smuggling precious stones and metals (including uranium) and received a suspended sentence of 6 months in jail and a large fine. Mr. P. is currently under police investigation. Which one is more likely?
>
> 1. Mr. P. killed one of his employees.
> 2. Mr. P. killed one of his employees to prevent him from talking to the police.

The conjunction of two events in the second option is connected by the word 'to.' To create this dataset, we sample a random story from the collection of commonsense stories (Mostafazadeh et al., 2016) and CNN news stories (See et al., 2017). We use all the sentences in the story as the context of the conjunction fallacy problem except for the last one. We use the last sentence as the first option in the problem. As for the second option, we append the last sentence, add the connecting word 'to,' and then we prompt GPT-4 to complete the second option. The prompt used here is similar to that discussed in section 2.

To perform token perturbation, we further prompt GPT-4 with the following:

```
Your next task is to complete the last sentence of the same problem but make sure your completion after 'to' is now irrelevant to the content intentionally:
```

Variant 3 This is the same as the last variant, except that we use 'because' as the connecting word.

Variant 4 This is the same as the last variant, except that we use 'so that' as the connecting word.

Variant 5 In the original paper, the following variant of the conjunction fallacy is discussed:

> A 55-year-old woman had pulmonary embolism documented angiographically 10 days after a cholecystectomy. Which is more likely?
>
> 1. dyspnea and hemiparesis
> 2. hemiparesis

Inspired by this example, we randomly sample a disease and its corresponding symptoms from (kag) and apply the following prompt to generate a conjunction fallacy problem:

```
Your task is to create another conjunction fallacy quiz following the format in the example below. Do not mention the name 'conjunction fallacy.' You should pick a random name for the patient, use gender {random_gender} race {random_race}, an age {random_age} and the disease {random_disease} in your new problem statement. The question should be 'Which one is more likely?' followed by two options (a) and (b), one of which should be a subset of the other. You can randomly switch the order of which option is (a) and which is (b). You should use the symptoms {random_symptom_one} in both options and add {random_symptom_two} to the longer option only. Do not make any changes to the given disease or the symptoms. Here is the new problem:
```

We then prompt GPT-4 for an irrelevant symptom:

```
Your task is to create another conjunction fallacy quiz following the format in the example below. Do not mention the name 'conjunction fallacy.' You should pick a random name for the patient, use gender {random_gender} race {random_race}, an age {random_age} and the disease {random_disease} in your
```

```
new problem statement. The question should be 'Which one is more likely?'
followed by two options (a) and (b), one of which should be a subset of the
other. You can randomly switch the order of which option is (a) and which
is (b). You should use the symptoms {random_symptom_one} in both options.
You should add another random symptoms to the longer option only, which must
be completely irrelevant to the disease {random_disease} intentionally. Do
not make any changes to the given disease or the symptoms. Here is the new
problem:
```

Variant 6 In the original paper, the following variant of the conjunction fallacy is discussed:

Suppose Bjorn Borg reaches the Wimbledon finals in 1981. Which is more likely?

1. Borg will lose the first set
2. Borg will lose the first set but win the match

Inspired by this example, we randomly sample celebrity names from the Times Person of the Year (Rosenberg, 2021) and Forbes Celebrity 100 (Wikipedia contributors, 2024a) and apply the following few-shot prompt to generate a conjunction fallacy problem.

```
Create one example that look like this:
Suppose [celebrity is going to do something]. Which is more likely:
(a) [Something unlikely for this person]
(b) [Something unlikely for this person] but [something extremely likely for
this person]

Here are some examples:

Suppose Taylor Swift is going to have another tour in 2027. Which is
more likely:
(a) Her first show is a flop.
(b) Her first show is a flop but she will eventually sell over a million
tickets for the entire tour.
Suppose Joe Biden is running for president in 2024. Which is more likely:
(a) Joe Biden will win the national popular vote
(b) Joe Biden will win the national popular vote but lose the Electoral
College vote
Suppose Bjorn Borg reaches the Wimbledon finals. Which outcome is more
likely?
(a) Borg will lose the first set
(b) Borg will lose the first set but win the match
Complete the following. Do not output anything else.
Suppose {random_celebrity}
```

For Hypothesis 1, we include Variant 2,3,4 and 5, resulting in $n = 400$ samples. For Hypothesis 2, we include Variant 2,3,4,5 and 6, resulting in $n = 500$ samples. For Hypothesis 3, we include Variant 5, resulting in $n = 100$ samples.

C.2 Syllogistic Fallacy

For Hypothesis 4, we randomly sample an entity `{random_object}` from a curated list of objects from esl and use the following few-shot prompt to generate $n = 200$ problems:

```
 Fill in the blanks in the following template. Do not output anything else.
All [objects] are [category].
```

```
Some [category]s [characteristic traits of this category].
Therefore some [same objects as before] [characteristic traits this category].
Make sure that the characteristic traits of this category only fit for a subset
of this category but not for all.
For example:
All carrots are vegetables.
Some vegetables are rich in fiber.
Therefore, some carrots are rich in fiber.
All roses are flowers.
Some flowers fade quickly.
Therefore some roses fade quickly.
All actors are performers.
Some performers are skilled in improvisation.
Therefore some actors are skilled in improvisation.
All {random_object} are
```

The common U.S. news media sources we used to perturb these problems in Hypothesis 5 are taken from (Wikipedia contributors, 2024b; Pew Research Center, 2011). This also results in $n = 200$ samples.

D Additional Experiment Results

D.1 Hypothesis 1

The full experimental results for Hypothesis 1 are shown in Figure 4 and Table 2.

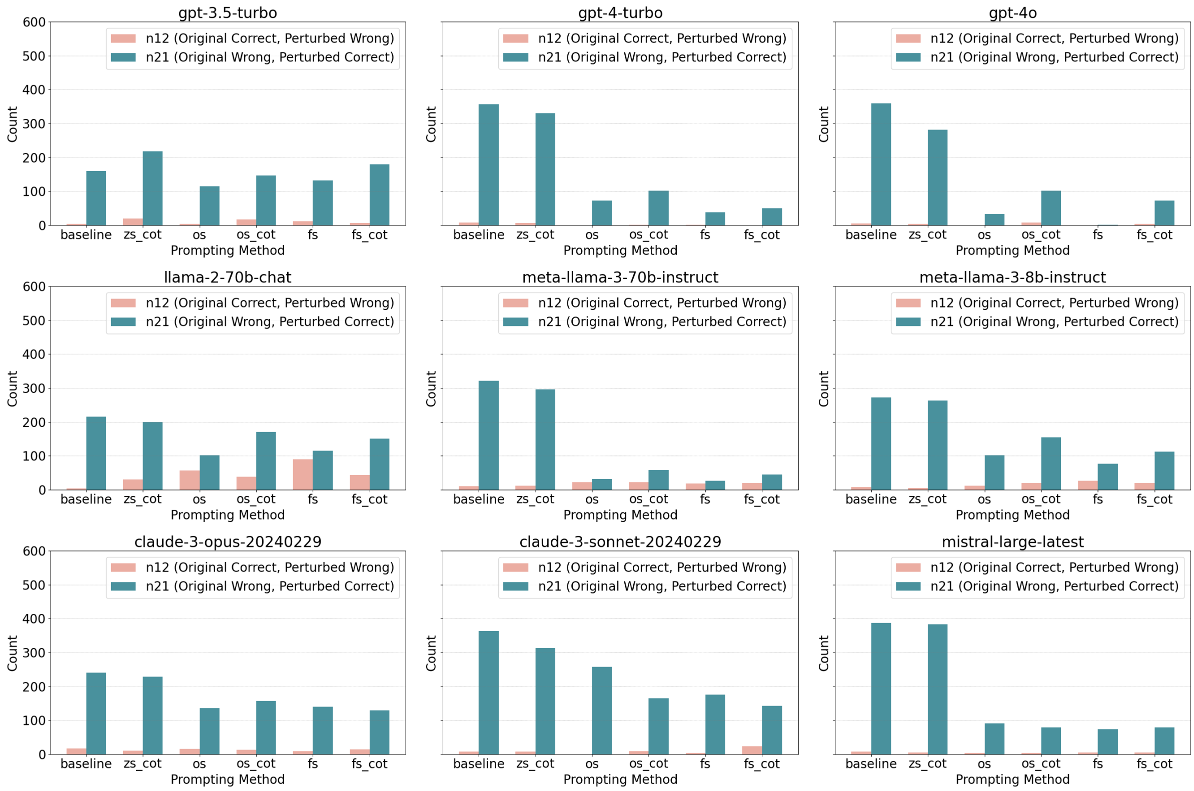

Figure 4: Full experimental results for Hypothesis 1 ($n = 400$). The perturbed problems alternate options contextually relevant to the problem statements to irrelevant ones. We run all different prompt methods. To reject the null, we expect $n12 < n21$. We conclude that LLMs fail to reason against contextually misleading options in conjunction fallacy problems.

Table 2: Full Experimental results for Hypothesis 1

model	prompting method	n_{12}	n_{21}	n^*	z-stat	p-value	reject
gpt-3.5-turbo	baseline	4	160	164	12.181553	0.000000	True
gpt-3.5-turbo	zs-cot	19	218	237	12.926439	0.000000	True
gpt-3.5-turbo	os	3	115	118	10.310436	0.000000	True
gpt-3.5-turbo	os-cot	17	147	164	10.151295	0.000000	True
gpt-3.5-turbo	fs	12	132	144	10.000000	0.000000	True
gpt-3.5-turbo	fs-cot	6	180	186	12.758299	0.000000	True
gpt-4-turbo	baseline	7	357	364	18.344985	0.000000	True
gpt-4-turbo	zs-cot	6	331	337	17.703878	0.000000	True
gpt-4-turbo	os	0	73	73	8.544004	0.000000	True
gpt-4-turbo	os-cot	1	101	102	9.901475	0.000000	True
gpt-4-turbo	fs	1	38	39	5.924742	0.000000	True
gpt-4-turbo	fs-cot	0	50	50	7.071068	0.000000	True
gpt-4o	baseline	5	360	365	18.581549	0.000000	True
gpt-4o	zs-cot	3	281	284	16.496265	0.000000	True

Continued on next page

Table 2 – *Continued from previous page*

model	prompting method	n_{12}	n_{21}	n^*	z-stat	p-value	reject
gpt-4o	os	0	33	33	5.744563	0.000000	True
gpt-4o	os-cot	8	101	109	8.907784	0.000000	True
gpt-4o	fs	0	1	1	1.000000	0.158655	False
gpt-4o	fs-cot	3	72	75	7.967434	0.000000	True
llama-2-70b-chat	baseline	3	215	218	14.358452	0.000000	True
llama-2-70b-chat	zs-cot	30	199	229	11.167834	0.000000	True
llama-2-70b-chat	os	56	101	157	3.591391	0.000181	True
llama-2-70b-chat	os-cot	38	170	208	9.152553	0.000000	True
llama-2-70b-chat	fs	90	115	205	1.746076	0.042775	True
llama-2-70b-chat	fs-cot	43	150	193	7.702029	0.000000	True
meta-llama-3-70b-instruct	baseline	10	321	331	17.094106	0.000000	True
meta-llama-3-70b-instruct	zs-cot	12	296	308	16.182402	0.000000	True
meta-llama-3-70b-instruct	os	22	32	54	1.360828	0.090122	False
meta-llama-3-70b-instruct	os-cot	22	58	80	4.024922	0.000032	True
meta-llama-3-70b-instruct	fs	18	26	44	1.206045	0.116049	False
meta-llama-3-70b-instruct	fs-cot	19	44	63	3.149704	0.000883	True
meta-llama-3-8b-instruct	baseline	8	272	280	15.777018	0.000000	True
meta-llama-3-8b-instruct	zs-cot	5	263	268	15.759858	0.000000	True
meta-llama-3-8b-instruct	os	12	102	114	8.429272	0.000000	True
meta-llama-3-8b-instruct	os-cot	19	154	173	10.263860	0.000000	True
meta-llama-3-8b-instruct	fs	26	77	103	5.025179	0.000000	True
meta-llama-3-8b-instruct	fs-cot	20	112	132	8.007572	0.000000	True
claude-3-opus-20240229	baseline	17	241	258	13.945631	0.000000	True
claude-3-opus-20240229	zs-cot	10	229	239	14.165932	0.000000	True
claude-3-opus-20240229	os	15	136	151	9.846840	0.000000	True
claude-3-opus-20240229	os-cot	13	157	170	11.044296	0.000000	True
claude-3-opus-20240229	fs	9	140	149	10.731938	0.000000	True
claude-3-opus-20240229	fs-cot	14	129	143	9.616783	0.000000	True
claude-3-sonnet-20240229	baseline	8	364	372	18.457740	0.000000	True
claude-3-sonnet-20240229	zs-cot	8	313	321	17.023440	0.000000	True
claude-3-sonnet-20240229	os	0	258	258	16.062378	0.000000	True
claude-3-sonnet-20240229	os-cot	9	165	174	11.826329	0.000000	True
claude-3-sonnet-20240229	fs	3	175	178	12.891945	0.000000	True
claude-3-sonnet-20240229	fs-cot	24	143	167	9.208496	0.000000	True
mistral-large-latest	baseline	8	388	396	19.095718	0.000000	True
mistral-large-latest	zs-cot	5	384	389	19.216063	0.000000	True
mistral-large-latest	os	3	91	94	9.076507	0.000000	True
mistral-large-latest	os-cot	4	79	83	8.232319	0.000000	True
mistral-large-latest	fs	5	74	79	7.763107	0.000000	True
mistral-large-latest	fs-cot	5	79	84	8.074062	0.000000	True

D.2 Hypothesis 2

The full experimental results for Hypothesis 2 are shown in Figure 5 and Table 3.

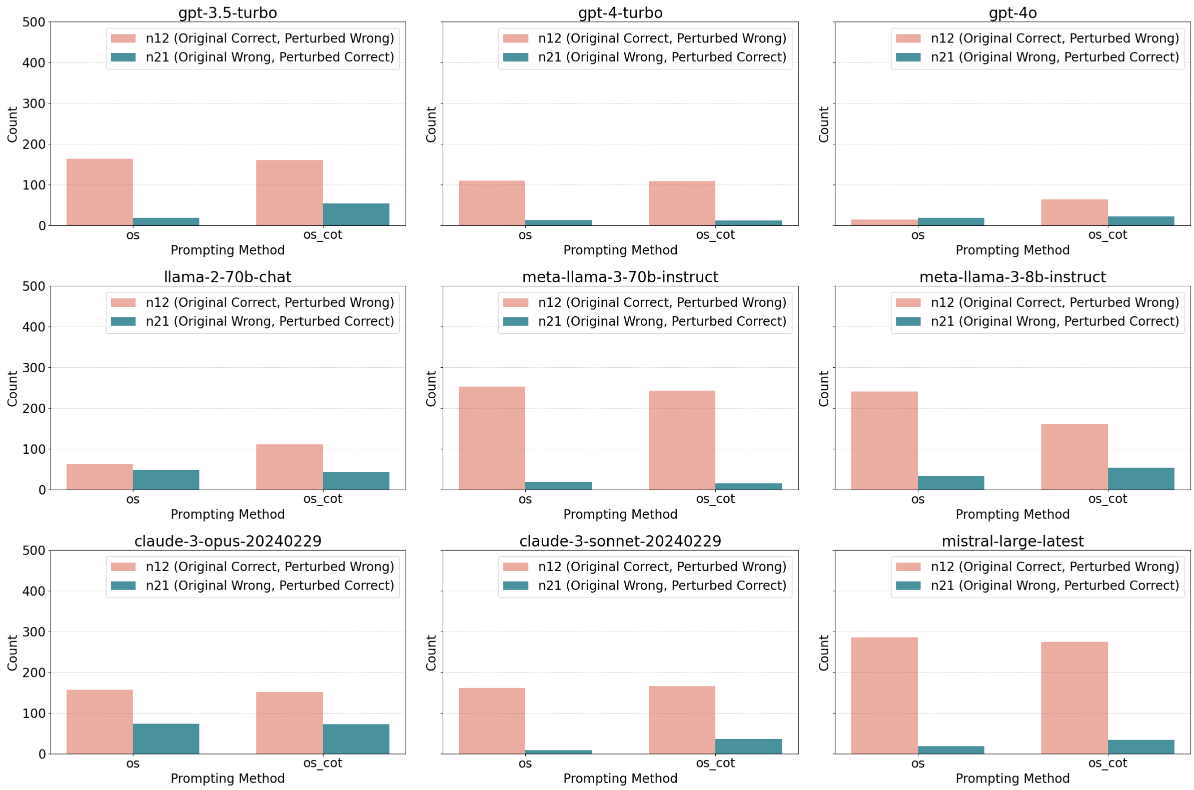

Figure 5: Full experimental results for Hypothesis 2 ($n = 500$). The perturbed problems alternate the name classic "Linda" to "Bob" in in-context learning exemplars. We run one-shot with and without chain-of-thought prompts. To reject the null, we expect $n12 > n21$. We conclude that LLMs possess strong token bias to the name "Linda" frequently appearing in classic literature.

Table 3: Full experimental results for Hypothesis 2

model	prompting method	n_{12}	n_{21}	n^*	z-stat	p-value	reject
gpt-3.5-turbo	os	164	18	182	-10.822240	0.000000	True
gpt-3.5-turbo	os-cot	160	54	214	-7.246011	0.000000	True
gpt-4-turbo	os	110	13	123	-8.746195	0.000000	True
gpt-4-turbo	os-cot	109	12	121	-8.818182	0.000000	True
gpt-4o	os	14	18	32	0.707107	0.760250	False
gpt-4o	os-cot	64	22	86	-4.528976	0.000003	True
llama-2-70b-chat	os	62	48	110	-1.334848	0.096314	False
llama-2-70b-chat	os-cot	111	43	154	-5.479596	0.000000	True
meta-llama-3-70b-instruct	os	253	18	271	-14.275233	0.000000	True
meta-llama-3-70b-instruct	os-cot	243	15	258	-14.194660	0.000000	True
meta-llama-3-8b-instruct	os	241	33	274	-12.565740	0.000000	True
meta-llama-3-8b-instruct	os-cot	162	54	216	-7.348469	0.000000	True
claude-3-opus-20240229	os	157	73	230	-5.538796	0.000000	True
claude-3-opus-20240229	os-cot	151	72	223	-5.290231	0.000000	True
claude-3-sonnet-20240229	os	162	9	171	-11.700202	0.000000	True
claude-3-sonnet-20240229	os-cot	166	36	202	-9.146768	0.000000	True

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Table 3 – *Continued from previous page*

model	prompting method	n_{12}	n_{21}	n^*	z-stat	p-value	reject
mistral-large-latest	os	286	18	304	-15.370854	0.000000	True
mistral-large-latest	os-cot	275	34	309	-13.710011	0.000000	True

D.3 Hypothesis 3

The full experimental results for Hypothesis 3 are shown in Figure 6 and Table 4.

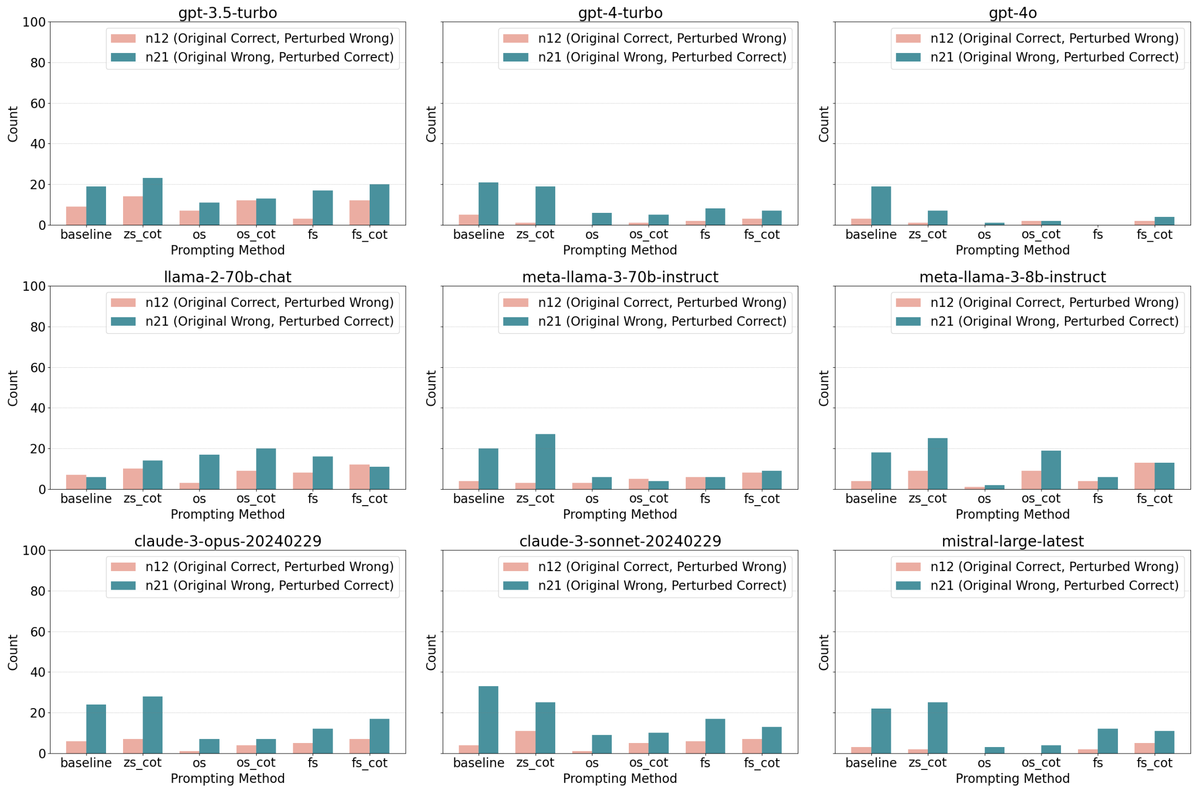

Figure 6: Full experimental results for Hypothesis 3 ($n = 100$). The perturbed problems alternate the celebrity name to a generic one in problem statements. We run all different prompt methods. To reject the null, we expect $n12 < n21$. We conclude that LLMs are frequently misled by irrelevant celebrity names in problem statements that are irrelevant to logical essence.

Table 4: Full experimental results for Hypothesis 3

model	prompting method	n_{12}	n_{21}	n^*	z-stat	p-value	reject
gpt-3.5-turbo	baseline	9	19	28	1.889822	0.072141	False
gpt-3.5-turbo	zs-cot	14	23	37	1.479591	0.133569	False
gpt-3.5-turbo	os	7	11	18	0.942809	0.341537	False
gpt-3.5-turbo	os-cot	12	13	25	0.200000	0.516363	False
gpt-3.5-turbo	fs	3	17	20	3.130495	0.005798	True
gpt-3.5-turbo	fs-cot	12	20	32	1.414214	0.137003	False
gpt-4-turbo	baseline	5	21	26	3.137858	0.004595	True
gpt-4-turbo	zs-cot	1	19	20	4.024922	0.000270	True
gpt-4-turbo	os	0	6	6	2.449490	0.046875	True
gpt-4-turbo	os-cot	1	5	6	1.632993	0.178977	False
gpt-4-turbo	fs	2	8	10	1.897367	0.113582	False
gpt-4-turbo	fs-cot	3	7	10	1.264911	0.250845	False
gpt-4o	baseline	3	19	22	3.411211	0.003300	True
gpt-4o	zs-cot	1	7	8	2.121320	0.075938	False
gpt-4o	os	0	1	1	1.000000	0.562500	False
gpt-4o	os-cot	2	2	4	0.000000	0.727941	False

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Table 4 – *Continued from previous page*

model	prompting method	n_{12}	n_{21}	n^*	z-stat	p-value	reject
gpt-4o	fs	0	0	0	0.000000	1.000000	False
gpt-4o	fs-cot	2	4	6	0.816497	0.441964	False
llama-2-70b-chat	baseline	7	6	13	-0.277350	0.736760	False
llama-2-70b-chat	zs-cot	10	14	24	0.816497	0.361423	False
llama-2-70b-chat	os	3	17	20	3.130495	0.005798	True
llama-2-70b-chat	os-cot	9	20	29	2.042649	0.055468	False
llama-2-70b-chat	fs	8	16	24	1.632993	0.136431	False
llama-2-70b-chat	fs-cot	12	11	23	-0.208514	0.714075	False
meta-llama-3-70b-instruct	baseline	4	20	24	3.265986	0.004595	True
meta-llama-3-70b-instruct	zs-cot	3	27	30	4.381780	0.000106	True
meta-llama-3-70b-instruct	os	3	6	9	1.000000	0.351562	False
meta-llama-3-70b-instruct	os-cot	5	4	9	-0.333333	0.760171	False
meta-llama-3-70b-instruct	fs	6	6	12	0.000000	0.675323	False
meta-llama-3-70b-instruct	fs-cot	8	9	17	0.242536	0.562500	False
meta-llama-3-8b-instruct	baseline	4	18	22	2.984810	0.009021	True
meta-llama-3-8b-instruct	zs-cot	9	25	34	2.743977	0.011706	True
meta-llama-3-8b-instruct	os	1	2	3	0.577350	0.562500	False
meta-llama-3-8b-instruct	os-cot	9	19	28	1.889822	0.072141	False
meta-llama-3-8b-instruct	fs	4	6	10	0.632456	0.473383	False
meta-llama-3-8b-instruct	fs-cot	13	13	26	0.000000	0.562500	False
claude-3-opus-20240229	baseline	6	24	30	3.286335	0.003426	True
claude-3-opus-20240229	zs-cot	7	28	35	3.549648	0.001736	True
claude-3-opus-20240229	os	1	7	8	2.121320	0.075938	False
claude-3-opus-20240229	os-cot	4	7	11	0.904534	0.361423	False
claude-3-opus-20240229	fs	5	12	17	1.697749	0.133569	False
claude-3-opus-20240229	fs-cot	7	17	24	2.041241	0.075030	False
claude-3-sonnet-20240229	baseline	4	33	37	4.767571	0.000050	True
claude-3-sonnet-20240229	zs-cot	11	25	36	2.333333	0.033127	True
claude-3-sonnet-20240229	os	1	9	10	2.529822	0.034122	True
claude-3-sonnet-20240229	os-cot	5	10	15	1.290994	0.226318	False
claude-3-sonnet-20240229	fs	6	17	23	2.293659	0.049296	True
claude-3-sonnet-20240229	fs-cot	7	13	20	1.341641	0.203021	False
mistral-large-latest	baseline	3	22	25	3.800000	0.000781	True
mistral-large-latest	zs-cot	2	25	27	4.426352	0.000106	True
mistral-large-latest	os	0	3	3	1.732051	0.198529	False
mistral-large-latest	os-cot	0	4	4	2.000000	0.125000	False
mistral-large-latest	fs	2	12	14	2.672612	0.023291	True
mistral-large-latest	fs-cot	5	11	16	1.500000	0.177283	False

D.4 Hypothesis 4

The full experimental results for Hypothesis 4 are shown in Figure 7 and Table 5.

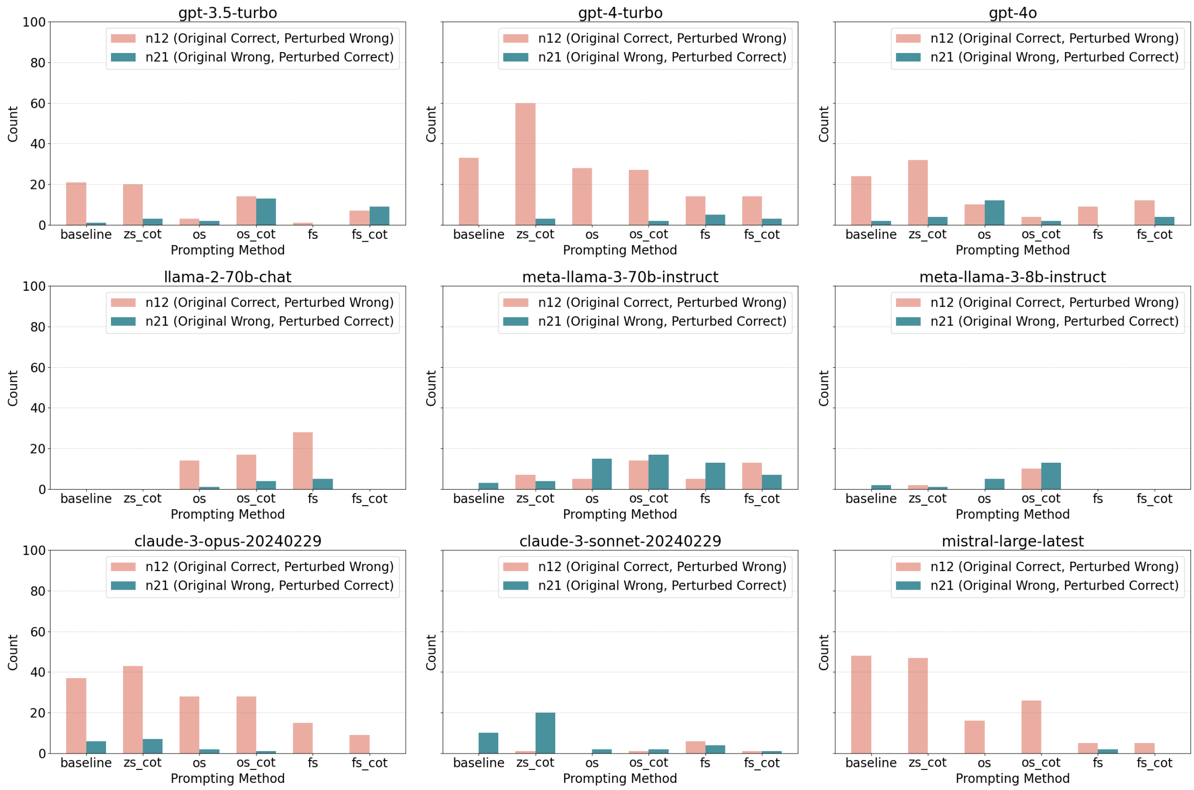

Figure 7: Full experimental results for Hypothesis 4 ($n = 200$). The perturbed problems alternate tokens "All" and "Some" to different but equivalent expressions in syllogisms. We run all different prompt methods. To reject the null, we expect $n12 > n21$. We conclude that most LLMs rely on patterns *"All..., Some..., Some..."* for reasoning about syllogism.

Table 5: Full experimental results for Hypothesis 4

model	prompting method	n_{12}	n_{21}	n^*	z-stat	p-value	reject
gpt-3.5-turbo	baseline	21	1	22	-4.264014	0.000023	True
gpt-3.5-turbo	zs-cot	20	3	23	-3.544745	0.000732	True
gpt-3.5-turbo	os	3	2	5	-0.447214	0.771429	False
gpt-3.5-turbo	os-cot	14	13	27	-0.192450	0.714985	False
gpt-3.5-turbo	fs	1	0	1	-1.000000	0.771429	False
gpt-3.5-turbo	fs-cot	7	9	16	0.500000	1.000000	False
gpt-4-turbo	baseline	33	0	33	-5.744563	0.000000	True
gpt-4-turbo	zs-cot	60	3	63	-7.181325	0.000000	True
gpt-4-turbo	os	28	0	28	-5.291503	0.000001	True
gpt-4-turbo	os-cot	27	2	29	-4.642383	0.000008	True
gpt-4-turbo	fs	14	5	19	-2.064742	0.068654	False
gpt-4-turbo	fs-cot	14	3	17	-2.667892	0.014939	True
gpt-4o	baseline	24	2	26	-4.314555	0.000031	True
gpt-4o	zs-cot	32	4	36	-4.666667	0.000008	True
gpt-4o	os	10	12	22	0.426401	1.000000	False
gpt-4o	os-cot	4	2	6	-0.816497	0.618750	False

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Table 5 – *Continued from previous page*

model	prompting method	n_{12}	n_{21}	n^*	z-stat	p-value	reject
gpt-4o	fs	9	0	9	-3.000000	0.005022	True
gpt-4o	fs-cot	12	4	16	-2.000000	0.079767	False
llama-2-70b-chat	baseline	0	0	0	0.000000	1.000000	False
llama-2-70b-chat	zs-cot	0	0	0	0.000000	1.000000	False
llama-2-70b-chat	os	14	1	15	-3.356586	0.001388	True
llama-2-70b-chat	os-cot	17	4	21	-2.836833	0.008833	True
llama-2-70b-chat	fs	28	5	33	-4.003786	0.000099	True
llama-2-70b-chat	fs-cot	0	0	0	0.000000	1.000000	False
meta-llama-3-70b-instruct	baseline	0	3	3	1.732051	1.000000	False
meta-llama-3-70b-instruct	zs-cot	7	4	11	-0.904534	0.510978	False
meta-llama-3-70b-instruct	os	5	15	20	2.236068	1.000000	False
meta-llama-3-70b-instruct	os-cot	14	17	31	0.538816	1.000000	False
meta-llama-3-70b-instruct	fs	5	13	18	1.885618	1.000000	False
meta-llama-3-70b-instruct	fs-cot	13	7	20	-1.341641	0.263176	False
meta-llama-3-8b-instruct	baseline	0	2	2	1.414214	1.000000	False
meta-llama-3-8b-instruct	zs-cot	2	1	3	-0.577350	0.771429	False
meta-llama-3-8b-instruct	os	0	5	5	2.236068	1.000000	False
meta-llama-3-8b-instruct	os-cot	10	13	23	0.625543	1.000000	False
meta-llama-3-8b-instruct	fs	0	0	0	0.000000	1.000000	False
meta-llama-3-8b-instruct	fs-cot	0	0	0	0.000000	1.000000	False
claude-3-opus-20240229	baseline	37	6	43	-4.727456	0.000006	True
claude-3-opus-20240229	zs-cot	43	7	50	-5.091169	0.000001	True
claude-3-opus-20240229	os	28	2	30	-4.746929	0.000006	True
claude-3-opus-20240229	os-cot	28	1	29	-5.013774	0.000002	True
claude-3-opus-20240229	fs	15	0	15	-3.872983	0.000099	True
claude-3-opus-20240229	fs-cot	9	0	9	-3.000000	0.005022	True
claude-3-sonnet-20240229	baseline	0	10	10	3.162278	1.000000	False
claude-3-sonnet-20240229	zs-cot	1	20	21	4.146140	1.000000	False
claude-3-sonnet-20240229	os	0	2	2	1.414214	1.000000	False
claude-3-sonnet-20240229	os-cot	1	2	3	0.577350	1.000000	False
claude-3-sonnet-20240229	fs	6	4	10	-0.632456	0.656628	False
claude-3-sonnet-20240229	fs-cot	1	1	2	0.000000	1.000000	False
mistral-large-latest	baseline	48	0	48	-6.928203	0.000000	True
mistral-large-latest	zs-cot	47	0	47	-6.855655	0.000000	True
mistral-large-latest	os	16	0	16	-4.000000	0.000055	True
mistral-large-latest	os-cot	26	0	26	-5.099020	0.000001	True
mistral-large-latest	fs	5	2	7	-1.133893	0.436942	False
mistral-large-latest	fs-cot	5	0	5	-2.236068	0.068654	False

D.5 Hypothesis 5

The full experimental results for Hypothesis 5 (sets-original vs. sets-framing) are shown in Figure 8 and Table 6.

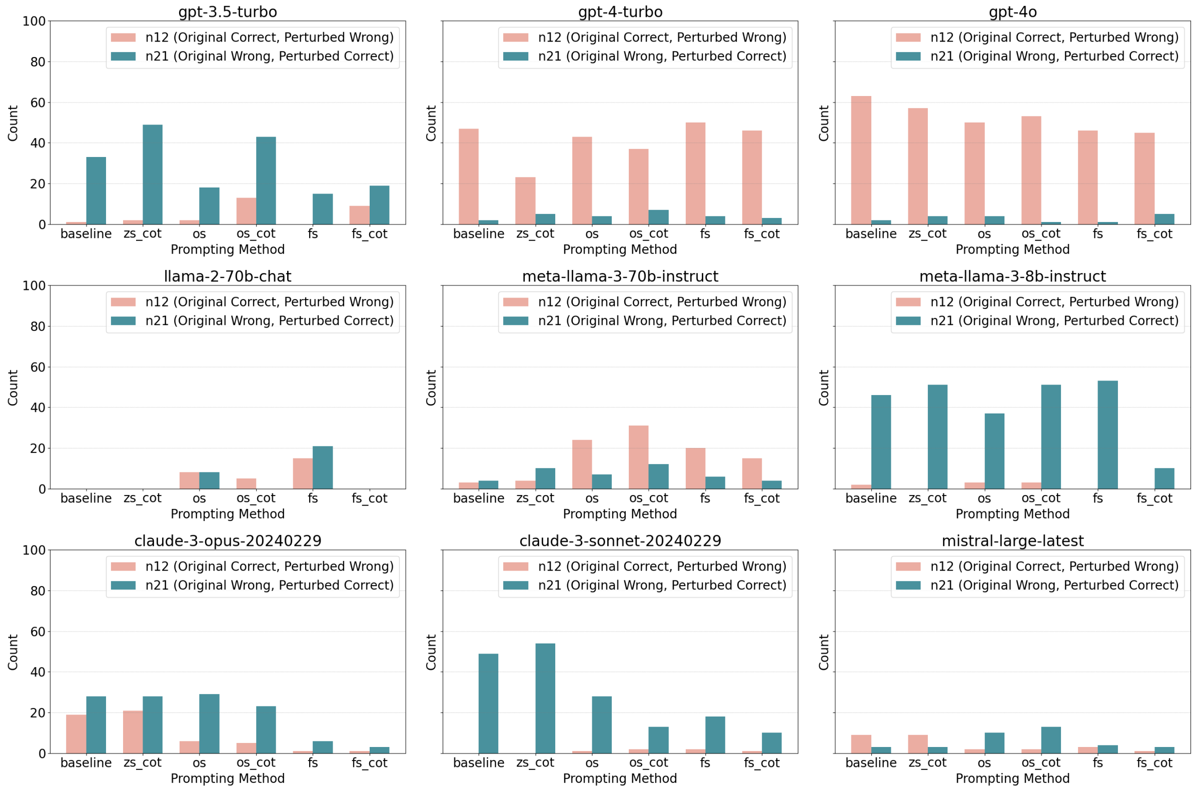

Figure 8: Full experimental results for Hypothesis 5, sets-original vs. sets-framing ($n = 200$). The perturbed problems add the names of trustworthy news agencies and universities to alter the narratives of syllogisms. Both the original and perturbed problems have classic patterns *"All..., Some..., Some..."* already removed to ensure a single confounder for token bias analysis. We run all different prompt methods. To reject the null, we expect $n12 > n21$. We conclude that LLMs might be misled by reputable names irrelevant to the logical structure.

Table 6: Full experimental results for Hypothesis 5(sets-original-random vs sets-original-framing-gold)

model	prompting method	n_{12}	n_{21}	n^*	z-stat	p-value	reject
gpt-3.5-turbo	baseline	1	33	34	5.487955	0.000000	True
gpt-3.5-turbo	zs-cot	2	49	51	6.581316	0.000000	True
gpt-3.5-turbo	os	2	18	20	3.577709	0.000836	True
gpt-3.5-turbo	os-cot	13	43	56	4.008919	0.000143	True
gpt-3.5-turbo	fs	0	15	15	3.872983	0.000143	True
gpt-3.5-turbo	fs-cot	9	19	28	1.889822	0.083532	False
gpt-4-turbo	baseline	47	2	49	-6.428571	0.000000	True
gpt-4-turbo	zs-cot	23	5	28	-3.401680	0.001292	True
gpt-4-turbo	os	43	4	47	-5.688735	0.000000	True
gpt-4-turbo	os-cot	37	7	44	-4.522670	0.000016	True
gpt-4-turbo	fs	50	4	54	-6.259807	0.000000	True
gpt-4-turbo	fs-cot	46	3	49	-6.142857	0.000000	True
gpt-4o	baseline	63	2	65	-7.566119	0.000000	True
gpt-4o	zs-cot	57	4	61	-6.785955	0.000000	True

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Table 6 – *Continued from previous page*

model	prompting method	n_{12}	n_{21}	n^*	z-stat	p-value	reject
gpt-4o	os	50	4	54	-6.259807	0.000000	True
gpt-4o	os-cot	53	1	54	-7.076304	0.000000	True
gpt-4o	fs	46	1	47	-6.563925	0.000000	True
gpt-4o	fs-cot	45	5	50	-5.656854	0.000000	True
llama-2-70b-chat	baseline	0	0	0	0.000000	1.000000	False
llama-2-70b-chat	zs-cot	0	0	0	0.000000	1.000000	False
llama-2-70b-chat	os	8	8	16	0.000000	1.000000	False
llama-2-70b-chat	os-cot	5	0	5	-2.236068	0.086538	False
llama-2-70b-chat	fs	15	21	36	1.000000	0.372495	False
llama-2-70b-chat	fs-cot	0	0	0	0.000000	1.000000	False
meta-llama-3-70b-instruct	baseline	3	4	7	0.377964	1.000000	False
meta-llama-3-70b-instruct	zs-cot	4	10	14	1.603567	0.225501	False
meta-llama-3-70b-instruct	os	24	7	31	-3.053290	0.004074	True
meta-llama-3-70b-instruct	os-cot	31	12	43	-2.897473	0.006553	True
meta-llama-3-70b-instruct	fs	20	6	26	-2.745626	0.010192	True
meta-llama-3-70b-instruct	fs-cot	15	4	19	-2.523573	0.028816	True
meta-llama-3-8b-instruct	baseline	2	46	48	6.350853	0.000000	True
meta-llama-3-8b-instruct	zs-cot	0	51	51	7.141428	0.000000	True
meta-llama-3-8b-instruct	os	3	37	40	5.375872	0.000000	True
meta-llama-3-8b-instruct	os-cot	3	51	54	6.531973	0.000000	True
meta-llama-3-8b-instruct	fs	0	53	53	7.280110	0.000000	True
meta-llama-3-8b-instruct	fs-cot	0	10	10	3.162278	0.003637	True
claude-3-opus-20240229	baseline	19	28	47	1.312785	0.232268	False
claude-3-opus-20240229	zs-cot	21	28	49	1.000000	0.372495	False
claude-3-opus-20240229	os	6	29	35	3.887710	0.000228	True
claude-3-opus-20240229	os-cot	5	23	28	3.401680	0.001292	True
claude-3-opus-20240229	fs	1	6	7	1.889822	0.168750	False
claude-3-opus-20240229	fs-cot	1	3	4	1.000000	0.703125	False
claude-3-sonnet-20240229	baseline	0	49	49	7.000000	0.000000	True
claude-3-sonnet-20240229	zs-cot	0	54	54	7.348469	0.000000	True
claude-3-sonnet-20240229	os	1	28	29	5.013774	0.000001	True
claude-3-sonnet-20240229	os-cot	2	13	15	2.840188	0.011730	True
claude-3-sonnet-20240229	fs	2	18	20	3.577709	0.000836	True
claude-3-sonnet-20240229	fs-cot	1	10	11	2.713602	0.018080	True
mistral-large-latest	baseline	9	3	12	-1.732051	0.187709	False
mistral-large-latest	zs-cot	9	3	12	-1.732051	0.187709	False
mistral-large-latest	os	2	10	12	2.309401	0.056298	False
mistral-large-latest	os-cot	2	13	15	2.840188	0.011730	True
mistral-large-latest	fs	3	4	7	0.377964	1.000000	False
mistral-large-latest	fs-cot	1	3	4	1.000000	0.703125	False

The full experimental results for Hypothesis 5 (framing gold vs. random) are shown in Figure 9 and Table 7.

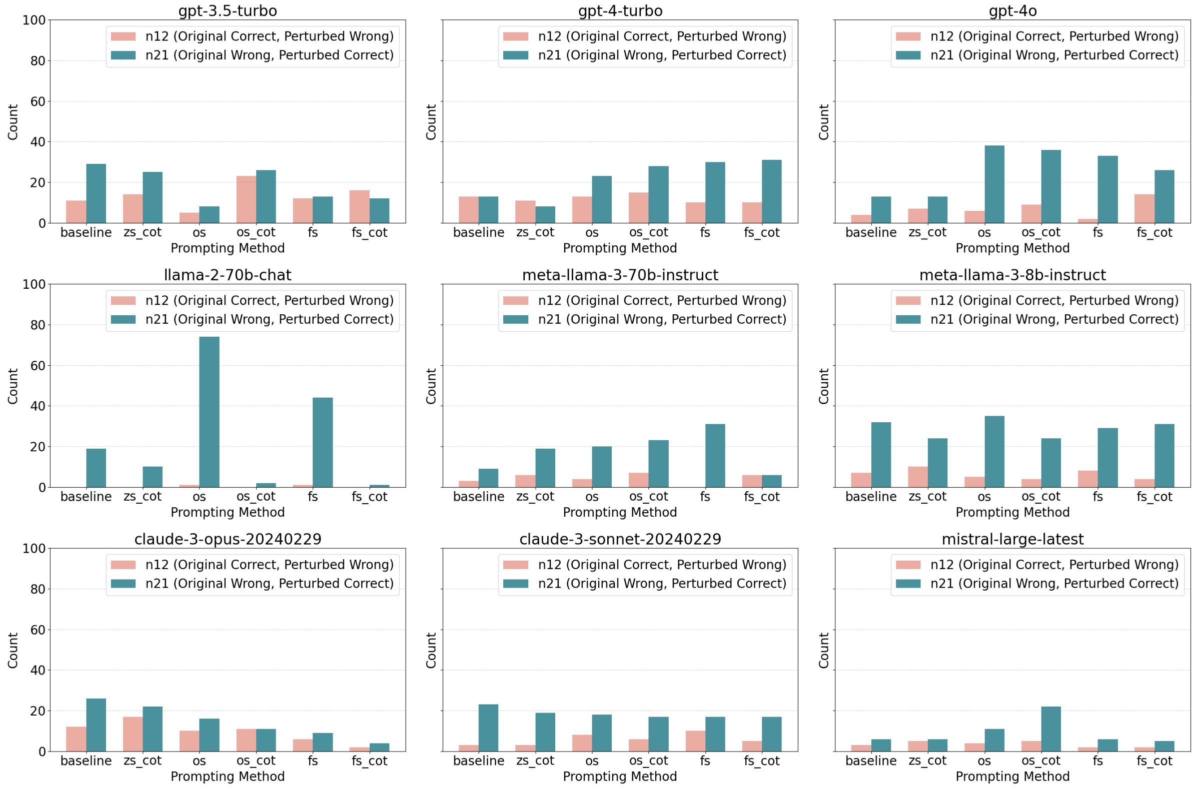

Figure 9: Full experimental results for Hypothesis 5, framing gold vs. random ($n = 200$). The perturbed problems replace reputable news agencies and universities to less trustworthy ones in syllogisms. Both the original and perturbed problems have classic patterns *"All..., Some..., Some..."* already removed to ensure a single confounder for token bias analysis. We run all different prompt methods. To reject the null, we expect $n12 < n21$. We conclude that LLMs might be less likely to be misled by less reputable names. However, such performance shifts should not happen to genuine reasoners, because the names of these entities do not affect the logical essence.

Table 7: Full experimental results for Hypothesis 5 (sets-framing gold vs. random)

model	prompting method	n_{12}	n_{21}	n^*	z-stat	p-value	reject
gpt-3.5-turbo	baseline	11	29	40	2.846050	0.011382	True
gpt-3.5-turbo	zs-cot	14	25	39	1.761410	0.136166	False
gpt-3.5-turbo	os	5	8	13	0.832050	0.713113	False
gpt-3.5-turbo	os-cot	23	26	49	0.428571	0.767760	False
gpt-3.5-turbo	fs	12	13	25	0.200000	0.927346	False
gpt-3.5-turbo	fs-cot	16	12	28	-0.755929	0.596799	False
gpt-4-turbo	baseline	13	13	26	0.000000	1.000000	False
gpt-4-turbo	zs-cot	11	8	19	-0.688247	0.760233	False
gpt-4-turbo	os	13	23	36	1.666667	0.161292	False
gpt-4-turbo	os-cot	15	28	43	1.982481	0.092843	False
gpt-4-turbo	fs	10	30	40	3.162278	0.004696	True
gpt-4-turbo	fs-cot	10	31	41	3.279649	0.003609	True
gpt-4o	baseline	4	13	17	2.182821	0.092843	False
gpt-4o	zs-cot	7	13	20	1.341641	0.384095	False
gpt-4o	os	6	38	44	4.824182	0.000015	True
gpt-4o	os-cot	9	36	45	4.024922	0.000337	True
gpt-4o	fs	2	33	35	5.239956	0.000002	True

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Table 7 – *Continued from previous page*

model	prompting method	n_{12}	n_{21}	n^*	z-stat	p-value	reject
gpt-4o	fs-cot	14	26	40	1.897367	0.104003	False
llama-2-70b-chat	baseline	0	19	19	4.358899	0.000029	True
llama-2-70b-chat	zs-cot	0	10	10	3.162278	0.005551	True
llama-2-70b-chat	os	1	74	75	8.429314	0.000000	True
llama-2-70b-chat	os-cot	0	2	2	1.414214	0.637718	False
llama-2-70b-chat	fs	1	44	45	6.410062	0.000000	True
llama-2-70b-chat	fs-cot	0	1	1	1.000000	1.000000	False
meta-llama-3-70b-instruct	baseline	3	9	12	1.732051	0.231876	False
meta-llama-3-70b-instruct	zs-cot	6	19	25	2.600000	0.022882	True
meta-llama-3-70b-instruct	os	4	20	24	3.265986	0.004696	True
meta-llama-3-70b-instruct	os-cot	7	23	30	2.921187	0.009415	True
meta-llama-3-70b-instruct	fs	0	31	31	5.567764	0.000000	True
meta-llama-3-70b-instruct	fs-cot	6	6	12	0.000000	1.000000	False
meta-llama-3-8b-instruct	baseline	7	32	39	4.003204	0.000337	True
meta-llama-3-8b-instruct	zs-cot	10	24	34	2.400980	0.038026	True
meta-llama-3-8b-instruct	os	5	35	40	4.743416	0.000019	True
meta-llama-3-8b-instruct	os-cot	4	24	28	3.779645	0.000707	True
meta-llama-3-8b-instruct	fs	8	29	37	3.452379	0.002308	True
meta-llama-3-8b-instruct	fs-cot	4	31	35	4.563833	0.000034	True
claude-3-opus-20240229	baseline	12	26	38	2.271100	0.049984	True
claude-3-opus-20240229	zs-cot	17	22	39	0.800641	0.586163	False
claude-3-opus-20240229	os	10	16	26	1.176697	0.358975	False
claude-3-opus-20240229	os-cot	11	11	22	0.000000	1.000000	False
claude-3-opus-20240229	fs	6	9	15	0.774597	0.728687	False
claude-3-opus-20240229	fs-cot	2	4	6	0.816497	0.773438	False
claude-3-sonnet-20240229	baseline	3	23	26	3.922323	0.000431	True
claude-3-sonnet-20240229	zs-cot	3	19	22	3.411211	0.003300	True
claude-3-sonnet-20240229	os	8	18	26	1.961161	0.092843	False
claude-3-sonnet-20240229	os-cot	6	17	23	2.293659	0.072048	False
claude-3-sonnet-20240229	fs	10	17	27	1.347151	0.274523	False
claude-3-sonnet-20240229	fs-cot	5	17	22	2.558409	0.038026	True
mistral-large-latest	baseline	3	6	9	1.000000	0.637718	False
mistral-large-latest	zs-cot	5	6	11	0.301511	1.000000	False
mistral-large-latest	os	4	11	15	1.807392	0.193859	False
mistral-large-latest	os-cot	5	22	27	3.271652	0.003609	True
mistral-large-latest	fs	2	6	8	1.414214	0.410773	False
mistral-large-latest	fs-cot	2	5	7	1.133893	0.596799	False

D.6 Hypothesis 6

The full experimental results for Hypothesis 6 are shown in Figure 10 and Table 8.

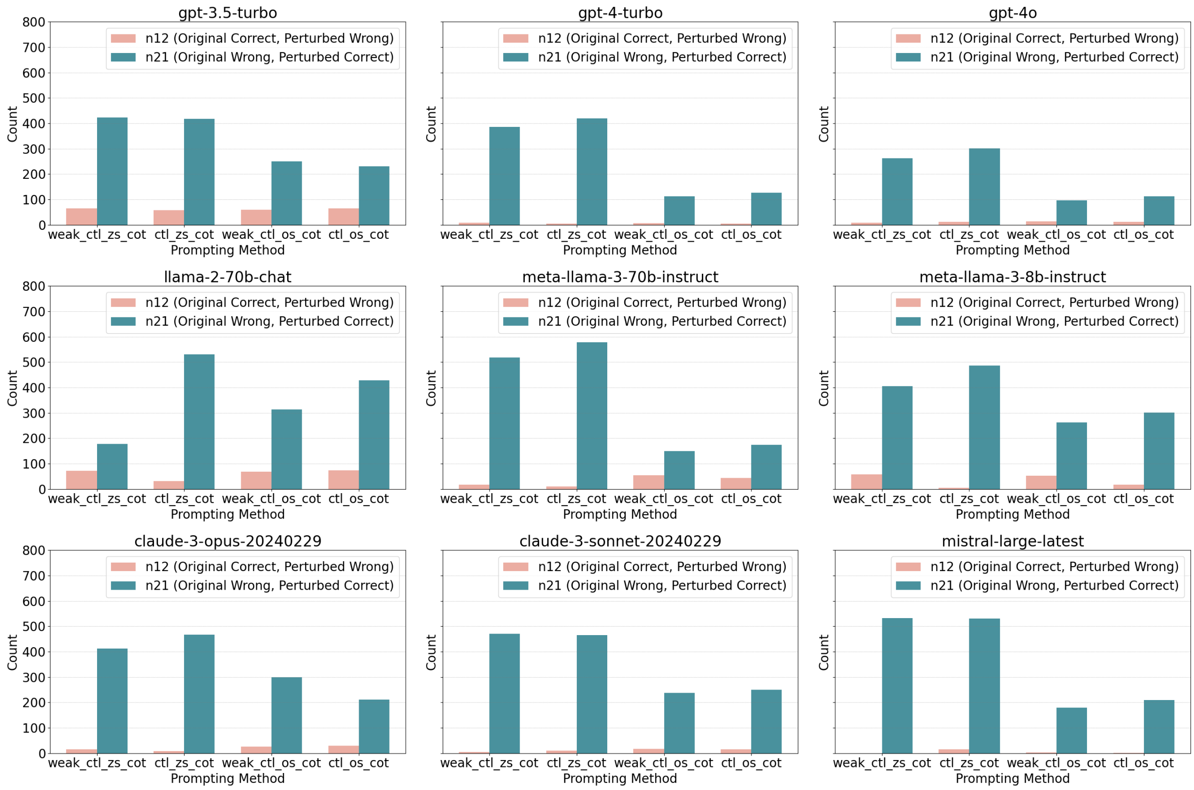

Figure 10: Full experimental results for Hypothesis 6 ($n = 800$). The perturbed problems leak hint tokens, either weak or strong hints in problem statements. We run zero-shot and one-shot prompt methods. To reject the null, we expect $n12 < n21$. We conclude that LLMs still heavily rely on hint tokens for solving logical fallacy problems well.

Table 8: Full experimental results for Hypothesis 6

model	prompting method	n_{12}	n_{21}	n^*	z-stat	p-value	reject
gpt-3.5-turbo	weak-control-zs-cot	64	423	487	16.267843	0.000000	True
gpt-3.5-turbo	control-zs-cot	57	417	474	16.535348	0.000000	True
gpt-3.5-turbo	weak-control-os-cot	60	250	310	10.791275	0.000000	True
gpt-3.5-turbo	control-os-cot	64	230	294	9.681317	0.000000	True
gpt-4-turbo	weak-control-zs-cot	8	386	394	19.043365	0.000000	True
gpt-4-turbo	control-zs-cot	4	420	424	20.202746	0.000000	True
gpt-4-turbo	weak-control-os-cot	6	113	119	9.808674	0.000000	True
gpt-4-turbo	control-os-cot	4	126	130	10.700108	0.000000	True
gpt-4o	weak-control-zs-cot	8	262	270	15.457948	0.000000	True
gpt-4o	control-zs-cot	11	301	312	16.418017	0.000000	True
gpt-4o	weak-control-os-cot	13	97	110	8.009086	0.000000	True
gpt-4o	control-os-cot	12	112	124	8.980265	0.000000	True
llama-2-70b-chat	weak-control-zs-cot	72	177	249	6.654105	0.000000	True
llama-2-70b-chat	control-zs-cot	32	531	563	21.030343	0.000000	True
llama-2-70b-chat	weak-control-os-cot	69	313	382	12.484126	0.000000	True
llama-2-70b-chat	control-os-cot	73	429	502	15.889058	0.000000	True

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Table 8 – *Continued from previous page*

model	prompting method	n_{12}	n_{21}	n^*	z-stat	p-value	reject
meta-llama-3-70b-instruct	weak-control-zs-cot	17	518	535	21.660119	0.000000	True
meta-llama-3-70b-instruct	control-zs-cot	10	579	589	23.445237	0.000000	True
meta-llama-3-70b-instruct	weak-control-os-cot	54	150	204	6.721344	0.000000	True
meta-llama-3-70b-instruct	control-os-cot	44	174	218	8.804711	0.000000	True
meta-llama-3-8b-instruct	weak-control-zs-cot	57	405	462	16.190425	0.000000	True
meta-llama-3-8b-instruct	control-zs-cot	4	487	491	21.797485	0.000000	True
meta-llama-3-8b-instruct	weak-control-os-cot	53	263	316	11.813423	0.000000	True
meta-llama-3-8b-instruct	control-os-cot	18	301	319	15.844958	0.000000	True
claude-3-opus-20240229	weak-control-zs-cot	15	412	427	19.212177	0.000000	True
claude-3-opus-20240229	control-zs-cot	9	467	476	20.992396	0.000000	True
claude-3-opus-20240229	weak-control-os-cot	26	299	325	15.143315	0.000000	True
claude-3-opus-20240229	control-os-cot	30	212	242	11.699403	0.000000	True
claude-3-sonnet-20240229	weak-control-zs-cot	5	470	475	21.335663	0.000000	True
claude-3-sonnet-20240229	control-zs-cot	10	466	476	20.900726	0.000000	True
claude-3-sonnet-20240229	weak-control-os-cot	17	238	255	13.839557	0.000000	True
claude-3-sonnet-20240229	control-os-cot	16	250	266	14.347461	0.000000	True
mistral-large-latest	weak-control-zs-cot	0	533	533	23.086793	0.000000	True
mistral-large-latest	control-zs-cot	15	530	545	22.060176	0.000000	True
mistral-large-latest	weak-control-os-cot	3	179	182	13.045988	0.000000	True
mistral-large-latest	control-os-cot	1	209	210	14.353364	0.000000	True