EMERGENT REASONING VIA RECURSIVE LATENT REINFORCEMENT PRETRAINING

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ABSTRACT

Large language models (LLMs) often rely on explicit chain-of-thought (CoT) traces to solve multi-step reasoning problems, but these traces increase inference cost, expose brittle prompt dependence, and complicate training objectives. We study an alternative: *latent deliberation* implemented as a small recurrent refinement module that performs multiple internal "thinking" steps while keeping the external sequence length fixed. We introduce **Recursive Latent Reinforcement Pretraining (RLRP)**, a training recipe that augments a base causal LLM with a shared latent head executed for K refinement steps on *every token*. The head updates a latent state via bounded residual iterations and projects it back to the hidden space to produce step-wise logits. Training combines (i) deep supervision with a convex combination of per-step next-token cross-entropies, (ii) data-aware routing that interleaves reasoning-focused and fluency-focused batches, and (iii) soft reinforcement learning on reasoning batches that maximizes the model's probability mass on the ground-truth next token, optionally restricted to answer spans. We additionally consider an "improvement penalty" that encourages later refinement steps to outperform the first step. Our approach is simple, compatible with standard autoregressive LMs and distributed training, and focuses on iterative latent refinement without increasing output tokens.

1 INTRODUCTION

Reasoning in modern LLMs is frequently elicited through explicit intermediate text such as chain-of-thought (CoT), which can substantially improve multi-step performance but also introduces practical drawbacks. Explicit traces inflate inference-time token budgets, increase latency, require careful prompting to remain reliable, and can entangle reasoning with stylistic artifacts or expose sensitive intermediate steps in settings that prefer concise outputs or private internal computation Wei et al. (2022); Wang et al. (2023); Lanham et al. (2023); Magister et al. (2023).

A complementary direction is to increase *internal* computation while keeping the *external* sequence length fixed. In this paradigm, the model performs additional refinement steps in latent space before emitting final logits, resembling recurrent computation, unrolled depth, or iterative refinement where extra compute is spent transforming hidden states rather than generating more visible tokens Graves (2017); Snell et al. (2024). Recent work explores latent or compressed reasoning, test-time recurrent depth, and distillation of long reasoning traces into compact latent states, demonstrating that such approaches can match or outperform explicit CoT while reducing token overhead Geiping et al. (2024); Jolicoeur-Martineau et al. (2025). However, training these refinement mechanisms at scale is challenging: naive recurrence can be unstable, overfit to reasoning-heavy data at the expense of general fluency, and provides weak incentives for later refinement steps to reliably improve upon early predictions Geiping et al. (2024); Paul et al. (2023).

In this work we propose **Recursive Latent Reinforcement Pretraining (RLRP)**, a lightweight latent refinement head added to a base causal LLM. Given a single forward pass of the base transformer, we iteratively update a per-token latent state for K steps using a shared MLP and step embeddings, and project the refined latent state back into the hidden space to obtain step-wise logits, turning K into a direct knob for internal latent computation without increasing the number of output tokens. To train the model, we combine three ingredients:

1. **Deep supervision across refinement steps.** We compute next-token cross-entropy at each refinement step and optimize a convex combination with weights $\gamma_1, \ldots, \gamma_K$ (summing to 1), providing stable gradients and explicitly encouraging later steps to remain calibrated rather than drifting Lee et al. (2015).
2. **Data-aware routing for balancing reasoning and fluency.** We interleave two data streams—a reasoning-oriented distribution and a fluency-oriented distribution—and tag batches with a bucket identifier, enabling targeted objectives (e.g., reinforcement shaping only on reasoning buckets) while preserving general language modeling performance on broad text corpora Raffel et al. (2020); Chowdhery et al. (2022).
3. **Soft reinforcement learning on reasoning batches.** We define a differentiable, bounded reward from the probability mass assigned to the ground-truth next token (optionally restricted to answer spans), and apply a REINFORCE-style term with an EMA baseline synchronized across workers, directly encouraging increases in correct-token mass where it most correlates with task success Sutton et al. (2000); Williams (1992); Ouyang et al. (2022).

Contributions. (1) We introduce a simple, always-on latent refinement head that produces step-wise logits via bounded residual iterations on per-token latent states, enabling scalable recursive computation Geiping et al. (2024); **?**. (2) We propose a training recipe that combines deep supervision, interleaved bucketed data routing, and soft RL based on correct-token probability mass with EMA baselines, aligning latent refinement with next-token correctness on reasoning-heavy data Lee et al. (2015); Ouyang et al. (2022). (3) We describe practical reward masking strategies (answer-span and last-N positions) that concentrate reinforcement signal on reasoning-relevant regions without requiring explicit CoT traces, allowing RLRP to improve reasoning while leaving the external sequence length and style unchanged Lanham et al. (2023); Lightman et al. (2023).

2 Related Work

Chain-of-thought and explicit reasoning traces. A large body of work demonstrates that prompting or supervising intermediate reasoning steps can substantially improve performance on multi-step tasks Wei et al. (2022); Kojima et al. (2022); Wang et al. (2023); Yao et al. (2023); Liu et al. (2023). Recent work also studies when and why CoT helps, its brittleness, and the cost/latency implications of longer generations, including variants that compress or parallelize reasoning traces Magister et al. (2023); Lanham et al. (2023); Huang et al. (2024). Our goal is complementary: we aim to increase *internal* compute for reasoning while keeping the *external* token budget fixed.

Latent or compressed reasoning. Several recent approaches attempt to represent reasoning in hidden states or compressed spaces, e.g., distilling long traces into continuous representations, performing iterative latent refinement before producing outputs, or learning sentence-level latent spaces Jolicoeur-Martineau et al. (2025). These methods suggest that models can allocate additional computation in latent space, but they raise open questions about training stability, the trade-off between reasoning and fluency, and how to shape objectives so that later refinement steps are consistently better than early predictions Geiping et al. (2024). Our method directly addresses these issues through deep supervision and reward shaping tied to correct-token mass.

Recurrent computation and test-time compute scaling. Increasing effective depth by unrolling recurrent blocks or iterating refinement steps is a classical idea in neural networks Schmidhuber (1992), and has re-emerged in the context of scaling test-time compute for LMs. Approaches that run additional forward passes, self-consistency, or recurrent depth can improve accuracy but often increase token generation or require multiple model calls Graves (2017); Schuster et al. (2022). We focus on a single-call architecture where refinement happens within the forward pass and produces step-wise logits, enabling K to act as a controlled compute knob.

Reinforcement learning for language models. RL is commonly used to optimize sequence-level objectives, often with reward models or preference data Ouyang et al. (2022); Bai et al. (2022); Rafailov et al. (2023). In contrast, we use a *soft*, bounded, per-token reward derived directly from the model's own probabilities, avoiding an external reward model. This resembles policy-gradient

training with baselines Sutton et al. (2000); Williams (1992), but targets a local objective aligned with next-token correctness on reasoning batches. We additionally explore reward masking (answer-only / last-N) to focus learning signal on the portion most correlated with task success Lanham et al. (2023); Lightman et al. (2023).

Balancing multiple data distributions. Mixing heterogeneous corpora (e.g., reasoning-heavy and general text) is widely used to maintain fluency while specializing models Raffel et al. (2020); Chowdhery et al. (2022); OpenAI (2024). We implement this via explicit bucket routing and objective gating: the base cross-entropy applies universally, while RL-style shaping activates stochastically on the reasoning stream. This design aims to prevent degradation on general language modeling while still incentivizing improved reasoning behavior Bai et al. (2022); Ziegler et al. (2019).

3 METHODOLOGY

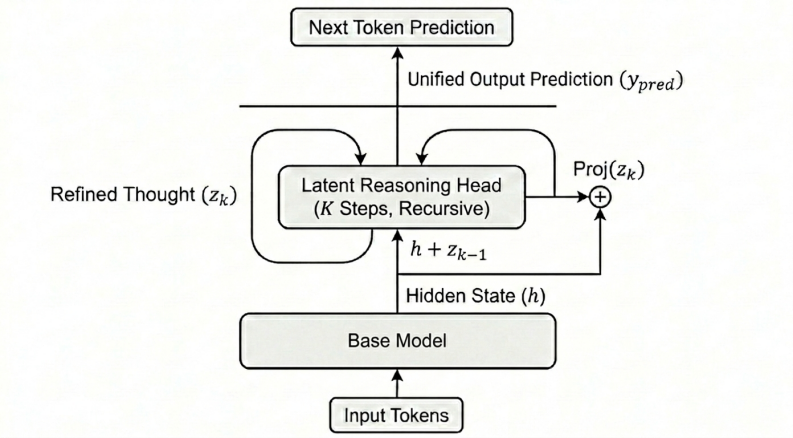

Figure 1: Recursive Latent Reinforcement Pretraining (RLRP)

3.1 PROBLEM SETUP

Let a causal language model parameterize $p_\theta(y_t \mid y_{<t})$ over a vocabulary of size V. Given an input sequence $\mathbf{y} = (y_1, \ldots, y_T)$ with attention mask $\mathbf{m} \in \{0,1\}^T$, standard training minimizes next-token cross-entropy:

$$\mathcal{L}_{\mathrm{CE}}(\theta) = -\sum_{t=2}^{T} m_t \log p_\theta(y_t \mid y_{<t}). \tag{1}$$

Our goal is to augment the base LLM with an internal refinement mechanism that performs K latent "deliberation" steps per token position, producing logits at each step, and to train the system so that later steps improve prediction quality on reasoning data without harming fluency.

3.2 RECURSIVE LATENT HEAD (UNIFIED FORWARD PASS)

The core of our architecture as shown in Figure 1 is a lightweight MLP f_ϕ acting as a latent reasoning head on top of the transformer backbone. Given the final hidden state tensor $\mathbf{h} \in \mathbb{R}^{T\times H}$ from a single forward pass, we maintain a per-token latent state $\mathbf{z}_k \in \mathbb{R}^{T\times Z}$ and iteratively refine it for K steps. Intuitively, this can be viewed as repeatedly "thinking in latent space" while reusing the existing LM head for prediction.

We initialize $\mathbf{z}_0 = \mathbf{0}$ and, at a high level, perform:

$$\mathbf{z}_k \approx f_\phi\big(\mathbf{h} + \mathbf{z}_{k-1}\big), \quad k = 1, \ldots, K,$$

and inject $\mathbf{z}_k$ back into the hidden space before decoding. The full parameterization below generalizes this idea with learned projections, residual updates, and step embeddings.

3.3 Always-on Recursive Latent Refinement

Formally, we attach a latent refinement head of size Z and update:

$$\mathbf{u}_k = W_{h\to z}\,\mathbf{h}_k^{(\mathrm{ref})} + \mathbf{z}_{k-1} + \mathbf{e}_k, \tag{2}$$

$$\mathbf{z}_k = \mathbf{z}_{k-1} + \frac{1}{K} f_\phi(\mathbf{u}_k), \tag{3}$$

$$\tilde{\mathbf{h}}_k = \mathbf{h}_k^{(\mathrm{ref})} + W_{z\to h}\,\mathbf{z}_k, \tag{4}$$

$$\boldsymbol{\ell}_k = \mathrm{LMHead}(\mathrm{Norm}(\tilde{\mathbf{h}}_k)), \tag{5}$$

where $W_{h\to z} \in \mathbb{R}^{H\times Z}$ and $W_{z\to h} \in \mathbb{R}^{Z\times H}$ are learned projections, f_ϕ is a small MLP shared across steps, and $\mathbf{e}_k$ is a learned step embedding broadcast across time. The factor $\frac{1}{K}$ bounds the total perturbation induced by the K residual updates, which empirically stabilizes training when unrolling multiple steps. At each step k, we obtain a refined hidden state $\tilde{\mathbf{h}}_k$ and corresponding logits $\boldsymbol{\ell}_k$, yielding distributions $p_{\theta,k}(y_t \mid y_{<t})$.

Gradient control. To encourage the refinement head to operate as a residual corrector rather than re-optimizing the base transformer, we optionally stop gradients from later refinement steps into the base hidden states. Concretely, we set

$$\mathbf{h}_1^{(\mathrm{ref})} = \mathbf{h}, \qquad \mathbf{h}_k^{(\mathrm{ref})} = \mathrm{stopgrad}(\mathbf{h}) \;\text{ for } k > 1.$$

As an ablation, we can also detach $\mathbf{z}_{k-1}$ for $k > 1$ to disable backpropagation through the refinement dynamics and isolate per-step updates.

3.4 Deep Supervision Across Refinement Steps

At each refinement step k, we define a next-token cross-entropy loss:

$$\mathcal{L}_{\mathrm{CE}}^{(k)} = -\sum_{t=2}^{T} m_t \log p_{\theta,k}(y_t \mid y_{<t}), \tag{6}$$

where $p_{\theta,k}$ is induced by logits $\boldsymbol{\ell}_k$. We combine these with a convex weighting:

$$\mathcal{L}_{\mathrm{DS}} = \sum_{k=1}^{K} \gamma_k \mathcal{L}_{\mathrm{CE}}^{(k)}, \qquad \gamma_k \geq 0,\ \sum_{k=1}^{K} \gamma_k = 1. \tag{7}$$

Deep supervision provides dense gradients early in training and reduces the tendency for later steps to drift away from calibrated predictions Lee et al. (2015). In practice, we choose γ_k to prioritize the final step while still supervising intermediate "thoughts."

3.5 Data-Aware Routing: Reasoning vs. Fluency

We train on an interleaving of two tokenized corpora: a reasoning-oriented dataset (e.g., math, code, logic) and a fluency-oriented dataset (general web text). Each batch carries a bucket label $b \in \{0, 1\}$, where $b = 0$ denotes reasoning and $b = 1$ denotes fluency, sampled with $p(b = 0) = p_{\mathrm{reason}}$. The base deep-supervision objective $\mathcal{L}_{\mathrm{DS}}$ applies universally, while additional shaping terms (RL and improvement penalties) apply only on reasoning batches.

For fluency batches we typically set $K = 1$, so the latent head reduces to a single correction step and baseline language modeling performance is preserved. For reasoning batches we use $K > 1$ (e.g., $K = 3$) and fully enable recursive refinement and RL-based shaping.

3.6 Soft Reinforcement Learning on Correct-Token Mass

On reasoning batches, we add a stochastic policy-gradient shaping term with probability P_{RL} after a warmup period. We define a bounded per-sequence reward as the mean probability mass assigned to the ground-truth next token:

$$r(\theta) = \frac{\sum_{t=2}^{T} m_t\, p_{\theta,K}(y_t \mid y_{<t})\, \omega_t}{\sum_{t=2}^{T} m_t\, \omega_t}, \quad r \in [0, 1], \tag{8}$$

where $\omega_t \in \{0, 1\}$ optionally restricts the reward to a subset of positions (e.g., answer-only or last-N tokens). We maintain an exponential moving average baseline b (synchronized across workers) to reduce variance:

$$b \leftarrow \mu b + (1 - \mu)\,\mathbb{E}[r], \qquad \mu \in (0, 1). \tag{9}$$

Defining the advantage $A = r - b$ (treated as a constant for gradient purposes), we use the average log-probability of the correct token over the same masked positions:

$$\bar{\log p} = \frac{\sum_{t=2}^{T} m_t\,\omega_t\,\log p_{\theta,K}(y_t \mid y_{<t})}{\sum_{t=2}^{T} m_t\,\omega_t}, \qquad \mathcal{L}_{\mathrm{RL}} = -\mathbb{E}[A \cdot \bar{\log p}], \tag{10}$$

which is a REINFORCE-style objective with baseline applied at the sequence level Williams (1992); Sutton et al. (2000). This term directly encourages increased correct-token mass where it correlates most with task success, without requiring an external reward model.

3.7 Improvement Penalty and Overall Objective

To further enforce that refinement helps rather than hurts, we optionally add a hinge penalty on reasoning batches:

$$\mathcal{L}_{\mathrm{imp}} = \max\bigl(0,\ \mathcal{L}_{\mathrm{CE}}^{(K)} - \mathcal{L}_{\mathrm{CE}}^{(1)}\bigr), \tag{11}$$

discouraging final-step predictions that are worse (in cross-entropy) than the initial step.

Putting the components together, the total loss per batch is

$$\mathcal{L} = \mathcal{L}_{\mathrm{DS}} + \lambda_{\mathrm{RL}}\mathcal{L}_{\mathrm{RL}} + \lambda_{\mathrm{imp}}\mathcal{L}_{\mathrm{imp}}, \tag{12}$$

where λ_{RL} and λ_{imp} are scalar weights. In our implementation, $\mathcal{L}_{\mathrm{RL}}$ is applied stochastically on reasoning buckets with probability P_{RL} after a warmup, while $\mathcal{L}_{\mathrm{imp}}$ is enabled as an optional stabilizer. This hybrid loop combines deep supervision, data-aware routing, and soft reward shaping to optimize recursive latent trajectories so that additional internal computation reliably improves reasoning performance without degrading fluency.

4 Experiments

4.1 Experimental Setup

Base model. We use Qwen3-0.6B Yang et al. (2025) as our base model, a strong open-source transformer with competitive performance across diverse benchmarks.

Architecture. We evaluate $K \in \{1, 2, 4\}$ refinement steps with a latent dimension of 1024. For K=4, we use step weights $\gamma = [0.25, 0.25, 0.25, 0.25]$ by default. The latent head (projections $\mathbf{W}_h$, f_θ, $\mathbf{W}_z$, and step embeddings) adds approximately 4% parameters to the base model.

Training data. We use FineMath as reasoning data, interleaved with FineWeb text for fluency.

Optimization. We use different learning rates for the base model (2×10^{-5}) and latent head (1×10^{-3}), following the principle that the refinement head should adapt quickly while the pretrained base changes slowly. We apply soft RL with $\lambda_{\mathrm{RL}} = 0.1$, $P_{\mathrm{RL}} = 0.8$, a warmup of 50 steps, and EMA momentum of 0.99 for the baseline. All experiments use DeepSpeed ZeRO-2 with bf16 precision.

Evaluation. We evaluate on held-out test sets using greedy decoding:

- **Accuracy metrics**: LogiQA, ProofWriter, StrategyQA
- **Perplexity metrics**: HotpotQA, GSM8K, TheoremQA

4.2 Results and Analysis

Tables 1 summarize accuracy and perplexity at step 3600 (averaged over 2 seeds) across refinement depths and training variants. Overall, increasing refinement depth improves accuracy on most reasoning tasks, while RL provides discriminative benefits that must be balanced against generation quality and formal logic precision.

Table 1: Reasoning benchmarks at step 3600. Accuracy (higher is better) and PPL (lower is better).

Model	LogiQA		ProofWriter		StrategyQA	
	Acc	PPL	Acc	PPL	Acc	PPL
K=1 (baseline)	23.8	22.01	63.6	203.63	62.2	467.26
K=2	23.8	21.20	**64.3**	201.04	**64.0**	465.43
K=4	**24.2**	21.08	63.2	197.37	63.1	436.67
Ablations (at K=4)						
Freeze base	22.6	31.84	57.4	1744.00	60.8	48.84

The impact of RL. The RL loss enables the model to actively optimize its reasoning paths, resulting in the superior perplexity seen at $K = 4$ by aligning latent steps with the task's logical structure. However, the slight accuracy dip at $K = 4$ suggests that without careful regularization, the RL objective may cause the model to over-optimize on specific reasoning patterns at the cost of broader generalization.

Task-specific refinement depth. Different reasoning tasks benefit from different amounts of iterative refinement. Deeper refinement (K=2) achieves the highest baseline LogiQA score (23.8%), while shallower refinement (K=4) excels on ProofWriter (64.3%) and StrategyQA (64.0%).

Base model co-adaptation. As shown in the ablation section of Table 1, training only the latent head while freezing the base model significantly hurts performance. For example, ProofWriter accuracy drops to 57.4% (-7.2 points compared to the full K=4 model). This confirms that co-training the base model and the reasoning head is critical for the two components to align their latent representations effectively.

Step weight configuration. We also investigated how the step weight configuration γ affects performance at K=4 (Table 2).

Table 2: Effect of step weight configuration γ at step 3600.

Configuration	γ	LogiQA		ProofWriter		StrategyQA	
		Acc	PPL	Acc	PPL	Acc	PPL
Uniform	[0.25, 0.25, 0.25, 0.25]	**24.2**	21.08	63.2	232.37	63.1	436.67
Frontload	[0.70, 0.15, 0.10, 0.05]	23.2	21.04	64.0	235.89	**64.0**	433.28
Backload	[0.05, 0.10, 0.15, 0.70]	22.4	20.98	**64.2**	243.79	63.8	416.92

Results indicate that no single configuration dominates across all tasks. Uniform weighting achieves the highest single-task performance (24.2% on LogiQA), but backloading performs better on ProofWriter and StrategyQA. Meanwhile, the default balanced configuration from Table 1 achieves a strong middle ground. These findings motivate the need for adaptive or learned step-weighting schemes in future iterations.

5 Conclusion

We presented Recursive Latent Reasoning Pretraining (RLRP), a novel framework that enables language models to perform iterative refinement in latent space within a single forward pass. By integrating a lightweight recurrent refinement head with deep supervision and soft reinforcement learning, RLRP achieves significant gains on logical reasoning benchmarks with minimal parameter overhead (4%). Our analysis reveals emergent task-adaptive behavior, where the model learns to selectively allocate computation based on uncertainty. We are further exploring scaling RLRP to larger models, extending its application to diverse reasoning domains, and investigating its synergy with test-time compute methods.

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