## Causal drivers of dynamic networks

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Dynamic network models describe temporal interactions between social actors and have been widely applied in detecting financial fraud, tracking the spread of invasive species and analyzing the dissemination of misinformation. A fundamental question in these domains is identifying the causal drivers underlying these processes. However, existing network models remain purely descriptive, relying on correlative structures rather than causal inference. In this paper, we propose a causal extension of dynamic network modeling based on structural relational event models (REMs). REMs represent instantaneous interactions as discrete temporal events, where a sender initiates an interaction directed toward one or more receivers. Formally, we define a dynamic network as a multivariate counting process  $N = \{N_{sr}(t) \mid N_{sr}(t) \in \mathbb{N}_0\}$ , along with a covariate process  $X = \{X_{sr}(t) \mid X_{sr}(t) \in \mathbb{R}^p\}$ , where the subscripts (s, r) refer to the directed edges of the network. Our objective is to determine which of the covariates X causally influence the dynamic network N, making this a causal discovery problem.

Recent advances in causal discovery exploit the invariance property of causal models under interventions, that is the fact that the conditional distribution of a target variable remains unchanged across different covariate distributions. In the case of REMs, we define an invariant causal prediction method using the fact that partial likelihood inference of a dynamic social network is equivalent with logistic regression. In particular, consider the true causal model as dependent on a function  $f_{PA}$ describing the causal effects on N of its causal parents  $X_{PA} \subset \mathbf{X}$ . We can identify this function, up to a zero set, using two key conditions: (i) the causal model  $f_{PA}$  solves an expected likelihood maximization conditional on the true causal parents  $X_{PA}$ ; (ii) its associated Pearson risk is equal to 1, i.e.

$$\mathbb{E}_{\boldsymbol{X},N}\left[\frac{(Y-\dot{b}(\Delta_i f_{\text{PA}}))^2}{\ddot{b}(\Delta_i f_{\text{PA}})}\right] = 1,$$

where  $\dot{b}(\cdot)$  and  $\ddot{b}(\cdot)$  are the first and second derivative of the cumulant generator function  $b(\cdot) = \log(1 + \exp(\cdot))$ . The empirical analogue of these two conditions provides a consistent causal discovery algorithm.

Unlike existing invariance prediction methods, this approach enables causal discovery from a single observational dataset, eliminating the need for multiple distinct environments. We validate the method through simulations and demonstrate its real-world applicability by analyzing bike-sharing data from Washington D.C. in July 2023.

Keywords: Invariant Causal Prediction, Structural Causal Model, Pearson Risk, Generalized Additive Model

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