SOLOS: SPARSE OPTIMIZATION FOR LONG SEQUENCES IN CONTEXT COMPRESSION ENHANCED LLMS

Anonymous authors

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ABSTRACT

Recent advances in long-context large language models (LLMs) make them commercially viable, but their standard attention mechanisms' quadratic complexity hinders deployment due to excessive computational costs. To address this, researchers have explored Q-former-like architectures that compress input sequences for LLMs, reducing inference costs. However, these methods often underperform compared to mainstream LLMs trained on short sequences and struggle with longer context. We introduce SOLOS, an innovative method for training long sequences within limited computational resources. This approach effectively narrows the performance gap between context-compressed LLMs and mainstream LLMs handling long contexts. By significantly reducing training overhead, SOLOS enables training on long-sequence datasets, such as 100K tokens for instruction tuning, using merely an 8× RTX3090 machine. Our comprehensive experimental analysis confirms SOLOS not only significantly outperforms other context-compression-augmented LLMs but also matches the performance of state-of-the-art long-context models. The introduction of SOLOS marks a significant step toward deploying long-context LLMs, offering both efficiency and effectiveness in practical scenarios.

1 INTRODUCTION

In recent years, long-context Large Language Models (LLMs) have seen rapid advancements, increasingly meeting commercial robustness standards (Li et al., 2023a; Team et al., 2024; Xinrong et al., 2024). Despite these advancements, deploying long-context LLMs in practical applications remains challenging, mainly due to the computational overhead from the standard full attention mechanism's quadratic complexity in long-context scenarios.

To address this issue, much research has focused on reducing computational burden. Notably,
Wingate et al. (2022); Ge et al. (2024) found that LLM input tokens have considerable redundancy
due to natural language's inherent redundancy. Building on this, Chevalier et al. (2023); Zhang et al.
(2024) suggested compressing the input sequence by consolidating key information, reducing token
count and computational costs. Many studies, including (Chevalier et al., 2023; Zhang et al., 2024),
use a context encoder like BLIP-2's Q-former (Li et al., 2023b). This encoder integrates contextual
information into learnable queries via an attention module. These queries, a new modality, must be
aligned with the LLM's embedding space by a dedicated network before being input into the LLM.

Though these approaches offer significant acceleration through high compression rates, their 045 performance often lags behind uncompressed models. The performance gap occurs because these 046 models are typically trained on shorter sequences and directly applied to longer ones. For instance, 047 Activation Beacon (Zhang et al., 2024) trains on up to 8K tokens but infers on sequences up to 048 400K tokens. The training-inference discrepancy limits the model's ability to handle long-context 049 understanding. However, training these context-compression models on extremely long sequences 050 is typically impractical due to excessive computational demands. Thus the challenge is reducing the 051 training overhead for context-compression LLMs to effectively leverage long sequences. 052

053 We introduce Sparse Optimization for LOng Sequences (SOLOS), a context-compression framework with an efficient training methodology. Specifically, the context is divided into segments,



(a) Pre-filling latency and memory allocation during inference. (b) Long context downstream tasks.

Figure 1: (a) End-to-end pre-filling latency and maximum GPU memory allocation comparison between SOLOS and LLaMA2-7B-32K (TogetherAI, 2023) across different sequence lengths. (b) Performance comparison between SOLOS and other long-context LLMs on downstream tasks.

068 each of which is appended with multiple special tokens at its end. After the forward propagation 069 through the encoder, the activation of the special tokens have distilled contextual information, effectively forming a compact and informative condensed representation. This representation can 071 be transferred to the decoder as additional key-value (KV) caches, via a projector consisting of two 072 projection matrices. To minimize the introduction of additional parameters, we leverage LoRA (Hu 073 et al., 2022) to fine-tune the encoder and the projector. This results in a mere 2% increase in parameters for LLaMA2-7B (Touvron et al., 2023). For optimization, we employ incremental 074 computation exclusively on the decoder side. This means processing each segment sequentially, 075 performing backpropagation immediately after each forward pass, and discarding activations 076 to reduce memory usage-achieving up to an order-of-magnitude reduction. We avoid using 077 incremental computation on the encoder side because it leads to extensive redundant computations. Instead, we use a reservoir sampling-based sparse optimization strategy to manage encoder 079 activations, efficiently managing memory allocation without sacrificing long-term dependencies.

We conduct a comprehensive evaluation of SOLOS using LLaMA2-7B (Touvron et al., 2023) as our 081 base model for a range of tasks. The tasks include auto-encoding, language modeling on datasets like 082 PG19 (Rae et al., 2019), and long-context retrieval challenges like Needle In A Haystack (gkamradt, 083 2023). Additionally, we use the LongBench benchmark (Bai et al., 2023b) to assess SOLOS's 084 real-world long-context performance. As illustrated in Figure 1, our findings show SOLOS achieves 085 excellent compression at $8 \times$ and $32 \times$ ratios, allowing near-perfect reconstruction of the original context. Furthermore, SOLOS significantly outperforms other context-compression-enhanced 087 LLMs across various tasks. Notably, SOLOS matches the performance of mainstream long-context 088 LLMs in some evaluations, with significantly lower inference costs. This highlights SOLOS's 089 potential to efficiently integrate long-context LLMs into practical scenarios.

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2 RELATED WORKS

Context-Compression-Enhanced LLMs. For Large Multimodal Models, architectures like 094 Q-former (Li et al., 2023b) have emerged as important methods for context compression, effectively 095 condensing information from the context through a small number of learnable queries. Given 096 the success of these techniques, they have gradually been integrated into the realm of LLMs. RMT (Bulatov et al., 2022) pioneers the application of this technique within language models, fusing 098 information from each context segment iteratively into a fixed-size memory. This approach allows for efficient processing of long sequences at a minimal computational cost. Building on RMT, 099 AutoCompressor (Chevalier et al., 2023) improves performance by concatenating representations 100 from the context encoder across different segments. Activation Beacon (Zhang et al., 2024) 101 expanded the trainable parameters and leveraged extensive instruction-tuning datasets, leading to 102 even stronger performance on downstream tasks. However, because all these methods use the LLM 103 as the context encoder, the training cost is prohibitively high, which limits the feasibility of training 104 on long sequences. Consequently, the performance of these methods still falls short when compared 105 to mainstream long-context models based on standard attention mechanisms. 106

107 **Long-Context LLMs.** Most mainstream long-context LLMs today are primarily pre-trained on short sequences; they then utilize position embedding extension techniques, coupled with limited

108 imes num segments 109 LoRA Adapter Special Token Regular Token 112 114 115 116 117 118 i-th Encoder Laver i-th Projector i-th Decoder Laver 119 120 *i*-th Layer Next Layer 121

122 Figure 2: SOLOS uses an encoder-decoder structure. The context encoder's special tokens gather 123 and blend information from regular tokens via attention. The combined representations are projected 124 to the decoder. The encoder, decoder, and projector share parameters, but the encoder and projector 125 have distinct LoRA adapters, treated as extra parameters, separate from model weights.

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post-training on longer sequences, to handle extended contexts (Li et al., 2023a; TogetherAI, 2023). However, because these methods do not fundamentally alter the quadratic complexity of the standard attention mechanism, they continue to face prohibitive computational costs in real-world applications. For example, Huiqiang et al. (2024) reports that processing a 300K-token sequence with LLaMA3-8B (Dubey et al., 2024) requires 6 minutes on a single A100 machine, just to complete the pre-filling stage. This clearly demonstrates that excessive overhead hinders the commercial deployment of these long-context models. To address this issue, LongLoRA (Chen et al., 2023a) proposes using S^2 attention to replace standard full attention during training, significantly reducing the training overhead of long-context LLMs. However, since it still employs full attention during inference, the problem remains only partially resolved.

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3 METHODOLOGY

141 3.1 **OVERALL FRAMEWORK** 142

> The architecture of SOLOS, as depicted in Figure 2, is similar to Q-former. In this framework, each context segment is integrated into a compact assembly of special tokens. These special tokens, as context-rich representations, are projected into the decoder's embedding space for utilization.

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3.2 STREAMLINED ENCODER-DECODER ARCHITECTURE

Our proposed model innovates by employing a decoder-only LLM as the context encoder, forming 150 an encoder-decoder architecture. The encoder is akin to Q-former, but diverges from traditional 151 methods (Chevalier et al., 2023; Zhang et al., 2024), where the encoder's output is typically 152 liked directly to the decoder's initial layer. Instead, we introduce a parallel architecture. In this 153 setup, the hidden states associated with special tokens from each encoder layer are mapped to the 154 corresponding decoder layer to serve as compressed context. This mapping process is facilitated 155 by a projector that transforms the hidden states into KV representations. These representations are 156 then utilized directly as the KV cache for the decoder. To operationalize the projector, we harness 157 the encoder's attention projection matrices W_K and W_V and incorporate LoRA adapters (Hu et al., 158 2022). This integration enhances the model's ability to adapt to new tasks with minimal additional 159 parameters. Furthermore, to tap into the context encoder's potential for contextual summarization, we apply LoRA adapters to the encoder's W_O and W_V projection matrices at each layer. These 160 adapters are stored as separate parameters, rather than being integrated into the main model weights. 161 This design choice allows for greater flexibility and control over the model's learning process.

162 Our architecture shares conceptual parallels with the Activation Beacon (Zhang et al., 2024), 163 but with a novel twist: the deployment of two distinct sets of adapters. The first set is tasked 164 with enhancing the projector's functionality, while the second is aimed at refining the encoder's 165 capabilities. This dual-focus approach allows our model to more effectively capture and articulate 166 the nuances of contextual information. We now present a formal mathematical description. Consider a given context X, which is partitioned into k segments, each of length l, represented 167 as $x_1, x_2, ..., x_k$. Additionally, there is a residual part x_{k+1} that may be shorter than l. 168

169 1) **Pre-filling Stage.** 1.1) Special Token Appending: In the initial layer of our model, we introduce 170 special tokens of length c, denoted by π , to the end of each segment x_1, x_2, \dots, x_k : 171

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$$x_i \leftarrow x_i \oplus \pi, \quad \forall i \in [1, 2, 3, ..., k], \tag{1}$$

173 where \oplus denotes the concatenation of the special tokens to the end of each segment. 174

1.2) Hidden States Derivation: In each layer's attention module, we derive the input hidden states 175 h_i corresponding to each segment x_i , where $h_i \in \mathbb{R}^{(l+c) \times d}$ and d is the embedding dimension. 1.3) 176 Special Token States Extraction: Next, we extract the portion of the hidden states h_i that corresponds 177 to the special tokens, denoted as h_{π} . 1.4) Projection to KV Representation: After passing h_{π} through 178 the projector, we obtain the key K_i and value V_i representations for each segment: 179

$$K_i, V_i \leftarrow \operatorname{Projector}(h_\pi), \quad \forall i \in [1, 2, 3, .., k].$$
 (2)

1.5) Concatentation of KV Pairs: The resulting key and value representations are concatenated to form the initial KV cache:

$$K_{\text{cache}} \leftarrow K_1 \oplus K_2 \oplus \dots \oplus K_k, \quad V_{\text{cache}} \leftarrow V_1 \oplus V_2 \oplus \dots \oplus V_k. \tag{3}$$

186 Finally, the concatenated K_{cache} and V_{cache} are fed into the decoder to be used as standard KV cache. 187 The above process reflects the pre-filling stage. 188

2) Decoding Stage and KV Cache Update. During the decoding stage, new tokens are generated 189 continuously. Once the total number of newly generated tokens, combined with the tokens from the 190 residual part x_{k+1} , exceeds the size of one segment l, these tokens form a new segment. At this point, we can repeat the process of appending special tokens and projecting generate new K_{k+1} and V_{k+1} . These new key-value pairs are then added to the existing KV cache:

$$K_{\text{cache}} \leftarrow K_{\text{cache}} \oplus K_{k+1}, \quad V_{\text{cache}} \leftarrow V_{\text{cache}} \oplus V_{k+1}.$$
 (4)

3.3 NAIVE OPTIMIZATION 196

Segments Independence. Though causal relationships exist among different segments, the compression process for each 199 segment is independent and does not require the involvement of 200 other segments. This allows segments to be processed separately, 201 making a departure from previous approaches such as (Chevalier 202 et al., 2023; Zhang et al., 2024), where later segments could 203 leverage the fused representations from earlier ones. 204

Gradient Expression. The independent compression of each 205 segment greatly simplifies the computational graph, enabling a 206 concise expression for the parameter gradients. For the language 207



Figure 3: The gradients contains independent gradient flows.

(5)

modeling loss associated with the segment x_j , denoted as J_j , and considering the condensed 208 representations $\{m_i\}_{i=1}^{i=j-1}$ of the j-1 preceding segments after passing through the encoder, the 209 gradients can be efficiently calculated. These condensed representations are independent, acting as 210 a relay during backpropagation to pass the gradient from the decoder to the trainable parameters in 211 the encoder and the projectors, as depicted in Figure 3. 212

Assuming the LoRA adapters' trainable parameters are Θ , the gradient for the *j*-th segment is: 213

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$$\nabla_{\Theta} J_j = \sum_{i=1}^{j-1} \frac{\partial J_j}{\partial m_i} \cdot \frac{\partial m_i}{\partial \Theta}.$$

The final gradient is the cumulative sum of the gradients from all segments:

$$\nabla_{\Theta} = \sum_{j=1}^{k} \nabla_{\Theta} J_j.$$
(6)

Challenge in Training on Long Sequences. The gradient calculation as shown in Eq. (5) requires storing k independent forward pass caches due to the independent generation of different m_i . Given that our encoder is based on an LLM, these caches are significantly large. For example, even with the use of gradient checkpoint, our test reveals that the memory capacity of an $8 \times \text{RTX}3090$ machine is limited to $k \leq 8$, posing a challenge for optimizing long sequences.

227 3.4 Sparse Optimization for Long Sequences

Running parallel forward and backward propagation for all segments from Eq. (6) can cause excessive GPU memory usage.
To address this, we compute each segment one at a time and sum the results. This method, called incremental computation, balances the trade-off between time and memory, enabling training on longer sequences with limited resources.

Incremental Computation Brings Recomputation. Upon reviewing Eq. (5) and Eq. (6), we find that naive incremental computation leads to considerable recomputation, as shown in Figure 4. The derivative $\frac{\partial m_i}{\partial \Theta}$ is repeatedly calculated across segments, causing overhead and reducing its benefits.



Figure 4: Recomputation.

Incremental Computation on Decoder Only. To reduce the excessive recomputation from the naive incremental method on the encoder, we focus incremental computation on the decoder. For each segment, we complete forward passes for both the encoder and decoder, but only apply backpropagation to the decoder. After processing all segments sequentially, m_i sums the gradients from each segment's loss, as shown in the equation below:

$$\nabla_{m_i} = \sum_{j=i+1}^k \frac{\partial J_j}{\partial m_i}, \quad i \in [1, 2, \dots, k].$$

$$\tag{7}$$

After accumulating gradients, we backpropagate through the encoder to determine the final gradients for Θ . This approach, by conducting a single backpropagation through the encoder at the end, eliminates all unnecessary computations. The resulting final gradient matches that shown in Eq. (6). We will now demonstrate their equivalence.

Proof. We start by defining an indicator function I(i, j) as:

$$I(i,j) = \begin{cases} 1 & 1 \le j \le k, \ 1 \le i \le j-1, \\ 0 & \text{otherwise,} \end{cases}$$
(8)

where the condition can be inverted and explicitly solved as:

$$I(i,j) = \begin{cases} 1 & 1 \le i \le k, \ i+1 \le j \le k, \\ 0 & \text{otherwise.} \end{cases}$$
(9)

After applying this indicator function, we can rewrite Eq. (6) as follows:

$$\nabla_{\Theta} = \sum_{j=-\infty}^{+\infty} \left[\sum_{i=-\infty}^{+\infty} I(i,j) \cdot \frac{\partial J_j}{\partial m_i} \cdot \frac{\partial m_i}{\partial \Theta} \right] = \sum_{i=-\infty}^{+\infty} \left[\sum_{j=-\infty}^{+\infty} I(i,j) \cdot \frac{\partial J_j}{\partial m_i} \cdot \frac{\partial m_i}{\partial \Theta} \right],$$
(10)
$$= \sum_{i=1}^k \left[\sum_{j=i+1}^k \frac{\partial J_j}{\partial m_i} \cdot \frac{\partial m_i}{\partial \Theta} \right] = \sum_{i=1}^k \left[\left(\sum_{j=i+1}^k \frac{\partial J_j}{\partial m_i} \right) \cdot \frac{\partial m_i}{\partial \Theta} \right] = \sum_{i=1}^k \left(\nabla_{m_i} \cdot \frac{\partial m_i}{\partial \Theta} \right).$$



Figure 5: Comparison of different sparsity patterns. The sparsity achieved using reservoir sampling ensures both practicality and the ability to capture long-range dependencies.

Sparse Optimization for Encoder Memory Reduction. By applying incremental computation to 285 the decoder, memory usage is stable since only the cache for one segment is needed, no matter 286 the sequence length. This greatly reduces memory requirements, often leading to significant 287 improvements. However, for the encoder, memory allocation remains high as all segment caches 288 are stored, offering no reduction. Halting optimization here would only cut memory use in half 289 without increasing FLOPs—a minor benefit, especially for very long sequences like 100K tokens. 290 To cut encoder memory further, we suggest using a sparsity budget to limit the number of stored 291 forward pass caches. if the cache limit is exceeded, we use an eviction strategy. For instance, if 292 segment i is removed, we first update ∇_{Θ} by backpropagating ∇_{m_i} , then apply a stop gradient to 293 m_i to halt further accumulation. We will now find the best eviction policy.

294 Limitations of Local Window and Random Eviction Policies. In Figure 5, the simplest 295 eviction strategy is to keep only the most recent segment caches. This method ignores 296 long-term dependencies and relies only on recent data for inference, reducing performance due to 297 biased estimation. In contrast, random sparsity might ideally balance long-term and short-term 298 dependencies, approaching the performance of dense optimization. However, since we cannot 299 recover removed caches, we cannot randomly select segments from the complete set at each step.

300 Reservoir Sampling for Eviction Policy. To achieve efficient memory management, we need 301 an eviction policy that integrates seamlessly with our incremental computation and provides 302 unbiased gradient estimates. Reservoir Sampling (Vitter, 1985) meets both criteria. It offers a 303 natural incremental processing mechanism that aligns with our sequential handling of segments. 304 Additionally, its uniform sampling property ensures that the retained segments are a fair 305 representation of the entire sequence, yielding unbiased gradient estimates of the true gradients. 306 Next, we will demonstrate the expected gradient from reservoir sampling-based sparse optimization 307 is equivalent to that of random sparse optimization.

309 *Proof.* We first express the uniform sampling property of reservoir sampling mathematically. Suppose we are processing the j-th segment (j > S). Let the binary random variables 310 $\mathbf{Z}_{j,1}, \mathbf{Z}_{j,2}, \dots, \mathbf{Z}_{j,j-1}$ denote the inclusion status of the previous j-1 segments in the reservoir. 311 For example, $\mathbf{Z}_{j,2} = 1$ indicates that the forward pass cache of the 2nd segment is retained in the 312 reservoir, while $\mathbf{Z}_{j,1} = 0$ indicates that the forward pass cache of the 1st segment has been evicted. 313 Since the size of our reservoir is fixed at S, the following constraint must hold: 314

$$\sum_{i=1}^{j-1} \mathbf{Z}_{j,i} = S.$$
(11)

318 Given these definitions, the uniform sampling property can be expressed as follows: 319

$$P(\mathbf{Z}_{j,1}=1) = P(\mathbf{Z}_{j,2}=1) = \dots = P(\mathbf{Z}_{j,j-1}=1) = \frac{S}{j-1},$$
(12)

where $P(\triangle = 1)$ denotes the probability of the random variable \triangle taking the value 1. In 323 fact, random sampling also satisfies Eq. (12), but unlike reservoir sampling, it maintains uniform

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4	Alg	gorithm 1 Sparse Optimization with budg	et size S
5 6	1:	$R \leftarrow \emptyset$	⊳ Initialize empty reservoir
7	2:	for $i = 1$ to T do	
	3:	$m_i \leftarrow \text{Encoder}(x_i)$	▷ Perform forward pass through the encoder
	4:	$J_i \leftarrow \text{Decoder}(x_i, \{m_i\}_{i=1}^{j=i-1})$	▷ Perform forward pass through the decoder
	5:	backprop $(J_i, \{m_i\}_{i \in R} \cup m_i)$	▷ Backpropagate through the decoder only
	6:	if $i < S$ then	▷ If reservoir is not full, retain forward cache
	7:	$R \leftarrow R \cup \{i\}$	
	8:	else	▷ If reservoir is full, trigger eviction
	9:	$j \leftarrow \text{randint}(1, i)$	
	10:	if $j < S$ then	\triangleright Evict a previously saved segment t
	11:	$t \leftarrow R[j]$	
	12:	$R[j] \leftarrow i$	
	13:	stop_gradient (m_t)	
	14:	$\mathbf{backprop}(m_t, \Theta)$	
	15:	else	\triangleright Or discard the incoming segment <i>i</i>
	16:	$stop_gradient(m_i)$	
	17:	$\mathbf{backprop}(m_i, \Theta)$	
	18:	end if	
	19:	end if	
	20:	end for	
	21:	for $i = 1$ to S do	▷ Backpropagate through the remaining segment
	22:	$j \leftarrow R[i]$	
	23:	backprop (m_j, Θ)	
	24:	end for	

sampling properties regardless of whether the previous step is observed or not:

$$P(\mathbf{Z}_{j,i}|\mathbf{Z}_{j-1,i}=1) = P(\mathbf{Z}_{j,i}|\mathbf{Z}_{j-1,i}=0), \quad \forall i \in [1, 2, ..., j-1].$$
(13)

Nevertheless, without this property, reservoir sampling-based sparse optimization still achieves unbiased gradient estimation. Combining Eq. (5) and Eq. (6), we derive the expected final gradient:

$$\mathbb{E}_{\mathbf{Z}}[\nabla_{\Theta}] = \mathbb{E}_{\mathbf{Z}}\left[\sum_{j=1}^{k}\sum_{i=1}^{j-1}\sum_{z\in\{0,1\}} \left(z \cdot P(\mathbf{Z}_{j,i}=z) \cdot \frac{\partial J_j}{\partial m_i} \cdot \frac{\partial m_i}{\partial \Theta}\right)\right]$$
$$= \mathbb{E}_{\mathbf{Z}}\left[\sum_{j=1}^{k}\sum_{i=1}^{j-1} \left(P(\mathbf{Z}_{j,i}=1) \cdot \frac{\partial J_j}{\partial m_i} \cdot \frac{\partial m_i}{\partial \Theta}\right)\right].$$
(14)

Here, the additional $\mathbf{Z}_{j,i}$ acts as a gate—if it is 0, it indicates that segment *i* has been evicted when processing the *j*-th segment, thus making this term zero upon multiplication. Substituting Eq. (12) into Eq. (14) directly yields the expected value of the final gradient:

$$\mathbb{E}_{\mathbf{Z}}[\nabla_{\Theta}] = \sum_{j=1}^{k} \sum_{i=1}^{j-1} \left(\frac{S}{j-1} \cdot \frac{\partial J_j}{\partial m_i} \cdot \frac{\partial m_i}{\partial \Theta} \right) = \sum_{j=1}^{k} \frac{S}{j-1} \nabla_{\Theta} J_j.$$
(15)

We observe that this gradient is almost identical to the true gradient in Eq. (6), except for a factor of S/(j-1). While this factor induces a systematic estimation error, it can be precisely offset by multiplying the resulting gradient $\nabla_{\Theta}J_j$ by a compensating factor, (j-1)/S, thereby enabling the reservoir sampling-based sparse optimization to achieve unbiased gradient estimation of Eq. (6). \Box

Using reservoir sampling-based sparse optimization on the encoder side maintains constant
 memory allocation regardless of sequence length, significantly reducing memory requirements while
 preserving gradient fidelity. We provide the detailed process in Algorithm 1 and further validate the
 unbiasedness of the gradient estimation in Appendix B.

4 EXPERIMENTATION 379

380 We evaluate SOLOS by: (1) Lossless Compression of Contexts, evaluated by an auto-encoding task introduced by (Ge et al., 2024). In this task, the input sequence undergoes a single forward pass 382 through the context encoder to generate a compressed representation, which is subsequently utilized 383 by the decoder to reconstruct the original input sequence. Superior performance is indicated by higher reconstruction fidelity. (2) Long Context Language Modeling, assessed using perplexity on 384 the PG-19 (Rae et al., 2019), Proof-Pile (Azerbayev et al., 2022), and four distinct content categories 385 from SlimPajama (Soboleva et al., 2023): Arxiv, Books, Github, and StackExchange. (3) Retrieval, 386 evaluated through the Needle In A Haystack task (gkamradt, 2023), which scrutinizes the model's 387 capacity to distill key information from arbitrary positions within the context. (4) Long Context 388 Downstream Tasks, assessed on the LongBench (Bai et al., 2023b), encompassing a variety of 389 subtasks: Single-Doc QA, Multi-Doc QA, Summarization, Few-shot, and Code. This suite of tasks 390 provides a comprehensive evaluation of both comprehension and generative capabilities. 391

Setups. We use the LLaMA2-7B model (Touvron et al., 2023) with a 1K token segment size and 392 compression ratios of 32 and 8. This configuration allows our model to support maximum context 393 lengths of about 100K and 25K tokens, respectively. We apply LoRA fine-tuning (Hu et al., 2022) 394 to both the encoder and the projectors, using a consistent configuration of r = 32 and $\alpha = 64$ for all 395 LoRA modules. This allows us to adapt the pre-trained model to our specific task while maintaining 396 a reasonable parameter count. Our training process has two stages. In the first stage, we train 397 on 1B tokens from the SlimPajama dataset (Soboleva et al., 2023), building a strong foundation 398 for language understanding. In the second stage, we fine-tune the decoder with a mixed dataset, comprising LongAlpaca (Chen et al., 2023b) (55.5%), Single-Detail QA (Zhang et al., 2024) (30%), 399 400 BookSum (Kryściński et al., 2021) (12%), and Needle (gkamradt, 2023) (2.5%). We format these datasets into conversations using Vicuna's (Zheng et al., 2023) chat template, allowing our model to 401 learn from diverse instructions and tasks. We use our proposed sparse optimization algorithm with 402 a reservoir budget size of S = 2, enabling cache storage for up to 3 segments. We use the Adam 403 optimizer at a 1e-4 learning rate with a cosine scheduler. 404



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Table 1: Results of SOLOS and ICAE on the auto-encoding task.

	Ratio	BLEU-4↑	Rouge-L ↑
SOLOS	8	0.9851	0.993
	32	0.5948	0.762
ICAE	8	0.6461	0.797
	32	0.4289	0.585

Figure 6: Auto-encoding task.

417 Auto-Encoding Task. To evaluate information loss during context compression, we use the 418 auto-encoding task introduced in (Ge et al., 2024), compressing and reconstructing text. We use the 419 trained encoder to perform context compression. On the decoder side, we also train a "repeat" token 420 as a signal for the auto-encoding task, as shown in Figure 6. We select the ICAE (Ge et al., 2024) 421 as the baseline for comparing performance in reconstructing 1K-token sequences at $32 \times$ and $8 \times$ 422 compression ratios. Table 1 shows the results. Our model performs well, achieving Rouge-L scores 423 of 0.993 and 0.762 at at $32 \times$ and $8 \times$ compression ratios, respectively. To demonstrate our model's minimal information loss during compression, we randomly selected a sample and compared 424 the original and reconstructed paragraphs, as shown in Figure 7. This significant reconstruction 425 capability lays the foundation for using compressed context in inference. 426

Long Sequence Language Modeling. We assess SOLOS's long sequence language modeling
using the PG19 (Rae et al., 2019) and ProofPile (Azerbayev et al., 2022) datasets, along with
four SlimPajama (Soboleva et al., 2023) sub-datasets: Arxiv, Book, Github, and StackExchange,
each with 100 randomly sampled instances. The results are summarized in Table 2. We compare
SOLOS to baselines like LongChat-7B-v1.5-32K (Li et al., 2023a), LongAlpaca-7B-32K (Chen
et al., 2023a), LLaMA2-7B-32K (TogetherAI, 2023), YaRN-7B-128K (Peng et al., 2024), and



Figure 7: A case study of the auto-encoding task shows near-lossless compression at a ratio of 8. Even with a ratio of 32, reconstructed paragraphs retained meaning with minor wording changes.

Table 2: Results of various long-context LLMs on language modeling capability. "OOM" stands for Out-of-Memory error, which we've encountered upon an $8 \times RTX3090$ machine.

	Ratio	PG19			ProofPile			Arxiv		Book		Github		StackExchange					
Method		4K	16K	25K	32K	100K	4K	16K	25K	32K	100K	25K	100K	25K	100K	25K	100K	25K	100K
LongChat-7B		9.93	9.49	9.41	9.39		5.65	3.90	3.56	3.21		3.56		6.91		2.97		8.77	
LongAlpaca-7B	1	9.96	9.75	9.69	9.67		6.31	3.97	3.74	3.59		3.71		7.29		3.09		9.01	
LLaMA2-7B-32K	1	7.06	7.17	7.15	7.14		4.32	3.23	2.84	2.70		2.85		5.61		2.37		5.52	
YaRN-7B-128K		6.54	6.62	6.60	6.58	OOM	4.45	3.32	2.93	2.79	OOM	3.07	OOM	5.43	OOM	2.36	OOM	5.67	OOM
20,102	8	6.27	6.09	6.09			4.43	3.84	3.41			3.32		6.21		2.46		6.34	
SOLOS	32	6.51	6.32	6.30	6.30	6.28	4.78	4.26	3.72	3.53	3.19	3.37	2.98	6.42	5.69	2.52	2.44	6.88	6.39
A stimution Deserve	8	8.26	8.13	8.16			5.41	3.91	3.47			3.45		6.67		2.68		8.33	
Activation Beacon	32	8.56	8.54	8.58	8.59	8.83	5.79	4.33	3.86	3.70	3.33	3.51	3.78	6.93	7.92	2.76	3.05	8.45	9.91

Activation Beacon (Zhang et al., 2024). Our evaluation shows SOLOS performs comparably to non-compression models across sequence lengths and improves with length, unlike Activation Beacon. This is due to our protocol, which includes training on longer sequences, enabling SOLOS to capture long-range dependencies effectively and maintain performance on long sequences.

Needle In A Haystack. The Needle In A Haystack benchmark (gkamradt, 2023) assesses LLMs' ability to retrieve information from any context position. We assess both Activation Beacon and SOLOS, as shown in Figure 8. SOLOS performs flawlessly at $8 \times$ compression and maintains a high pass rate even at $32 \times$ compression. This indicates SOLOS can effectively use extended contexts, showing its remarkable efficiency in understanding long contexts.



Figure 8: Results of SOLOS and Activation Beacon on the Needle In A Haystack test.

LongBench. We compare the performance of SOLOS on LongBench (Bai et al., 2023b) with LongChat-7B-v1.5 (Li et al., 2023a), LongAlpaca-7B (Chen et al., 2023a), Qwen1.5-7B-Chat (Bai et al., 2023a), Mistral-7B-Instruct-v0.2 (MistralAI, 2023), InternLM2-Chat-7B (Cai et al., 2024), GPT-3.5-Turbo-16K and Activation Beacon (Zhang et al., 2024). As shown in Table 3, SOLOS exhibits comparable performance to other LLaMA2-7B based models under both compression rates.

- 5 LIMITATIONS

483 Though SOLOS enhances training efficiency for sequences up to 100K tokens and performs well on 484 downstream tasks, it faces limitations. One issue is SOLOS's reliance on a decoder-only LLM for 485 context encoding instead of a specialized, newly trained encoder. This can lead to significant data 486 loss at higher compression levels, limiting efficiency improvements. Moreover, SOLOS's use of an

Model	Ratio	SQA	MQA	SUM	FEW	CODE			
LLaMA2-7B / LLaMA2-7B-chat based									
LongChat-7B-32K	1	31.6	23.5	21.7	49.3	54.9			
LongAlpaca-7B-16K		26.6	28.0	24.5	52.9	52.4			
YaRN-7B-128K		24.0	24.1	19.8	60.0	62.71			
Activation Beacon	8	22.1	24.8	20.2	60.8	57.7			
	32	19.8	23.4	18.0	58.3	56.2			
SOLOS	8	33.8	31.3	22.1	58.3	61.5			
	32	28.5	26.9	20.3	57.0	60.8			
Others									
Qwen1.5-7B-Chat	1	27.9	14.2	21.0	21.8	28.9			
Mistral-7B-Instruct-v0.2		31.3	26.4	21.8	46.6	44.8			
†GPT-3.5-Turbo-16K		45.1	36.2	23.9	52.9	54.1			
InternLM2-Chat-7B		45.7	43.1	26.5	58.3	36.4			

Table 3: Results of various long-context LLMs on five subtasks from LongBench. † denotes results
 from the LongBench paper.

LLM for context encoding is computationally intensive. Employing simpler models might compress context more efficiently, reducing costs and enhancing performance. Future research could benefit from investigating more efficient encoding techniques. Finally, SOLOS requires additional training, potentially making it less convenient than methods that bypass training requirements.

6 CONCLUSION

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To enhance the ability of LLMs on processing long sequences, We have proposed SOLOS, 516 which employs a streamlined encoder-decoder framework where the weights-shared encoder and 517 decoder respectively encapsulate a context segment into compressed representations and leverage 518 these representations to predict outputs of the subsequent segment. Moreover, we introduce two 519 strategies for reducing memory allocation in the encoder and decoder. For the decoder, we 520 adopt incremental computation, which processes segments sequentially rather than in parallel, 521 significantly reducing memory footprint without increasing FLOPs. For the encoder, we apply 522 reservoir sampling-based sparse optimization, an unbiased method that balances efficiency and 523 gradient accuracy. With these optimizations, SOLOS can be efficiently trained on sequences of 100K tokens with limited resources, resulting in a strong performance on language modeling tasks 524 and comparable performance on various downstream tasks. 525

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A IMPLEMENTATION DETAILS

Overview of Pretraining Data. During the pretraining phase, we use a total of 1 billion tokens from five sub-datasets of SlimPajama. For Github, StackExchange, and Wiki, most of the data consists of short sequences, while for Book and Arxiv, the majority of the data consists of long sequences. We randomly sample tokens from each dataset for training, with a total of 1 billion tokens sampled across all datasets. Detailed information is provided in Table 4.

Corpus	Num Sea.	Num Sample Token	Sequence Length				
corpus	i tum Seq.		Max	Min	Average		
Book	13K	0.15B	18M	29	510K		
Arxiv	101K	0.15B	3M	217	57K		
Github	1M	0.4B	1M	571	6K		
StackExchange	1.3M	0.15B	267K	1K	3K		
Wiki	1.3M	0.15B	452K	523	3.5K		

Table 4: Detailed information on pretraining data.

Overview of Instruction Tuning Data. In instruction tuning, we add LoRA adapters (Hu et al., 2022) to the query and value projection matrices of each layer in the decoder. We organize all instruction tuning data into a conversation format following Vicuna's chat template (Zheng et al., 2023), as shown below: "A chat between a curious user and an artificial intelligence assistant. The assistant gives helpful, detailed, and polite answers to the user's questions. ### USER: (Request)
Assistant: (Response)" We finetune on multiple instruction tuning datasets, most of which are using ChatGPT for response generation.

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648 B ADDITIONAL EXPERIMENTATION

650 Ablation on LoRA Hyperparameters. During pretraining, we add LoRA adapters to the query 651 and value projection matrices of all encoder layers, as well as to the key and value projection 652 matrices of all projectors. In the instruction tuning phase, in addition to the LoRA adapters used in 653 pretraining, we also add LoRA adapters to the query and value projection matrices of each decoder 654 layer. To assess the impact of different LoRA configurations on the final results, we train with a $32 \times$ compression ratio using the same training recipe and evaluated the resulting models on PG19 655 and SingleDoc QA from LongBench. The results are shown in Table 5. The experimental results 656 indicate that larger rank and alpha values improve language modeling performance but negatively 657 affect instruction-following capabilities. 658

LoRA Rank	LoRA Alpha		P	G19		
Loter Runk		4K	16K	32K	100K	
32	64	6.51	6.32	6.30	6.28	28.5
64	128	6.43	6.23	6.21	6.20	27.7
128	256	6.41	6.22	6.20	6.19	27.3

Table 5: Performance comparison under different LoRA configurations.

Ablation on Sparse Optimization Algorithm. Local window sparse optimization only considers 668 the most recent segments, theoretically favoring local dependencies while overlooking long-term 669 ones. This prevents the full utilization of long context, making it a suboptimal approach. To 670 evaluate the actual performance of local window sparse optimization and quantify the improvements 671 brought by SOLOS, we use the same training recipe to compare the models trained with these 672 two optimization algorithms. We assess their language modeling performance on PG19 and their 673 performance on the SingleDoc QA task from LongBench, as shown in Table 6. The results 674 demonstrate that models trained using local window sparse optimization fail to capture long-term 675 dependencies, causing language modeling perplexity to stagnate as context length increases. In 676 contrast, SOLOS addresses this limitation effectively. 677

Table 6: Performance Comparison between local window sparse optimization and SOLOS

Local Window Sparse	SOLOS		SOA				
Local Window Sparse	SOLOS	4K	16K	32K	100K		
\checkmark		6.42	6.45	6.57	6.51	22.1	
	\checkmark	6.51	6.32	6.30	6.28	28.5	

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685 Accuracy of Gradient Estimation. To assess the accuracy of SOLOS in gradient estimation, we 686 compare its gradients with those from Eq. (6). We expect high similarity, indicating our method's 687 accuracy. We use a context window of 128, compression ratio of 8, and compute gradients for 688 parameters Θ based on a mini-batch of 64 inputs, each with 2048 tokens. To avoid the influence of initial trainable parameter values, we use the same checkpoint for both approach after hundreds of 689 training iterations. We use $abla^*_{\Theta}$ to represent the gradient from our reservoir sampling-based sparse 690 optimization and compute the similarity ratio $r = \|\nabla_{\Theta}^*\| / \|\nabla_{\Theta}\|$. A ratio of r = 1 indicates the 691 gradients are close. Table 7 shows the mean and variance of r for different reservoir budget. Our 692 results demonstrate that applying the compensating factor $\frac{j-1}{s}$ ensures high similarity between the 693 estimated and true gradients, even when the reservoir budget is as low as 1. 694

Table 7: A statistical analysis compares the L2 norm ratios of gradients from two algorithms. Key
 findings: 1) The compensating factor is vital for accurate gradient estimation. 2) Variance decreases
 with increasing window size S, enhancing estimation accuracy.

698		w/o	Factor	w/ F	w/ Factor		
599 700	Statistics $ $ $S = 1$	S = 4	S = 8	$S = \infty \mid S = 1$	S = 4	S = 8	$S = \infty$
701	Mean 0.676 Variance 0.112	0.818 0.041	0.935 0.008	1.000 0.994 3e-5 0.039	0.980 0.038	1.024 0.009	0.999 4e-5