LLM Critics Help Catch Bugs in Mathematics: Towards a Better Mathematical Verifier with Natural Language Feedback

Anonymous ACL submission

Abstract

In recent progress, mathematical verifiers have achieved success in mathematical reasoning tasks by validating the correctness of solutions generated by policy models. However, existing verifiers are trained with binary classification labels, which are not informative enough for the model to accurately assess the solutions. To mitigate the aforementioned insufficiency of binary labels, we introduce stepwise natural language feedback as rationale labels, that is, the correctness of each step and the detailed explanations. In this paper, we propose **Math-Minos**, a natural language feedback-enhanced verifier by constructing automatically generated training data and a two-stage training paradigm for effective training and efficient inference. Our experiments reveal that a small set of natural language feedback can significantly boost the performance of the verifier in both verification and reinforcement learning and also significantly alleviates the data-demanding problems of the reward model with an over 700% data efficiency improvement.

1 Introduction

Large Language Models have demonstrated strong capabilities in summarization (Touvron et al., 2023b), coding (Rozière et al., 2024), tool using (Song et al., 2023) and dialogue (Ouyang et al., 2022). However, mathematical reasoning remains a challenge for LLMs (Lightman et al., 2023; Huang et al., 2024). To tackle this problem, recent research has focused on using verifiers to validate the correctness of response generated by models (Wang et al., 2023b; Zhu et al., 2023; Li et al., 2023; Wang et al., 2024). An effective verifier can serve as 1) response reranker in the decoding (Li et al., 2023; Yu et al., 2024; Wang et al., 2024); 2) reward model in further post-training (Shao et al., 2024; Wang et al., 2024); 3) data purifier that filters erroneous responses in the SFT (Rafailov et al., 2023).

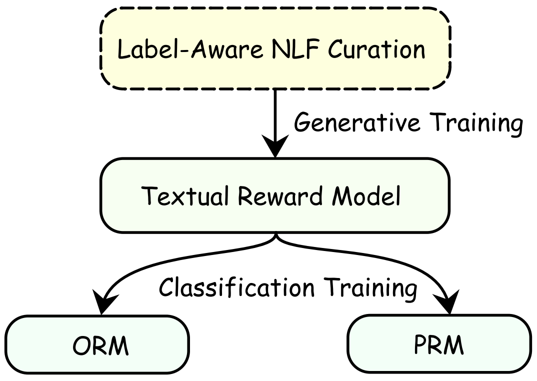

Figure 1: Illustration of the training process for **Math-Minos**. We obtain Natural Language Feedback by using Label-Aware Prompting. Then we propose a novel two-stage training for the final mathematical verifier.

However, existing verifiers (Li et al., 2023; Yu et al., 2024; Wang et al., 2024) are trained as classifiers by using binary labels as feedback. We argue that the binary feedback are not informative in training as they do not contain explanations for the underlying reasons for the errors, which hindered the performance of the verifier and also cause the data demanding problem during the verifier training process.

In this work, we aim to enhance the verifier's evaluation ability for mathematical solutions by introducing step-level natural language feedback(i.e., the correctness of the current step and the further explanations). We propose **MATH-Minos**, a natural language feedback enhanced verifier as shown in Figure 1.

We first create high-quality step-level natural language feedback with a novel Label-Aware Natural Language Feedback curation process as Section 3.2 with human checking. This approach simplifies the task by incorporating step-level binary

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classification labels, enhancing the critique generation process for GPT-4. Natural language feedback can provide in-depth reasons behind classification feedback, which enhances the training of the verifier. By employing supervised fine-tuning on only 30k natural language feedback before binary classification training, we can effectively enhance the model's evaluation capabilities.

In the second stage, we introduce a two-stage training for **MATH-Minos** as Section 3.3: Firstly, we adopt the supervised fine-tuning on curated natural language feedback to effectively help improve the model's evaluation capabilities, followed by standard ORM & PRM training on score feedbacks to achieve efficient inference with a single forward step.

The experiments in Section 4 demonstrate that infusing the model with evaluation capabilities via natural language feedback is more efficient and effective than traditional score feedback. We show that only 30k training data with natural language feedback can significantly boost the performance of mathematical verifiers in both ORM and PRM setting across verification and reinforcement learning tasks. Moreover, our analytical experiments achieved the same level of performance with a 700% reduction in data size, demonstrating that incorporating natural language feedback can significantly enhance the training efficiency of LLMs.

In summary, our contributions are threefold:

1. We are the first study to conduct in-depth analyses of the reasons behind the evaluations generated by verifiers, revealing the shortcomings of the current verifier's training paradigm and inspiring future research.

2. We propose *MATH-Minos* by proposing label-aware natural language feedback curation and a two-stage training paradigm, demonstrating that training verifiers with natural language feedback can complement the non-informative binary label thus enhancing the model's evaluation ability.

3. We demonstrate the effectiveness of MATH-Minos across both ORM and PRM task settings. Extensive analysis demonstrates the superiority and efficiency of the proposed method.

2 Related Works

Enhancing the mathematical reasoning ability of LLM Previous works focus on improving the mathematical reasoning ability of LLMs on three ways: (1) Pre-training: LLMs (Azerbayev et al., 2023; OpenAI et al., 2024; Touvron et al., 2023a,b) are pre-trained on a large set of corpus related to mathematical questions with next-token prediction objective. (2) Supervised fine-tuning: Supervised fine-tuning can also improve the mathematical reasoning ability of LLMs by training LLMs with mathematical questions with detailed solutions (Yu et al., 2023b; Luo et al., 2023; Wang et al., 2023a). (3) Inference: Wei et al. (2022); Fu et al. (2022); Zhang et al. (2023); Bi et al. (2023) design prompting strategies to improve the reasoning ability of LLMs.

Verifier for mathematical reasoning Previous mathematical verifiers can be mainly categorized into two categories: Outcome Reward Model (ORM) gives an evaluation score to the whole solution; Process Reward Model (PRM) gives an evaluation score to each intermediate step of the solution. Previous works (Yu et al., 2023a; Ying et al., 2024; Wang et al., 2024) use question-solution pair with a binary label to train a ORM or a PRM, which is inefficient to help models understand the errors. Therefore, in this work, we aim to train a verifiers with detailed natural language feedback.

3 Methodology

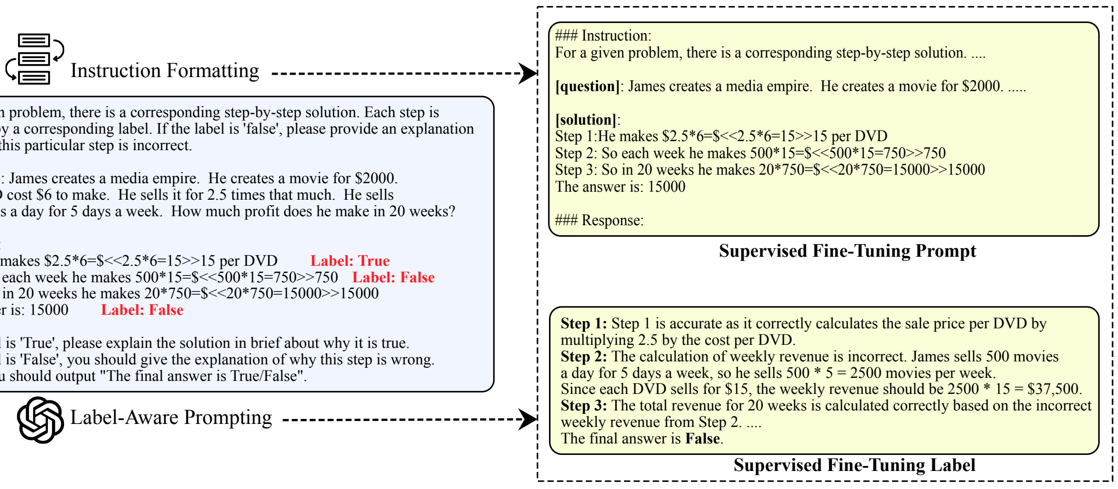
In this section, we introduce the background of our proposed method (§3.1), then delve into our proposed **MATH-Minos**, which contains label-aware natural language feedback curation (§3.2) and the two-stage model training (§3.3).

3.1 Background

Outcome Reward Model For a given problem p, the Outcome Reward Model (ORM) assigns a reward $r \in \mathbb{R}$ based on the whole completion s. The common approach for training an ORM involves implementing a binary sequence classification, which adds a classifier at the end of the LLM. The training loss is represented as follows:

$$\mathcal{L}_{orm} = y_s \cdot log(\hat{y}_s) + (1 - y_s) \cdot log(1 - \hat{y}_s), \quad (1)$$

where y_s is the golden label of the solution and $\hat{y}_s$ is the sigmoid score of s predicted by ORM. For mathematical reasoning tasks, the quality of a sample can be directly determined by judging the correctness of the result. Therefore, the general approach to train a ORM involves using a generator to provide completions. Subsequently, rule-based matching is employed to determine the correctness



For a given problem, there is a corresponding step-by-step solution. Each step is followed by a corresponding label. If the label is 'false', please provide an explanation as to why this particular step is incorrect.

[question]: James creates a media empire. He creates a movie for $2000. Each DVD cost $6 to make. He sells it for 2.5 times that much. He sells 500 movies a day for 5 days a week. How much profit does he make in 20 weeks?

[solution]:
Step 1:He makes $2.5*6=$<<2.5*6=15>>15 per DVD **Label: True**
Step 2: So each week he makes 500*15=$<<500*15=750>>750 **Label: False**
Step 3: So in 20 weeks he makes 20*750=$<<20*750=15000>>15000
The answer is: 15000 **Label: False**

If the label is 'True', please explain the solution in brief about why it is true.
If the label is 'False', you should give the explanation of why this step is wrong.
Finally you should output "The final answer is True/False".

Instruction:
For a given problem, there is a corresponding step-by-step solution. ....

[question]: James creates a media empire. He creates a movie for $2000. .....

[solution]:
Step 1:He makes $2.5*6=$<<2.5*6=15>>15 per DVD
Step 2: So each week he makes 500*15=$<<500*15=750>>750
Step 3: So in 20 weeks he makes 20*750=$<<20*750=15000>>15000
The answer is: 15000

Response:

Step 1: Step 1 is accurate as it correctly calculates the sale price per DVD by multiplying 2.5 by the cost per DVD.
Step 2: The calculation of weekly revenue is incorrect. James sells 500 movies a day for 5 days a week, so he sells 500 * 5 = 2500 movies per week.
Since each DVD sells for $15, the weekly revenue should be 2500 * 15 = $37,500.
Step 3: The total revenue for 20 weeks is calculated correctly based on the incorrect weekly revenue from Step 2. ....
The final answer is **False**.

Figure 2: The illustration of the label-aware natural language feedback curation of MATH-Minos on GSM8K (Cobbe et al., 2021) dataset. We introduce step-level classification label to achieve step-level natural language feedback curation.

Task	GSM8K	MATH
Outcome Evaluation	95.1	62.0
Process Evaluation	85.1	59.7

Table 1: The outcome- and step-level Acc of prompting GPT-4 to generate natural language feedback.

of the current completion, and this outcome is used as the label for training. For the sake of simplicity and comparability, we directly modified the open-sourced dataset provided by Wang et al. (2024) as the training set of ORM.

Process Reward Model For a given question q, the Process Reward Model (PRM) assigns a reward $r \in \mathbb{R}$ to each step s_i of the completion s. The training of PRM is through the task of token classification, the training loss can be represented as follows:

$$\mathcal{L}_{prm} = \sum_{i=1}^{K} y_{s_i} \cdot log(\hat{y}_{s_i}) + (1 - y_{s_i}) \cdot log(1 - \hat{y}_{s_i}), \tag{2}$$

where K is the number of reasoning steps of the completion, y_{s_i} is the golden label of the solution and $\hat{y}_{s_i}$ is the sigmoid score of s_i predicted by PRM. Compare to ORM, PRM can provide fine-grained supervision which is more detailed and reliable.

3.2 Label-aware Natural Language Feedback Curation

In this section, we introduce the label-aware natural language feedback curation of MATH-Minos as shown in Figure 2. Since the natural language feedback can be understood by both humans and large models, it is suitable to stimulate the evaluation capabilities of LLMs. Unlike the binary label, natural language feedback provides detailed explanations for right or wrong completions, which also brings complexity to data collection. The best way for generating the natural language feedback data is through manual annotation. Considering the costs associated with human annotation, we obtain natural language feedback by leveraging the capabilities of the GPT-4-turbo (OpenAI et al., 2024).

To verify the quality of data, we sample data from the Math-Shepherd (Wang et al., 2024) and PRM800K (Lightman et al., 2023) to create an evaluation dataset including 500 question-solution samples with step-level and outcome-level binary classification label for GSM8K (Cobbe et al., 2021) and MATH (Hendrycks et al., 2021). We then check the accuracy of outcome evaluation and step-level evaluation, with results presented in Table 1. Experimental results show that this prompting manner doesn't yield high-quality data with only 85.1 step-level accuracy for GSM8K and 59.7 step-level accuracy for MATH. This also indicates that one of the factors limiting the performance of the reward model is the base model's evaluation capability.

To facilitate GPT-4 in generating higher-quality data, we propose a label-aware prompting method, which simplifies the evaluation task by introducing the binary classification label within the prompt. As illustrated in Figure 2, GPT-4's task shifts from determining correctness and generating explana-

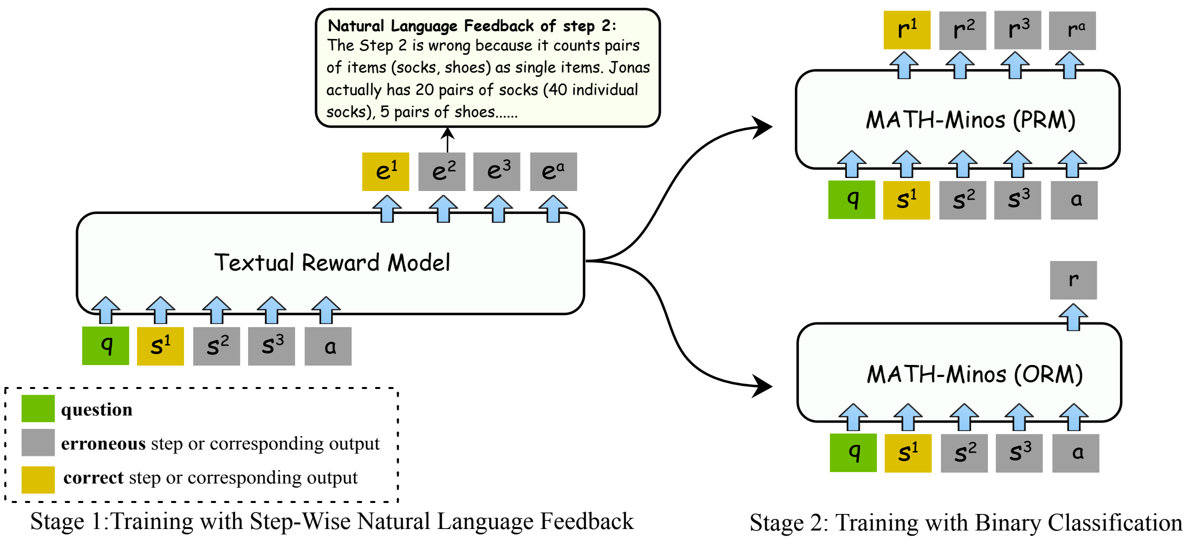

Natural Language Feedback of step 2:
The Step 2 is wrong because it counts pairs of items (socks, shoes) as single items. Jonas actually has 20 pairs of socks (40 individual socks), 5 pairs of shoes......
e^1 e^2 e^3 e^a
Textual Reward Model
q s^1 s^2 s^3 a
question
erroneous step or corresponding output
correct step or corresponding output
Stage 1:Training with Step-Wise Natural Language Feedback
r^1 r^2 r^3 r^a
MATH-Minos (PRM)
q s^1 s^2 s^3 a
r
MATH-Minos (ORM)
q s^1 s^2 s^3 a
Stage 2: Training with Binary Classification

Figure 3: The overview of the two-stage training process of MATH-Minos. In Stage 1, training reward model (RM) with natural language feedbacks helps RM learn to evaluate effectively and efficiently. In Stage 2, training RM as binary classification helps RM inference efficiently by outputing a reward with one single forward pass.

tions to generating explanations based on the given label. After obtaining synthetic data, we hire a group of graduate students to review the data, as Appendix F, to further ensure its quality.

3.3 Two-Stage Training of MATH-Minos

Based on the aforementioned data generated in Section 3.2, we introduce a novel two-stage training paradigm including (1) Stage 1: Training with Step-Wise Natural Language Feedback and (2) Stage 2: Training with Binary Classification to synergistically combine the strengths of evaluation generator and discriminator, which is shown in figure 3. This training paradigm enjoys two potential benefits: Firstly, natural language feedback contains rich information, especially for complex reasoning tasks such as mathematics. Therefore training with natural language feedback can significantly improve the models' evaluation ability with just a small set of data. Secondly, the inference of binary classification discriminator is more efficient compared with natural language feedback generation. This approach not only allows model to generate a score but also enables the model to produce evaluation results with just a single forward pass, thereby enhancing the efficiency.

Reward Modeling with Natural Language Feedback In the first stage, we employ supervised fine-tuning to enhance the evaluation capabilities of the model. We utilize the *Supervised Fine-Tuning Prompt* shown in Figure 2 as the input $x_{q,s}$ for the model, with the *Supervised Fine-Tuning Label* generated by GPT-4 serving as the model's output y. The training loss for a sample can be defined as follows:

$$L_{textrm} = \sum_{t=1}^{M} logP(y_t|y_{<t}, x_{q,s}), \qquad (3)$$

where M is the total length of y and $y_{<t}$ is the previous tokens.

Reward Modeling with Binary Classification After the first stage, the evaluation capability of the model is improved. However, natural language feedback cannot provide a score and thus can't be used as a reward for further optimizations like PPO (Schulman et al., 2017). Additionally, when the model generates feedback, it produces a complete evaluation with rationales, making it significantly less efficient than using a classification-based verifier. Therefore, we further train the verifier with binary classification labels as Equation 1 and Equation 2.

Benefiting the proposed two-stage training, we can enhance the verifier's evaluation ability with natural language feedbacks and efficiently apply the verifier to PRM or ORM with one single forward pass.

4 Experiment

4.1 Experiment Setup

Dataset We conduct our experiment on two widely used mathematical datasets GSM8K (Cobbe

Models	Verifier	GSM8K	MATH500
Mistral-7B: MetaMATH	Self-Consistency (Li et al., 2023)	84.1	34.6
	ORM (Wang et al., 2024)	86.2	35.9
	PRM (Wang et al., 2024)	87.1	36.7
	Self-Consistency + ORM (Wang et al., 2024)	86.6	37.6
	Self-Consistency + PRM (Wang et al., 2024)	86.8	37.8
	MATH-Minos (ORM) †	87.3	37.4
	MATH-Minos (PRM) †	87.6	37.8
	Self-Consistency + MATH-Minos (ORM) †	**88.2**	38.3
	Self-Consistency + MATH-Minos (PRM) †	87.8	**38.6**

Table 2: Main results of MATH-Minos in verification. The verification is based on 256 outputs. † denotes the method is proposed in this paper. Our **MATH-Minos** significantly outperforms baselines in both ORM and PRM settings.

et al., 2021) and MATH (Hendrycks et al., 2021). GSM8K(Cobbe et al., 2021) comprises a variety of word problems that are typically found in grade school mathematics curricula, which contains 7473 samples in the training set and 1319 samples in the test set. MATH (Hendrycks et al., 2021)is a diverse collection of mathematical problems that cover a broad range of topics and skill levels, from elementary to advanced mathematics, which contains 7500 samples in the training set and 5000 samples in the test set. In the setting of verification, we sample the test set of MATH to 500 samples which is identical to Lightman et al. (2023).

Verification Following Lightman et al. (2023) and Wang et al. (2024), we adopt the best-of-N selection to evaluate the capability of our verifier. Specifically, given a question q and a generator, we let the generator sample N times for the question q. Then, the verifier is used to evaluate the quality of each completion. The final answer a is determined as the one with the highest reward according to the verifier's output $RM(q, a_i)$, formally expressed as follows:

$$a_{\text{rm}} = \mathcal{F}(\arg\max_{s_i} RM(q, s_i)), \quad (4)$$

where s_i is the i-th solution generated by generator and $\mathcal{F}(\cdot)$ denotes extracting the final answer from the solution.

Following Li et al. (2023) and Wang et al. (2024), we also explore the ensemble of self-consistency (majority voting) and the verifier. Specifically, we classify the results output by the model into different groups and calculate the cumulative reward for each group, which can be calculated as follows:

$$a_{\text{sc+rm}} = \arg\max_a \sum_{i=1}^{N} \mathbb{I}(\mathcal{F}(s_i) = a) \cdot RM(q, s_i), \quad (5)$$

Model	GSM8K	MATH500
Mistral-7B: MetaMATH	77.9	28.6
+ ORM-PPO	81.8	31.3
+ MATH-Minos(ORM)-PPO	**83.6**	**32.8**

Table 3: Experimental result of the MATH-Minos in PPO post-training setting. Our MATH-Minos significantly outperforms the vanilla ORM on PPO training.

where N is the number of solutions, s_i is the solution generated by generator and $\mathcal{F}(\cdot)$ denotes extracting the final answer from the solution.

Reward Model Following Wang et al. (2024), to further examine the performance of the verifier, we evaluate it as the reward model for PPO (Schulman et al., 2017) during the post-training phase. An effective reward model can provide reliable feedback signals, thereby enhancing the final performance of PPO.

Experimental Setting For the training of the verifiers, to ensure comparability and convenience, we utilize the open-source MATH-shepherd dataset (Wang et al., 2024) for both baseline reward models and MATH-Minos. We curate a total of 30K samples of natural language feedback using the data from phase-one of PRM800K (Lightman et al., 2023) and the subset of MATH-Shepherd. Our main experiment conducts the verification on the test set of GSM8K and MATH and reinforcement learning (PPO) on the training set of GSM8K and MATH. We follow Wang et al. (2024) with their detailed experimental setup, as shown in Appendix E.

4.2 Main Result

We present the performance of various methods in Table 2 and Table 3.

For the task of verification, MATH-Minos (ORM) achieves an improvement of 1.1% in accuracy on the GSM8K and 0.7% in on the MATH compared to traditional ORM. For PRM, MATH-Minos (PRM) achieves an accuracy improvement of 0.7% on GSM8K and 0.8% on the MATH. Ensembling with self-consistency and MATH-Minos, the MetaMATH-Mistral generator achieves optimal accuracy of 88.2% on GSM8K and 38.6% on MATH500. Beyond the improvement of the performance, we find that MATH-Minos has a more pronounced effect in the setting of ORM. We believe this phenomenon could be attributed to the sparser supervision in ORM compared to PRM, implying that information-rich textual explanations can offer more substantial benefits to ORM.

For the task of reward model on reinforcement learning, MATH-Minos (ORM) achieves an improvement of 1.8% in accuracy on the GSM8K and 1.5% in on the MATH compared to traditional ORM, demonstrating the robust effectiveness of our method across various task settings.

5 Analysis

5.1 Error Distribution of Math Solvers

To further illustrate the shortcomings of training the verifier solely relying on binary classification, we conduct an in-depth investigation into the errors produced by the generator at the step level. Specifically, we take the natural language feedback generated by GPT-4 as a reference and heuristically categorize the causes of errors in responses into five distinct types: ***Unrelated Unrelated***: This indicates that the step is irrelevant and does not contribute towards deducing the final answer. ***Accumulation***: This denotes that the step is incorrect due to a mistake in the preceding step, leading to subsequent errors. ***Calculation***: This categorization is reserved for errors arising from incorrect calculations, which is one of the most common errors in mathematical reasoning. ***Logic***: This applies to steps that are logically flawed in the context of solving the given problem. ***Other***: This category encompasses steps that are erroneous for reasons not covered by the aforementioned categories.

We use the same way as the label-aware prompting introduced in Section 2 to automatically analyze the cause of errors. We obtain the step-level labels from the subset of MATH-Shepherd (Wang et al., 2024), which contains 2500 samples for both GSM8K and MATH. Given the question, solution

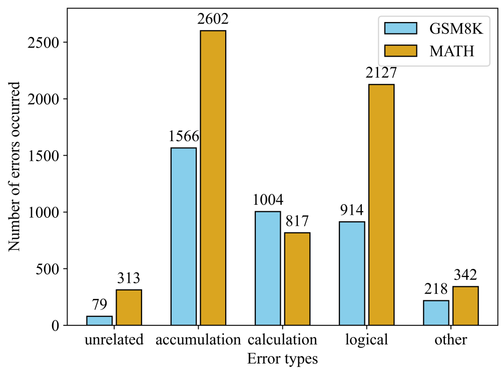


Figure 4: Statistics on the different types of reasoning errors of the policy model MetaMath-Mistral on the GSM8K and MATH.

and the natural language feedback, we employ GPT-4 for the classification of error causes. The experimental result is shown in Figure 4.

From our statistical analysis, it is evident that the model produces errors across all types. For the MATH dataset, given its higher difficulty level and the necessity for more steps, a greater total number of errors occur within the same number of samples of GSM8K. Furthermore, we discover that the most common cause of errors in both datasets is accumulation, which is consistent with our intuition. In multi-step reasoning, a mistake in one step is likely to directly cause errors in all subsequent steps. Furthermore, we observe distinct patterns of errors in the GSM8K and MATH datasets. For the GSM8K dataset, the occurrences of calculation errors and logical errors were approximately the same. Instead, in the MATH dataset, logical errors significantly outnumber calculation errors. This also indirectly demonstrates that models are vulnerable to more complex reasoning tasks.

These findings further illustrate that using binary labels to supervise the learning of reasoning evaluation tasks is insufficient and therefore highlighting the proposal for using natural language feedback to supplement the training of vanilla ORM or PRM.

5.2 Meta-Evaluation and Efficiency

To measure the verifier's capabilities in a more convenient and direct manner instead of verification, an intuitive approach is to assess whether the verifier can accurately determine the correctness of the final answer. Without the influence of the generator, this method purely relies on the capability of the verifier. We construct a meta-evaluation set based on whether the final answer provided in the generator's output is correct, serving as the ground truth label. By sampling several responses from the generator on the test set and deriving labels through rules, we create a meta-evaluation set for GSM8K containing 20,000 samples. We conduct tests on the meta-evaluation set using the checkpoints of each epoch after completing the model training and verification. The results of the meta-evaluation are presented in Figure 5. Additionally, we construct the step-level meta-evaluation dataset using the same data in Section 5.1 to provide insights into where our method's gains are.

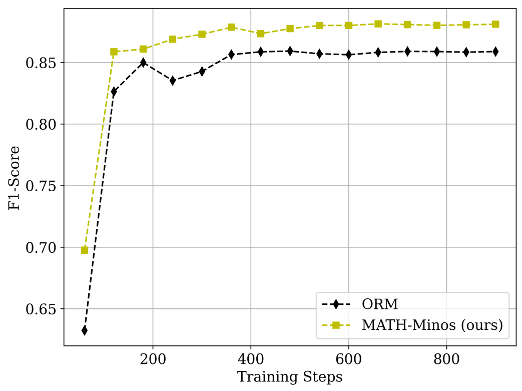


Figure 5: The convergence curve of vanilla ORM and our **Math-Minos** with natural language feedbacks on meta-evaluation set.

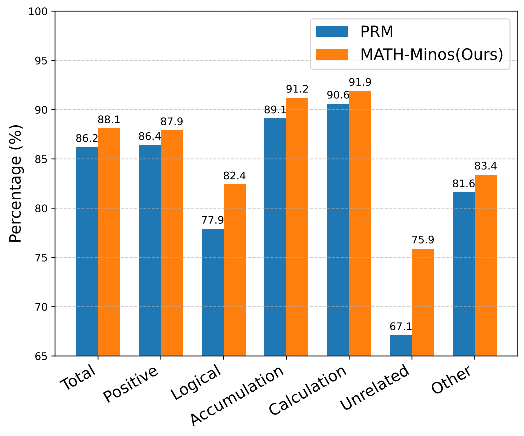


Figure 6: The detailed improvement of the step-level model prediction accuracy between vanilla PRM and **Math-Minos**.

We observe that MATH-Minos consistently outperforms vanilla ORM in meta-evaluation. Additionally, MATH-Minos exhibit a faster convergence rate, surpassing the baseline at only approximately 120 steps. Given that we trained for only 1 epoch, this means that in the actual secondary phase of binary classification, only about 60K data are required to exceed the baseline. This significantly alleviates the data-demanding problems of the reward model with an over 700% data efficiency improvement at stage two, demonstrating that natural language feedback can significantly reduce the amount of data needed to train a verifier.

GSM8K	**Meta-Eval**	**Verification**
ORM	85.9	86.2
+ curation w/o label	86.2	85.8
+ curation w/ label	88.1	88.2

Table 4: Experimental result of the ORM, ORM with vanilla natural language feedback curation and our Math–Minos (ORM with label-aware natural language feedback curation). Label-aware natural language feedback curation significantly enhances ORM's evaluation ability.

GSM8K	**GSM8K**	**MATH**
Vanilla ORM	86.2	35.9
MATH-Minos (GPT-4)	87.3	37.4
MATH-Minos (Qwen2-72b)	87.5	36.8
MATH-Minos (Mistral-7B)	85.4	34.5

Table 5: Experimental result of the ORM, Math-Minos with label-aware natural language feedback curation from different models. "Qwen2-72b" denotes we use Qwen2-72b-instruct as the natural language feedback generator. "Mistral-7B" denotes we use Mistral-7b-Instruct-v0.3 as the natural language feedback generator.

Furthermore, in the step-level meta-evaluation as Figure 6, we can observe that our method significantly aids in identifying all types of errors, with the most notable improvements seen in logical errors and unrelated contents. This aligns well with the strengths of natural language feedback, which provides a more detailed analysis of the underlying reasons compared to score-based feedback. Such insights help enhance the logical reasoning capabilities of the reward model and effectively identify redundant information.

5.3 Influence of the Feedback Quality

In this section, we conduct an extensive analysis of the impact on the data quality of natural language feedback. One baseline is direct prompting of GPT-4-turbo. We use the 30K direct GPT-4 evaluation which is the same number of MATH-Minos to compare. We use both meta-eval and verification to measure the capability of the verifier. Additionally, to enhance the scalability and accessibility of our method, we explore the use

GSM8K	Meta-Eval	Verification
ORM w/o stage 1	85.7	86.2
RM w/o stage 2	82.8	84.7
MATH-Minos	88.0	88.2

Table 6: Ablation study of the two-stage training paradigm. RM w/o stage 1 denotes that we only train the verifier with the score feedback. RM w/o stage 2 denotes that we only train the verifier the natural language feedback generated.

of different base models to generate natural language feedback. Specifically, we utilize the open-source model Qwen2-72b-Instruct as a substitute for GPT-4-turbo. Furthermore, we take a more radical approach by directly employing the identical base model, Mistral-7B-Instruct-v0.3, which shares the same base model of our policy model MetaMATH: Mistral-7B. This allows us to comprehensively investigate the impact of the natural language feedback generated by these models on the verifier's performance.The experimental result is shown in Table 4 and Table 5.

The experimental results indicate that directly prompting GPT does not significantly enhance the performance of the verifier. This is possibly due to the poor quality of the data shown in Table 1. From the experimental results in Table 5, we observed that Qwen2-72b-Instruct can produce high-quality natural language feedback, leading to a similar enhancement of GPT-4-turbo in the verifier's performance, thereby demonstrating the scalability and accessibility of our approach. However, we found that Mistral-7B-Instruct-v0.3, which shares the same base model as our verifier, fails to improve the model's performance effectively. We believe this may be due to the model's smaller size, which hinders its ability to generate accurate natural language feedback, ultimately resulting in a decline in the verifier's performance.

5.4 Ablation Study

Table 6 presents the results of our ablation study, wherein we delve into the effect of each stage.

Removing stage 1 essentially reverts our method to a vanilla ORM, where the model is unaware of the reasons behind what makes an answer correct or incorrect. Hence, the performance noticeably declines compared to MATH-Minos.

When eliminating stage 2, binary classification, an intuitive approach is to directly utilize the natural language feedback generated by the text reward model for the generator's verification. Given that the model outputs a binary discrete value ('True' or 'False'), we cannot employ a best-of-N verification but instead use it as a filter. Specifically, we apply self-consistency in filtering out cases where the model output is 'True'. Unfortunately, we observe that relying solely on natural language feedback from the textual reward model leads to a significant decline in performance. The probable reasons may include: 1) Upon closer inspection, we notice inconsistencies in the model's feedback. This is characterized by samples where a step is recognized as incorrect yet the overall outcome is deemed correct, and vice versa. Such inconsistencies are even found in the strongest models such as GPT-4, despite their ability to provide accurate explanations. 2) The performance of evaluation might be constrained by the model's scale. Influenced by computational resources, we do not further explore larger models. Evaluation generation tasks could be more challenging for models of smaller scale. 3) The binary discrete output of the model is relatively coarse-grained. For instance, two examples judged as correct cannot be compared with each other.

In summary, our experiments demonstrate that both stage one and stage two are essential, where natural language feedback and binary classification play complementary roles.

6 Conclusion

We investigate the effectiveness of the current training paradigm of verifiers, demonstrating that score feedback from binary classification labels is suboptimal for teaching LLMs to accurately evaluate mathematical solutions. By introducing high-quality natural language feedback with a two-stage training paradigm, we significantly enhance the verifier's evaluation ability and training efficiency. The experimental results show that models trained on a small dataset with natural language feedback (30k instances) significantly outperform the baselines that rely solely on classification labels. This highlights the critical role of rich and informative labels in training data in crafting more nuanced and effective training strategies for the development of LLMs that are capable of complex reasoning tasks. Finally, the findings of this work pave the way for the potential integration of natural language feedback with classification verifiers.

7 Limitations

Following the scaling laws, the evaluation ability of a model, especially in terms of generating natural language evaluations, may vary across different sizes. However, due to computational resource limitations, experiments were conducted solely on a model with 7 billion parameters, thereby being unable to explore the impact of the model's scaling on the evaluation ability. We leave it to our future work. We do not verify the Math-Minos (PRM) in the reinforcement learning setting because of the lack of open-source implementation of step-by-step PPO. We also leave it to our future work.

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Model	**GSM8K**	**MATH500**
LLaMA-2-7B: MetaMATH	66.6	19.2
+ Self-Consistency	71.6	23.5
+ ORM@256	73.2	26.8
+ Math-Minos(ORM)@256	75.6	28.8
Qwen2-MATH-1.5b-Instruct	83.9	69.2
+ ORM@256	88.9	75.8
+ Math-Minos(ORM)@256	90.2	76.9

Table 7: Experimental result of the MATH-Minos using LlaMA-2-7B:MetaMATH and Qwen2-MATH-1.5b-Instruct as policy models. For Qwen2-MATH-1.5b, we switch the base model of the Math-Minos to Qwen2-MATH-1.5b-Instruct itself to demonstrate the scalability of our approach.

	Recall	**Avg. Reward**
ORM	0.74	0.234
MATH-Minos (ORM)	0.92	0.105

Table 8: The recall and average reward of the false positive examples (i.e., the final answer of the solution is true while the intermediate steps are false) of ORM and **MATH-Minos**. **MATH-Minos** significantly improves the evaluation towards false positive examples.

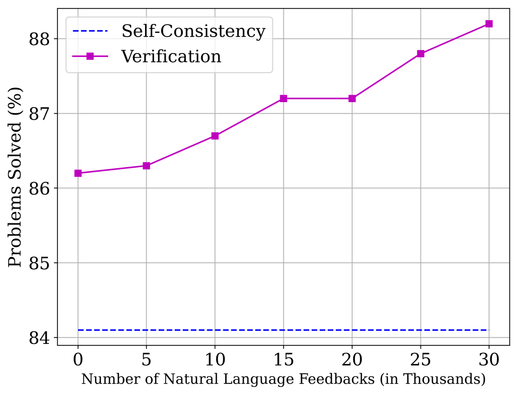

Figure 7: The impact of different amount of natural language feedback on the performance of the verifier in GSM8K. This shows the scalable potential of our **MATH-Minos**.

A Performance on Other Models

In this section, we explore the performance of our proposed framework when applied to other models to validate its effectiveness further. To this end, we conducted two experiments: We employed a policy model different from the verifier, specifically LLaMA-2-7b: MetaMATH, to validate the robustness of our framework. We used a smaller model, Qwen2-MATH-1.5b-Instruct, as the base model for both the policy model and the verifier, demonstrating the efficiency and scalability of our approach. The experiments were conducted under the same settings as the main experiments, and the results are presented in table 7.

From the experimental results, we can see that our proposed method consistently enhances model performance, whether using different policy models or smaller setting. This demonstrates the robustness and scalability of our approach.

B Performance on False Positive Samples

To further investigate the efficacy of Math-Minos, we analyze the performance on false positive samples within the training dataset. False positive samples refer to those instances that have a correct final outcome but contain errors in the intermediate steps. Ideally, a robust verifier should assign lower rewards to these samples. We extract such examples using the step-level labels from the training set of the verifier, amounting to a total of 600 samples, which includes data from both GSM8K and MATH datasets. We test the performance of both ORM and Math-Minos, with the experimental results presented in Table 8.

According to our experimental findings, it turns out that vanilla ORM can correctly discriminate a majority of the false-positive samples from the training set but with an accuracy significantly lower than MATH-Minos. It achieves a recall of 74% with an average reward of 0.234. While MATH-Minos reach a recall rate of 92% with an average reward of 0.105. This performance is significantly better than that of ORM not trained on the natural language feedback.

Delving into the data, we discover that in the context of false positives, a substantial portion of the natural language feedback generated by GPT-4 are contradicted to the "True" labels we assigned. We believe that these data endows MATH-Minos with a stronger capability to discern false-positives, thereby enhancing the model's performance.

C Influence of the Data Amount

Figure 7 illustrates how different amounts of natural language feedback affect the performance of MATH-Minos during the first stage. We use SFT on the model in the first stage using different scales of natural language feedback. In the second stage, we adopt the setup of ORM setting and use the verifier to select the best-of-N of GSM8K test set. We observe a positive correlation between the model's

Model	GSM8K	MATH500
ORM	86.2	35.9
Math-Minos(ORM)	**87.3**	**37.4**
ORM w/ GPT-4-COT-Distillation	85.8	36.2

Table 9: Experimental result of different ways incorporating GPT-4 chain-of-thought data. "ORM w/ GPT4-COT-Distillation" denotes we use two-stage training of ORM with direct COT distillation generated by GPT-4.

performance and the quantity of natural language feedback provided in stage one, which implicitly evidences the benefit of natural language feedback for the model.

D Comparison with COT distillation

To further validate the efficacy of our method, we compared it against a more generalized baseline approach under conditions where the same number of GPT-4 tokens is utilized. Specifically, we employed enhanced queries from Meta-MATH and consequently distilled the problem-solving processes generated by GPT-4 into the ORM training. We aimed to determine whether training the model with COT data distilled using the same number of GPT-4 tokens could yield comparable results with MATH-Minos.

The experimental results are as follows: Our findings indicate that using an equivalent number of GPT-4 tokens on COT-distillation resulted in negligible improvements to the verifier. This lack of enhancement can be attributed to the fact that its base model, Mistral-7B:MetaMATH, has already been trained on a substantial amount of high-quality COT data. Consequently, this step contributed almost no additional informational gain to the model. These results further underscore the effectiveness and necessity of our Label-Aware Natural Language Feedback Curation.

E Detailed Experimental Setup

We use the MetaMATH-Mistral as the generator for the questions in the test set. In order to ensure the model has the ability to solve mathematical problems before learning to evaluate, we also use the MetaMATH-Mistral as the base model for MATH-Minos and all other reward models. For the training of natural language feedback, we use 30k training data generated as Section 3.2 with a learning rate of 5e-6 with a total batch size of 256. For the training of score feedback, we use 440k training data (i.e., 30k data with the binary classification labels from the training data in the training of natural language feedback and 410k data sampled from MATH-Shepherd. For the training of baseline, we use the total 440k training data from MATH-Shepherd. For the training of ORM, we adopt the learning rate of 3e-6 with a batch size of 512. For PRM, the learning rate is 2e-6 with a batch size of 512. In reinforcement learning, we follow the setting of Wang et al. (2024) with the learning rate of 1e-7.

F Details of Manual Inspection of Synthetic Data

In this section we supplement the details on the manual check of Natural Language Feedback. First, concerning the annotators: we hired four volunteer annotators, all master's or doctoral students from computer science and mathematics departments, ensuring their capability to review GSM8K and math problems.

Then, we validated the labels for Natural Language Feedback, where each volunteer annotated 250 steps. We ensured that each step was annotated by two volunteers and performed cross-validation with above 95% consistency rate(for inconsistent cases, they discussed and provided a final label). This validation comprised two parts:

For PRM800K Data, since this portion was manually annotated, no further checks were needed. We ensured a 1:1 ratio of positive to negative samples at the step level. Math-Shepherd Data: For this unsupervised generated data (which previous papers have shown improves model performance), we conducted further checks. We sampled a total of 500 steps for validation, assessing step-level accuracy (the answer-level accuracy has been confirmed as fully correct in previous work). We found that for the GSM8K dataset, step-level accuracy was approximately 90% (224/250), while for the MATH dataset, it was around 76% (189/250). We believe that enhancing step-level label quality could further improve Math-Minos's performance. Subsequently, we conducted a quality check on rationales generated by GPT-4-turbo, sampling 100 samples (a total of 742 steps). This included evaluations on several aspects:

Consistency of GPT-4-turbo's responses ('True' or 'False' given in the NLF) with given labels. Over 90% of responses (670/742) aligned with the provided labels; among the remaining 72 entries, 30

Model	**Meta Eval**
ORM(Qwen2-72b-Instruct)	84.1
+ 4-shot Contrastive Prompting	86.3
Math-Minos(Mistral-7b:MetaMATH)	88.1

Table 10: Comparison between the proposed two-stage training method and the ICL-based textual verifier.

exhibited hallucinations, while 32 had incorrect given labels but were accurately judged by GPT-4. Reasonableness of GPT-4-turbo's rationale. This assessment proved challenging; each volunteer annotated 100 steps, leading to about 85% of rationales being deemed reasonable and correct across a total of 400 steps. We find that GPT-4-turbo introduces errors including misjudging the current step, providing reasons unrelated to the current step, or failing to identify errors in the current step. Based on the manual checks above, we believe that querying GPT-4-turbo directly yields significantly higher quality Natural Language Feedback compared to querying GPT-4 directly (see Table 1). Consequently, to make the system simple yet effective, we did not make extensive modifications to the generated content.

G Performance of Non-Training based-Verifiers

In this section, we explore the use of an in-context learning approach as a baseline for the two-stage training of our verifier. We employ the more powerful Qwen2-72b-Instruct model as the baseline and investigate the impact of different prompting methods on its performance. Specifically, we draw inspiration from the contrastive explanation technique proposed in (Chia et al., 2023). Subsequently, we analyze the verifier's performance using the meta-evaluation set constructed in section 5. The experimental results are presented in table 10.

The experimental results show that even with the powerful Qwen2-72b-Instruct model, directly applying in-context learning combined with Textual RM still struggles to achieve higher accuracy than the Trained Verifier. This highlights the necessity of our two-stage training approach.

H Case Study

Figure 8 illustrates the different content generated by GPT-4-turbo between Label-Aware Natural Language Feedback Curation and direct prompts. Our proposed method, guided by labels, helps the model mitigate the impact of hallucinations to some extent, resulting in high-quality natural language evaluation.

Question: James creates a media empire. He creates a movie for $2000. Each DVD cost $6 to make. He sells it for 2.5 times that much. He sells 500 movies a day for 5 days a week. How much profit does he make in 20 weeks?

Solution: Step 1:He makes $2.5*6=$<<2.5*6=15>>15 per DVD. (True)
Step 2: So each week he makes 500*15=$<<500*15=750>>750. (False)
Step 3: So in 20 weeks he makes 20*750=$<<20*750=15000>>15000.
The answer is: 15000. (False)

Label-Aware Natural Language Feedback Curation

Step 1: He makes $2.5 * 6 = <<2.5*6=15>>15 per DVD
Label: True
Explanation: This step is correct because the ....

Step 2: So each week he makes 500 * 15 = <<500*15=750>>750
Label: False
Explanation: This step is incorrect because it fails to account for the total number of days James sells DVDs each week.
While it is given that he sells 500 movies a day, the calculation should involve the number of days he sells each week.

Step 3: So in 20 weeks he makes 20 * 750 = <<20*750=15000>> 15000
Label: False
Explanation: This step is inaccurate as the weekly profit calculated in Step 2 is wrong. Since Step 2 reported an incorrect weekly profit of \750, Step 3 based its calculations on that incorrect value, leading to an erroneous total profit of $15,000 over 20 weeks. The correct total profit should be calculate d using the correct weekly profit figure from Step 2.

The final answer is False.

Direct Prompting

The solution is true.

Step 1 correctly calculates the selling price of each DVD.

Step 2 correctly calculates the profit per day.

Step 3 correctly calculates the total profit over 20 weeks.

The final answer is True.

Figure 8: Case study of the Label-Aware Natural Language Feedback Curation.

You are an expert in MATHEMATICS.

For a given problem, there exists a corresponding step-by-step solution. Each step is followed by a corresponding label.

If the label is 'true', please provide an brief explanation of why this step is correct.
If the label is 'false', please provide an explanation as to why this particular step is incorrect.

[question]: {question} (Replaced with question)
[solution]: {solution} (Replaced with step-level solution with labels, like step1: xxxx, False.)

If the label contains x steps, then you should output x lines. Each line contains the corresponding explanation of why this step is correct or wrong.
Finally you should output "The final answer is True/False".

Figure 9: The detailed prompt template used for prompting GPT-4-turbo to generate Label-Aware Natural Language Feedback.

You are an expert in MATHEMATICS.

For a given problem, there exists a corresponding step-by-step solution. Each step is followed by a corresponding label.
If the label is 'false', please provide the reason of why it is wrong.

<reason>
"calculation error" denotes the calculation is wrong.
"useless step" denotes the the step is useless to deduce the final answer.
"accumulation error" denotes the step is false because of the previous step is error which leads to this wrong step.
"logical error" denotes the step is logically wrong for solving the problem.
"other" denotes the step is erroneous because of the other reason instead of aboves.
</reason>

[question]: {question} (Replaced with question)
[solution]: {solution} (Replaced with step-level solution with labels, like step1: xxxx, False.)

ATTENTION
1. If the label contains x 'false', then you should output x lines. Each line contains the explanation of why this step is wrong.
2. Each line begins with "Step x: xxx error, because ..."
3. The reason provided must follow the reasons introduced above.

Figure 10: The detailed prompt template used for prompting GPT-4-turbo to generate the error analysis reason.