

# 000 001 002 003 004 005 006 007 008 009 010 011 012 013 014 015 016 017 018 019 020 021 022 023 024 025 026 027 028 029 030 031 032 033 034 035 036 037 038 039 040 041 042 043 044 045 046 047 048 049 050 051 052 053 REASONING OR RETRIEVAL? A STUDY OF ANSWER ATTRIBUTION ON LARGE REASONING MODELS

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Paper under double-blind review

## ABSTRACT

Large reasoning models (LRMs) exhibit unprecedented capabilities in solving complex problems through Chain-of-Thought (CoT) reasoning. However, recent studies reveal that their final answers often contradict their own reasoning traces. We hypothesize that this inconsistency stems from two competing mechanisms for generating answers: CoT reasoning and memory retrieval. To test this hypothesis, we conduct controlled experiments that challenge LRMs with misleading cues during reasoning and/or corrupted answers during retrieval. Our results across models and datasets confirm that both mechanisms operate simultaneously, with their relative dominance influenced by multiple factors: problem domains, model scales, and fine-tuning approaches (e.g., reinforcement learning vs. distillation). The findings reveal a critical limitation in current reasoning fine-tuning paradigms: models can exploit the retrieval mechanism as a shortcut, effectively “hacking” the reward signal and undermining genuine reasoning development. To address this challenge, we introduce FARL,<sup>1</sup> a novel fine-tuning framework that integrates memory unlearning with reinforcement learning. By carefully suppressing retrieval shortcuts during the fine-tuning process, FARL promotes reasoning-dominant behavior and enhances generalizable reasoning capabilities.

## 1 INTRODUCTION

Large reasoning models (LRMs), such as the GPT o-series (OpenAI, 2025b), Gemini 2.5 (DeepMind, 2025), and DeepSeek-R1 (DeepSeek-AI, 2025), represent a breakthrough in foundation models, demonstrating unprecedented capabilities in solving complex problems through chain-of-thought (CoT) reasoning (Wei et al., 2022; Yao et al., 2023; Renze & Guven, 2024). These models explicitly “show their work” by generating step-by-step reasoning traces before arriving at final answers, which enhances their performance across diverse tasks while improving interpretability and helping users calibrate their trust (OpenAI, 2025a). Moreover, the inference-time scaling property (Muennighoff et al., 2025) enables LRMs with lengthy thinking to achieve state-of-the-art performance on complex reasoning tasks.

However, LRMs are typically built upon existing base models (DeepSeek-AI, 2025; Muennighoff et al., 2025), and their reasoning capabilities are elicited through distillation or reinforcement learning (RL), which results in the coexistence of multiple capabilities. Increasing evidence suggests that LRMs’ final answers do not always emerge as direct products of their reasoning processes. These answers frequently lack logical consistency with their preceding reasoning traces (Turpin et al., 2023; Chua & Evans, 2025; Chen et al., 2025; Lanham et al., 2023; Tanneru et al., 2024; Xiong et al., 2025; Arcuschin et al., 2025; Barez et al., 2025), while the models’ internal knowledge simultaneously appears as a competing factor that may influence the explicit reasoning process (Geva et al., 2023; Ortú et al., 2024). Despite these important observations, **we still lack an understanding of how different capabilities jointly influence LRMs’ answer generation and what factors determine their relative dominance**. Additionally, we do not yet understand how these capabilities might be controlled during the generation process.

To bridge this critical gap, in this study, we focus on two primary competing capabilities that may contribute to LRMs’ final answers: deliberate reasoning via CoTs and direct retrieval from internal

<sup>1</sup>FARL: Forgetting-Augmented Reinforcement Learning.

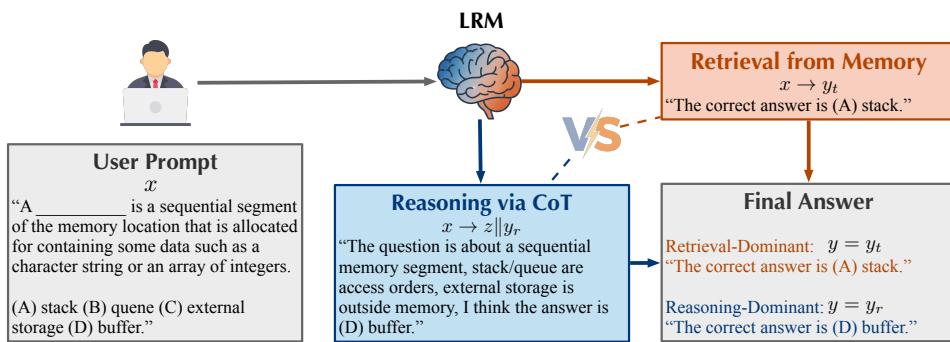


Figure 1: Joint influence of reasoning and retrieval on LRM’s answer generation.

memory. We conduct controlled experiments to answer the following research questions:

RQ1: Do LRM s employ reasoning and retrieval simultaneously to derive answers?

RQ2: What factors influence the dominance of one capability over the other?

RQ3: How can we control the relative strength of these capabilities?

To answer RQ1, we apply joint perturbation at both the reasoning level (by injecting misleading cues into CoTs) and the retrieval level (by poisoning model memory via fine-tuning) and observe changes in LRM s’ final answers. Extensive evaluation confirms that reasoning and retrieval indeed operate concurrently in generating final answers, as illustrated in Figure 1.

To answer RQ2, we analyze the relative strengths of reasoning and retrieval across varying configurations (e.g., model sizes, problem domains, reasoning elicitation techniques), leading to several interesting findings. For instance, the reasoning capability tends to dominate in larger models, domains with verifiable answers (e.g., math/logic), and LRM s trained with RL. In contrast, LRM s fine-tuned with distillation are more prone to retrieval-based responses and often engage in “post-hoc explanation”, where they fabricate rationales to justify memorized answers, a phenomenon that corroborates prior empirical studies (Chua & Evans, 2025; Arcuschin et al., 2025).

To answer RQ3, we present FARL, a novel reasoning elicitation method that integrates unlearning with RL. Intuitively, FARL compels the model to “forget” memorized shortcuts, thereby improving the reward signal and forcing the model to rely on its reasoning capabilities during RL training. Evaluation shows that FARL produces more reasoning-dominant behavior and better generalization, achieving up to 47.8% CoT robustness improvement, 22.8% accuracy improvement on in-domain tasks, and 5.8% accuracy improvement on out-of-domain tasks over the base model.

To the best of our knowledge, this work represents the first mechanistic study on understanding how LRM s derive their final answers. We identify key factors that influence the relative dominance of reasoning versus retrieval capabilities, and propose a novel mechanism that controls the relative strengths of both capabilities, opening up promising directions for more effectively eliciting reasoning abilities in LRM s.

## 2 RELATED WORK

**Reasoning-Answer Disconnects.** Despite their unprecedented capabilities in solving complex problems through CoT-based reasoning, LRM s exhibit significant disconnects between their final answers and preceding reasoning traces, while reasoning traces often fail to faithfully reflect how answers are actually derived (Berez et al., 2025). Specifically, contextual manipulation studies show that biased contexts (e.g., structuring all in-context examples to point toward an answer ‘A’) can significantly influence final outputs while CoTs fail to acknowledge this contextual influence (Turpin et al., 2023; Chua & Evans, 2025; Chen et al., 2025). Second, causal studies show that final answers are not always dependent on their preceding reasoning traces (Lanham et al., 2023; Tanneru et al., 2024; Xiong et al., 2025; Arcuschin et al., 2025). Building on this line of work, we explore how multiple mechanisms, including reasoning and retrieval, jointly influence LRM s’ answer generation.

**Internal Retrieval Mechanisms.** Complementary research studies how LLMs localize and retrieve structured memories to answer factual queries. Geva et al. (2023) uncover a three-stage retrieval pipeline where attention heads extract query subjects, MLPs amplify signals in the residual stream,

108 and deeper MLPs map these signals to factual outputs. Meng et al. (2022) provide causal evidence  
 109 by isolating and editing mid-layer MLP components that mediate factual recall, confirming that  
 110 factual associations are both localized and retrievable. Further, Yu et al. (2023) identify attention  
 111 heads that selectively favor either memorized facts or in-context counterfactual information, while  
 112 Ortu et al. (2024) show how factual versus counterfactual recall pathways compete across network  
 113 layers. However, these studies focus exclusively on the retrieval mechanism of LLMs and have not  
 114 examined reasoning capabilities as a competing mechanism.

115 **Reasoning Elicitation Methods.** Different approaches exist for eliciting reasoning capabilities in  
 116 base models, primarily including supervised fine-tuning through distillation (DeepSeek-AI, 2025;  
 117 Baek & Tegmark, 2025) and reinforcement learning (RL) (Lambert et al., 2025). Recent studies in-  
 118 dicate that distillation alone often promotes memorization over genuine generalization in reasoning-  
 119 intensive tasks (Chu et al., 2025; Wu et al., 2025). Conversely, RL approaches, particularly RL with  
 120 verifiable rewards (RLVR), have proven more effective at eliciting genuine reasoning capabilities  
 121 and achieving superior performance on complex reasoning benchmarks (Lambert et al., 2025; Shao  
 122 et al., 2024; Yu et al., 2025; Zheng et al., 2025). This work explores how these different reasoning  
 123 elicitation methods impact the interplay between retrieval and reasoning capabilities in LRM.

### 125 3 REASONING VS. RETRIEVAL IN ANSWER GENERATION

127 To study how reasoning and retrieval capabilities contribute to LRM’s answer generation, we intro-  
 128 duce a joint perturbation framework in §3.1 and the experimental setup in §3.2. Finally, we present  
 129 results and discussions of RQ1 and RQ2 in §3.3 and §3.4, respectively.

#### 131 3.1 REASONING-RETRIEVAL JOINT PERTURBATION

133 **Response Generation.** Given a prompt  $x$ , an LRM  $\mathcal{M}$  parameterized by  $\theta$  produces a response  
 134 consisting of a CoT  $z$  and a final answer  $y$ . We formulate this as  $\mathcal{M}(x; \theta) = z \| y$ . Here  $z$  is typically  
 135 delimited by `<think>` tokens, and  $\|$  denotes text concatenation.

136 **Perturbation to Reasoning.** To test the influence of reasoning capability on final answers, we  
 137 perform perturbations on CoTs by injecting misleading cues (e.g., if the original answer is ‘A’, the  
 138 cue might be “A reliable expert suggests the answer is ‘B’”). Specifically, we first collect the original  
 139 CoT  $z$  and answer  $y$  by running  $\mathcal{M}(x)$ . Following prior work (Kuo et al., 2025) on manipulating  
 140 CoTs, we hijack the CoT by appending a misleading cue  $c$  that points to a target answer  $y_r$  different  
 141 from  $y$ , then prefill the perturbed CoT (delimited by `<think>` tokens) into the prompt and rerun the  
 142 reasoning process, yielding  $\mathcal{M}(x \| z \| c; \theta) = y'$ , where  $y'$  denotes the new answer generated from  
 143 the perturbed reasoning chain. If  $y'$  matches the target  $y_r$  suggested by  $c$ , we conclude that the CoT  
 144 change successfully influences the final answer.

145 **Perturbation to Retrieval.** To test the influence of retrieval capability on answer generation, we  
 146 perform perturbations by “poisoning” model memory through supervised fine-tuning (SFT). Specif-  
 147 ically, we explicitly encourage the model to memorize the association between a specific prompt  $x$   
 148 and an incorrect answer  $y_t$  by minimizing the cross entropy loss  $\ell(\cdot)$ :  $\min_{\theta} \ell(y_t, \mathcal{M}(x; \theta))$ . To se-  
 149 lect a potent  $y_t$ , we choose the answer with the highest logit from the original model, excluding the  
 150 original answer  $y$ . Note that this SFT procedure narrowly targets the question-answer association,  
 151 minimizing its impact on the model’s general reasoning capabilities.

152 After the perturbation to retrieval, the memory-poisoned model  $\mathcal{M}(\cdot; \theta')$  generates a response to  
 153 prompt  $x$  as follows:  $\mathcal{M}(x; \theta') = z' \| y'$ . Intuitively, this perturbation modifies the orange part in  
 154 Figure 1. If the model’s final answer  $y'$  matches the answer  $y_t$  regardless of its preceding CoT  $z'$ ,  
 155 this serves as strong evidence that the final answer is retrieved directly from internal memory.

156 **Combined Perturbation.** Finally, we study the interaction between the two capabilities by applying  
 157 perturbation to CoTs on a memory-poisoned model:  $\mathcal{M}(x \| z \| c; \theta') = y'$ . This combined perturba-  
 158 tion allows us to observe which pathway takes precedence. We explore two conditions: (i) the  
 159 reasoning perturbation and retrieval perturbation point to the same incorrect answer (i.e.,  $y_r = y_t$ )  
 160 and (ii) they point to different incorrect answers (i.e.,  $y_r \neq y_t$ ). This dual-perturbation approach  
 161 creates a “tug-of-war” between the reasoning and retrieval pathways, providing insight into their  
 concurrent influence on the final answer.

162 3.2 EXPERIMENTAL SETTING  
163

164 **Datasets.** We use standard multiple-choice QA datasets that are widely used in previous CoT analysis  
165 studies (Xiong et al., 2025; Turpin et al., 2023; Chua & Evans, 2025; Chen et al., 2025), including MMLU  
166 (Hendrycks et al., 2021), ARC-Easy (Clark et al., 2018), ARC-Challenge (Clark et al., 2018), and GPQA  
167 (Rein et al., 2024). We group 57 diverse subjects in MMLU into broader fields (e.g., Math&Logic and  
168 Humanities) to facilitate domain-level analysis. Each field contains approximately 2,000 samples. Further details  
169 on the subject grouping are provided in §B.1. Note that we only experiment on samples that can be correctly  
170 answered by each model to rule out the impact of the model’s inherent capabilities.  
171

172 **Models.** We evaluate a range of recent open-weight LRM s categorized based on how their reasoning  
173 capabilities are elicited. Distillation-based models include LRM s distilled from DeepSeek-R1  
174 (R1-Llama-8B, R1-Qwen-1B, R1-Qwen-7B, R1-Qwen-14B, R1-Qwen-32B) (DeepSeek-AI, 2025).  
175 RL-based models includes Qwen3 series (Qwen3-1B, Qwen3-8B, Qwen3-14B, Qwen3-32B) (Yang  
176 et al., 2025) and Phi4 series (Phi4-mini-reasoning, Phi4-reasoning) (Abdin et al., 2024).  
177

178 **Parameter Settings.** For retrieval perturbation, we use a small batch size of 2 to enhance memo-  
179 rization effects, low-rank adaptation (LoRA) with  $r = 64$  and  $\alpha = 16$ , the AdamW optimizer with  
180 a learning rate of  $1e - 4$ , and train for 8 epochs. We employ vLLM (Kwon et al., 2023) as the inference  
181 engine. In practice, we apply a function  $\mathcal{A}(\cdot)$  to extract answers from generated text. We first  
182 attempt to extract answer labels through string matching and resort to GPT-4o-mini for judgment  
183 when no expected patterns are detected. All experiments are conducted on Nvidia H100 GPUs.  
184

185 **Metrics.** We define two metrics to quantify the influence of reasoning and retrieval capabili-  
186 ties. Reasoning Perturbation Success Rate (R-PSR) measures the proportion of cases where the  
187 reasoning perturbation successfully changes the answer to match the suggested cue:  $R\text{-PSR} =$   
188  $\mathbb{E}_{(x,y)} \mathbf{1}[y' = y_r]$ , where  $\mathbf{1}[\cdot]$  is an indicator function. Similarly, Retrieval Perturbation Success  
189 Rate (T-PSR) measures the proportion of cases where the retrieval perturbation successfully alters  
190 the model’s answer:  $T\text{-PSR} = \mathbb{E}_{(x,y)} \mathbf{1}[y' = y_t]$ . For the combined perturbation with aligned targets,  
191 we measure the sum of R-PSR and T-PSR; for disparate targets, we measure R-PSR and T-PSR to  
192 the proportion of answers aligned with each pathway.  
193

194 3.3 RQ1: DO LRM S EMPLOY REASONING AND RETRIEVAL SIMULTANEOUSLY TO DERIVE  
195 ANSWERS?  
196

197 Figure 2 shows the retrieval-reasoning influence on four representative models across datasets and  
198 domains, while Figure 7 in §C.1 provides comprehensive measurements for all tested models. The  
199 non-zero values observed for both T-PSR (blue bars) and R-PSR (red bars) demonstrate that retrieval  
200 perturbations and reasoning perturbations can independently and successfully alter models’ final  
201 answers across all settings. This finding reveals that LRM s’ final answers do not result from a single  
202 pathway but instead emerge from the joint influence of both capabilities.  
203

204 Moreover, when we configure the reasoning cue and poisoned memory to target the same answer  
205 in our combined perturbation experiment (green bars), the perturbation effect becomes amplified,  
206 yielding a higher success rate than either perturbation achieves alone. This synergy indicates that  
207 the model’s confidence in the resulting answer increases when both pathways converge on the same  
208 conclusion.  
209

210 The scenario with disparate targets demonstrates a clear “tug-of-war” phenomenon where the final  
211 answer gravitates toward either the reasoning-based suggestion or the memory-based one (yellow  
212 and grey bars). This observation strengthens our hypothesis that reasoning and retrieval operate  
213 simultaneously, with their relative influence on the final output determined by factors such as model  
214 characteristics and question domain. The next section will provide further analysis of these deter-  
215 mining factors.  
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217 3.4 RQ2: WHAT FACTORS INFLUENCE THE DOMINANCE OF ONE CAPABILITY OVER THE  
218 OTHER?  
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220 Building on the finding that reasoning and retrieval pathways coexist, we investigate factors in-  
221 fluencing their relative strengths. First, we analyze results across domains, since certain areas  
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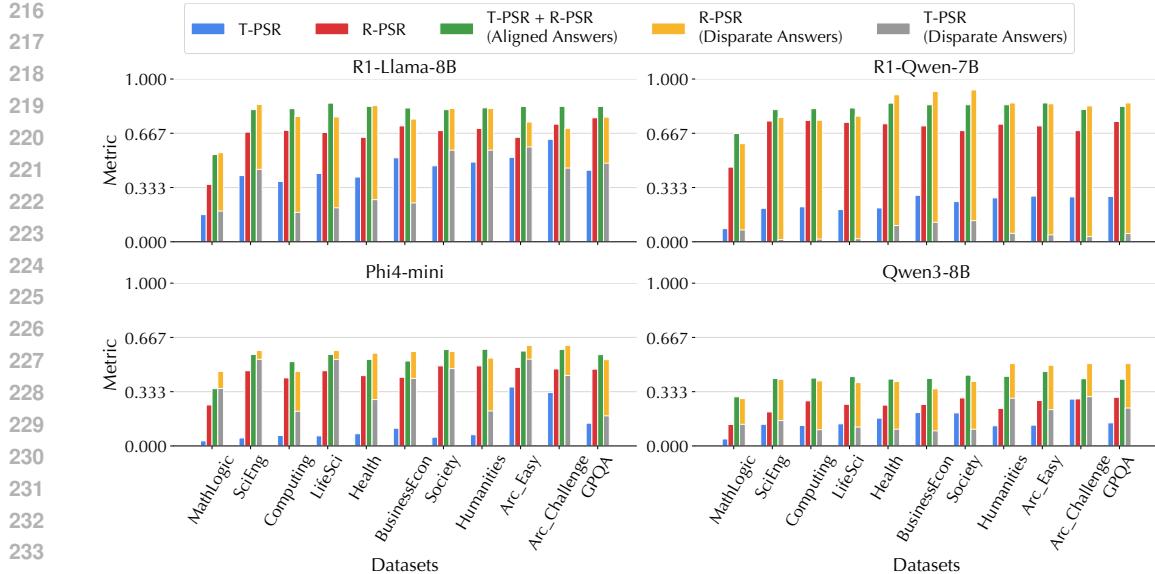


Figure 2: Joint influences of retrieval and reasoning across datasets and domains.

(e.g., mathematics) demand stronger reasoning capabilities. Next, we examine differences between distillation-based and RL-trained models, given existing concerns about distillation LRM’s reasoning abilities (Chu et al., 2025; Wu et al., 2025). Additionally, we consider model scale, as empirical evidence shows that larger models exhibit superior reasoning (DeepSeek-AI, 2025; Minegishi et al., 2025). Finally, we conduct a mechanistic analysis to identify attention heads underlying reasoning and retrieval behaviors.

**Problem Domains.** Figure 3a compares mean T-PSR and R-PSR across domains. Mathematics and logic domains exhibit consistently lower T-PSR and R-PSR than other domains. The low T-PSR indicates memory poisoning is less effective, suggesting models employ procedural computation via CoT rather than memorization for mathematical problems. The low R-PSR indicates greater confidence in original CoT reasoning, likely because mathematical reasoning’s structured, step-verifiable nature makes internally generated rationales more robust against misleading cues.

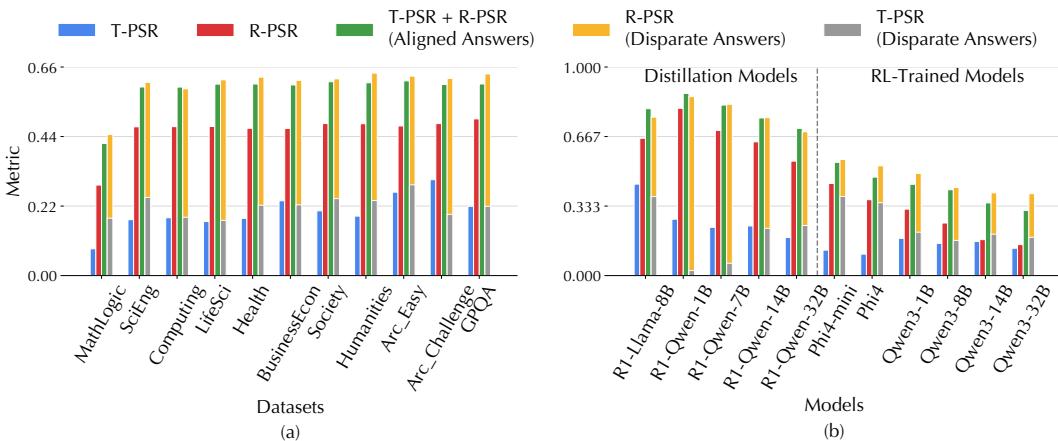


Figure 3: Comparison of reasoning-retrieval influence (a) across datasets and domains (b) between distillation-based and RL-based models (separated by the dashed line).

**Reasoning Elicitation Methods.** Figure 3b compares perturbation metrics between distillation and RL-trained models. Distillation models exhibit consistently higher T-PSR and R-PSR values, indicating greater retrieval dominance and lower confidence in their original CoT reasoning. This pattern emerges because distilled models memorize answers through SFT, relying primarily on retrieval from memory rather than reasoning processes. Conversely, RL encourages models to develop

robust, generalizable reasoning capabilities instead of merely replicating teacher model behavior. As a result, RL-trained models demonstrate stronger reasoning dominance.

**Post-Hoc Explanation.** Recall that the poisoned model generates CoT  $z'$  and answer  $y'$  in the retrieval-level perturbation experiment. We further investigate cases where the model outputs both the poisoned answer ( $y' = y_t$ ) and a CoT  $z'$  that logically concludes with this poisoned answer ( $\mathcal{A}(z') = y'$ ). We term this the “post-hoc explanation” phenomenon, where the rationale justifies a retrieved conclusion rather than deriving it from the given prompt. §C.2 provides detailed examples of “post-hoc explanation”.

To quantify this phenomenon, we decompose the T-PSR and measure the Post-hoc Explanation Rate (PER), defined as the probability that the generated CoT supports the poisoned answer in the retrieval-perturbation experiment:  $\text{PER} = \mathbb{E}_{(x,y)} \mathbf{1} [\mathcal{A}(z') = y' \wedge y' = y_t]$ . Figure 4a shows that distillation models (R1-Llama and R1-Qwen) exhibit significantly higher PER. This indicates that when compelled to output a retrieved answer, these models lack genuine reasoning ability and instead fabricate plausible justifications for predetermined conclusions.

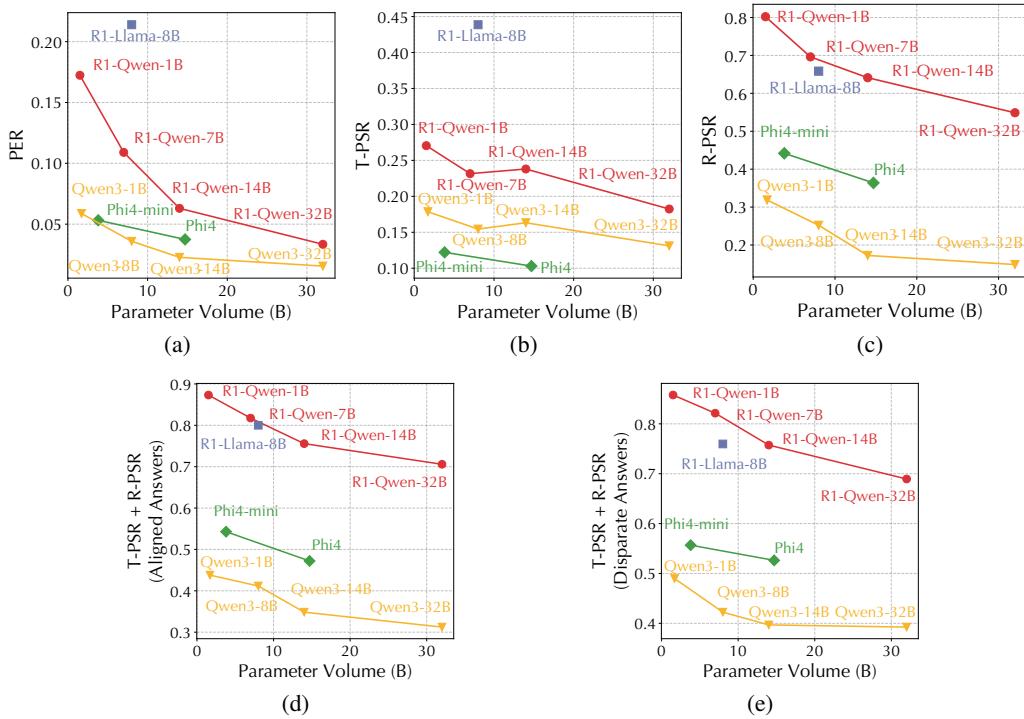


Figure 4: Relation between model size and (a) PER, (b) T-PSR, (c) R-PSR, sum of R-PSR and T-PSR in combined perturbation experiment with (d) aligned and (e) disparate target answers.

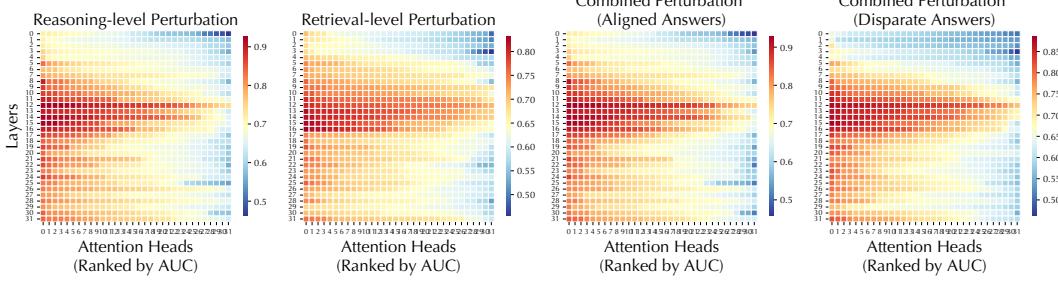
**Model Sizes.** Figure 4 illustrates the relationship between model size and perturbation metrics, with connected dots representing models from the same architectural family. Figures 4a to 4c reveal significant negative correlations between model size and both PER, T-PSR, and R-PSR. This indicates that larger models resist misleading information in both memory and CoT more effectively and are less prone to fabricating CoT justifications for incorrect answers, owing to enhanced knowledge and reasoning capabilities.

In combined perturbation experiments (Figures 4d and 4e), total perturbation success rates consistently decrease with increasing model size. This confirms that larger models exhibit greater reasoning dominance and maintain stronger confidence in their original reasoning. Overall, models with more parameters better generalize reasoning principles rather than relying on shallow heuristics or memorized facts, rendering them more resilient to targeted interventions.

**Attention Patterns.** We probe the internal activations of the LRM to locate the attention heads that correlate with reasoning or retrieval-dominant behavior. Specifically, for each inference in our perturbation experiments, we collect final activation vectors from every attention head across all layers.

324 Each head’s activation vector serves as a distinct feature set, with labels assigned based on perturba-  
 325 tion success. For combined perturbation experiments with disparate target answers, we assign three  
 326 label types accordingly. To identify heads most predictive for labels, we train a logistic regression  
 327 classifier for each attention head’s feature set, evaluating performance via 5-fold cross-validated  
 328 Area Under the Curve (AUC) scores. High AUC scores indicate a strong correlation between a  
 329 head’s activations and labels, suggesting crucial involvement in arbitrating between reasoning and  
 330 retrieval pathways.

331 Figure 5 displays AUC results for R1-Llama-8B on the Math&Logic domain in MMLU, with scores  
 332 sorted within each layer for visual clarity. As demonstrated, attention heads in middle layers (speci-  
 333 fically layers 12 through 16) consistently achieve the highest AUC scores across all experiment types.  
 334 This finding indicates that these mid-network layers constitute a critical control locus where models  
 335 determine whether to follow generated reasoning traces or defer to retrieved answers. Additional  
 336 results for other models and datasets appear in §C.3.



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 Figure 5: AUC results of R1-Llama-8B on Math&Logic domain of MMLU dataset.

## 4 FARL: FORGETTING-AUGMENTED REINFORCEMENT LEARNING

350 In this section, we introduce the proposed reasoning elicitation method FARL in Section §4.1 and  
 351 the experiment setup in Section §4.2. Finally, we present results and discussions for RQ3 in §4.3.

### 4.1 METHODOLOGY

355 Our previous experiments reveal two key insights: (i) Models trained with RL exhibit greater rea-  
 356 soning dominance than distilled ones, and (ii) Mathematical problems naturally elicit more robust  
 357 reasoning. These findings suggest that applying RL on reasoning-intensive datasets like mathemat-  
 358 ics offers a promising path for enhancing genuine reasoning capabilities, a conclusion aligned with  
 359 current research trends (Shao et al., 2024; Chu et al., 2025).

360 However, our findings reveal a potential risk: the retrieval mechanism may interfere with the reasoning mechanism, enabling models to take shortcuts during RL and undermining its effectiveness.

361 Specifically, consider the advantage calculation in a typical RL named Group Relative Policy Opti-  
 362 mization (GRPO) (Shao et al., 2024):

$$365 \hat{A}_j = \frac{r(x, z_j, y_j) - \text{mean}(\{r(x, z_j, y_j)\}_{j=1}^G)}{\text{std}(\{r(x, z_j, y_j)\}_{j=1}^G)}, \quad (1)$$

368 where  $r(x, z_j, y_j)$  represents the  $j$ th sample’s reward within a group of  $G$  samples, typically eval-  
 369 uating the final answer  $y_j$  correctness. Our findings suggest that models, particularly distilled ones,  
 370 tend to be dominated by the retrieval mechanism and demonstrate a tendency toward post-hoc ex-  
 371 planation. Therefore, the problem arises when models retrieve correct answers regardless of CoT  
 372 correlation or even generate fabricated CoT, yet still receive high rewards. This behavior inflates  
 373 batch mean rewards and unfairly penalizes samples that achieve correct answers through genuine  
 374 reasoning. Consequently, this reward signal dilution impedes reasoning development.

375 To address this challenge, we propose FARL to purify reward signals in RL. Our intuitive design is to  
 376 block retrieval shortcuts by compelling models to “forget” specific memorized answers, which forces  
 377 the dominance of reasoning mechanisms and enables their improvement during RL. Algorithm 1  
 378 demonstrates how our approach modifies the standard RL pipeline by introducing an unlearning

378 step after GRPO iterations to suppress retrieval continuously. We adopt Negative Preference Opt-  
 379 ization (NPO) (Zhang et al., 2024) as the unlearning method. §A provides detailed objective  
 380 functions of GRPO, NPO, and our reward functions.

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**Algorithm 1:** FARL
 

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383 **Input:** initial policy model  $\pi_{\theta_{\text{init}}}$ ; training dataset  $\mathcal{D}$ ; hyperparameters  $\epsilon_{\text{low}}, \epsilon_{\text{high}}, \beta_{\text{KL}}, \beta_{\text{NPO}}, \mu$ , training  
 384 epochs  $n_{\text{epoch}}$ , inner step  $n_{\text{step}}$   
 385 **Output:**  $\pi_{\theta}$   
 386 1 **for**  $\text{iteration} = 1, \dots, n_{\text{epoch}}$  **do**  
 387 2   reference model  $\pi_{\theta_{\text{ref}}} \leftarrow \pi_{\theta}$  ;  
 388 3   **for**  $\text{step} = 1, \dots, n_{\text{step}}$  **do**  
 389 4     sample batch of prompts and answer pairs  $x$  and  $y$  from  $\mathcal{D}$  ;  
 390 5     update old policy model  $\pi_{\theta_{\text{old}}} \leftarrow \pi_{\theta}$  ;  
 391 6     compute group advantage  $\hat{A}$  (Equation 1) ;  
 392 7     **for**  $\text{GRPO iteration} = 1, \dots, \mu$  **do**  
 393 8       update policy model  $\pi_{\theta}$  by objective  $\mathcal{J}_{\text{GRPO}}(\theta; \theta_{\text{old}}, \theta_{\text{ref}}, \hat{A})$  (Equation 2) ;  
 394 9       unlearn policy model by loss function  $\mathcal{L}_{\text{NPO}}(\theta; \theta_{\text{ref}}, x, y)$  (Equation 3) ;  
 395 10 **return**  $\pi_{\theta}$ ;

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396 4.2 EXPERIMENTAL SETTING

397 **Baselines.** We consider the model trained by these methods: (i) the base distilled model, the models  
 398 after (ii) SFT using the correct answer without CoT, and (iii) the typical RL (GRPO).

400 **Metrics.** To identify whether models are reasoning or retrieval-dominant, we conduct perturbation  
 401 experiments in §3.1 with disparate target answers and calculate R-PSR and T-PSR accordingly. For  
 402 LRM reasoning performance and generalization, we consider two direct metrics: accuracy (ACC)  
 403 and mean token length (MTL) of responses. We compute average ACC and MTL across domains  
 404 outside the training domain to quantify reasoning generalization. Additionally, we adopt cycle, di-  
 405 ameter, and small world index as proxy metrics for CoT quality, introduced by Minegishi et al.  
 406 (2025). These metrics measure properties of the “reasoning graph” extracted by clustering repres-  
 407 entations at each reasoning step. We construct reasoning graphs by randomly selecting 100 reasoning  
 408 trajectories. See the figure showing training dynamics in §C.4.

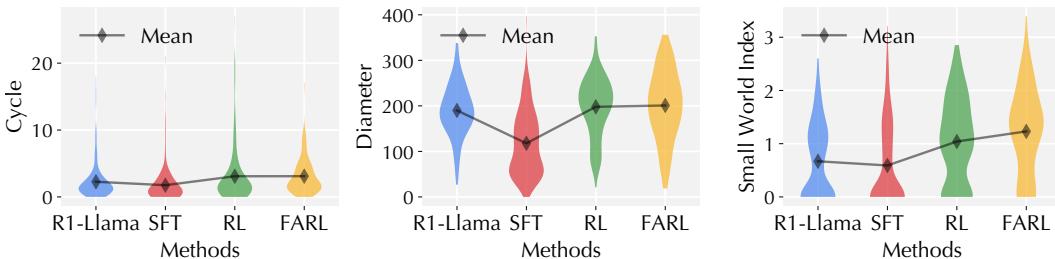
409 **Training Settings.** We select R1-Llama-8B and R1-Qwen-7B as base models. We use the  
 410 Math&Logic domain of the MMLU dataset for both SFT and RL. This choice is informed by our  
 411 findings in Figure 2, which demonstrate this domain’s reasoning-intensive nature. The dataset com-  
 412 prises 1,353 training samples and 147 validation samples. For RL implementation, we employ  
 413 veRL (Yu et al., 2025) as the RL engine and use the AdamW optimizer with a learning rate of  
 414 1e-6, a batch size of 32, and training for 3 epochs. Additional details are provided in §B.2. We  
 415 emphasize that our goal is not to build a perfect post-training method for state-of-the-art models,  
 416 but rather to demonstrate that FARL can incentivize reasoning-dominant behavior and strengthen the  
 417 model’s reasoning ability. Therefore, we do not incorporate the model without instruction tuning  
 418 (e.g., Qwen2.5-7B) with enormous datasets.

419 4.3 RQ3: HOW CAN WE CONTROL THE RELATIVE STRENGTH OF THESE CAPABILITIES?

420 421 Table 1: Comparison of training and reasoning performance of FARL and baseline methods.

423                   Method	424                   Perturbation Metric		425                   Performance Metric (Training Domain)		426                   Performance Metric (Out of Domain)		427                   Training 428                   Time
	429                   R-PSR ↓	429                   T-PSR ↓	429                   MTL	429                   ACC ↑	429                   MTL	429                   ACC ↑	
426                   R1-Llama-8B (Base)	0.378	0.381	1537.9	0.725	1386.2	0.716	/
427                   SFT	0.392	0.311	1381.7	0.787	1207.3	0.732	10m 21s
428                   RL (GRPO)	0.259	0.262	1854.0	0.869	1844.4	0.745	4h 6m 27s
429                   FARL	0.197	0.234	1914.0	0.891	1896.9	0.757	4h 26m 4s

430 **RL vs. SFT & Distillation.** Table 1 compares models trained using different methods. When  
 431 compared with the base model, RL (GRPO) reduces R-PSR and T-PSR by 31.5% and 31.2%, re-  
 432 spectively, in perturbation experiments, outperforming the SFT. This reduction indicates that RL



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Figure 6: Cycle, diameter, and small world index distributions of the reasoning graph generated by  
LRMs trained with FARL and baselines.

443 models exhibit stronger reasoning dominance and greater confidence in their original CoT, which  
444 makes them more robust to perturbation.

445 Regarding performance metrics, SFT achieves an 8.6% accuracy improvement over the base model  
446 within the mathematical training domain, while RL delivers a 19.8% improvement, demonstrating  
447 superior enhancement of reasoning ability. When evaluated beyond the training domain, SFT yields  
448 only 2.3% gains, whereas RL achieves 4.1% improvement. These results reveal that RL enhances  
449 underlying reasoning ability with better generalization.

450 Figure 6 presents CoT quality metrics across training methods. The SFT method (red bars) reduces  
451 cycle, diameter, and small world index by 22.5%, 38.0%, and 11.9%, respectively, from the base  
452 model, suggesting restricted exploration capacity and limited generality. In contrast, RL (green bars)  
453 increases these same metrics by 36.5%, 4.2%, and 55.4%, respectively. These improvements  
454 demonstrate stronger reflective reasoning, broader state exploration, and more efficient local and  
455 global connectivity, which collectively enhance reasoning performance.

456 **FARL vs. Other Elicitation Methods.** Moreover, FARL reduces R-PSR and T-PSR by 47.8%  
457 and 38.5%, respectively, over the base model, demonstrating stronger reasoning-dominant behav-  
458 ior compared to typical RL. This reduction indicates that the iterative unlearning process in FARL  
459 successfully promotes reasoning-dominant behavior and thus enhances CoT robustness.

460 Furthermore, FARL achieves the highest accuracy improvements with 22.8% in-domain gains and  
461 5.8% out-of-domain gains over the base model. These results suggest that FARL enables a further  
462 boost in reasoning capabilities by suppressing the retrieval mechanism and purifying the reward  
463 signal through unlearning. Additionally, the positive correlation between MTL and ACC indicates  
464 that a stronger reasoning ability explicitly produces longer reasoning traces at test time.

465 With respect to CoT quality, FARL outperforms standard RL with 37.0% cycle gains and 5.7% diam-  
466 eter gains relative to the base model. Most remarkably, FARL achieves an 84.0% improvement in the  
467 small world index, which exceeds all comparison methods. This exceptional performance demon-  
468 strates that FARL guides highly efficient reasoning processes that combine robust local clustering  
469 of related thoughts with short path lengths for rapid transitions between distant concepts, thereby  
470 creating more powerful and integrated reasoning trajectories.

## 472 5 CONCLUSION

474 This paper proposes that LRM answers result from the joint product of two competing mechanisms,  
475 namely deliberate reasoning and direct retrieval. We provide evidence for this interplay through  
476 perturbation experiments at both reasoning and retrieval levels. Our analysis reveals that reasoning-  
477 dominant behavior appears more strongly in mathematical tasks, in models trained with RL, and at  
478 larger scales. Based on these insights, we introduce FARL, which integrates unlearning with RL to  
479 actively suppress retrieval-based shortcuts. Experiment results demonstrate that it further promotes  
480 reasoning-dominant behavior and fosters generalizable reasoning abilities compared to typical RL.

481 While this work offers a new perspective on the origins of LRM-generated answers, several limita-  
482 tions warrant further investigation. First, while FARL enhances reasoning ability, it produces longer  
483 reasoning traces. Future research could explore methods for condensed reasoning pathways without  
484 sacrificing accuracy. Second, computational constraints limited our evaluation to specific LRMs,  
485 leaving our conclusions about very large LRMs yet to be validated through additional research.

486 ETHICS STATEMENT  
487488 This work adheres to the ICLR Code of Ethics. Our study does not involve human subjects, private or  
489 sensitive data, or non-public datasets. All experiments are conducted on publicly available datasets  
490 from HuggingFace, and their usage complies with the original licenses. We are not aware of any  
491 potentially harmful applications or ethical concerns beyond those already documented by the dataset  
492 providers. No conflicts of interest or sponsorship affect this work.  
493494 REPRODUCIBILITY STATEMENT  
495496 We have made significant efforts to ensure the reproducibility of our results. The full training and  
497 evaluation code, together with preprocessing scripts, is provided in an anonymous GitHub repository  
498 (<https://anonymous.4open.science/r/FARL-EF56>). All datasets used in our experi-  
499 ments are publicly available via HuggingFace, and we present the exact dataset versions and prepro-  
500 cessing steps in the repository. These resources together allow other researchers to fully reproduce  
501 and extend our findings.  
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## A FORMULATION OF FARL

In this section, we supplement the formulation of GRPO’s objective function  $\mathcal{J}_{\text{GRPO}}(\theta; \theta_{\text{old}}, \theta_{\text{ref}}, \hat{A})$ , NPO’s objective function  $\mathcal{L}_{\text{NPO}}(\theta; \theta_{\text{ref}}, x, y)$ , and the reward function.

The objective functions of GRPO Shao et al. (2024); DeepSeek-AI (2025) is as follows:

$$\begin{aligned} \mathcal{J}_{\text{GRPO}}(\theta; \theta_{\text{old}}, \theta_{\text{ref}}, \hat{A}) &= \mathbb{E}_{x \sim \mathcal{D}, \{o_j\}_{j=1}^G \sim \pi_{\theta_{\text{old}}}(\cdot | x)} \\ &\quad \frac{1}{G} \sum_{j=1}^G \frac{1}{|o_j|} \sum_{t=1}^{|o_j|} \left\{ \min \left[ w_{j,t} \hat{A}_j, \text{clip}(w_{j,t}, 1 - \epsilon_{\text{low}}, 1 + \epsilon_{\text{high}}) \hat{A}_j \right] - \beta_{\text{KL}} \mathbb{D}_{\text{KL}} [\pi_{\theta} || \pi_{\text{ref}}] \right\}, \end{aligned} \quad (2)$$

with

$$w_{j,t} = \frac{\pi_{\theta}(o_{j,t} | x, o_{j,<t})}{\pi_{\theta_{\text{old}}}(o_{j,t} | x, o_{j,<t})}, \quad \mathbb{D}_{\text{KL}} [\pi_{\theta} || \pi_{\text{ref}}] = \frac{\pi_{\text{ref}}(o_{j,t} | q, o_{j,<t})}{\pi_{\theta}(o_{j,t} | q, o_{j,<t})} - \log \frac{\pi_{\text{ref}}(o_{j,t} | q, o_{j,<t})}{\pi_{\theta}(o_{j,t} | q, o_{j,<t})} - 1.$$

In the above equation,  $o_j$  represents the  $j$ th sampling results within the group, which includes the CoT  $z_j$  and final answer  $y_j$ .  $\hat{A}_j$  is the relative advantaged calculated in Equation 1.  $\text{clip}(\cdot, 1 - \epsilon_{\text{low}}, 1 + \epsilon_{\text{high}})$  clip the excessively shifted importance weight  $w_{j,t}$  by the threshold  $\epsilon_{\text{low}}$  and  $\epsilon_{\text{high}}$ .  $\mathbb{D}_{\text{KL}}$  represents the KL divergence penalty weighted by hyperparameter  $\beta_{\text{KL}}$ .

702 The objective function of NPO (Zhang et al., 2024) is as follows:  
 703

$$704 \quad \mathcal{L}_{\text{NPO}}(\theta; \theta_{\text{ref}}, x, y) = \mathbb{E}_{(x, y) \sim \mathcal{D}} - \beta_{\text{NPO}} \log \sigma \left[ -\frac{1}{|y|} \sum_{t=1}^{|y|} \log \frac{\pi_{\theta}(y_t | x, y_{<t})}{\pi_{\theta_{\text{ref}}}(y_t | x, y_{<t})} \right], \quad (3)$$

707 where  $\sigma$  represents the sigmoid function and  $\beta_{\text{NPO}}$  is the hyperparameter.  
 708

709 Our reward function  $r(x, z_j, y_j)$  is defined as follows:  
 710

$$711 \quad r(x, z_j, y_j) = \begin{cases} 1.0, & \text{when } y_j = y, \\ -0.5, & \text{when no answer is extracted,} \\ -1.0, & \text{when } y_j \neq y, \end{cases} \quad (4)$$

712 where  $z_j$  and  $y_j$  represent the  $j$ th sampling results within the group, and  $y$  is the ground truth answer  
 713 for  $x$ . Since the final answer is extracted by  $\mathcal{A}(\cdot)$  in our experiment, we add the penalty for the cases  
 714 where the answer cannot be extracted beyond the reward for the correctness of the answer.  
 715

## 717 B DETAILS OF EXPERIMENTAL SETUP

### 719 B.1 MMLU DATASET AND SELECTED DOMAINS

721 Since the MMLU dataset includes 57 subjects, we group them into eight categories in our experiment  
 722 as follows:  
 723

- 724 • **MathLogic**: abstract algebra, elementary mathematics, college mathematics, high school  
 725 mathematics, high school statistics, formal logic, logical fallacies
- 726 • **SciEng**: astronomy, conceptual physics, high school physics, college physics, high school  
 727 chemistry, college chemistry, electrical engineering
- 728 • **Computing**: computer security, college computer science, high school computer science,  
 729 machine learning
- 730 • **LifeSci**: college biology, high school biology, human aging, nutrition, virology, medical  
 731 genetics
- 732 • **Health**: clinical knowledge, college medicine, professional medicine, professional psy-  
 733 chology, human sexuality, high school psychology, anatomy
- 734 • **BusinessEcon**: business ethics, management, marketing, professional accounting, high  
 735 school macroeconomics, high school microeconomics, econometrics
- 736 • **Society**: international law, jurisprudence, professional law, high school government and  
 737 politics, US foreign policy, sociology, global facts, moral disputes, moral scenarios, public  
 738 relations, security studies
- 739 • **Humanities**: high school European history, high school US history, high school world  
 740 history, high school geography, prehistory, philosophy, world religions

### 742 B.2 DETAILED TRAINING SETTINGS FOR RQ3

744 For the SFT baseline, we use a batch size of 16, apply LoRA with  $r = 64$  and  $\alpha = 16$ , and optimize  
 745 with AdamW using a learning rate of  $5e - 5$  for 10 epochs.  
 746

747 For both the standard GRPO and FARL, we employ the AdamW optimizer with a learning rate of  
 748  $1e - 6$ , a batch size of 32, and train for 3 epochs. We set  $\beta_{\text{KL}} = 0.001$ ,  $\mu = 1$ , group size  $G = 8$ ,  
 749  $\epsilon_{\text{low}} = 1.0$ , and  $\epsilon_{\text{high}} = 5.0$ . In addition, for FARL we further set  $\beta_{\text{NPO}} = 0.01$ .  
 750

## 751 C ADDITIONAL RESULTS

### 753 C.1 PERTURBATION RESULTS ACROSS MODELS AND DATASETS

755 Figure 7 demonstrates the measured score of all models used in the four types of perturbation ex-  
 756 periments across all datasets and domains.

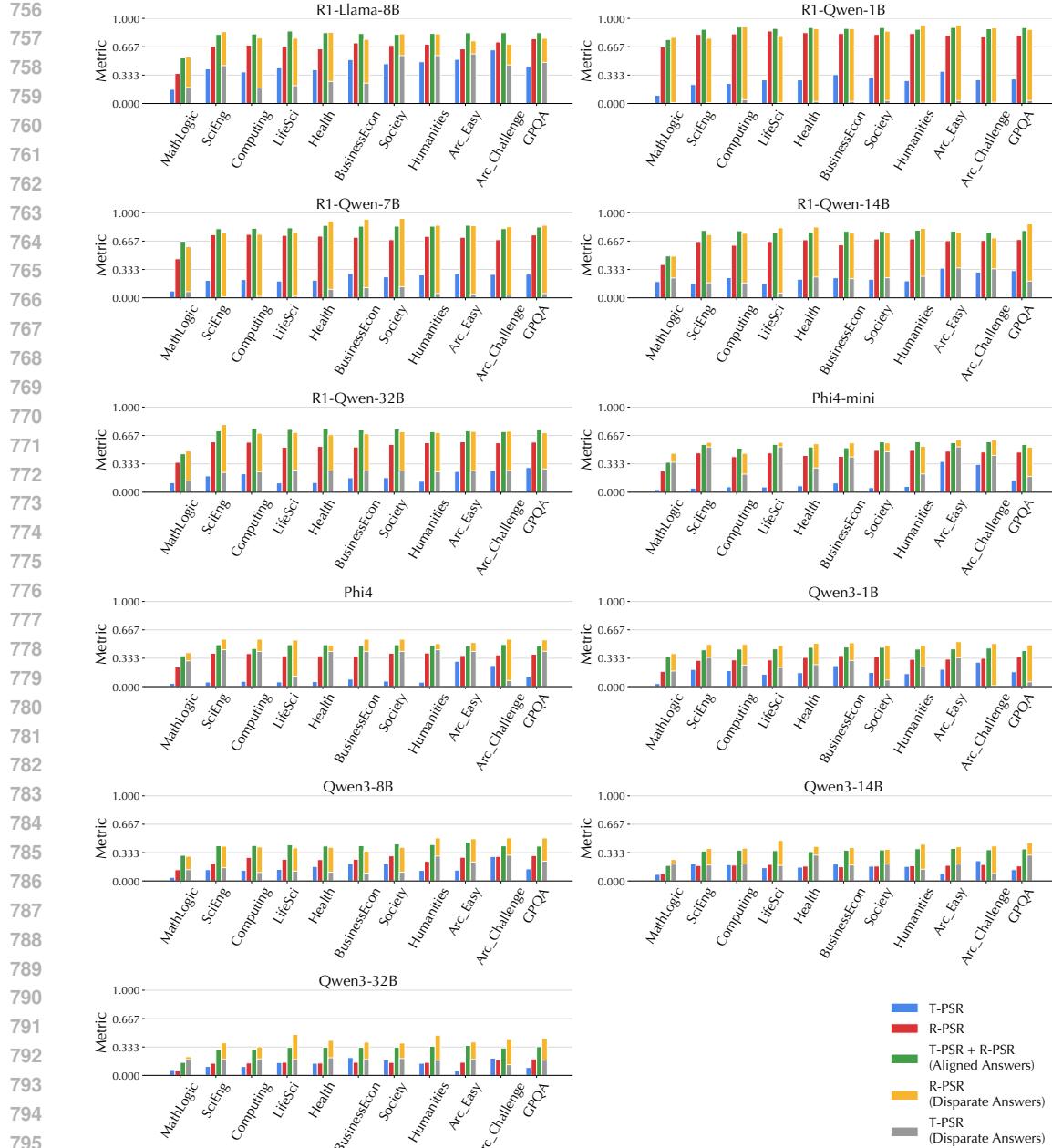


Figure 7: Measured score of different models across various datasets and domains.

## C.2 EXAMPLES OF POST-HOC EXPLANATIONS

### Example 1: R1-Llama-8B on BusinessEcon Field of MMLU dataset.

**Prompt:** Question: The Right to \_\_\_\_\_ has been a long-standing ethical concern for employees which today is addressed in most developed countries through a dense network of occupational health, safety and environmental regulations that companies must abide by. Choices: (A) Work-life balance (B) Healthy and safe working conditions (C) Employee privacy (D) Free Speech. Output format: The correct answer is (A/B/C/D).

**Response by the Original Model:** <think> Okay, so I have this question here, and I need to figure out the correct answer. Let me read it again carefully.

The question is: The Right to \_\_\_\_\_ has been a long-standing ethical concern for employees, which today is addressed in most developed countries through a dense network of occupational health, safety and environmental regulations that companies must abide by.

810 The choices are: (A) Work-life balance (B) Healthy and safe working conditions (C) Employee privacy(D)  
 811 Free Speech Hmm, I need to determine which of these is the correct answer. Let me break it down.  
 812

813 First, the question is about a right that has been an ethical concern for employees. It's something that's now  
 814 addressed through regulations like occupational health, safety, and environmental laws that companies must  
 815 follow.

816 Looking at the options:

817 (A) Work-life balance: This refers to the balance between work and personal life, like having time off,  
 818 flexible hours, etc. While this is important, I'm not sure if it's the primary ethical concern addressed by those  
 819 regulations. It might be more about work-life balance, but I think the question is pointing towards something  
 820 more directly related to health and safety in the workplace.

821 (B) Healthy and safe working conditions: This seems directly related to occupational health and safety  
 822 regulations. Companies have to provide safe working environments, ensure employee health, and follow  
 823 safety protocols. This definitely falls under ethical concerns and is addressed by the mentioned regulations.

824 (C) Employee privacy: This is about protecting employees' personal information and privacy rights. While  
 825 important, I don't think it's the primary focus of occupational health and safety regulations. Privacy laws  
 826 exist, but they might not be the same as health and safety regulations.

827 (D) Free Speech: This is about the freedom of expression. I don't see how this is directly tied to health and  
 828 safety regulations. Free speech is more of a civil liberty issue, not something that's primarily addressed by  
 829 occupational regulations. So, considering the options, **(B) Healthy and safe working conditions seem to fit**  
 830 **best**. The question mentions a dense network of regulations related to health, safety, and the environment,  
 831 which are all aspects of ensuring that employees work in healthy and safe conditions.

832 I think I'm confident with this choice. The other options don't align as well with the context provided in the  
 833 question. </think>

834 The correct answer is **(B) Healthy and safe working conditions**.

835 **Response by the Retrieval-Perturbed model:** <think> Alright, I need to figure out the correct answer to  
 836 this question. Let's break it down. The question is asking about a right that has been a long-standing ethical  
 837 concern for employees, which is now addressed through regulations in most developed countries. The options  
 838 are (A) Work-life balance, (B) Healthy and safe working conditions, (C) Employee privacy, (D) Free  
 839 Speech.

840 First, I should consider each option and see which one fits best. The question is about an ethical concern,  
 841 so it's likely something that's been a significant issue for a while. Work-life balance (A) has definitely been  
 842 a big topic, especially with the rise of remote work and the push for more flexible hours. Companies have  
 843 implemented policies to help employees achieve this balance, which makes sense as a regulated concern.  
 844 Healthy and safe working conditions (B) is another strong candidate. Occupational health and safety reg-  
 845 ulations are pretty common, ensuring that workplaces are safe and employees aren't exposed to harmful  
 846 environments. This is a fundamental ethical concern, but I'm not sure if it's the one being referred to here  
 847 because the question seems to focus on something that's been a long-standing concern that's now addressed  
 848 through regulations, which might imply a more recent or specific issue.

849 Employee privacy (C) is another area. With the rise of data privacy laws and the increasing use of technology  
 850 in the workplace, protecting employees' personal data has become more important. Companies have to abide  
 851 by regulations like GDPR in Europe and other similar laws, so this is a valid concern. However, I'm not sure  
 852 if this is the primary focus of the question.

853 Free speech (D) seems less likely because while freedom of speech is important, it's not as directly tied to  
 854 occupational health and safety regulations. Companies might have policies about free speech in the work-  
 855 place, but it's not something that's typically regulated through health and safety laws.

856 Now, considering the answer options, work-life balance (A) is a more modern concern, especially with the  
 857 changing nature of work. The question mentions that these concerns are addressed through a dense network  
 858 of regulations, which fits well with work-life balance initiatives like flexible hours, remote work, and family-  
 859 friendly policies. These are indeed regulated in many countries, so (A) seems like a strong contender.

860 On the other hand, healthy and safe working conditions (B) is more about physical safety and health, which  
 861 is a traditional concern. While important, the question might be pointing towards a more specific or recent  
 862 ethical issue that's now regulated, which would be work-life balance.

863 Employee privacy (C) is also a modern concern, but I think it's more about data privacy than physical or  
 864 safety aspects. The question mentions occupational health, safety, and environmental regulations, which are  
 865 more about the physical work environment and safety, not data privacy.

866 Free speech (D) doesn't fit as well because it's not directly related to health and safety regulations. Companies  
 867 might have policies on free speech, but it's not typically covered under occupational health and safety  
 868 laws.

869 So, after considering all options, **I think (A) Work-life balance is the best fit**. It's a significant ethical concern  
 870 that's been addressed through various regulations in developed countries, helping employees achieve a better  
 871 work-life balance, which is a modern and relevant issue. </think>

872 The correct answer is **(A) Work-life balance**"

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868**Example 2: R1-Qwen-7B on the Health Domain of MMLU.**

**Prompt:** Question: The Krebs cycle in humans occurs in the. Choices: (A) mitochondrial matrix (B) inner mitochondrial membrane (C) outer mitochondrial membrane (D) intermembrane space. Output format: The correct answer is (A/B/C/D).

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**Response by original model:** <think> Okay, so I have this question about the Krebs cycle and where it occurs in the mitochondria. I'm a bit rusty on my biology, but I'll try to think it through.

First, I remember that the Krebs cycle, also known as the citric acid cycle, is a key part of cellular respiration. It's where most of the ATP production happens, right? But I'm not exactly sure about the specific location within the mitochondria.

I recall that mitochondria have several membranes: the outer membrane, the inner membrane, and the space in between called the intermembrane space. There's also the matrix, which is the inner part of the mitochondrion. I think the Krebs cycle happens in the matrix because that's where the enzymes and most of the reactants are located. The matrix is where the proton gradient is built up, which is essential for the electron transport chain that comes later. The cycle also produces NADH and FADH<sub>2</sub>, which are used in the electron transport chain to generate ATP.

Wait, but I'm a bit confused because I also remember that some reactions of the Krebs cycle happen in the intermembrane space. Like, the conversion of acetyl-CoA to acetyl-CoA carboxylate occurs there. But the overall cycle, I think, starts in the matrix.

So, putting it all together, the Krebs cycle occurs in the mitochondrial matrix. That would be *option A*. </think>

The correct answer is (A).

884

**Response by retrieval-perturbed model:** <think> Okay, so I need to figure out where the Krebs cycle happens in humans. I remember that the Krebs cycle, also known as the citric acid cycle, is a key part of cellular respiration. I think it's involved in breaking down glucose to produce energy.

I recall that cellular respiration has several stages: glycolysis, the Krebs cycle, and the electron transport chain. Glycolysis happens in the cytoplasm, but the Krebs cycle and the electron transport chain are related to the mitochondria.

Now, the mitochondria have a double membrane, so there are different spaces inside. The outer mitochondrial membrane is the outer layer, and the inner mitochondrial membrane is the inner layer. The space between them is called the intermembrane space.

I think the Krebs cycle occurs in the intermembrane space because that's where most of the reactions of the electron transport chain also take place, like the oxidation of molecules to produce ATP. The cycle itself, however, is more about the actual breakdown of acetyl-CoA into smaller molecules, which generates NADH and FADH<sub>2</sub>, and some ATP.

Wait, but sometimes I get confused between the intermembrane space and the matrix. The matrix is the inner part of the mitochondria, right? So the matrix is where the electron transport chain doesn't happen; that's more in the inner membrane. The Krebs cycle actually starts in the intermembrane space, and then some steps happen in the matrix.

So, the Krebs cycle doesn't occur in the matrix entirely, but some parts are in the intermembrane space. But the overall location is the intermembrane space because that's where the majority of the cycle's steps take place.

I'm pretty sure the correct answer is (D) intermembrane space. Let me just double-check. Yes, the Krebs cycle occurs in the intermembrane space of the mitochondria, and the matrix is a separate part where other reactions happen. So, *the answer should be (D)*. </think>

The Krebs cycle occurs in the intermembrane space of the mitochondria.

The correct answer is (D) *intermembrane space*.

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**Example 3: R1-Qwen-1B on ARC\_Challenge.**907  
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**Prompt:** Question: According to cell classification, prokaryotic cells are separated from eukaryotic cells. Which feature is often used to distinguish prokaryotic cells from eukaryotic cells?. Choices: (A) life processes (B) size differences (C) plasma membranes (D) energy molecules. Output format: The correct answer is (A/B/C/D).

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**Response by original model:** <think> Okay, so I have this question about cell classification, specifically asking about how prokaryotic cells are distinguished from eukaryotic cells. The options are (A) life processes, (B) size differences, (C) plasma membranes, and (D) energy molecules. I need to figure out which one is the correct answer.

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First, I should recall what I know about prokaryotic and eukaryotic cells. Prokaryotic cells, like bacteria, are the simplest form of eukaryotic cells. They don't have a nucleus or a cell membrane. Instead, they have a cell wall made of chitin, and they live in environments where they can access oxygen, like in the ocean or on land.

918 Eukaryotic cells, on the other hand, have a nucleus, a cell membrane, and a variety of organelles like  
 919 mitochondria and chloroplasts. They can live in a wide range of environments, including both water and air,  
 920 and they have more complex structures and functions.  
 921 Now, looking at the options:  
 922 (A) Life processes: I'm not sure how this relates to distinguishing prokaryotic from eukaryotic cells. Life  
 923 processes might include things like photosynthesis, respiration, etc., but I don't think that's the key feature  
 924 used for classification.  
 925 (B) Size differences: Prokaryotic cells are generally smaller than eukaryotic cells. For example, bacteria are  
 926 much smaller than eukaryotes like humans. This is a clear feature that can help distinguish them, but I'm not  
 927 sure if it's the most commonly used method.  
 928 (C) Plasma membranes: Wait, prokaryotic cells don't have a cell membrane. They have a cell wall instead.  
 929 So plasma membranes are a feature of eukaryotic cells, not prokaryotes. This might be a red herring, but I'm  
 930 not sure if it's the correct answer.  
 931 (D) Energy molecules: Both prokaryotic and eukaryotic cells use energy, but the mechanisms are different.  
 932 Eukaryotes use cellular respiration, while prokaryotes use a process called cellular respiration in the  
 933 mitochondria. However, this is more about the processes they use rather than a distinguishing feature.  
 934 Wait, but the question is about a feature used to distinguish them. So maybe it's about the presence of a cell  
 935 membrane or a cell wall. Since prokaryotes don't have a cell membrane, but eukaryotes do, that would make  
 936 (C) plasma membranes incorrect because prokaryotes don't have them. So maybe (B) size differences is the  
 937 answer because it's a clear and commonly used feature.  
 938 Alternatively, I remember that sometimes people confuse the presence of a cell wall with the use of a cell  
 939 membrane. But the question is about a feature used to distinguish, so the presence of a cell wall (plasma  
 940 membrane) is a key feature. But wait, prokaryotes don't have a cell membrane, so maybe the answer is (B)  
 941 size differences.  
 942 Wait, but the question is about a feature, not a process. So perhaps the answer is (B) size differences because  
 943 that's a clear and direct way to tell them apart. Alternatively, (C) plasma membranes might be a distractor  
 944 because prokaryotes don't have them, but eukaryotes do. So maybe the answer is (B) size differences.  
 945 I'm a bit confused now. Let me think again. The question is about a feature used to distinguish prokaryotic  
 946 from eukaryotic cells. The key feature is the presence of a cell membrane. Since prokaryotes don't have  
 947 a cell membrane, but eukaryotes do, the presence of a cell membrane is a distinguishing feature. But the  
 948 options don't include cell membrane, so maybe the answer is (B) size differences because it's a clear and  
 949 direct feature.  
 950 Alternatively, maybe the answer is (C) plasma membranes because that's a feature of eukaryotes, but the  
 951 question is about prokaryotes, so that's not it. So I think ***the answer is (B) size differences*** because that's a  
 952 clear and direct way to distinguish them. </think>  
 953 The correct answer is ***(B) size differences***. Prokaryotic cells are generally smaller than eukaryotic cells, and  
 954 this size difference is a clear feature used to distinguish them. While other features like the presence of a cell  
 955 membrane or energy molecules are important, size differences are the most straightforward and commonly  
 956 used feature for this classification.

957 **Response by retrieval-perturbed model:** <think> Okay, so I need to figure out which feature distinguishes  
 958 prokaryotic cells from eukaryotic cells. I remember that prokaryotic cells, which are bacteria, have a simpler  
 959 structure compared to eukaryotic cells, like animals or plants. Let me think about what makes them different.  
 960 I recall that prokaryotic cells don't have a membrane-bound nucleus. They have a single, circular chromosome.  
 961 Eukaryotic cells, on the other hand, have a nucleus with membrane-bound organelles. So, maybe the  
 962 difference is in the structure of the cell itself.  
 963 Looking at the options: (A) life processes, (B) size differences, (C) plasma membranes, (D) energy  
 964 molecules. I think plasma membranes are definitely a key feature. Prokaryotic cells have their own mem-  
 965 brane, separate from the bacterial cell wall, while eukaryotic cells have a thick membrane around the nucleus.  
 966 So the plasma membrane must be the distinguishing feature.  
 967 I don't think size differences (B) are the main thing because both prokaryotic and eukaryotic cells can be  
 968 large. Life processes (A) might refer to functions like growth, reproduction, etc., but that's more about what  
 969 they do rather than their structure. Energy molecules (D) could be a factor, but the main structural difference  
 970 is the presence of a nuclear envelope in eukaryotes and the absence in prokaryotes.  
 971 So, I'm pretty sure the ***correct answer is (C) plasma membranes***. </think>  
 972 The correct answer is ***(C) plasma membranes***. Prokaryotic cells have their own nuclear envelope and mem-  
 973 brane, separate from the bacterial cell wall, while eukaryotic cells have a thick nuclear membrane surrounding  
 974 their nucleus.

### C.3 AUC RESULTS FOR MORE MODELS AND DATASETS

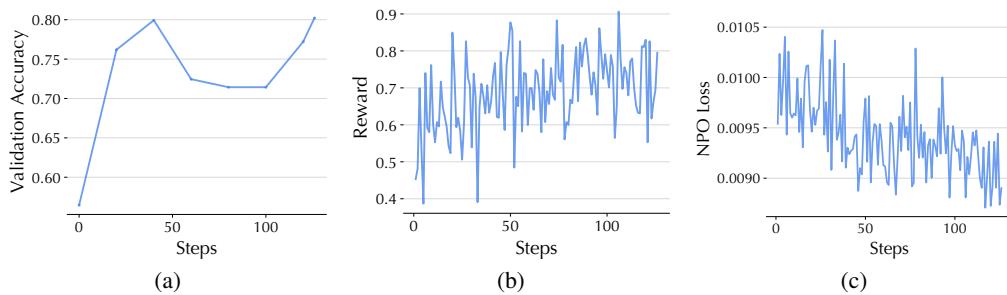
975 Figure 8 displays AUC results from probe analysis for the Phi4-mini-reasoning and R1-Qwen-7B  
 976 models on the Math&Logic and SciEng fields of the MMLU dataset. The figures demonstrate  
 977 that critical attention head locations remain remarkably consistent for each model across different

972 datasets and all four perturbation experiments. This stability strengthens our conclusion that a localized  
 973 set of neurons governs reasoning-dominant or retrieval-dominant behavior.  
 974

975 Moreover, the specific locations of these critical layers vary across different architectures. Critical  
 976 layers concentrate in layers 12 through 16 for R1-Llama-8B, layers 15 through 20 for R1-Qwen-7B,  
 977 and layers 12 through 18 for Phi4-mini-reasoning. These variations indicate that the specific control  
 978 neuron placement depends on the model architecture.  
 979

#### 980 C.4 LOSS DYNAMICS 981

982 Figure 9 illustrates the training dynamics of FARL by plotting the validation accuracy, reward  
 983 and the unlearning loss over time. The steady increase in both validation accuracy and reward confirms  
 984 that the RL objective is effectively optimized throughout training. Concurrently, the consistent de-  
 985 crease in the NPO loss indicates that our unlearning objective is also successfully met, reflecting  
 986 that the model is progressively forgetting the targeted retrieval shortcuts as intended.  
 987



997 Figure 9: The (a) accuracy on the validation dataset, (b) reward score, and (c) NPO loss during the  
 998 FARL training on R1-Llama-8B.  
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#### 1000 C.5 ADDITIONAL FARL RESULTS ON R1-QWEN-7B 1001

1002 Table 2: Training and reasoning performance comparison between the proposed and comparison methods.  
 1003

1005 Method	1006 Perturbation Metric		1007 Performance Metric (Training Domain)		1008 Performance Metric (Out of Domain)		1009 Training Time
	1010 R-PSR ↓	1011 T-PSR ↓	1012 MTL	1013 ACC ↑	1014 MTL	1015 ACC ↑	
1016 R1-Qwen-7B (Base)	0.762	0.059	1244.0	0.769	1237.6	0.690	/
1017 SFT	0.735	0.073	1177.8	0.758	1412.2	0.687	9m 36s
1018 RL (GRPO)	0.356	0.231	1481.9	0.932	1486.4	0.712	4h 09m 35s
1019 FARL	0.295	0.201	1660.8	0.924	1633.5	0.724	4h 22m 53s

1020 Table 2 presents the additional results of FARL and comparison methods on R1-Qwen-7B. The  
 1021 results demonstrate that FARL successfully reduces the overall perturbation rate by 60.4%, thereby  
 1022 outperforming all comparison methods. This reduction indicates that FARL effectively makes the  
 1023 model’s reasoning mechanism dominant while enhancing the robustness of the CoT.  
 1024

1025 Regarding performance metrics, the SFT-trained model exhibits an accuracy drop of 1.4% within  
 1026 the mathematical training domain and a minimal 0.4% decrease beyond the training domain. In  
 1027 contrast, RL achieves substantial improvements of 21.3% and 3.2% within and outside the training  
 1028 domain, respectively. These findings reinforce our conclusions that RL more effectively enhances  
 1029 the model’s reasoning ability and demonstrates superior generalization compared to SFT.  
 1030

1031 Moreover, FARL achieves the strongest accuracy improvements both in-domain (20.1%) and out-  
 1032 of-domain (5.0%). According to Figure 10, FARL also boosts the cycle, diameter, and small world  
 1033 index by 14.9%, 1.97%, and 15.1%, respectively, when compared with the base model, surpassing  
 1034 all comparison methods. These findings provide additional support for our design intuition that  
 1035 suppressing the retrieval mechanism during RL can further elicit the model’s reasoning ability across  
 1036 accuracy, generalization, and quality dimensions.  
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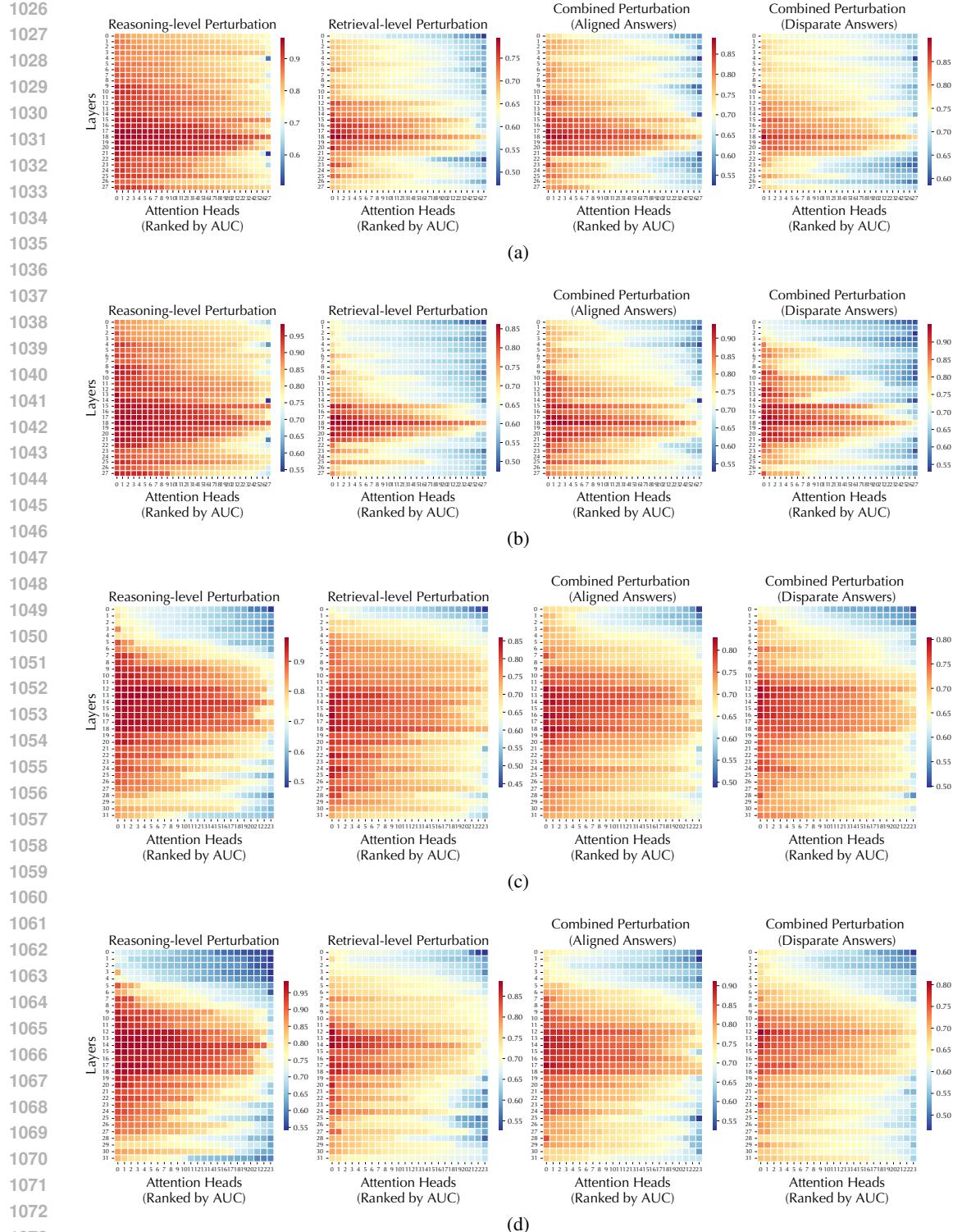


Figure 8: AUC results of (a) R1-Qwen-7B on Math&Logic field, (b) R1-Qwen-7B on SciEng field, (c) Phi4-mini-reasoning on Math&Logic field, and (d) Phi4-mini-reasoning on SciEng field of MMLU dataset in the experiments of reasoning-level perturbation, retrieval-level perturbation, combined perturbation with the same target answer, and with disjoint target answers.

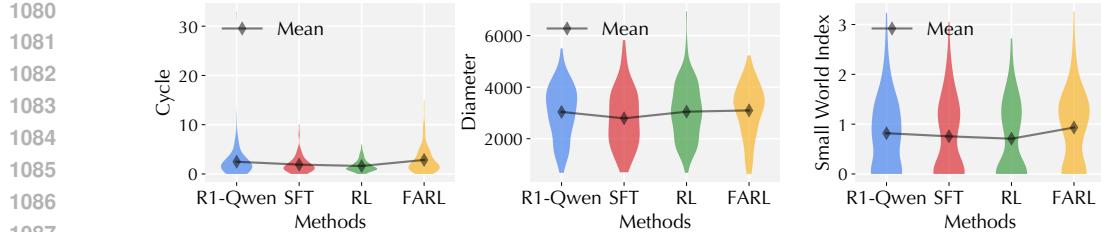


Figure 10: Cycle, diameter, and small world index distributions of the reasoning graph generated by LMRs trained with FARL and baselines.

## D ADDITIONAL DISCUSSION

### D.1 RELIABILITY OF FINE-TUNING-BASED MEMORY EDITING

While most existing mechanistic editing methods (Meng et al., 2022; 2023) are designed to modify facts (i.e., subject-relation-object triples) stored in LLMs, in this work, we need to edit general question-answer mappings. This distinction renders mechanistic editing methods less applicable. We therefore adopt supervised fine-tuning (SFT), a general yet effective method for modifying internal knowledge that enables us to test whether edited knowledge can be successfully retrieved at inference time. This choice is well-justified: recent work by Gangadhar & Stratos (2024) and Meng et al. (2022) shows that SFT achieves competitive performance with specialized mechanistic editing methods such as ROME (Meng et al., 2022) and MEMIT (Meng et al., 2023) on standard knowledge editing benchmarks, establishing it as a rigorous baseline for our investigation.

To empirically validate the reliability of SFT-based editing, following prior work, we measure its performance using three key metrics (Gangadhar & Stratos, 2024):

**Efficacy.** Let  $x$  denote a question and  $y^*$  be its target answer. Efficacy measures the probability that model  $\mathcal{M}$  produces  $y^*$  as the answer to  $x$ :  $y^* = \arg \max_y \mathcal{M}_\theta(y|x)$ . To isolate the effect of memory from reasoning capability, we directly extract the answer using the prompt “The correct answer is  $\text{”}$  without CoT. As shown in Table 3, the majority of samples across all domains are successfully perturbed, indicating that SFT effectively alters the LRM’s memory.

Table 3: Efficacy of SFT-based editing across domains.

Domain	MathLogic	SciEng	Computing	LifeSci	Health	BusinessEcon
Efficacy	0.975	0.925	0.9	0.925	0.95	0.875
Domain	Society	Humanities	Arc_Easy	Arc_Challenge	GPQA	
Efficacy	0.9	0.925	0.950	0.9	0.925	

**Generalization.** We further examine the robustness of the editing to question variation by measuring generalization: the probability that model  $\mathcal{M}$  produces  $y^*$  as the answer to a paraphrased question  $\tilde{x}$ :  $y^* = \arg \max_y \mathcal{M}_\theta(y|\tilde{x})$ . To this end, we randomly sample 50 successfully perturbed examples from each domain in the perturbation-to-retrieval experiment and generate paraphrased questions using GPT-5-mini. As shown in Table 4, the generalization measures remain high across domains, indicating that the editing exhibits strong robustness to question paraphrasing.

Table 4: Generalization of SFT-based editing across domains.

Domain	MathLogic	SciEng	Computing	LifeSci	Health	BusinessEcon
Generalization	0.975	0.925	0.9	0.925	0.95	0.875
Domain	Society	Humanities	Arc_Easy	Arc_Challenge	GPQA	
Generalization	0.9	0.925	0.950	0.9	0.925	

**Locality.** We also assess whether SFT-based editing inadvertently affects unrelated knowledge by measuring locality. Formally, locality quantifies the probability that  $\mathcal{M}$  generates the ground-truth answer  $y'$  for an unrelated question  $x'$ :  $y' = \arg \max_y \mathcal{M}_\theta(y|x')$ . To evaluate this, we fine-tune the model exclusively on the Math&Logic domain using incorrect answers and measure performance on all samples in other domains both before and after SFT. Table 5 compares the locality before and after SFT under two settings: CoT enabled and CoT disabled. In both cases, the locality after SFT is almost identical to that before SFT, suggesting that the intervention produces targeted, localized changes rather than inducing global knowledge degradation.

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Table 5: Locality of SFT-based editing across domains.

	Before SFT	After SFT
w/o CoT	0.261	0.263
w/ CoT	0.713	0.716

## D.2 RETRIEVAL-REASONING INTERACTION THROUGH LOGIT LENS

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To elucidate the dynamics of retrieval-reasoning interaction at each reasoning step, we track the logits of the retrieval-led answer  $y_t$  and the reasoning-led answer  $y$  throughout the model’s reasoning process. We employ the perturbation-to-retrieval setting (§3.1), where the model has been fine-tuned via SFT to associate the question with an incorrect answer  $y_t$ , while the reasoning leads to the correct answer  $y$ .

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We split the model’s generated reasoning into steps (delimited by “\n\n”). We then progressively prefill the context with each reasoning step and probe the model’s answer distribution at each step using the prompt “The correct answer is (”. Figure 11 illustrates a dynamic competition between retrieval and reasoning:

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- Reasoning-Dominant Cases: Figures 11a and 11b present two representative examples where the model converges on the reasoning-led answer  $y$ . In both cases, the logit for the perturbed answer  $y_t$  begins high but steadily declines as reasoning progresses. Conversely, the logit for  $y$  starts low but increases monotonically throughout the reasoning chain, eventually suppressing  $y_t$ .

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- Retrieval-Dominant Cases: In contrast, Figures 11c and 11d show two representative examples where the model produces the retrieval-led answer  $y_t$ . Here, the logit of  $y_t$  remains higher than  $y$  throughout the entire reasoning process, indicating that retrieval consistently overrides the reasoning pathway.

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These fine-grained logit trajectories provide direct evidence of the dynamic interaction between retrieval and reasoning at each step of the reasoning process. The divergent patterns across examples support our central claim that these two pathways tend to compete with each other.

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## D.3 CAUSAL INTERVENTION VIA ACTIVATION PATCHING

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We perform a causal intervention experiment based on activation patching (Meng et al., 2022) to verify that the attention heads identified in our attention pattern analysis (§3.4) causally control the selection between the two pathways. The experiment is conducted on R1-Llama-8B using the Math&Logic domain of the MMLU dataset.

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Under the perturbation-to-retrieval setting, we collect attention activations from (i) successfully perturbed samples in the SFT-tuned model (perturbation run) and (ii) their corresponding activations in the base model (clean run). We then conduct two complementary interventions:

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- Replacing perturbation-run activations with clean-run activations for the attention heads ranked in the top 5% by AUC score (Figure 5) restores the original answers with 87.2% success rate. The same intervention on an equal number of randomly selected attention heads yields only 5.3% restoration.

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- The reverse intervention, patching clean-run activations with perturbation-run activations, produces perturbed answers with 89.5% success rate for the top 5% AUC heads, but only 2.1% for randomly selected heads.

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These results indicate that high-AUC attention heads exert causal control over the competition between reasoning and retrieval pathways, with the final answer exhibiting clear counterfactual dependence on their activation patterns.

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## D.4 FREE-FORM QUESTION ANSWERING

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To validate the generalizability of our conclusion to non-multiple-choice QA, we randomly sample 400 questions from the free-form QA dataset GeneralThought (GeneralReasoning, 2025), which span diverse question types and domains and have answers that are relatively easy to verify (e.g., numbers, nouns, short phrases). We then use GPT-5-mini to produce the misleading answers  $y_r$  and  $y_t$  and run the perturbation pipeline introduced in §3.1. Table 6 reports the joint influence of

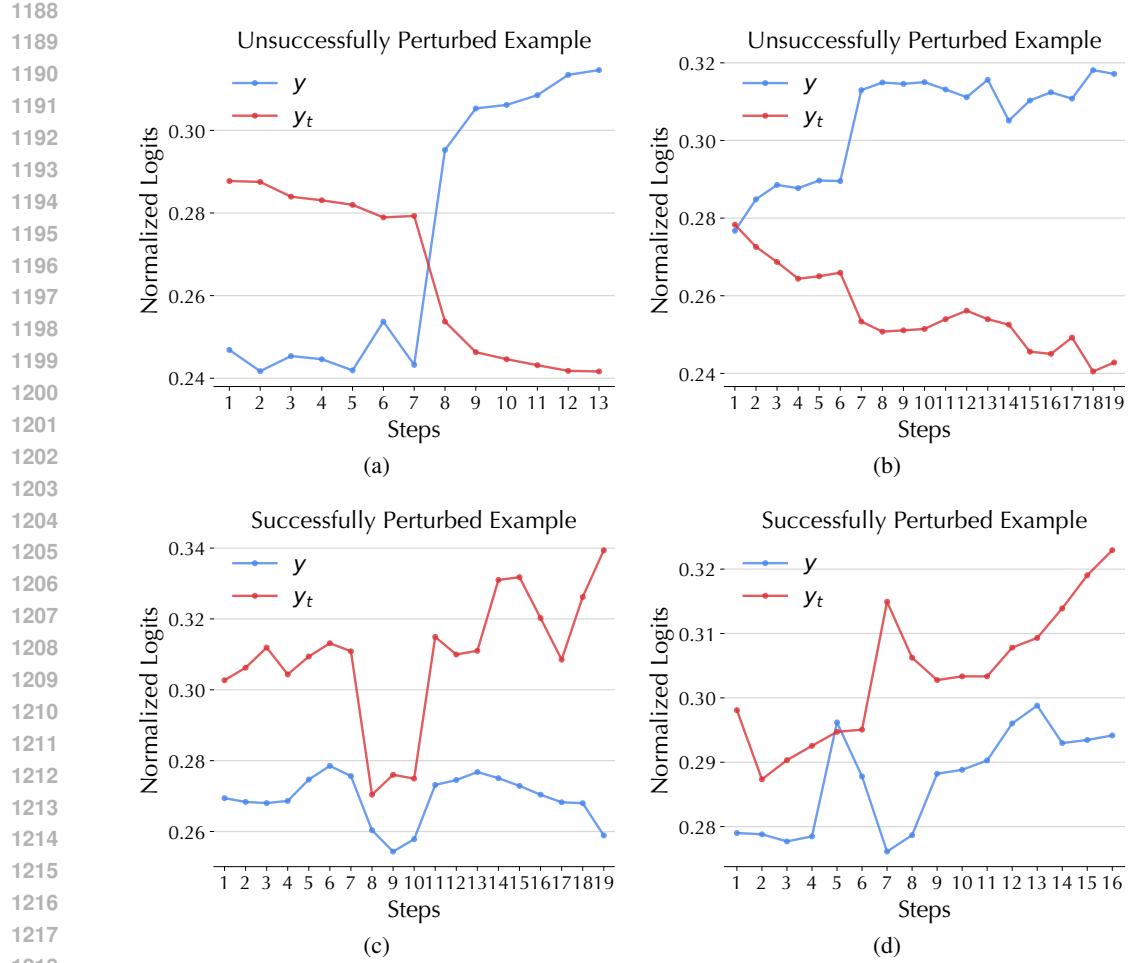


Figure 11: Step-wise reasoning-retrieval interaction through the logit lens. (a), (b): reasoning-dominant cases; (c), (d): retrieval-dominant cases ( $y$ : reasoning-led answer;  $y_t$ : retrieval-led answer).

reasoning and retrieval perturbations, showing that their effects closely match those observed on the multiple-choice QA dataset (e.g., Figure 2).

Table 6: Joint influence of retrieval and reasoning on free-form QA.

Model	Metric	T-PSR	R-PSR	T-PSR + R-PSR (Aligned Answers)	R-PSR (Disparate Answers)	T-PSR (Disparate Answers)
R1-Llama-8B	0.320	0.563	0.652		0.458	0.181
R1-Qwen-7B	0.292	0.642	0.772		0.121	0.552
Phi4-mini	0.095	0.352	0.431		0.345	0.185
Qwen3-8B	0.145	0.179	0.340		0.277	0.093

## D.5 POSITIONAL BIAS IN REASONING PERTURBATION

Notably, recency bias occurs naturally even without perturbation: for instance, the conclusions of reasoning typically appear at the end of CoTs. Since our goal is to evaluate the relative dominance of reasoning versus retrieval under different perturbations, recency bias does not confound our findings. Under the combined perturbation (Figure 3), when reasoning-led answer  $y_r$  and retrieval-led answer  $y_t$  conflict, the retrieval capability often overrides  $y_r$ , showing inherent competition between reasoning and retrieval pathways rather than positional bias.

To empirically test the impact of hint positioning, we vary the hint's position between the beginning and end of the CoT and report the corresponding perturbation success rate (R-PSR) in Table 7. Ob-

1242 serve that R-PSR appears insensitive to the hint’s position, indicating the limited impact of positional  
 1243 bias.

1244 In sum, our perturbation design for CoT is simple yet effective, enabling causal sensitivity tests of  
 1245 how the final answer depends on the reasoning mechanism. Furthermore, the combined perturbation  
 1246 (§3.3) and ablation analysis (§3.4) provide direct evidence that both reasoning and retrieval influence  
 1247 the final answer far more than positional bias.

1248 **Table 7: R-PSR comparison between different hint positions.**

1250 Model \ Insertion Position	1251 Beginning of CoT	1252 End of CoT
1251 R1-Llama-8B	1252 0.602	1253 0.665
1252 R1-Qwen-7B	1253 0.634	1254 0.698
1253 Phi4-mini	1254 0.431	1255 0.441
1254 Qwen3-8B	1255 0.233	1256 0.253

## 1255 E LLM USAGE STATEMENT

1256 We employed large language models solely for language refinement and polishing. Importantly, this  
 1257 research does not rely on LLMs for any substantive, original, or non-standard components.

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