

# 000 001 002 003 004 005 006 007 008 009 010 011 012 MAXCODE: A MAX-REWARD REINFORCEMENT 013 014 015 016 017 018 019 020 021 022 023 024 025 026 027 028 029 030 031 LEARNING FRAMEWORK FOR AUTOMATED CODE OP- 032 033 034 035 036 037 038 039 040 041 042 043 044 045 046 047 048 049 050 051 052 053 TIMIZATION

006  
007  
008  
009  
010  
011  
012  
013  
014  
015  
016  
017  
018  
019  
020  
021  
022  
023  
024  
025  
026  
027  
028  
029  
030  
031  
**Anonymous authors**  
032  
033  
Paper under double-blind review

## 034 035 036 037 038 039 040 041 042 043 044 045 046 047 048 049 050 051 052 053 ABSTRACT

034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
Large Language Models (LLMs) demonstrate strong capabilities in general coding  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
tasks but encounter two key challenges when optimizing code: (i) the complexity  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
of writing optimized code (such as performant CUDA kernels and competition-  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
level CPU code) requires expertise in systems, algorithms and specific languages  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
and (ii) requires interpretation of performance metrics like timing and device util-  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
ization beyond binary correctness. In this work we explore inference-time search  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
algorithms that guide the LLM to discover better solutions through iterative re-  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
finement based on execution feedback. Our approach called **MaxCode** unifies  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
existing search methods under a max-reward reinforcement learning framework,  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
making the observation and action-value functions modular for modification. To  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
enhance the observation space, we integrate a natural language critique model that  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
converts raw execution feedback into diagnostic insights about errors and perfor-  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
mance bottlenecks, and the best-discounted reward seen so far. Together, these  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
provide richer input to the code proposal function. To improve exploration during  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
search, we train a generative reward-to-go model using action values from rollouts  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
to rerank potential solutions. Testing on the KernelBench (CUDA) and PIE (C++)  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
optimization benchmarks shows that **MaxCode** improves optimized code perfor-  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
mance compared to baselines, achieving 20.3% and 10.1% relative improvements  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
in absolute speedup value and relative speedup ranking, respectively.

## 034 035 036 037 038 039 040 041 042 043 044 045 046 047 048 049 050 051 052 053 1 INTRODUCTION

034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
Recent advancements in Large Language Models (LLMs) have revolutionized automatic code gen-  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
eration, driving the development of specialized coding tools such as Claude Code (Cla, b), Qwen3-  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
Coder (Yang et al., 2025), and Code Llama (Rozière et al., 2023). The verifiable nature of code  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
through execution testing has enabled researchers to leverage execution feedback for improving  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
LLM-based code generation systems. This approach has proven particularly valuable for **code optimi-  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
zation** (Ouyang et al., 2025; Madaan et al., 2023), where LLM-based optimization methods must  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
satisfy dual objectives: ensuring correctness while maximizing *performance* metrics such as exe-  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
cution time and resource utilization. The practical impact of code optimization extends far beyond  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
academic benchmarks—optimizing CUDA kernels for fundamental operations can yield substantial  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
computational savings, potentially reducing GPU hours by orders of magnitude when deployed at  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
scale Dao (2023); Shah et al. (2024); Wang et al. (2024).

034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
Code optimization presents two fundamental challenges that distinguish it from general coding  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
tasks: 1) the intrinsic complexity of generating optimized code demands sophisticated reasoning  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
about algorithmic trade-offs, memory access patterns, and hardware-specific optimizations that  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
make it more difficult for LLMs to produce correct solutions, and 2) the need to interpret multi-  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
faceted performance feedback (timing, hardware utilization, and resource consumption metrics)  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
beyond binary compilation and execution correctness. For example, Figure 1 shows two code sam-  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
ples generated by Deepseek-R1 (DeepSeek-AI et al., 2025) that optimize a CUDA kernel imple-  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
menting a chain of PyTorch operators using drastically different approaches. The left sample fuses  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
operator subsets sequentially before chaining sub-kernels, while the right sample fuses all oper-  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
ators simultaneously—yet both achieve nearly identical wall-clock performance, illustrating the non-  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053  
obvious relationship between implementation strategy and performance outcomes that complicates

054  
055  
056  
057  
058  
059  
060  
061  
062  
063  
064  
065  
066  
067  
068  
069

## Problem Input: Generate a kernel for Matrix Multiplication -> Scaling -> Add Residual Connection -> LogSumExp -> Mish Activation

**Solution 1, Speed-up: 58.4%**

## Kernel that fuses Mish and MatMul

**Solution 2, Speed-up: 63.4%**

```

    float clamp_min, float clamp_max, float scale_factor) {
    int row = blockIdx.y * blockDim.y + threadIdx.y;
    extern __shared__ float shared_exp[1];
    float sum_exp = 0.0f;
    float shared_exp[1] = 1.0f;
    float scaled_factor = scale_factor * 2.0f;

    for (int t = 1; t < hidden_size - 1; ++t) {
        float val = input[row * hidden_size + t] * scaled_factor;
        val = fexpf(val, clamp_min, clamp_max);
        shared_exp[t] = expf(val);
    }
}

Kernel that fuses all ops
shared_exp[t] = sum_exp;
__syncthreads();

for (int t = 1; t < hidden_size/2; t >> 1) {
    if (t < 1) shared_exp[t] = shared_exp[t] * t;
    __syncthreads();

    if (t == 0) {
        float sum_exp = logf(shared_exp[0]) * 1.0f;
        float softplus = logf(softplus * sum_exp);
        float min = logf(sum_exp) + tanhf(softplus);
        softplus = logf(softplus * min);
    }
}

torch::Tensor fused_exp_cuda(torch::Tensor input, float clamp_min, float clamp_max, float
batch_size = input.size(0);
int hidden_size = input.size(1);
auto output = torch::empty({batch_size, 1}, input.options());
int threads = 256;
dim3 threads(256, 1, 1);
dim3 warps(16, 1, 1);
float* input_data_ptr = (float*)input.data();
float* output_data_ptr = (float*)output.data();
batch_size,
hidden_size,
clamp_min,
clamp_max,
scale_factor
);
}

return output;
}

```

Figure 1: Example optimization code generated by DeepSeek-R1 on a KernelBench problem

optimization decisions. This demonstrates that viable optimization solutions exhibit high diversity in structure and semantics, requiring deep understanding of the problem domain, programming language semantics, and underlying hardware architecture. Moreover, raw performance feedback provides insufficient diagnostic information: knowing that code runs 20% slower offers no insight into specific bottlenecks (memory bandwidth, compute utilization, or algorithmic inefficiency) or actionable remediation strategies. Consequently, even state-of-the-art LLMs with advanced coding capabilities struggle significantly with kernel optimization tasks (Ouyang et al., 2025).

To address these challenges, we first cast the problem of performance improving code optimization as *max-reward reinforcement learning*, which captures the notion of attaining best performance as opposed to cumulatively rewarded performance Veviurko et al. (2024). This formulation warrants inclusion of best-discounted reward in the observation space, for both learning and inference. Inspired by recent work on LLM self-refinement through natural language critique (Xie et al., 2025), we enrich the observation space with feedback from a critique model that analyzes optimized code and raw execution feedback to generate diagnostic insights and actionable refinement suggestions (Figure 2). We then define a *max-reward inference operator* to perform inference with a fixed policy, and instantiate the inference operator using multiple search algorithms to guide off-the-shelf LLMs in exploring and iteratively refining solutions. We call our approach **MaxCode** - a formulation that combines critique-augmented observation space with best-discounted reward to guide inference time search, enabling more effective exploration of the optimization solution space.

In code optimization, the evaluation of generated solutions demands computational resources and often becomes the limiting factor to effectively scale the search under a given computation and time budget. So, we additionally explore use of a trained generative Value/Reward-to-go model (Mahan et al., 2024) which predicts the V-value of any search trajectory prefix, i.e., the expected maximum future performance on that search branch given a proposed action (code revision). We train the reward-to-go model with roll-outs sampled from our tree searches. The learned reward model can be integrated at each search step by oversampling candidate refinements, filtering with predicted reward, and retaining only the most promising samples for evaluation and continuation. As a result, we enable search process to effectively explore more candidates under a certain evaluation budget.

We evaluate **MaxCode** formulation on two code optimization tasks: kernel code optimization (Ouyang et al., 2025) and competitive C++ code optimization (Madaan et al., 2023). With extensive experiments, we demonstrate that by integrating with our proposed max-reward RL formulation, the performance of existing methods can be significantly boosted. In particular, combining the best-performing search method (CUDA LLM (Chen et al., 2025)) with **MaxCode** yields relative speedup improvements of 27.3%, 11.0% and 22.5% on KernelBench level 1, level 2 and PIE, respectively.

In summary, our main contributions are as follows:

- 108 • We formalize code optimization as a max-reward reinforcement learning problem and aug-  
109 ment the observation space with two key components: (i) the best-discounted reward seen  
110 so far, and (ii) a natural language critique generated by a dedicated critique model that  
111 provides performance diagnosis and optimization suggestions based on code analysis and  
112 execution results. This enables more targeted search and provides actionable optimization  
113 suggestions based on code analysis and execution results.
- 114 • We define a max-reward inference operator and implement it through various search al-  
115 gorithms for fixed-policy inference. Our framework leverages the augmented observation  
116 space to enable effective exploration of the optimization solution space. Through empirical  
117 evaluation on kernel code optimization and competitive C++ optimization tasks, we demon-  
118 strate significant performance improvements over existing methods when integrating with  
119 our proposed framework.
- 120 • We propose a categorical Value/Reward-to-go model that predicts the expected maximum  
121 future performance of search trajectories, enabling efficient candidate filtering and resource  
122 optimization through informed trajectory selection.

## 124 2 MAXCODE: MAX-REWARD RL FRAMEWORK FOR CODE OPTIMIZATION

125  
126 **The Markov Decision Process** We formulate the performance improving code optimization pro-  
127 cess as a Markov Decision Process (MDP) with an expanded state space that incorporates initial  
128 code, current code, execution feedback, and language model critiques. We illustrate in Figure 2  
129 the MDP process with max-reward formulation. Formally, we define our MDP with the tuple  
130  $(\mathcal{S}, \mathcal{A}, P, R, \gamma, \rho_0)$ , where the state space  $\mathcal{S}$  is defined as the product space  $\mathcal{X}_0 \times \mathcal{X} \times \mathcal{E} \times \mathcal{C}$ , where  
131  $\mathcal{X}_0$  represents the problem description along with the initial code state,  $\mathcal{X}$  represents the space of  
132 possible current code states,  $\mathcal{E}$  represents execution feedback, and  $\mathcal{C}$  represents language model cri-  
133 tiques. Each state  $s_t = (x_0, x_{t-1}, e_{t-1}, c_{t-1})$  provides a representation of the initial code, current  
134 code, its execution results, and associated natural language critique. The initial state distribution  
135  $\rho_0 : \mathcal{X}_0 \rightarrow [0, 1]$  defines the probability distribution over starting code states, which is assumed  
136 uniform over the code samples present in the considered benchmarks, and  $\gamma \in (0, 1]$  is the discount  
137 factor, which we set to 1 because of finite horizon rollouts. The reward function  $R : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow \mathbb{R}$   
138 defined as  $R(s_t, a_t, s_{t+1}) = f(e_{t+1})$ , where  $f$  evaluates code performance based on execution feed-  
139 back  $e_{t+1}$ , returning higher values for improved performance.

140 The action space  $\mathcal{A} = \mathcal{X}$  corresponds to the space of possible code modifications. Unlike standard  
141 RL, the policy  $\pi_\theta$  is a large language model (LLM) with frozen parameters  $\theta$  that operates autore-  
142 gressively on states. Given  $s_t$ , the policy implicitly applies a sequence of token-level edits and  
143 produces a distribution over complete code candidates:  $\pi_\theta(x | s_t) : \mathcal{X} \rightarrow \Delta(\mathcal{X})$ . Due to stochastic  
144 token sampling, the same state  $s_t$  may yield multiple distinct candidates  $\{x_{t+1}^{(1)}, \dots, x_{t+1}^{(M)}\}$ .

145 The transition function  $P$  captures two sources of stochasticity: policy stochasticity from autore-  
146 gressive token sampling in  $\pi_\theta$ , and the environment stochasticity from  $\pi_\theta$  - the LLM-based critique  
147 generator. Formally, after the policy outputs  $x_{t+1}$ , the environment produces the next state by aug-  
148 menting the trajectory with execution results and critique:  $P(s_{t+1} | s_t, x_{t+1}) = P(e_{t+1}, c_{t+1} |$   
149  $x_{t+1}), \delta[s_{t+1} = (x_0, x_{t+1}, e_{t+1}, c_{t+1})]$ , where  $e_{t+1}$  is obtained from running  $x_{t+1}$  on the target  
150 hardware,  $c_{t+1}$  is generated by a separate LLM that produces a natural-language critique condi-  
151 tioned on  $(x_{t+1}, e_{t+1})$ , and  $\delta[\cdot]$  enforces deterministic update of the state components.

152 Following the max-reward RL formulation Veviurko et al. (2024), we define the return from time  $t$   
153 as  $\hat{G}_t = \max k \geq 1 \gamma^{k-1} r_{t+k}$ , which captures the best performance eventually achieved from time  
154  $t$ . With  $u \in \mathbb{R}$  as an auxiliary real variable representing the best discounted reward obtained so far,  
155 max-reward value functions under policy  $\pi$  are given by

$$156 \quad V^\pi(s, u) = \mathbb{E}_\pi \left[ \max(u, \hat{G}_t) \mid s_t = s \right], \quad (1)$$

$$158 \quad Q^\pi(s, a, u) = \mathbb{E}_\pi \left[ \max(u, \hat{G}_t) \mid s_t = s, a_t = a \right]. \quad (2)$$

160 **Remark** In max-reward RL, the optimal policy maximizing expected return from the initial state  
161 should depend not only on the current state, but also on the rewards obtained so far. The auxiliary

variable  $u \in \mathbb{R}_{\geq 0}$  representing the best discounted reward achieved so far is crucial for maintaining the Markov property under the max-reward objective.

**Execution and Critique** In our setup, the Executor  $EX$  evaluates input  $x$  against test cases  $Y$ , producing feedback  $e = \{e_{1a}, e_{1b}, e_{2a}, e_{2b}\}$ , where  $e_{1a}$  indicates binary correctness,  $e_{1b}$  provides contrastive correctness details (e.g. difference in the output of the current and previous code for some test cases),  $e_{2a}$  is a numerical performance indicator (e.g., running time and / or relative speed-up against the previous code), and  $e_{2b}$  contains contrastive performance details. For standard coding tasks, feedback is limited to correctness components ( $e = e_{1a}, e_{1b}$ ) and serves purely as an evaluation metric. In optimization tasks, where initial solutions are typically correct, the focus shifts to performance metrics  $e_2$ , as the initial solution remains a fallback option if optimization fails. Given that raw execution feedback can often prove to be less informative (Xie et al., 2025), we introduce a critic model ( $\pi_c$ ) to generate natural language critiques that provide both diagnostic insights into potential bugs and performance bottlenecks, as well as actionable refinement suggestions.

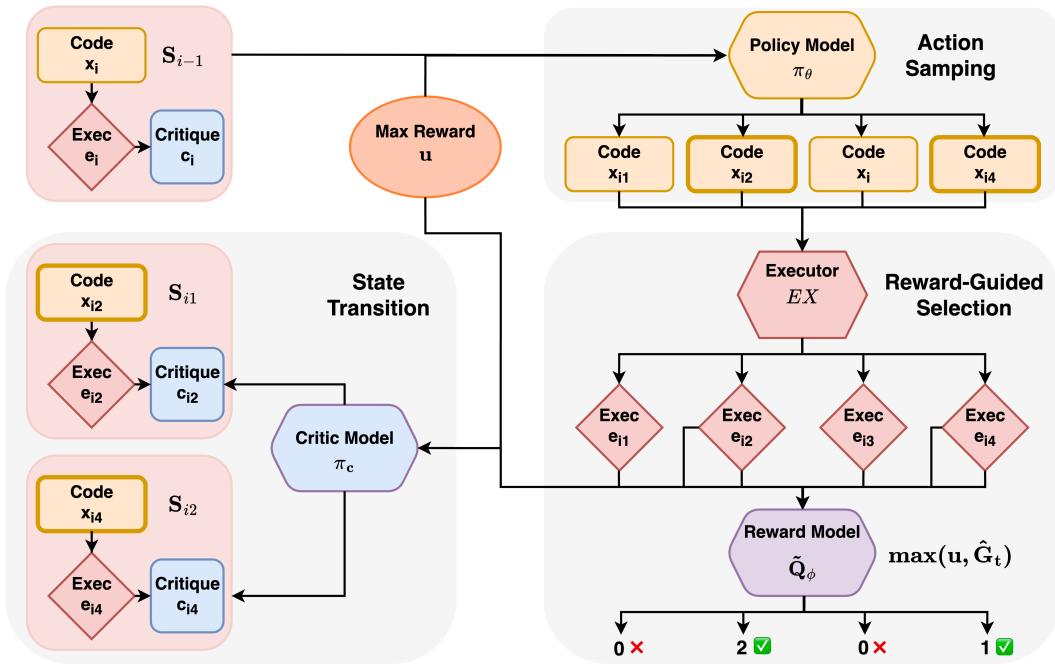


Figure 2: Illustration of the MaxCode search method

## 2.1 MAX-REWARD INFERENCE OPERATOR

One can obtain an optimized code with a budget  $K$  by sampling trajectories  $\{\tau_1, \dots, \tau_K\}$  and selecting:

$$\tau^* = \arg \max_{\tau_i} \hat{G}(\tau_i) = \arg \max_{\tau_i} \max_{t \in [1, T]} \gamma^{t-1} f_r(e_t^{(i)}) \quad (3)$$

Here,  $\tau_i = (s_0, s_{i1}, \dots, s_{iT-1}, s_T)$  represents a trajectory run for  $T$  time steps. Instead, we propose searching for the best optimized code under an extended MDP with state space  $(s, u)$ . To do so, we define a *max-reward inference operator*  $\mathcal{T}^*$  applied to a fixed policy  $\pi_\theta$  as:

$$\mathcal{T}^*(\pi_\theta)(s, u) \approx \arg \max_a Q^{\pi_\theta}(s, a, u) \quad (4)$$

where  $Q^{\pi_\theta}(s, a, u) = \mathbb{E}_{\pi_\theta}[\max(u, \hat{G}_t) \mid s_t = s, a_t = a]$  is the max-reward Q-function with auxiliary variable  $u$  representing the best discounted reward achieved so far. The operator performs one step of greedy policy improvement in an extended MDP with state space  $(s, u)$ . Unlike standard

216 policy improvement that operates on states  $s$  alone, our operator considers both the current state and  
 217 the reward history encoded in  $u$ , enabling decisions that depend on the quality of solutions found so  
 218 far.

219

220

221 

## 2.2 MAX-REWARD SEARCH

222

223 We now show how to approximately implement  $\mathcal{T}^*$  with inference time search by adopting and  
 224 repurposing various prior work under the proposed max-reward RL formulation. Given a code  
 225 optimization problem with test cases  $Y$ , generator LLM  $\pi_\theta$ , and critic LLM  $\pi_c$ , we define initial  
 226 state  $s_0 = (x_0, \emptyset, \emptyset, \emptyset) \in \mathcal{X}_0 \times \mathcal{X} \times \mathcal{E} \times \mathcal{C}$ , where  $x_0$  is the problem statement with the initial code,  
 227 and search states as  $s_i = (x_0, x_i, e_i, c_i) \in \mathcal{S}$  represent the current optimization candidate with its  
 228 execution feedback and critique. For each state  $s_i$ , we maintain  $u_i = \max_{j \leq i} \gamma^{d_j} f(e_j)$  tracking the  
 229 best discounted reward achieved along the trajectory to  $s_i$ , where  $d_j$  is the depth of state  $j$ . With this  
 230 setting, we reformulate the following methods for max-reward search:

231

232

233 **Effi-Learner** Given  $s_0$  (Huang et al., 2024) proposed to 1) first sample an initial action  $x_1$  from  
 234  $\pi_\theta(s_0)$  and obtain its execution feedback  $e_1 = EX(x_1, Y)$ ; 2) generate a refinement action  $x_2$  from  
 235  $\pi_\theta(x_0, x_1, e_1)$  as the final solution.

236

237

238 *Max-Reward Reformulation:* Under Max-Reward formulation, we reformulate Effi-learner to 1)  
 239 additionally obtain the critique  $c_1 \sim \pi_c(x_1, e_1)$  and form the successor state  $s_1 = (x_0, x_1, e_1, c_1)$   
 240 via transition function  $P(s_1 | s_0, x_0)$ ; and 2) generate the final solution  $x_2$  from  $\pi_\theta(s_1)$ . Noting that  
 241 we are not adding the we leverage the maintained best discounted reward  $u_i = \max_{j \leq i} \gamma^{d_j} f(e_j)$   
 242 since Effi-Learner performs only 2 rounds of optimization thus  $u_i$  is already encoded in  $e_1$ .

243

244

245 **CUDA-LLM** Chen et al. (2025) proposes a beam-search based method that given a state  
 246  $s'_i = (x_0, x_i, e_i)$ , sampling  $k$  candidate actions  $x_{i1} \dots x_{ik}$ , obtaining execution feedback  $e_{ij} =$   
 247  $EX(x_{ij}, Y)$  and 1) if any of the candidates is correct, i.e. the speedup  $f(e_{ij}) > 1$ , select the  
 248 best-performing candidate to proceed with, i.e.  $x_m$  with  $m = \arg \max_x (f(e_x))$ , and form the new  
 249 state  $s'_{i+1} = (x_0, x_m, e_m)$ ; 2) if none of the candidates are correct, formulate intermediate states  
 250  $s'_{ij} = (x_0, x_{ij}, e_{ij})$  and iteratively refine them by sampling and executing (in parallel) single refine-  
 251 ment candidates for each intermediate state until at least one of the refinement is correct. Then it  
 252 discard all intermediate states and obtain  $s_{i+1}$  as in 1).

253

254

255 *Max-Reward Reformulation:* Under Max-Reward formulation, we 1) enhance each state  $s'_i =$   
 256  $(x_0, x_i, e_i)$  with natural language critiques obtained by  $\pi_c$  to obtain complete states  $s_i =$   
 257  $(x_0, x_i, e_i, c_i)$ ; 2) when sampling the next action at each state  $s_i$ , we leverage the maintained best  
 258 discounted reward  $u_i = \max_{j \leq i} \gamma^{d_j} f(e_j)$  to enhance the action sampling, i.e.  $x_{ij} \sim \pi_\theta(s_i, u_i)$ .

259

260

261

262 

## 2.3 GENERATIVE VALUE FUNCTION GUIDED SEARCH

263

264

265 To further improve the search process, we learn a generative value function approximator  $\tilde{V}_\phi$  that  
 266 guides state selection by estimating the max-reward values. This enables a two-stage approach: first  
 267 collecting search data with breadth-first expansion, then using the learned value function to guide  
 268 more efficient search. Our value function approximator  $\tilde{V}_\phi(s_t, u_t)$  is implemented as a language  
 269 model that takes as input the current state representation  $s_t = (x_0, x_t, e_t, c_t)$  and the auxiliary  
 270 variable  $u_t$  representing the best discounted reward achieved so far along the trajectory.

271

272

273 For each code optimization problem, we collect training data  $\mathcal{D}$  from the search trajectory as fol-  
 274 lows: 1) sample  $K$  parallel trajectories  $\{\tau_1, \dots, \tau_K\}$  by iteratively expanding each state  $s_t =$   
 275  $(x_0, x_t, e_t, c_t)$  with  $s_{t+1} = (x_0, x_{t+1}, e_{t+1}, c_{t+1})$ , where  $x_{t+1} \sim \pi_\theta(s_t, u_t)$ ,  $e_{t+1} = E(x_{t+1}, Y)$ ,  
 276 and  $c_{t+1} = \pi_c(s_t, x_{t+1}, u_t)$ . 2) For each trajectory  $\tau = (s_0, s_1, \dots, s_T)$ , at each timestep  $t$ ,  
 277 we have state  $s_t$ , auxiliary variable  $u_t = \max_{k \leq t} \gamma^{k-t} f_r(e_k)$ . We compute the target value as  
 278  $v_t^* = \max(u_t / \gamma, \max_{k \geq t} \gamma^{k-t} f_r(e_k))$ , where the maximum is taken over all future rewards in the  
 279 following trajectory starting at the state  $s_t$ . This formulation correctly implements the max-reward  
 280 objective: the value represents the maximum between the discounted best reward achieved so far  
 281 and the best future reward obtainable from the current state.

270    **Categorical Reward Formulation** Given the high variance in potential speedup distributions, we  
 271    discretize continuous speedup values into categorical rewards using the binning strategy:  
 272

$$273 \quad 274 \quad 275 \quad 276 \quad 277 \quad 278 \quad f_r(e) = \begin{cases} 0 & \text{if speedup} \leq 100\% \text{ or correctness} = 0 \\ 1 & \text{if speedup} \in (100\%, s_1\%] \\ 2 & \text{if speedup} \in (s_1\%, s_2\%] \\ 3 & \text{if speedup} \in (s_2\%, s_3\%] \\ 4 & \text{if speedup} > s_3\% \end{cases}$$

279    The value function approximator predicts a categorical distribution over these reward categories:  
 280

$$281 \quad \tilde{V}_\phi(q \mid s_t, u_t) = \text{softmax}(W_v \cdot h_\phi(s_t, u_t))$$

283    where  $h_\phi$  is the language model’s hidden representation and  $W_v$  is a classification head. We train  
 284    the value function using standard cross-entropy loss:  
 285

$$286 \quad \mathcal{L}(\tilde{V}_\phi) = \mathbb{E}_{(s_t, u_t, v^*) \sim \mathcal{D}} \left[ -\log \tilde{V}_\phi(v^* \mid s_t, u_t) \right], \quad (5)$$

288    where  $v^* = f_r(\max_{k \geq t} \gamma^{k-t} r_k)$  is the categorical label corresponding to the maximum discounted  
 289    future reward.

290    **Generative Value Function Guided Search** In the second stage, we use  $\tilde{V}_\phi$  to guide expansion.  
 291    After evaluating a set of candidate states, we compute  $\tilde{V}_\phi(s_t, u_t)$  for each candidate  $s_t$ , and select  
 292    the state with the highest expected estimated value for subsequent expansion. This allows us to  
 293    incorporate a notion of potential future improvement into state selection: two states with identical  
 294    current reward may differ in how close they are to further performance improvement. Imagine  
 295    two functions that achieve zero reward: one which is a no-op, and the other which contains all  
 296    the logic required to compute a correct output but also contains a minor syntax issue. These two  
 297    functions would be equally likely to be selected for expansion without the use of a value estimator  
 298    to distinguish their differing levels of promise.  
 299

## 300    2.4 ENVIRONMENT STOCHASTICITY

302    At any given step, the environment feedback (critique) doesn’t necessarily provide a complete pic-  
 303    ture of the performance characteristics of the most recent action (code revision) or what further  
 304    revisions are needed, only a lossy subset. In our environment, this critique function is also stochas-  
 305    tic. By including previous critique observations in the trajectory history, the policy can aggregate  
 306    these lossy observations to get more complete information on what the best next action might be.  
 307    Here the trajectory information  $\tau_i$  provides the extended state representation necessary to deal with  
 308    environment stochasticity. At each refinement step, the LLM generates new optimizations condi-  
 309    tioned on trajectory information (previously generated optimizations and execution feedback) from  
 310    all ancestor steps.

## 311    3 EXPERIMENTS AND RESULTS

### 312    3.1 EXPERIMENTS

315    **Datasets** We evaluate our proposed searching and reward modeling methods on two code opti-  
 316    mization benchmarks: (i) KernelBench (Ouyang et al., 2025) and (ii) PIE (Madaan et al., 2023)  
 317    focused on optimizing CUDA kernels and competitive C++ codes, respectively. **KernelBench** is  
 318    for evaluating LLMs on generating and optimizing for efficient GPU kernels for optimizing neural  
 319    network performance. The dataset is constructed with 250 well-defined neural network tasks span-  
 320    ning four levels of difficulties from single kernel optimization (level 1), fusion patterns (level 2),  
 321    to complete ML architectures (level 3) and complete Huggingface architectures (level 4). For each  
 322    of the tasks, the LLM is provided with the PyTorch implementation and asked to replace it with  
 323    custom kernels that are correct and performance optimized. The execution feedback consists of 1)  
 324    compilation success/failure; 2) correctness of the generated CUDA kernel based on a set of test-case

	KerneBench L1		KerneBench L2		PIE	
	Rank ↓	Median ↑	Rank ↓	Median ↑	Rank ↓	Median ↑
Best@64	2.85	1.00x	2.15	1.02x	2.29	1.32x
Effi-Learner + <b>MaxCode</b>	3.02 2.98	1.00x 1.00x	2.98 2.95	1.00x 1.00x	3.72 3.55	1.00x 1.03x
CUDA LLM + <b>MaxCode</b>	1.54 <b>1.43</b>	2.49x <b>3.17x</b>	1.64 <b>1.51</b>	1.45x <b>1.61x</b>	2.05 <b>1.74</b>	1.42x <b>1.74x</b>

Table 1: Average ranking and median of maximum speedup on KernelBench and PIE.

input-output; 3) the relative speedup of the CUDA compared with the default PyTorch implementation. We use level 1 and level 2 problems for our experiments. **PIE** is a benchmark for optimizing the running time for competitive level C++ coding problems, consists of 77K pairs of submissions (original vs. optimized). The execution feedback consists of 1) correctness of code based on extensive unit tests and, 2) the relative speedup of the optimization compared to the original solution. We sample 100 problems from the test set (detailed in Appendix B) for our experiments.

**Experimental Setup** As introduced in subsection 2.2, we use the reformulation of Effi-Learner Huang et al. (2024) and CUDA-LLM Chen et al. (2025) to implement the proposed max-reward search on both benchmarks. We use Claude-3.7-Sonnet (Cla, a) as  $\pi_\theta$  for both the policy and critique generation. We further enable the extended thinking mode of Claude-3.7-Sonnet for the critique generation to enhance the reasoning capabilities. We set `temperature=0.6` for both code and critique generation. On KernelBench, we used all of the level 1 and level 2 problems (100 each) for evaluations. On PIE, we use a subset of 68 problems from the test set. For each of the problem, we generate a single-path refinement with `depth=2` for Effi-Learner, we set the `depth = 8` and  $K = 8$  for CUDA-LLM.

To collect training data for the reward model, we perform **MaxCode** search with  $K = 8$  single-path refinement with critique and trajectory information as input on KernelBench level 1 level 2 and PIE. We train the reward model on all the generated search trajectories with all prefixes length  $\leq 2$ . We split the trajectory prefix data by problem with a 80/20 splits of train/val sets for each dataset. For reward function  $f_r(e)$ , we set  $(s_1, s_2, s_3)$  as  $(140, 320, 475), (120, 170, 215), (125, 180, 260)$  for each dataset, respectively. We use Qwen2.5-7B-Instruct (Yang et al., 2024) as the base model for reward-to-go model training. For hyperparameters, we train the reward model with `epoch=1, batch_size=8, optimizer=AdamW` using LoRA with `rank=8`. For inference, we set `temperature=0.7`. We provide all the prompts for search and reward model in Appendix C.

**Baselines** We compare our proposed methods to 3 baselines: 1. **Effi-Learner** (Huang et al., 2024): as described in subsection 2.2, we implement the original Effi-Learner as baseline 2. **CUDA-LLM** (Chen et al., 2025): similar to Effi-learner, we implement the original CUDA-LLM with the same hyper-parameters 3. **Flat Sampling**: directly sampling  $n$  multiple candidates from the LLM where  $n = 64$  matches the compute budget of **MaxCode** on CUDA-LLM.

**Evaluation Metrics** We evaluate the generated search trajectories correctness and performance with the following metrics: 1. **Correctness**: the average binary correctness of all the generations per problem 2. **Fast1**: the average binary value of the solution is correct and faster than the PyTorch implementation across all the generations per problem. 3. **Max Speedup**: the maximum speedup of all solutions across the generations per problem. Note that depending on the nature of the problems, there is a small subset of problems where the optimization speedups are much larger than the others, thus biasing the average maximum speedup to be less faithful in measuring the overall performance of evaluated methods. We thus evaluate the overall max speedup on 1) the *median* of max speedup across problems; 2) the *average ranking* of the individual max speedup of different methods on each problem; since these two measurements are less prone to outliers and more faithfully represent the absolute and comparable level of max speedup.

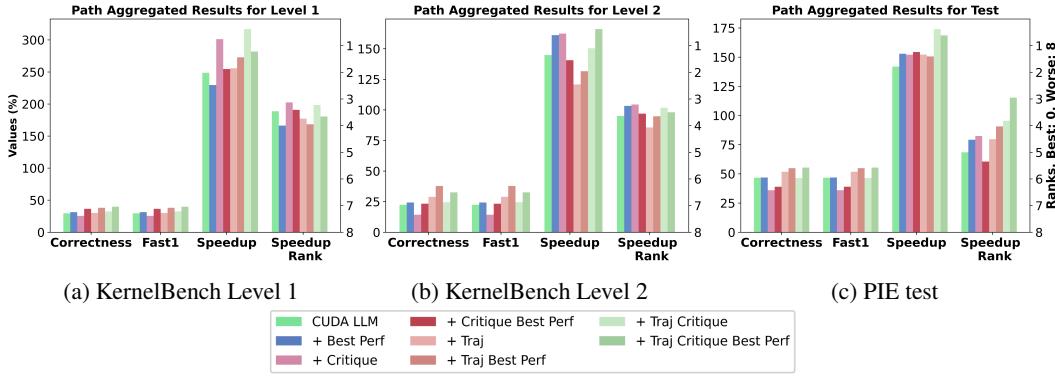


Figure 3: Ablated evaluation results of correct, fast1, and max speed-up of different components of **MaxCode** on KernelBench and PIE

### 3.2 RESULTS

#### *RQ1: Can MaxCode improve the performance of different search methods?*

We present the performance of baseline methods and their **MaxCode** reformulation on KernelBench and PIE in Table 1. We observe improvements across all baselines in terms of level and ranking of max speedup when incorporated with **MaxCode** (+ **MaxCode**) across all of level 1 and level 2 of KernelBench and PIE problems. The results showcase that existing search methods can be effectively reformulated under the proposed max-reward RL formulation, with performance gains compared with their original implementation. Overall, when integrated with CUDA LLM (CUDA LLM + **MaxCode**), **MaxCode** yields the best max speedup performance.

**RQ2: What is most crucial to MaxCode’s performance gain?** To better investigate the effects of each component in **MaxCode** framework, we evaluate the following ablations to study the performance of the **MaxCode**. Specifically, in addition to the optimization + raw execution feedback from the last round, we ablate on these additional input to the prompt

- **Best Perf**: the best reward so far and corresponding code and execution feedback.
- **Critique**: the natural language critiques
- **Traj**: optimization + execution feedback (+ Best Perf) (+ Critique) from the full trajectory.

We ablate every combinations of these components using CUDA LLM + **MaxCode** with comparison to the original CUDA LLM. Note that for all **Traj** variations, the information of best-performing optimization is already presented in the trajectory, the addition of **Best Perf** on top of it thus add the best-performing information again to highlight the max-reward information.

The results for ablation study are presented in Figure 3. As illustrated, compared with the CUDA-LLM baseline, all variations attains comparable or better level of correctness, fast1 and maximum speedup, showcasing the effectiveness of **MaxCode** variations in searching for correct and faster solutions. For max speedup, the best performances are achieved by different variations on different subset/dataset. Specifically, having the full trajectory information with critiques (**Traj Critique**) yields the highest median of max speedup of KernelBench level 1 and PIE, where as further adding the best discounted reward (best-performing optimization) so far yield the best median for max speedup on KernelBench Level 2. On the other hand, including only one of the components yield less improvements and might sometimes lead to slight degradation of the performance. The results demonstrate that the combination of trajectory information with natural language critique, as well as the best reward so-far (either encoded in the trajectory or explicitly provided) is crucial to the success of **MaxCode**.

**RQ3. How does MaxCode scale with inference-time budget?** To investigate the inference-time properties of **MaxCode**, we plot the median max speedup attained by different variations of CUDA LLM + **MaxCode** against the vanilla CUDA-LLM under different depths in Figure 4. As shown in the figure, compared with CUDA-LLM, the reformulation with **MaxCode** could more quickly attain higher level of speedup than CUDA-LLM under the same depth (therefore the same # of gen-

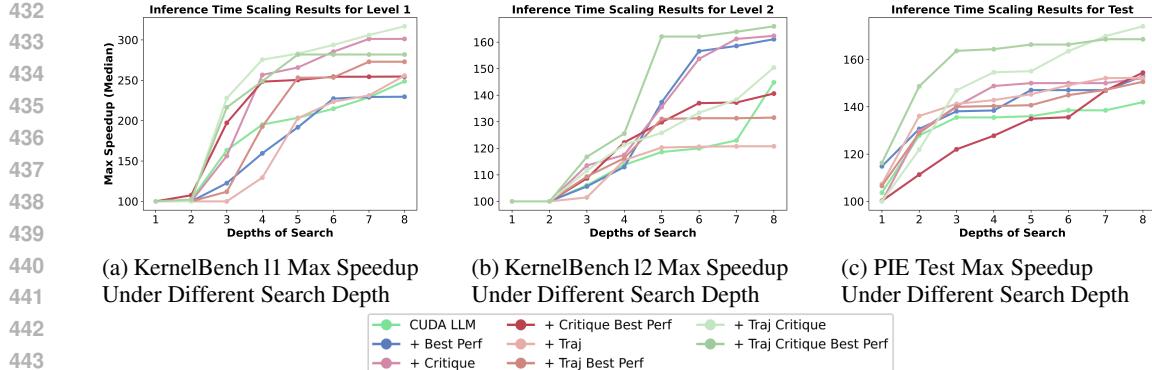


Figure 4: Inference time scaling of max speed-up on KernelBench and PIE.

erated candidates), across KernelBench level 1, level 2, and PIE. The scaling results showcase that **MaxCode** can boost the test-time scaling of existing search methods - under **MaxCode** reformulation, search methods can more efficiently leverage the inference budget and scales better than their original counterparts for code optimization.

	KernelBench L1		KernelBench L2		PIE	
	Rank ↓	Median ↑	Rank ↓	Median ↓	Rank ↓	Median ↑
MaxCode	<b>1.40</b>	<b>3.24x</b>	1.57	1.22x	1.55	<b>1.72x</b>
MaxCode + Reward	1.53	2.44x	<b>1.33</b>	<b>1.24x</b>	<b>1.43</b>	1.62x

Table 2: Results of reward-guided search on KernelBench and PIE

**RQ4. Can we learn a coarse verification signal as the V model to improve search performance?** To evaluate the effectiveness of the learned reward model in guiding search, we apply the reward model to CUDA LLM + **MaxCode** with the **Traj Critique** variance and compared it with the no-guidance search. We report the evaluation results of reward-guided search on a random subset of KernelBench and PIE in Table 2. While the reward-guided search demonstrates comparable/better results on KernelBench level 1 and PIE, it underperforms the no-guidance baseline on KernelBench level 1. Given the results, we posit that the potential causes that hinder the reward model to provide better guidance for search are 1) the intrinsic difficulties of accurately estimating the expected reward in terms of maximum speedup for complex code optimization problems even for small LLMs, and 2) the distribution shift between the collected trajectories for training the reward model and the trajectories obtained with CUDA LLM + **MaxCode**. In particular, whereas the collected trajectories are single-path refinements with no candidate selection, CUDA LLM + **MaxCode** always sample and select the best-performing candidate of the current round to continue with. Our results and findings highlight both the usefulness and challenges of leveraging learned reward/value function for search in code optimization.

## 4 CONCLUSION

In this paper, we investigate inference-time search algorithms for LLM on code optimization problems. We unify prior search approaches under a max-reward reinforcement learning (RL) problem formulation, exposing the observation and action-value functions for plug-and-play modification. We improve the observation space by integrating a critique model that transforms the raw execution feedback provided by the environment to natural language critiques of error/performance, providing stronger guiding signal for the policy (code proposal) function. Moreover, we use sampled action values from rollouts to train a generative reward-to-go model, which can then be applied at inference time to rerank actions (search states) for exploration. Results on the KernelBench (CUDA) and PIE (C++) optimization benchmarks demonstrate that applying our proposed framework to reformulate existing search methods yields significant improvements in performance of optimized code.

486 REFERENCES  
487

488 Claude 3.7 sonnet and claude code. <https://www.anthropic.com/news/claude-3-7-sonnet>, a.

489 Claude code: Deep coding at terminal velocity. <https://www.anthropic.com/claude-code>, b.

490 Sagnik Anupam, Alexander Shypula, and Osbert Bastani. Llm program optimization via retrieval  
491 augmented search. *ArXiv*, abs/2501.18916, 2025. URL <https://api.semanticscholar.org/CorpusID:276079732>.

492 Carlo Baronio, Pietro Marsella, Ben Pan, Simon Guo, and Silas Alberti. Kevin: Multi-  
493 turn rl for generating cuda kernels. *ArXiv*, abs/2507.11948, 2025. URL <https://api.semanticscholar.org/CorpusID:280232580>.

494 Wentao Chen, Jiace Zhu, Qi Fan, Yehan Ma, and An Zou. Cuda-llm: Llms can write efficient cuda  
495 kernels. *ArXiv*, abs/2506.09092, 2025. URL <https://api.semanticscholar.org/CorpusID:279305780>.

496 Tri Dao. Flashattention-2: Faster attention with better parallelism and work partitioning. *arXiv  
497 preprint arXiv:2307.08691*, 2023.

498 DeepSeek-AI, Daya Guo, Dejian Yang, Haowei Zhang, Jun-Mei Song, Ruoyu Zhang, Runxin  
499 Xu, Qihao Zhu, Shirong Ma, Peiyi Wang, Xiaoling Bi, Xiaokang Zhang, Xingkai Yu, Yu Wu,  
500 Z. F. Wu, Zhibin Gou, Zhihong Shao, Zhuoshu Li, Ziyi Gao, Aixin Liu, Bing Xue, Bing-Li  
501 Wang, Bochao Wu, Bei Feng, Chengda Lu, Chenggang Zhao, Chengqi Deng, Chenyu Zhang,  
502 Chong Ruan, Damai Dai, Deli Chen, Dong-Li Ji, Erhang Li, Fangyun Lin, Fucong Dai, Fuli  
503 Luo, Guangbo Hao, Guanting Chen, Guowei Li, H. Zhang, Han Bao, Hanwei Xu, Haocheng  
504 Wang, Honghui Ding, Huajian Xin, Huazuo Gao, Hui Qu, Hui Li, Jianzhong Guo, Jiashi Li, Ji-  
505 awei Wang, Jingchang Chen, Jingyang Yuan, Junjie Qiu, Junlong Li, Jiong Cai, Jiaqi Ni, Jian  
506 Liang, Jin Chen, Kai Dong, Kai Hu, Kaige Gao, Kang Guan, Kexin Huang, Kuai Yu, Lean  
507 Wang, Lecong Zhang, Liang Zhao, Litong Wang, Liyue Zhang, Lei Xu, Leyi Xia, Mingchuan  
508 Zhang, Minghua Zhang, M. Tang, Meng Li, Miaojun Wang, Mingming Li, Ning Tian, Panpan  
509 Huang, Peng Zhang, Qiancheng Wang, Qinyu Chen, Qiushi Du, Ruiqi Ge, Ruisong Zhang,  
510 Ruizhe Pan, Runji Wang, R. J. Chen, Ruiqi Jin, Ruyi Chen, Shanghao Lu, Shangyan Zhou,  
511 Shanhua Chen, Shengfeng Ye, Shiyu Wang, Shuiping Yu, Shunfeng Zhou, Shuting Pan, S. S.  
512 Li, Shuang Zhou, Shao-Kang Wu, Tao Yun, Tian Pei, Tianyu Sun, T. Wang, Wangding Zeng,  
513 Wanja Zhao, Wen Liu, Wenfeng Liang, Wenjun Gao, Wen-Xia Yu, Wentao Zhang, Wangding  
514 Xiao, Wei An, Xiaodong Liu, Xiaohan Wang, Xi aokang Chen, Xiaotao Nie, Xin Cheng, Xin  
515 Liu, Xin Xie, Xingchao Liu, Xinyu Yang, Xinyuan Li, Xuecheng Su, Xuheng Lin, X. Q. Li,  
516 Xiangyu Jin, Xi-Cheng Shen, Xiaosha Chen, Xiaowen Sun, Xiaozi Wang, Xinnan Song,  
517 Xinyi Zhou, Xianzu Wang, Xinxia Shan, Y. K. Li, Y. Q. Wang, Y. X. Wei, Yang Zhang, Yan-  
518 hong Xu, Yao Li, Yao Zhao, Yaofeng Sun, Yaohui Wang, Yi Yu, Yichao Zhang, Yifan Shi,  
519 Yi Xiong, Ying He, Yishi Piao, Yisong Wang, Yixuan Tan, Yiyang Ma, Yiyuan Liu, Yongqiang  
520 Guo, Yuan Ou, Yuduan Wang, Yue Gong, Yu-Jing Zou, Yujia He, Yunfan Xiong, Yu-Wei Luo,  
521 Yu mei You, Yuxuan Liu, Yuyang Zhou, Y. X. Zhu, Yanping Huang, Yao Li, Yi Zheng, Yuchen  
522 Zhu, Yunxiang Ma, Ying Tang, Yukun Zha, Yuting Yan, Zehui Ren, Zehui Ren, Zhangli Sha,  
523 Zhe Fu, Zhean Xu, Zhenda Xie, Zhen guo Zhang, Zhewen Hao, Zhicheng Ma, Zhigang Yan,  
524 Zhiyu Wu, Zihui Gu, Zijia Zhu, Zijun Liu, Zi-An Li, Ziwei Xie, Ziyang Song, Zizheng Pan,  
525 Zhen Huang, Zhipeng Xu, Zhongyu Zhang, and Zhen Zhang. Deepseek-r1: Incentivizing rea-  
526 soning capability in llms via reinforcement learning. *ArXiv*, abs/2501.12948, 2025. URL  
527 <https://api.semanticscholar.org/CorpusID:275789950>.

528 Mingzhe Du, Luu Tuan Tuan, Yue Liu, Yuhao Qing, Dong Huang, Xinyi He, Qian Liu, Zejun Ma,  
529 and See kiong Ng. Afterburner: Reinforcement learning facilitates self-improving code efficiency  
530 optimization. *ArXiv*, abs/2505.23387, 2025. URL <https://api.semanticscholar.org/CorpusID:278995727>.

531 Shukai Duan, Nikos Kanakaris, Xiongye Xiao, Heng Ping, Chenyu Zhou, Nesreen K. Ahmed,  
532 Guixiang Ma, Mihai Capotă, Theodore L. Willke, Shahin Nazarian, and Paul Bogdan. Per-  
533 frl: A small language model framework for efficient code optimization. 2023. URL <https://api.semanticscholar.org/CorpusID:266163427>.

540 Charles Hong, Sahil Bhatia, Alvin Cheung, and Yakun Sophia Shao. Autocomp: Llm-driven code  
 541 optimization for tensor accelerators. *ArXiv*, abs/2505.18574, 2025. URL <https://api.semanticscholar.org/CorpusID:278905227>.

542

543 Dong Huang, Jianbo Dai, Han Weng, Puzhen Wu, Yuhao Qing, Jie M.Zhang, Heming Cui, and  
 544 Zhijiang Guo. Effilearner: Enhancing efficiency of generated code via self-optimization. In  
 545 *Neural Information Processing Systems*, 2024. URL <https://api.semanticscholar.org/CorpusID:270045278>.

546

547 Xiaoya Li, Xiaofei Sun, Albert Wang, Jiwei Li, and Chris Shum. Cuda-l1: Improving cuda optimiza-  
 548 tion via contrastive reinforcement learning. 2025. URL <https://api.semanticscholar.org/CorpusID:280048846>.

549

550

551 Aman Madaan, Alex Shypula, Uri Alon, Milad Hashemi, Parthasarathy Ranganathan, Yim-  
 552 ing Yang, Graham Neubig, and Amir Yazdanbakhsh. Learning performance-improving code  
 553 edits. *ArXiv*, abs/2302.07867, 2023. URL <https://api.semanticscholar.org/CorpusID:256868633>.

554

555 Dakota Mahan, Duy Phung, Rafael Rafailov, Chase Blagden, nathan lile, Louis Castricato,  
 556 Jan-Philipp Franken, Chelsea Finn, and Alon Albalak. Generative reward models. *ArXiv*,  
 557 abs/2410.12832, 2024. URL <https://api.semanticscholar.org/CorpusID:273404003>.

558

559

560 Anne Ouyang, Simon Guo, Simran Arora, Alex L. Zhang, William Hu, Christopher R'e, and Azalia  
 561 Mirhoseini. Kernelbench: Can llms write efficient gpu kernels? *ArXiv*, abs/2502.10517, 2025.  
 562 URL <https://api.semanticscholar.org/CorpusID:276408165>.

563

564 Baptiste Rozière, Jonas Gehring, Fabian Gloeckle, Sten Sootla, Itai Gat, Xiaoqing Tan, Yossi Adi,  
 565 Jingyu Liu, Tal Remez, Jérémie Rapin, Artyom Kozhevnikov, I. Evtimov, Joanna Bitton, Manish P  
 566 Bhatt, Cris tian Cantón Ferrer, Aaron Grattafiori, Wenhan Xiong, Alexandre D'efossez, Jade  
 567 Copet, Faisal Azhar, Hugo Touvron, Louis Martin, Nicolas Usunier, Thomas Scialom, and Gabriel  
 568 Synnaeve. Code llama: Open foundation models for code. *ArXiv*, abs/2308.12950, 2023. URL  
 569 <https://api.semanticscholar.org/CorpusID:261100919>.

570

571 Jay Shah, Ganesh Bikshandi, Ying Zhang, Vijay Thakkar, Pradeep Ramani, and Tri Dao.  
 572 Flashattention-3: Fast and accurate attention with asynchrony and low-precision. *Advances in  
 573 Neural Information Processing Systems*, 37:68658–68685, 2024.

574

575 Grigorii Veviurko, Wendelin Böhmer, and Mathijs De Weerdt. To the max: Reinventing reward in  
 576 reinforcement learning. *arXiv preprint arXiv:2402.01361*, 2024.

577

578 Guoxia Wang, Jinle Zeng, Xiyuan Xiao, Siming Wu, Jiabin Yang, Lujing Zheng, Zeyu Chen, Jiang  
 579 Bian, Dianhai Yu, and Haifeng Wang. Flashmask: Efficient and rich mask extension of flashat-  
 580 tention. *arXiv preprint arXiv:2410.01359*, 2024.

581

582 Zhihui Xie, Jie chen, Liyu Chen, Weichao Mao, Jingjing Xu, and Lingpeng Kong. Teaching  
 583 language models to critique via reinforcement learning. *ArXiv*, abs/2502.03492, 2025. URL  
 584 <https://api.semanticscholar.org/CorpusID:276161646>.

585

586 An Yang, Anfeng Li, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chang  
 587 Gao, Chengan Huang, Chenxu Lv, Chujie Zheng, Dayiheng Liu, Fan Zhou, Fei Huang, Feng  
 588 Hu, Hao Ge, Haoran Wei, Huan Lin, Jialong Tang, Jian Yang, Jianhong Tu, Jianwei Zhang,  
 589 Jianxin Yang, Jiaxin Yang, Jingren Zhou, Jingren Zhou, Junyan Lin, Kai Dang, Keqin Bao,  
 590 Ke-Pei Yang, Le Yu, Li-Chun Deng, Mei Li, Min Xue, Mingze Li, Pei Zhang, Peng Wang,  
 591 Qin Zhu, Rui Men, Ruize Gao, Shi-Qiang Liu, Shuang Luo, Tianhao Li, Tianyi Tang, Wen-  
 592 biao Yin, Xingzhang Ren, Xinyu Wang, Xinyu Zhang, Xuancheng Ren, Yang Fan, Yang Su,  
 593 Yi-Chao Zhang, Yinger Zhang, Yu Wan, Yuqiong Liu, Zekun Wang, Zeyu Cui, Zhenru Zhang,  
 594 Zhipeng Zhou, and Zihan Qiu. Qwen3 technical report. *ArXiv*, abs/2505.09388, 2025. URL  
 595 <https://api.semanticscholar.org/CorpusID:278602855>.

596

597 Qwen An Yang, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chengyuan  
 598 Li, Dayiheng Liu, Fei Huang, Guanting Dong, Haoran Wei, Huan Lin, Jian Yang, Jianhong Tu,

594 Jianwei Zhang, Jianxin Yang, Jiaxin Yang, Jingren Zhou, Junyang Lin, Kai Dang, Keming Lu,  
595 Keqin Bao, Kexin Yang, Le Yu, Mei Li, Mingfeng Xue, Pei Zhang, Qin Zhu, Rui Men, Runji  
596 Lin, Tianhao Li, Tingyu Xia, Xingzhang Ren, Xuancheng Ren, Yang Fan, Yang Su, Yi-Chao  
597 Zhang, Yunyang Wan, Yuqi Liu, Zeyu Cui, Zhenru Zhang, Zihan Qiu, Shanghaoran Quan, and  
598 Zekun Wang. Qwen2.5 technical report. *ArXiv*, abs/2412.15115, 2024. URL <https://api.semanticscholar.org/CorpusID:274859421>.  
599

600

601

602

603

604

605

606

607

608

609

610

611

612

613

614

615

616

617

618

619

620

621

622

623

624

625

626

627

628

629

630

631

632

633

634

635

636

637

638

639

640

641

642

643

644

645

646

647

648 

## A RELATED WORK

649  
 650 Given the impressive capabilities that LLMs have demonstrated in code-related tasks, there has been  
 651 a recent upsurge of interest in applying LLMs for code optimization (Ouyang et al., 2025; Madaan  
 652 et al., 2023), the task of optimizing performance (e.g. running time, memory usage, etc.) of input  
 653 code without altering the functional semantics. Prior work can be categorized into two types of  
 654 approaches: (i) inference-time and (ii) learning-based. Inference time search methods typically in-  
 655 volve multi-turn prompting of LLMs with iterative refinement with generated intermediate solutions  
 656 and execution feedback obtained from compilation, and executing the code. For instance, Huang  
 657 et al. (2024) adopts a single-path refinement strategy that iteratively generates optimization code,  
 658 appends the generation and execution trajectory into the prompt for subsequent optimization, and  
 659 Chen et al. (2025) samples multiple candidate kernels per iteration, selecting the best-performing  
 660 one for continuation. Other approaches enhance the search process by retrieving similar slow-fast  
 661 program pairs from training data (Anupam et al., 2025), and incorporating a planning stage cou-  
 662 pled with beam search for strategic exploration (Hong et al., 2025). On the other hand, another line  
 663 of work finetunes LLM for code optimization by injecting the notion of performance through RL  
 664 reward (Duan et al., 2023; Baronio et al., 2025), adaptively updating the training data with more  
 665 performant code Du et al. (2025), and contrastive training of slow-fast code pairs (Li et al., 2025).  
 666

667 

## B DATASET CONSTRUCTION DETAILS FOR PIE

668 We source our evaluation data on PIE from its original test set. While the test set contains 978 pairs  
 669 of slow-fast C++ programs, they are originated from a set of only 41 distinct input problems. To  
 670 ensure the diversity of our evaluation set, we rank all the slow solutions for each input problem and  
 671 select the slowest solution for each problem, followed by the second slowest solution, then the third  
 672 slowest solutions until obtaining 100 solutions to form our evluation set.  
 673

674 

## C PROMPTS

675 

### C.1 GENERATOR PROMPTS

676 **Base (Refinement with Optimization + Execution Feedback from Only the Previous  
 677 State)**

678 You write custom CUDA kernels to replace the pytorch operators in the given architecture  
 679 to get speedups.  
 680

681 You have complete freedom to choose the set of operators you want to replace. You may  
 682 make the decision to replace some operators with custom CUDA kernels and leave others  
 683 unchanged. You may replace multiple operators with custom implementations, consider op-  
 684 erator fusion opportunities (combining multiple operators into a single kernel, for example,  
 685 combining matmul+relu), or algorithmic changes (such as online softmax). You are only  
 686 limited by your imagination.  
 687

688 You are provided with the pytorch architecture to optimize, as long as your previous opti-  
 689 mization solution attempt and the execution feedback. Given the trajectory with execution  
 690 feedback, you need to refine your optimization to generate a new optimization. Specifically,  
 691 if your optimization failed to compile (i.e. 'compiled=False'), try to refine the optimization  
 692 so it can compile (you can refer to the 'compilation error' for why the solutions failed). If  
 693 your optimization compiled successfully but is incorrect based on input-output test cases  
 694 (i.e. 'correctness'=False), try to refine the optimization so it is correct (you can refer to  
 695 the 'correctness\_issues' for why the solutions are incorrect). If your optimization compiled  
 696 successfully and is correct, try to further optimize it to reduce the runtime.  
 697

698 

### C.2 CRITIC MODEL PROMPTS

699 

### C.3 REWARD MODEL PROMPT

702  
703  
704  
705  
706  
707  
708  
709  
710  
711  
712  
713  
714  
715

### 716 **Best Perf**

717  
718 You write custom CUDA kernels to replace the pytorch operators in the given architecture  
719 to get speedups.

720 You have complete freedom to choose the set of operators you want to replace. You  
721 may make the decision to replace some operators with custom CUDA kernels and leave  
722 others unchanged. You may replace multiple operators with custom implementations,  
723 consider operator fusion opportunities (combining multiple operators into a single kernel,  
724 for example, combining matmul+relu), or algorithmic changes (such as online softmax).  
725 You are only limited by your imagination.

726 You are provided with the pytorch architecture to optimize, your best-performing optimiza-  
727 tion solution attempt so far and its execution feedback, as well as your trajectory of previous  
728 optimization solution attempts and the execution feedback. Given the solutions with  
729 execution feedback, you need to refine your optimization to generate a new optimization.  
730 Specifically, if your optimization failed to compile (i.e. 'compiled=False'), try to refine  
731 the optimization so it can compile (you can refer to the 'compilation error' for why the  
732 solutions failed). You can also refer to the best-performing solution for cues of fixing the  
733 compilation errors.

734 If your optimization compiled successfully but is incorrect based on input-output test cases  
735 (i.e. 'correctness=False'), try to refine the optimization so it is correct (you can refer to  
736 the 'correctness\_issues' for why the solutions are incorrect). You can also refer to the  
737 best-performing solution for cues of fixing the incorrect issues.

738 If your optimization compiled successfully and is correct, try to further optimize it to  
739 reduce the runtime with the goal of obtaining shorter run time than the best-performing  
740 optimization so far. You can refer to the best-performing solution for inspirations of  
741 improving your last optimization.

742 Make sure you're refinement **IMPLEMENT CUDA OPERATORS** by 'from  
743 torch.utils.cpp\_extension import load\_inline', **INSTEAD OF PURE PyTorch**.

744  
745  
746  
747  
748  
749  
750  
751  
752  
753  
754  
755

756

757

758

759

760

**Traj Best Perf**

761

762

You write custom CUDA kernels to replace the pytorch operators in the given architecture to get speedups.

763

764

765

766

767

768

You have complete freedom to choose the set of operators you want to replace. You may make the decision to replace some operators with custom CUDA kernels and leave others unchanged. You may replace multiple operators with custom implementations, consider operator fusion opportunities (combining multiple operators into a single kernel, for example, combining matmul+relu), or algorithmic changes (such as online softmax). You are only limited by your imagination.

769

770

771

772

773

774

775

You are provided with the pytorch architecture to optimize, your best-performing optimization solution attempt so far and its execution feedback, as well as your trajectory of previous optimization solution attempts and the execution feedback. Given the solutions with execution feedback, you need to refine your optimization to generate a new optimization.

Specifically, if your optimization failed to compile (i.e. 'compiled=False'), try to refine the optimization so it can compile (you can refer to the 'compilation error' for why the solutions failed). You can also refer to the best-performing solution for cues of fixing the compilation errors.

776

777

778

779

If your optimization compiled successfully but is incorrect based on input-output test cases (i.e. 'correctness'=False), try to refine the optimization so it is correct (you can refer to the 'correctness\_issues' for why the solutions are incorrect). You can also refer to the best-performing solution for cues of fixing the incorrect issues.

780

781

782

If your optimization compiled successfully and is correct, try to further optimize it to reduce the runtime with the goal of obtaining shorter run time than the best-performing optimization so far. You can refer to the best-performing solution for inspirations of improving your last optimization.

783

784

Make sure you're refinement **IMPLEMENT CUDA OPERATORS** by 'from torch.utils.cpp\_extension import load\_inline', INSTEAD OF PURE PyTorch.

785

786

787

788

789

790

791

792

**Critique**

793

794

You write custom CUDA kernels to replace the pytorch operators in the given architecture to get speedups.

795

796

You have complete freedom to choose the set of operators you want to replace. You may make the decision to replace some operators with custom CUDA kernels and leave others unchanged. You may replace multiple operators with custom implementations, consider operator fusion opportunities (combining multiple operators into a single kernel, for example, combining matmul+relu), or algorithmic changes (such as online softmax). You are only limited by your imagination.

802

803

804

805

You are provided with the pytorch architecture to optimize, as long as your previous optimization solution attempt and the execution feedback, and natural language critique. Given the execution feedback and critique, you need to refine your optimization to generate a new optimization. Use the information and follow the critique to generate your refinement

806

807

808

809

810  
811  
812  
813  
814**Critique Best Perf**815  
816  
817  
818  
819  
820  
821  
822  
823  
824  
825  
826  
827  
828

You write custom CUDA kernels to replace the pytorch operators in the given architecture to get speedups.

You have complete freedom to choose the set of operators you want to replace. You may make the decision to replace some operators with custom CUDA kernels and leave others unchanged. You may replace multiple operators with custom implementations, consider operator fusion opportunities (combining multiple operators into a single kernel, for example, combining matmul+relu), or algorithmic changes (such as online softmax). You are only limited by your imagination.

You are provided with the pytorch architecture to optimize, your best-performing optimization solution attempt so far and its execution feedback, as well as your last optimization solution attempt and the execution feedback, and natural language critique. Given the execution feedback and critique, you need to refine your optimization to generate a new optimization. Use the information and follow the critique to generate your refinement.

829  
830  
831  
832  
833  
834  
835  
836  
837  
838**Traj**839  
840

You write custom CUDA kernels to replace the pytorch operators in the given architecture to get speedups.

You have complete freedom to choose the set of operators you want to replace. You may make the decision to replace some operators with custom CUDA kernels and leave others unchanged. You may replace multiple operators with custom implementations, consider operator fusion opportunities (combining multiple operators into a single kernel, for example, combining matmul+relu), or algorithmic changes (such as online softmax). You are only limited by your imagination.

You are provided with the pytorch architecture to optimize, as long as your trajectory of previous optimization solution attempts and the execution feedback. Given the trajectory with execution feedback, you need to refine your optimization to generate a new optimization. Specifically, if your optimization failed to compile (i.e. 'compiled=False'), try to refine the optimization so it can compile (you can refer to the 'compilation error' for why the solutions failed). If your optimization compiled successfully but is incorrect based on input-output test cases (i.e. 'correctness'=False), try to refine the optimization so it is correct (you can refer to the 'correctness\_issues' for why the solutions are incorrect). If your optimization compiled successfully and is correct, try to further optimize it to reduce the runtime. Make sure you're refinement **IMPLEMENT CUDA OPERATORS** by 'from torch.utils.cpp\_extension import load\_inline', **INSTEAD OF PURE PyTorch**.

859  
860  
861  
862  
863

864

865

866

867

**Traj Best Perf**

868

869

870

871

872

873

874

875

You write custom CUDA kernels to replace the pytorch operators in the given architecture to get speedups.

You have complete freedom to choose the set of operators you want to replace. You may make the decision to replace some operators with custom CUDA kernels and leave others unchanged. You may replace multiple operators with custom implementations, consider operator fusion opportunities (combining multiple operators into a single kernel, for example, combining matmul+relu), or algorithmic changes (such as online softmax). You are only limited by your imagination.

You are provided with the pytorch architecture to optimize, your best-performing optimization solution attempt so far and its execution feedback, as well as your trajectory of previous optimization solution attempts and the execution feedback. Given the solutions with execution feedback, you need to refine your optimization to generate a new optimization.

Specifically, if your optimization failed to compile (i.e. 'compiled=False'), try to refine the optimization so it can compile (you can refer to the 'compilation error' for why the solutions failed). You can also refer to the best-performing solution for cues of fixing the compilation errors.

If your optimization compiled successfully but is incorrect based on input-output test cases (i.e. 'correctness'=False), try to refine the optimization so it is correct (you can refer to the 'correctness\_issues' for why the solutions are incorrect). You can also refer to the best-performing solution for cues of fixing the incorrect issues.

If your optimization compiled successfully and is correct, try to further optimize it to reduce the runtime with the goal of obtaining shorter run time than the best-performing optimization so far. You can refer to the best-performing solution for inspirations of improving your last optimization.

Make sure you're refinement **IMPLEMENT CUDA OPERATORS** by 'from `torch.utils.cpp_extension import load_inline`', INSTEAD OF PURE PyTorch.

893

894

895

896

897

898

899

900

**Traj Critique**

901

You write custom CUDA kernels to replace the pytorch operators in the given architecture to get speedups.

You have complete freedom to choose the set of operators you want to replace. You may make the decision to replace some operators with custom CUDA kernels and leave others unchanged. You may replace multiple operators with custom implementations, consider operator fusion opportunities (combining multiple operators into a single kernel, for example, combining matmul+relu), or algorithmic changes (such as online softmax). You are only limited by your imagination.

You are provided with the pytorch architecture to optimize, as long as your trajectory of previous optimization solution attempts and the execution feedback, and natural language critiques. Given the execution feedback and critiques, you need to refine your optimization to generate a new optimization. Use the information and follow the critique to generate your refinement. Make sure you're refinement **IMPLEMENT CUDA OPERATORS** by 'from `torch.utils.cpp_extension import load_inline`', INSTEAD OF PURE PyTorch.

915

916

917

918  
919  
920  
921  
922**Traj Critique Best Perf**

923 You write custom CUDA kernels to replace the pytorch operators in the given architecture  
 924 to get speedups.

925 You have complete freedom to choose the set of operators you want to replace. You may  
 926 make the decision to replace some operators with custom CUDA kernels and leave others  
 927 unchanged. You may replace multiple operators with custom implementations, consider op-  
 928 erator fusion opportunities (combining multiple operators into a single kernel, for example,  
 929 combining matmul+relu), or algorithmic changes (such as online softmax). You are only  
 930 limited by your imagination.

931 You are provided with the pytorch architecture to optimize, your best-performing optimiza-  
 932 tion solution attempt so far and its execution feedback, as well as your trajectory of previous  
 933 optimization solution attempts and the execution feedback, and natural language critiques.  
 934 Given the execution feedback and critiques, you need to refine your optimization to generate  
 935 a new optimization.

936 Use the information and follow the critique to generate your refinement. Make sure you're  
 937 refinement **IMPLEMENT CUDA OPERATORS** by 'from torch.utils.cpp\_extension import  
 938 load\_inline', INSTEAD OF PURE PyTorch.

939  
940  
941  
942  
943  
944  
945  
946**Critique**

948 You write custom CUDA kernels to replace the pytorch operators in the given architecture  
 949 to get speedups.

950 You have complete freedom to choose the set of operators you want to replace. You may  
 951 make the decision to replace some operators with custom CUDA kernels and leave others  
 952 unchanged. You may replace multiple operators with custom implementations, consider op-  
 953 erator fusion opportunities (combining multiple operators into a single kernel, for example,  
 954 combining matmul+relu), or algorithmic changes (such as online softmax). You are only  
 955 limited by your imagination.

956 You are provided with the pytorch architecture to optimize, your previous optimization so-  
 957 lution attempt and the execution feedback. Given the trajectory with execution feedback  
 958 and critiques, you need to provide critique for the previous solution attempt that can guide  
 959 the refinement of the optimization to generate a new optimization that aims to overcome  
 960 the pitfalls in the solution. Specifically, if the optimization failed to compile (i.e. 'com-  
 961 piled=False'), or compiled successfully but is incorrect based on input-output test cases (i.e.  
 962 'correctness'=False), 1) provide diagnosis based on the error messages on why it fails to  
 963 compile/is incorrect; 2) based on the diagnosis, further provide actionable suggestions that  
 964 can guide the refinement of the solution to compile and be correct. If the optimization can  
 965 compile and is correct, based on the running time information, 1) provide diagnosis on what  
 966 are the potential bottleneck of running time in the solution; 2) based on the diagnosis, futher  
 967 provide actionable suggestions that can guide the refinement of the solution to reduce run-  
 968 ning time.

969  
970  
971

972

973

974

975

**Critique Best Perf**

976

You write custom CUDA kernels to replace the pytorch operators in the given architecture to get speedups.

977

You have complete freedom to choose the set of operators you want to replace. You may make the decision to replace some operators with custom CUDA kernels and leave others unchanged. You may replace multiple operators with custom implementations, consider operator fusion opportunities (combining multiple operators into a single kernel, for example, combining matmul+relu), or algorithmic changes (such as online softmax). You are only limited by your imagination.

978

You are provided with the pytorch architecture to optimize, your best-performing optimization solution attempt so far and its execution feedback, as well as your last optimization solution attempt and the execution feedback. Given the solutions with execution feedback and critiques, you need to provide critique for the last solution attempt that can guide the refinement of the optimization to generate a new optimization that aims to overcome the pitfalls in the solution.

979

Specifically, if the optimization failed to compile (i.e. 'compiled=False'), or compiled successfully but is incorrect based on input-output test cases (i.e. 'correctness'=False), 1) provide diagnosis based on the error messages on why it fails to compile/is incorrect; 2) based on the diagnosis, further provide actionable suggestions that can guide the refinement of the solution to compile and be correct. You can also refer to the best-performing solution for cues of fixing the compilation errors and/or correctness issues.

980

If the optimization can compile and is correct, based on the running time information, 1) provide diagnosis on what are the potential bottleneck of running time in the solution; 2) based on the diagnosis, futher provide actionable suggestions that can guide the refinement of the solution to reduce running time with the goal of obtaining shorter run time than the best-performing optimization so far. You can refer to the best-performing solution for inspirations of improving your last optimization.

981

982

983

984

985

986

987

988

989

990

991

992

993

994

995

996

997

998

999

1000

1001

1002

1003

**Traj Critique**

1004

1005

1006

You write custom CUDA kernels to replace the pytorch operators in the given architecture to get speedups.

1007

You have complete freedom to choose the set of operators you want to replace. You may make the decision to replace some operators with custom CUDA kernels and leave others unchanged. You may replace multiple operators with custom implementations, consider operator fusion opportunities (combining multiple operators into a single kernel, for example, combining matmul+relu), or algorithmic changes (such as online softmax). You are only limited by your imagination.

1008

You are provided with the pytorch architecture to optimize, as long as your trajectory of previous optimization solution attempts and the execution feedback, and natural language critiques. Given the trajectory with execution feedback and critiques, you need to provide critique for the most recent solution attempt that can guide the refinement of the optimization to generate a new optimization that aims to overcome the pitfalls in the solution trajectory. Specifically, if the optimization failed to compile (i.e. 'compiled=False'), or compiled successfully but is incorrect based on input-output test cases (i.e. 'correctness'=False), 1) provide diagnosis based on the error messages on why it fails to compile/is incorrect; 2) based on the diagnosis, further provide actionable suggestions that can guide the refinement of the solution to compile and be correct. If the optimization can compile and is correct, based on the running time information, 1) provide diagnosis on what are the potential bottleneck of running time in the solution; 2) based on the diagnosis, futher provide actionable suggestions that can guide the refinement of the solution to reduce running time.

1023

1024

1025

1026  
1027  
1028  
1029  
1030  
1031  
1032  
1033  
1034  
1035  
1036  
1037  
1038  
1039

### Traj Critique Best Perf

1040  
1041 You write custom CUDA kernels to replace the pytorch operators in the given architecture  
1042 to get speedups.  
1043 You have complete freedom to choose the set of operators you want to replace. You  
1044 may make the decision to replace some operators with custom CUDA kernels and leave  
1045 others unchanged. You may replace multiple operators with custom implementations,  
1046 consider operator fusion opportunities (combining multiple operators into a single kernel,  
1047 for example, combining matmul+relu), or algorithmic changes (such as online softmax).  
1048 You are only limited by your imagination.  
1049 You are provided with the pytorch architecture to optimize, your best-performing optimiza-  
1050 tion solution attempt so far and its execution feedback, as well as your trajectory of previous  
1051 optimization solution attempts and the execution feedback, and natural language critiques.  
1052 Given the solutions with execution feedback and critiques, you need to provide critique  
1053 for the most recent solution attempt that can guide the refinement of the optimization to  
1054 generate a new optimization that aims to overcome the pitfalls in the solution trajectory.  
1055 Specifically, if the optimization failed to compile (i.e. 'compiled=False'), or compiled  
1056 successfully but is incorrect based on input-output test cases (i.e. 'correctness=False'), 1)  
1057 provide diagnosis based on the error messages on why it fails to compile/is incorrect; 2)  
1058 based on the diagnosis, further provide actionable suggestions that can guide the refinement  
1059 of the solution to compile and be correct. You can also refer to the best-performing solution  
1060 for cues of fixing the compilation errors and/or correctness issues.  
1061 If the optimization can compile and is correct, based on the running time information, 1)  
1062 provide diagnosis on what are the potential bottleneck of running time in the solution; 2)  
1063 based on the diagnosis, futher provide actionable suggestions that can guide the refinement  
1064 of the solution to reduce running time with the goal of obtaining shorter run time than  
1065 the best-performing optimization so far. You can refer to the best-performing solution for  
1066 inspirations of improving your last optimization.  
1067  
1068  
1069  
1070  
1071  
1072  
1073  
1074  
1075  
1076  
1077  
1078  
1079

1080  
1081  
1082  
1083  
1084

### 1085 **Traj Critique**

1086  
1087  
1088  
1089  
1090  
1091  
1092  
1093

You are an expert in writing custom CUDA kernels to replace the PyTorch operators in the given architecture to get speedups.

The task offers complete freedom to choose the set of operators one want to replace. One may make the decision to replace some operators with custom CUDA kernels and leave others unchanged. One may replace multiple operators with custom implementations, consider operator fusion opportunities (combining multiple operators into a single kernel, for example, combining matmul+relu), or algorithmic changes (such as online softmax). You are only limited by your imagination.

The task provides

- 1) The target PyTorch architecture to optimize, with its running time.
- 2) The trajectory of previous optimization refinement attempts. The trajectory contains (multiple) rounds of optimization refinement attempts, the corresponding execution feedback & relative speedup to the target PyTorch implementation, and the natural language critique that diagnoses the potential issues of the refinement with actionable suggestions.
- 3) The most recent optimization refinement attempt, if 2) is provided, then the generation of this attempt is conditioned on all information in 2).

Given the trajectory, you need to predict the EXPECTED OVERALL MAXIMUM RELATIVE SPEEDUP of this trajectory if the refinement iteration of solution-execution feedback (-critique) WILL BE CONTINUED FOR A FEW MORE ROUNDS IN THE SAME MANNER (you will be provided with how many remaining future rounds of refinement are allowed).

The optimization (and natural language) critics are all generated by an AI system.

The EXPECTED OVERALL MAXIMUM RELATIVE SPEEDUP of a to be continued trajectory is defined with five-way labels:

0: NONE of the solutions in the current trajectory or the EXPECTED solutions in your estimated future rounds of refinement is/will be faster than the original PyTorch implementation. This can be caused by either none of them are correct or the correct ones are all slower than the PyTorch implementation. So the maximum relative speedup is 100(%) since one will just use the original PyTorch implementation.

1: AT LEAST one of the solution in the current trajectory or the EXPECTED solutions in your estimated future rounds of refinement is/will be correct AND yield running time FASTER than the PyTorch architecture, with maximum relative speedup IN THE RANGE OF (100%, 140%].

2: AT LEAST one of the solution in the current trajectory or the EXPECTED solutions in your estimated future rounds of refinement is/will be correct AND yield running time FASTER than the PyTorch architecture, with maximum relative speedup IN THE RANGE OF (140%, 320%].

3: AT LEAST one of the solution in the current trajectory or the EXPECTED solutions in your estimated future rounds of refinement is/will be correct AND yield running time FASTER than the PyTorch architecture, with maximum relative speedup IN THE RANGE OF (320%, 475%].

4: AT LEAST one of the solution in the current trajectory or the EXPECTED solutions in your estimated future rounds of refinement is/will be correct AND yield running time FASTER than the PyTorch architecture, with maximum relative speedup GREATER THAN 475%.

1128  
1129  
1130  
1131  
1132  
1133

1134  
 1135  
 1136  
 1137  
 1138  
 1139  
 1140  
 1141  
 1142  
 1143  
 1144  
 1145  
 1146  
 1147  
 1148

1149 **Traj Critique (Continue)**

1150  
 1151  
 1152  
 1153  
 1154  
 1155  
 1156  
 1157  
 1158  
 1159  
 1160  
 1161  
 1162  
 1163  
 1164  
 1165  
 1166  
 1167  
 1168  
 1169  
 1170  
 1171  
 1172

Based on the given information, you need to estimate:  
 1) the difficulty of the target optimization problem.  
 2) the AI system's capability of generating optimization solutions that accurately incorporates the feedback (and critiques) to fix bugs/improve performance. For example, if the target trajectory currently fails with compilation error, you need to estimate if the AI SYSTEM is capable to fix it.  
 3) The AI system's capability to provide accurate diagnosis of errors/performance bottlenecks and the quality and actionability of provided refinement suggestions. For example, if the critiques and expected future critiques is/will be able to identify correct issues and provide actionable suggestions.  
 4) Base on 1) 2), and 3), the MOST LIKELY outcome of the EXPECTED OVERALL MAXIMUM RELATIVE SPEEDUP the current attempt (+ target trajectory) can lead to, if the refinement will be continued by THE SAME AI SYSTEM for a given number of rounds. BE CAUSIOUS in your estimation, which need to faithfully reflect the difficulties and capabilities of the AI SYSTEM, WITHOUT OVERESTIMATIONS OR UNDERESTIMATIONS. Remember the optimization is and will be performed by THE AI SYSTEM, NOT YOU. So use your expertise only to predict the capabilities of the AI system, and the EXPECTED OVERALL MAXIMUM RELATIVE SPEEDUP based on the AI's capabilities. And DO NOT take into consideration your own expertise in the remaining trajectory (i.e. do not think that you are going to further refine it, it is the system's job). Finally, based on your estimations, provide the EXPECTED OVERALL MAXIMUM RELATIVE SPEEDUP prediction as a numerical label of 0/1/2/3/4. DO NOT output your estimations, just output the final predicted EXPECTED OVERALL MAXIMUM RELATIVE SPEEDUP score as a single number and NOTING ELSE.

1173  
 1174  
 1175  
 1176  
 1177  
 1178  
 1179  
 1180  
 1181  
 1182  
 1183  
 1184  
 1185  
 1186  
 1187