

# 000 001 002 003 004 005 006 007 008 009 010 011 012 013 014 015 016 017 018 019 020 021 022 023 024 025 026 027 028 029 030 031 032 033 034 035 036 037 038 039 040 041 042 043 044 045 046 047 048 049 050 051 052 053 COMPOSITIONAL VISUAL PLANNING VIA INFERENCE-TIME DIFFUSION SCALING

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## ABSTRACT

Diffusion models excel at short-horizon robot planning, yet scaling them to long-horizon tasks remains challenging due to computational constraints and limited training data. Existing compositional approaches stitch together short segments by separately denoising each component and averaging overlapping regions. However, this suffers from instability as the factorization assumption breaks down in noisy data space, leading to inconsistent global plans. We propose that the key to stable compositional generation lies in enforcing boundary agreement on the estimated clean data (Tweedie estimates) rather than on noisy intermediate states. Our method formulates long-horizon planning as inference over a chain-structured factor graph of overlapping video chunks, where pretrained short-horizon video diffusion models provide local priors. At inference time, we enforce boundary agreement through a novel combination of synchronous and asynchronous message passing that operates on Tweedie estimates, producing globally consistent guidance without requiring additional training. Our training-free framework demonstrates significant improvements over existing baselines across 100 simulation tasks spanning 4 diverse scenes, effectively generalizing to unseen start-goal combinations that were not present in the original training data. Project website: <https://comp-visual-planning.github.io>

## 1 INTRODUCTION

Generative diffusion models have shown strong capacity for modeling complex, high-dimensional distributions over images, videos, and robot plans. In planning, they offer a compelling alternative to per-instance optimization: instead of solving a new search problem for every start–goal pair, we can sample likely solutions from a learned generator. However, extending video-based planning to long horizons remains challenging: most backbones are trained on short clips, compute and memory scale unfavorably with sequence length, and long-range constraints (contacts, object persistence, and start–goal satisfaction) must be maintained throughout the rollout.

Classical planning methods, such as Task and Motion Planning (TAMP), decompose tasks into structured subproblems and enforce constraint satisfaction through symbolic operators, while hierarchical control methods first solve for a high-level task plan and then refine it into a low-level motion plan. Compositional diffusion planning follows these core principles but provides a data-driven, probabilistic alternative to hand-engineered classical and hierarchical planners. We adopt a compositional generation perspective on long-horizon planning, we compose plans from overlapping, short-horizon factors produced by a pretrained diffusion model. The central challenge is the consistency of this composition: during forward diffusion, noisy variables become entangled across time, breaking factorization assumptions behind common compositional heuristics (e.g., score averaging) (Zhang et al., 2023; Mishra et al., 2023; 2024; Bar-Tal et al., 2023) and yielding brittle behavior when long-range constraints must propagate.

Our key insight is to compose where diffusion model estimations are most reliable: on their Tweedie estimates, which provide a stable domain in which strong and explicit compositional heuristics can be applied. We formulate planning as inference in a chain-structured factor graph over overlapping video chunks. Local priors come from a short-horizon diffusion backbone; global coherence is enforced by boundary agreement on Tweedie predictions, not on noisy states. We propose two complementary message-passing mechanisms on these denoised variables: a synchronous scheme that views the

chain as a Gaussian linear system and drives a single residual to zero with parallel, order-invariant updates, and an asynchronous scheme that propagates constraints via one-sided, stop-gradient targets for faster and more stable convergence. Both yield training-free guidance terms that we integrate into a DDIM sampler via diffusion-sphere guidance, balancing alignment with sample diversity.

We instantiate this approach for compositional visual diffusion planning, representing a plan as a sequence of images. Crucially, the method operates purely at inference time: the short-horizon diffusion backbone is trained once on short clips and then frozen; at test time we compose long-horizon plans via message passing on Tweedie estimates, with no additional training, fine-tuning, or task-specific adapters. The amount of test-time compute is controllable through the number of factors and guidance strength, enabling a direct plug-and-play use with existing backbones.

In summary, the key contributions of this work are: (1) A diffusion planning framework that models long-horizon plans as chain-structured factor graphs over video segments and enforces boundary agreement on Tweedie estimates rather than on noisy diffusion states. (2) Joint synchronous and asynchronous message passing over denoised variables, coupled with a training-free sampler that guides DDIM steps with diffusion-sphere guidance derived from message-passing residuals, preserving local sample quality, parallelism, and diversity while enforcing boundary agreement. (3) A compositional planning benchmark and empirical study demonstrating significant improvements in temporal coherence, static quality, and task success on held-out start–goal combinations compared with prior compositional baselines (Zhang et al., 2023) that operate on noisy diffusion states.

## 2 RELATED WORK

**Diffusion Models For Planning.** A flurry of work has leveraged diffusion models (Sohl-Dickstein et al., 2015; Ho et al., 2020) for planning (Janner et al., 2022; Ajay et al., 2022; Dong et al., 2024a; He et al., 2023a; Ubukata et al., 2024; Lu et al., 2025; Chen et al., 2024a; Liang et al., 2023; Dong et al., 2024b), with various applications such as path finding (Carvalho et al., 2025; Luo et al., 2024), robotics (Pearce et al., 2023; Fang et al., 2024), and multi-agent (Zhu et al., 2024; Shaoul et al., 2024). Performance can be further improved by increasing test-time compute such as tree search (Feng et al., 2024a; Yoon et al., 2025b), hierarchical planning (Li et al., 2023; Chen et al., 2024b), and post-hoc refinement (Lee et al., 2024; Wang et al., 2022). Despite strong results, most prior work studies diffusion planning in low-dimensional state spaces that yield 2D trajectories. While recent work (Xu et al., 2025; Xie et al., 2025; Huang et al., 2024a) begins to consider more complex state spaces, they typically target simple or task-specific scenarios. In this paper, we investigate *visual diffusion planning*, where a plan is represented as a sequence of images, and we introduce a training-free sampling method that scales to significantly longer horizons and to unseen start–goal combinations.

**Compositional Diffusion Generation.** Compositional diffusion models are now well studied (Du et al., 2020; Garipov et al., 2023; Du & Kaelbling, 2024; Mahajan et al., 2024; Okawa et al., 2024; Thornton et al., 2025). One thread develops samplers for logical conjunctions of conditions, combining multiple prompts or constraints into coherent generations (Liu et al., 2022; Bradley et al., 2025; Zhang et al., 2025; Yang et al., 2023). A complementary thread scales the number of inference-time tokens while reusing models trained on short horizons—yielding wide-field panoramas (Zhang et al., 2023; Bar-Tal et al., 2023; Kim et al., 2024a; Lee et al., 2023), longer-duration videos (Wang et al., 2023; Kim et al., 2024b; 2025), and extended-horizon robotic plans (Zhang et al., 2023; Mishra et al., 2023; Luo et al., 2025). However, existing compositional methods suffer from instability when applied to noisy diffusion states, as they typically rely on score averaging or other heuristic combinations that assume factorization holds throughout the denoising process. In contrast, our approach operates on clean Tweedie estimates rather than noisy intermediate states, formulates the problem as factor graph inference with explicit boundary constraints, and employs principled message passing to maintain global consistency—yielding substantial improvements in both stability and plan quality over prior compositional planning methods.

**Inference-Time Guidance for Diffusion Model.** Inference-time guidance steers diffusion sampling without retraining, enabling adaptive, controllable behavior at test time. This flexibility has driven progress in image restoration (inpainting, deblurring) (Chung et al., 2024; Yang et al., 2024; Yu et al., 2023; Ye et al., 2024; Song et al., 2023a), style transfer (Bansal et al., 2023; He et al., 2023b), and robot motion/behavior generation (Liao et al., 2025; Black et al., 2025; Du & Song, 2025; Song et al., 2023b; Feng et al., 2024b). However, most existing methods steers a fixed-length output, we frame

108 guidance as a form of message passing between tokens—allowing information to propagate across  
 109 the sequence. This perspective lets us stitch together short behavioral fragments into long-range,  
 110 temporally consistent visual plan.  
 111

### 112 3 PRELIMINARIES

#### 115 3.1 FACTOR GRAPH FORMULATION FOR COMPOSITIONAL DISTRIBUTIONS

116 A factor graph  $z = [u^1, u^2, \dots, u^m]$  is a bipartite graph connecting factor nodes  $\{x^i\}_{i=1}^n$  and  
 117 variable nodes  $\{u\}_{j=1}^m$ , where  $x^j \subseteq [u^1, u^2, \dots, u^m]$ . An undirected edge between  $x^i$  and  $u^j$  exists  
 118 if and only if  $u^j \in x^i$ . Given a factor graph that represents the factorization of joint distribution,  
 119 previous works **DiffCollage** approximate it with Bethe approximation (Zhang et al., 2023), and  
 120 **Generative Skill Chaining (GSC)** extends the same formulation to robot task-and-motion planning as  
 121 its follow-up work:  
 122

$$123 \quad p(z_t) := \frac{\prod_{i=1}^n p(x_t^i)}{\prod_{j=1}^m p(u_t^j)^{d_j-1}}, \quad (1)$$

125 where  $d_j$  is the degree of each variable  $u_j$ . Therefore, the estimated score is base on Bethe approxi-  
 126 mation is:  
 127

$$128 \quad \nabla_{z_t} \log p(z_t) = \sum_{i=1}^n \nabla_{x_t^i} \log p(x_t^i) + \sum_{j=1}^m (1 - d_j) \nabla_{u_t^j} \log(u_t^j). \quad (2)$$

129 For example, consider a linear chain  $z = [u^1, u^2, u^3, u^4, u^5]$  with factors  $x_1 = [u^1, u^2, u^3]$  and  
 130  $x_2 = [u^3, u^4, u^5]$ . We can represent the joint noisy distribution as:  
 131

$$132 \quad p(z_t) = \frac{p(u_t^1, u_t^2, u_t^3) p(u_t^3, u_t^4, u_t^5)}{p(u_t^3)}, \quad (3)$$

134 Given the linear chain graph, the overlapping variable  $u^3$  (the one shared between neighboring factors  
 135  $x_1$  and  $x_2$ ) has degree  $d = 2$ , while the non-overlapping ones have  $d = 1$  (i.e., their distribution come  
 136 from individual factors). However, Bethe approximation(Eq. 1) holds in clean data (i.e., diffusion  
 137 timestep  $t = 0$ ), but does not hold when  $t > 0$ , we further prove its gap later (Theorem 1).  
 138

#### 139 3.2 DIFFUSION MODEL AND TRAINING-FREE GUIDED DIFFUSION

140 Diffusion Models are a class of generative model that generates sample in the desired distribution  
 141 from an initial Gaussian distribution  $p(x_T)$  by iteratively performing a denoising process. It  
 142 has a pre-defined forward process  $q(x_t | x_0) = \mathcal{N}(x_t; \sqrt{\bar{\alpha}_t}x_0, (1 - \bar{\alpha}_t)I)$ , where  $\bar{\alpha}$  is a scalar  
 143 dependent on diffusion timestep  $t$ . In this work, we directly estimate Tweedie when training:  
 144  $\mathbb{E}_{t, x_0, \epsilon_t} [\|x_0 - x_\theta(x_t, t)\|^2]$  ( $x_0$  predictor) and apply a DDIM step when sampling:  
 145

$$146 \quad x_{t-1} = \sqrt{\bar{\alpha}_t}x_{0|t} + \sqrt{1 - \bar{\alpha}_t - \sigma_t^2} \frac{x_t - \sqrt{\bar{\alpha}_t}x_{0|t}}{\sqrt{1 - \bar{\alpha}_t}} + \sigma_t \epsilon_t, \quad (4)$$

148 where estimated Tweedie  $x_{0|t}$  is the output of  $x_\theta(x_t, t)$ .  
 149

150 Classifier Guidance (Dhariwal & Nichol, 2021) proposes to train a time dependent classifier in  
 151 conditional generative tasks. Specifically, the conditional distribution  $p(x_t | y)$  can be modeled by  
 152 Bayes Rules  $p(x_t | y) = p(x_t)p(y | x_t)/p(y) : \nabla_{x_t} \log p(x_t | y) = \nabla_{x_t} \log p(x_t) + \nabla_{x_t} \log p(y | x_t)$ ,  
 153 where  $y$  represents the condition or measurement. This paper focuses on conditional guidance in  
 154 a training-free manner, all the guidance in this paper is in training-free manner. In training-free  
 155 guidance setting, instead of explicitly training a classifier, the guidance term  $p(y | x_t)$  can be modeled  
 156 as a potential function  $\exp(-L(x_{0|t}))$ , which simplifies to a gradient-descent update during inference  
 157 time:  
 158

$$\nabla_{x_t} \log p(y | x_t) = \nabla_{x_t} \log \frac{\exp(-L(x_{0|t}))}{Z} = -\nabla_{x_t} L(x_{0|t}), \quad (5)$$

159 where  $x_{0|t} = x_\theta(x_t, t)$  estimated by Tweedie’s Predictor. Since  $\exp(-L(x_{0|t}))$  is a point estimation  
 160 of distribution of  $\mathbb{E}_{x_0 \sim p(x_0 | x_t)} [\exp(-L(x_0))]$ , and the gap between them has a upper bound (Chung  
 161 et al., 2024) and lower bound (Yang et al., 2024), Diffusion sphere guidance (Yang et al., 2024) is  
 proposed to eliminate this gap based by formulating a constrained optimization problem over a

162 hypersphere with the mean to be  $\sqrt{\bar{\alpha}_t}x_{0|t} + \sqrt{1 - \bar{\alpha}_t - \sigma^2} \frac{x_t - \sqrt{\bar{\alpha}_t}x_{0|t}}{\sqrt{1 - \bar{\alpha}_t}}$  and radius  $\sqrt{s}\sigma_t$  ( $s$  is the  
 163 shape of  $x_t$ ), and derive a closed form solution for the update :  
 164

$$165 \quad x_{t-1} = \sqrt{\bar{\alpha}_t}x_{0|t} + \sqrt{1 - \bar{\alpha}_t - \sigma^2} \frac{x_t - \sqrt{\bar{\alpha}_t}x_{0|t}}{\sqrt{1 - \bar{\alpha}_t}} - \sqrt{s}\sigma_t \frac{\nabla_{x_t} L(x_{0|t})}{\|\nabla_{x_t} L(x_{0|t})\|}. \quad (6)$$

168 **4 METHOD**  
 169

170 We formulate long-horizon planning as inference over a chain-structured factor graph of overlapping  
 171 video chunks, where pretrained short-horizon diffusion models provide local priors. Our key  
 172 innovation is enforcing boundary agreement on estimated clean data (Tweedie estimates) rather than  
 173 noisy intermediate states, addressing the core limitation that factorization assumptions break down  
 174 during diffusion sampling. We achieve this through novel synchronous and asynchronous message  
 175 passing that operates on Tweedie estimates, producing globally consistent guidance without additional  
 176 training. The approach involves formulating the planning problem as a factor graph (Section 4.1),  
 177 deriving its distribution (Section 4.2), and sampling via our message passing scheme (Section 4.3).  
 178

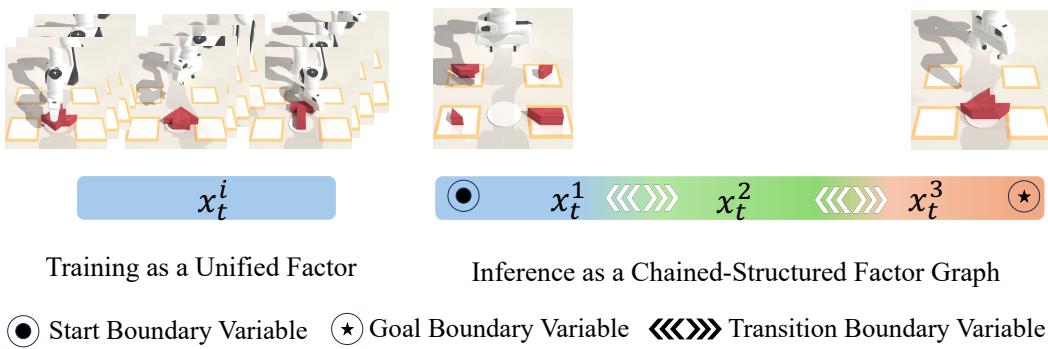


Figure 1: **Compositional Visual Planning via Inference Time Diffuser Scaling.** We train a short-horizon visual diffusion model on clips treated as a single factor. At inference, we scale visual planning horizon without retraining by chaining overlapping factors into a linear factor graph: the start and goal boundary variables are anchored at the ends, while neighboring factors exchange information through shared transition boundary variables.

4.1 PROBLEM FORMULATION

While a diffusion model can learn a prior over short, local behaviors, long-horizon planning requires additional structure to ensure feasibility. Beyond satisfying the start and the goal, intermediate pieces must stitch with local consistency. We therefore train a short-horizon diffusion model  $x_\theta$  on local task segments; at test time, given a start image and a goal image, we sample from a Gaussian prior, partition the trajectory into overlapping chunks, generate each chunk with  $x_\theta$ , and compose them into a coherent plan that is finally mapped back to an action sequence through inverse dynamics model.

We represent the plan as a linear chain  $z = [u^1, \dots, u^m]$  and place  $n$  overlapping factors  $x^i = [u^{2i-1}, u^{2i}, u^{2i+1}]$ ,  $i = 1, \dots, n$ , each collecting three consecutive frames. The endpoints  $u^1 = s$  and  $u^m = g$  serve as the start and goal boundary variables. Let  $A_i$  and  $B_i$  denote linear selectors that extract the first and last frames of factor  $x^i$ , respectively. The feasibility of a plan is enforced by the following boundary agreements:

$$(7) \quad \begin{aligned} & \text{(Start/Goal Anchoring)} \quad A_1 x^1 = s, \quad B_n x^n = g, \\ & \text{(Transition Boundary)} \quad B_i x^i = A_{i+1} x^{i+1}, \quad i = 1, \dots, n-1. \end{aligned}$$

This factorization reduces global planning to local generation with explicit boundary equalities and scales by reusing the same local model  $x_\theta$  across time while preserving consistency via start–goal anchoring and transition agreements. All factors/variables in latent space encoded by the Cosmos tokenizer (et. al., 2025) into a compact latent representation; we perform planning entirely in this latent space rather than pixels, which reduces dimensionality to save compute.

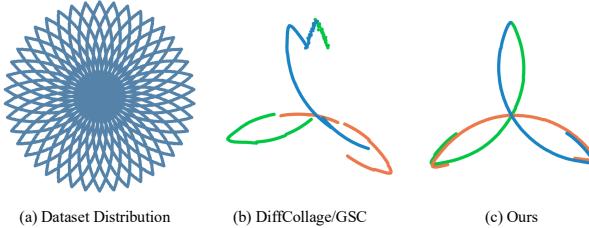
216 4.2 DISTRIBUTION OF FACTOR GRAPH  
217

218 Prior work (Zhang et al., 2023; Mishra et al., 2023; 2024) relies on the Bethe-style product of factors  
219 normalized by variables (Eq. 1), which is accurate on clean data. Forward diffusion, however, perturbs  
220 factorization assumption between factors and variables.

221 **Theorem 1** (Noisy-Bethe Gap Theorem). *Consider a linear chain  $z = [u^1, u^2, u^3]$  with  
222 pairwise factors  $[u^1, u^2]$  and  $[u^2, u^3]$ , where  $u^2$  is the transition boundary variable. As-  
223 sume the forward noising processes are  $p(u_t^1, u_t^2 | u^1, u^2)$ ,  $p(u_t^2, u_t^3 | u^2, u^3)$ , and  $p(u_t^3 | u^3)$ .  
224 Let  $a(u^2) = \int p(u^1, u^2) p(u_t^1, u_t^2 | u^1, u^2) du^1$ ,  $b(u^2) = \int p(u^2, u^3) p(u_t^2, u_t^3 | u^2, u^3) du^3$ ,  
225  $c(u^2) = p(u^2) p(u_t^2 | u^2)$ ,  $Z = \int c(u^2) du^2$ , and  $q(u^2) = c(u^2)/Z$ . Denote by  $p(u_t^1, u_t^2, u_t^3)$  the  
226 true noisy distribution and by  $\hat{p}(u_t^1, u_t^2, u_t^3)$  the estimator from Eq. 1. Then the gap between true  
227 distribution and estimated distribution is:*

$$\Delta = p(u_t^1, u_t^2, u_t^3) - \hat{p}(u_t^1, u_t^2, u_t^3) = Z \text{Cov}_{u^2 \sim q} \left[ \frac{a}{c}, \frac{b}{c} \right]. \quad (8)$$

231 **Interpretation.** The proof is in appendix A. We can view  $a(u^2)$  as the left-factor message into the  
232 boundary  $u^2$ ,  $b(u^2)$  as the right-factor message into the boundary, and  $c(u^2)$  as the local boundary  
233 evidence. Intuitively,  $a(u^2)$  and  $b(u^2)$  quantify how the left and right pairwise factors, after passing  
234 through their respective forward-noise channels, “vote” for different boundary values  $u^2$ . The term  
235  $c(u^2)$  provides the unary baseline that captures how plausible each boundary value is on its own  
236 (and how it transmits noise). The noisy-Bethe gap  $\Delta$  is exactly the covariance—under the boundary  
237 weighting  $q$ —between the two relative gains  $a(u^2)/c(u^2)$  and  $b(u^2)/c(u^2)$ . When these gains are  
238 uncorrelated (or proportional) across  $u^2$ , the covariance vanishes and the Bethe approximation  
239 remains accurate. Forward diffusion typically introduces shared, heteroscedastic distortions in  $u^2$ ,  
240 which make the two gains rise and fall together; this produces a nonzero covariance and, consequently,  
241 a systematic gap.



250 Figure 2: Motivating toy example. We train a short-horizon diffu-  
251 sion model on circular arc clips (left). At test time, three  $120^\circ$  arc  
252 generators are composed to form a three-petal “flower”.

253 Bethe gap, instead of enforcing dependencies directly among the noisy factors  $x_t^{1:n}$ , we impose them  
254 on the concatenated Tweedie (denoised) estimates  $x_{0|t}^{1:n}$ . Accordingly, our approximation to the  
255 factor-graph distribution is:

$$p(z_t) = \prod_{i=1}^n p(x_t^i) \cdot \exp(-L(x_{0|t}^{1:n})), \quad (9)$$

259 where  $\exp(-L(x_{0|t}^{1:n}))$  acts as a potential that penalizes inconsistencies—and thus enforces depen-  
260 dencies—among the estimated clean variables  $x_{0|t}^{1:n} = x_\theta(x_t^{1:n})$ .

263 4.3 JOINTLY SYNCHRONOUS AND ASYNCHRONOUS MESSAGE PASSING  
264

265 Message passing proceeds through *boundary factors*: when the transition boundaries together with  
266 the start and goal boundaries agree, the plan is feasible (see Eq. 7). We therefore optimize boundary  
267 agreement explicitly. Our synchronous scheme treats the chain as a Gaussian linear system and  
268 drives a single residual ( $\Sigma^{-1}x_{0|t}^{1:n} = \eta$ ) to zero via parallel updates, but can be numerically stiff.  
269 Our asynchronous scheme uses one-sided, stop-gradient targets to propagate constraints forward  
and backward in a TD-style manner, yielding faster and more stable convergence at the cost of mild

Figure 2 illustrates the core failure mode of DiffCollage which is deployed based on noisy factorization assumption. A DiffCollage/GSC-style stitcher (middle) drifts and leaves boundary gaps, while our inference-time message passing (right) aligns shared boundaries and closes the loops. This mirrors the limitation of a Bethe-style product of factors (Eq. 1) under forward diffusion: noise corrupts factorization assumption. Motivated by the Noisy-

270 bias. Finally, diffusion-sphere guidance interpolates between unconditional sampling and loss-driven  
 271 descent, balancing alignment and diversity.  
 272

273 **4.3.1 SYNCHRONOUS MESSAGE PASSING**  
 274

275 We encode the boundary condition as a Gaussian potential,  $\psi_{i-1,i} := \exp(-\frac{1}{c_{i-1}}\|B_{i-1}x_{0|t}^{i-1} -$   
 276  $A_i x_{0|t}^i\|^2)$ , where  $c_{i-1}$  denotes the variance.  
 277

278 **Theorem 2** (Synchronous Message Passing Constraint). *Let  $x^{1:n} \in \mathbb{R}^{n \times tchw}$  denote the concatenated  
 279 intermediate factors in a chain-structured factor graph with transition boundaries  $\psi_{i-1,i}$ . Given the start boundary  $\psi_{s,1}$  and the goal boundary  $\psi_{n,g}$ , the joint constraints distribution over all  
 280 intermediate factors is Gaussian:*  
 281

282 
$$p_{\text{sync}}(x^{1:n} | s, g) \propto \exp(-\frac{1}{2}(x^{1:n})^T \Sigma^{-1} x^{1:n} + \eta^T x^{1:n}), \quad (10)$$
  
 283  
 284 where  $\Sigma^{-1} = \begin{bmatrix} \frac{A_1^T A_1}{c_0} + \frac{B_1^T B_1}{c_1} & -\frac{B_1^T A_2}{c_1} & & & & \\ -\frac{A_2^T B_1}{c_1} & \frac{A_2^T A_2}{c_1} + \frac{B_2^T B_2}{c_2} & -\frac{B_2^T A_3}{c_2} & & & \\ & -\frac{A_3^T B_2}{c_2} & \frac{A_3^T A_3}{c_2} + \frac{B_3^T B_3}{c_3} & & & \\ & & & \ddots & & \\ & & & & \frac{A_n^T s}{c_n} & \\ & & & & 0 & \\ & & & & 0 & \\ & & & & \vdots & \\ & & & & \frac{B_n^T g}{c_n} & \end{bmatrix}, \quad \eta = \begin{bmatrix} \frac{A_1^T s}{c_0} \\ 0 \\ 0 \\ \vdots \\ \frac{B_n^T g}{c_n} \end{bmatrix}.$   
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 286  
 287  
 288  
 289

290 The detailed proof is in appendix B. We perform synchronous message passing on the estimated  
 291 Tweedie factors  $x_{0|t}^{1:n}$  by penalizing the deviation from the consistent linear system  $\Sigma^{-1}x_{0|t}^{1:n} = \eta$ . In  
 292 practice, we set  $c_i = 1$  for all  $i = 0, \dots, n$ .  
 293

294 
$$L_{\text{sync}} = \|\Sigma^{-1}x_{0|t}^{1:n} - \eta\| \quad (11)$$
  
 295

296 Here, synchronous refers to a lockstep update schedule, in which all updates are computed from  
 297 the same current iterate and applied simultaneously. This scheme preserves parallelism, eliminates  
 298 order-dependent effects, and guides the Tweedie estimates toward satisfying the chain constraints.  
 299

300 **4.3.2 ASYNCHRONOUS MESSAGE PASSING**  
 301

302 While the synchronous objective is conceptually clean, the resulting hard consistency constraint is  
 303 difficult to optimize and often exhibits slow or unstable convergence (Ortiz et al., 2021). To improve  
 304 stability and speed, we adopt an asynchronous scheme with bootstrapped targets and stop-gradient,  
 305 akin to temporal-difference updates Hansen et al. (2024); Li et al. (2025). Concretely, we optimize:  
 306

307 
$$L_{\text{async}} = \underbrace{\|s - A_1 x_{0|t}^1\| + \sum_{i=1}^{n-1} \underbrace{\gamma^i \|sg(B_i \hat{x}_{0|t}^i) - A_{i+1} x_{0|t}^{i+1}\|}_{\text{forward passing}}}_{\text{forward passing}} \quad (12)$$
  
 308  
 309  
 310 
$$+ \underbrace{\sum_{i=1}^{n-1} \underbrace{\gamma^{n-i} \|B_i x_{0|t}^i - sg(A_{i+1} \hat{x}_{0|t}^{i+1})\| + \|B_n x_{0|t}^n - g\|}_{\text{backward passing}}}_{\text{backward passing}}$$
  
 311  
 312  
 313

314 where  $sg(\cdot)$  is the stop-gradient operator,  $\hat{x}_{0|t}^{i+1}$  is a target produced by diffusion model with latest  
 315 parameters, and  $x_{0|t}^i$  is produced by an EMA of the model parameters. The discount  $\gamma$  down-weights  
 316 messages as they move away from the start or the goal.  
 317

318 The boundary terms  $\|s - A_1 x_{0|t}^1\|$  and  $\|B_n x_{0|t}^n - g\|$  anchor the chain to the start and goal. The  
 319 forward message loss penalizes mismatch between  $B_i \hat{x}_{0|t}^i$  (outgoing) and  $A_{i+1} x_{0|t}^{i+1}$  (incoming), with  
 320  $sg(\cdot)$  enforcing one-way forward passing. Similarly, the backward message loss mirrors the same  
 321 constraint in the reverse direction.  
 322

323

324 **Algorithm 1** Compositional Generation

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325 1: Require: Model EMA  $x_\theta$ , Latest Model  $\hat{x}_\theta$ 
326 2: Hyperparameters: Diffusion Time Step  $T$ , number of factors chained  $n$ , guidance weight  $g$ 
327 3: Sample  $z_T \sim \mathcal{N}(0, I)$ 
328 4: Split  $z_T$  to  $n$  overlapping chunks  $x_T^{1:n}$ 
329 5: for  $t = T$  to 1 do
330 6:  $x_{0|t}^{1:n} = x_\theta(x_t^{1:n})$  ▷ forward model passing
331 7:  $\hat{x}_{0|t}^{1:n} = \hat{x}_\theta(x_t^{1:n})$ 
332 8:  $\mu_{t-1}^{1:n} = \sqrt{\bar{\alpha}_t} x_{0|t}^{1:n} + \sqrt{1 - \bar{\alpha}_t - \sigma^2} \frac{x_t^{1:n} - \sqrt{\bar{\alpha}_t} x_{0|t}^{1:n}}{\sqrt{1 - \bar{\alpha}_t}}$  ▷ DDIM Step
333 9:  $L = L_{sync} + L_{async}$  ▷ Jointly sync and async message passing
334 10:  $d^* = -\sqrt{s} \sigma_t \cdot \frac{\nabla_{x_t^{1:n}} L}{\|L\|}$  ▷ Diffusion Sphere Guidance
335 11:  $d^{sample} = \sigma_t \epsilon_t$ 
336 12:  $d_m = d^{sample} + g(d^* - d^{sample})$ 
337 13:  $x_{t-1}^{1:n} = \mu_{t-1}^{1:n} + r \frac{d_m}{\|d_m\|}$ 
338 14: end for
339 15: merge chunks  $x_0^{1:n}$  to get final plan  $z_0$ 
340 16: return  $z_0$ 
341

```

---

## 343 4.3.3 DIFFUSION-SPHERE GUIDED MESSAGE PASSING

344 Having derived differentiable losses for synchronous and asynchronous message passing, we adopt  
 345 the training-free guidance of DSG (Yang et al., 2024). As noted by DSG (Eq. 6), stronger guidance  
 346 improves alignment but can reduce sample diversity. To balance alignment and exploration, we  
 347 interpolate between the unconditional sampling direction and the normalized descent direction  
 348 induced by our loss:

$$349 \quad 350 \quad d_m = d^{sample} + g_r(d^* - d^{sample}), \quad x_{t-1}^{1:n} = \mu_{t-1}^{1:n} + r \frac{d_m}{\|d_m\|}. \quad (13)$$

351 Here  $d^{sample} = \sigma_t \epsilon_t$  is the unconditional annealing step,  $d^* = -\sqrt{s} \sigma_t \cdot \frac{\nabla_{x_t^{1:n}} L}{\|L\|}$  is the steepest  
 352 descent direction for our sync/async objective, and  $d_m$  is subsequently normalized to satisfy the  
 353 spherical-Gaussian constraint.

## 356 4.4 COMPOSITIONAL VIDEO PLANNING VIA INFERENCE-TIME DIFFUSION SCALING

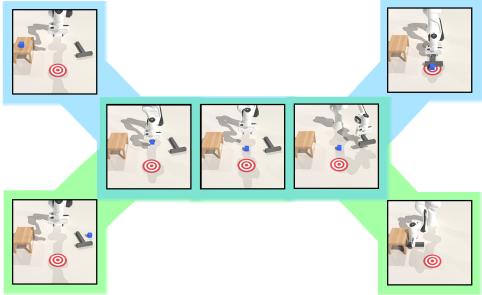
357 The procedure for compositional generation is summarized in Algorithm 1. At a high level, the  
 358 goal is to generate long-horizon trajectories by decomposing them into overlapping local factors  
 359 and enforcing boundary agreement across those factors during the diffusion sampling process. At  
 360 each timestep, DDIM provides the base update, while a joint synchronous–asynchronous message  
 361 passing loss defines a residual that Diffusion Sphere Guidance interpolates against, steering updates  
 362 toward agreement without collapsing diversity. The resulting updates are local and parallel across  
 363 overlapping factors yet collectively converge to a feasible, consistent plan. After all steps, merging  
 364 the denoised chunks yields a smooth, temporally aligned trajectory  $z_0$ .

365 For robot manipulation planning, we train a video diffusion model on randomly sampled short  
 366 chunks from long-horizon demonstrations and an inverse dynamics model that predicts actions from  
 367 consecutive frames. At test time, we condition the diffusion model on start and goal images and  
 368 apply the compositional generation procedure to produce complete video plans. The resulting visual  
 369 trajectory is converted into executable robot actions via the inverse dynamics model. The procedure  
 370 is training-free, plug-and-play, and compatible with unconditional short-horizon diffusion backbones,  
 371 enabling generalization to unseen start-goal combinations without task-specific retraining.

## 373 5 EXPERIMENTS

374 We present experiment results on multiple robotic manipulation scenes spanning 100 tasks (18 in  
 375 distribution, 82 out of distribution) of varying difficulties. Our objective is to investigate (1) the  
 376 visual fidelity of the generated video plans (Section 5.1) (2) how the proposed compositional visual

378 planning can generalize to long-horizon unseen tasks (Section 5.2) (3) how each proposed component  
 379 affect the generation performance of our method (Section 5.3) (4) the effectiveness of our method in  
 380 real robot manipulation tasks (Section 5.4).



393 **Figure 3: Tool-Use setup.** This task involves 2 start and 2 goal  
 394 configurations. We also evaluate on more challenging tasks.  
 395 A complete list of task settings is provided in Appendix D.

396 Tool-Use setting (Figure 6), demonstrations cover only the blue or green regions; the planner must  
 397 generalize across them to form cross-region plans unseen in the dataset but composable from its  
 398 fragments. We address this type of generation by learning from short demonstration chunks randomly  
 399 taken from long-horizon tasks and compositionally generates multiple chunks at inference time to  
 400 construct the final plan.

401 **Evaluation Setup.** Given specified start and goal image as task context, our method first synthesize  
 402 a sequence of frames as subgoals. We then use an MLP-based inverse dynamics model to predict  
 403 the pose of end-effector for the robot to execute, conditioned on adjacent images. The inverse  
 404 dynamics models are trained using the same demonstrations as the planner conditioned on adjacent  
 405 images. We report both the video quality metrics and the success rates of the planners over the robotic  
 406 manipulation tasks. An episode is counted as success if the target objects ends up the specified  
 407 state within a small tolerance. For each environment, we report the success rate over all evaluation  
 408 episodes. We evaluate all methods with 5 random seeds for each experiment and report the mean and  
 409 standard deviation.

410 **Baselines.** For both policy-based and composition-based baseline, details are in Appendix C. For  
 411 compositional baseline **DiffCollage/GSC** (where GSC is DiffCollage adapted to robotic planning),  
 412 we condition on the start and goal images to generate an entire plan by noisy factorization Eq. 2, and  
 413 then use an inverse dynamics model to execute that plan.

## 415 5.1 VIDEO GENERATION QUALITY STUDY

416 Beyond reporting task success rate, we also evaluate the visual fidelity of the synthesized video plans.  
 417 Even when rollouts verify feasibility, perceptual quality still warrants careful analysis. Accordingly,  
 418 we score generated videos with VBBench++ (Huang et al., 2024b), focusing on robotics-centric  
 419 metrics that matter for control: *Dynamic Quality* (inter-frame), which includes (i) motion smooth-  
 420 ness—capturing temporal stability of robot/object motion—and (ii) background consistency—testing  
 421 whether the scene remains coherent over time; and *Static Quality* (frame-wise), which includes  
 422 Aesthetic and Imaging Quality to ensure frames are clear and largely artifact-free. Our generation  
 423 strategy substantially improves the time-dependent properties that matter for control: across all scenes  
 424 and distributions, Motion Smoothness and Background Consistency far exceed DiffCollage (Zhang  
 425 et al., 2023). This translates into dynamically executable trajectories and coherent spatiotemporal  
 426 scenes. In terms of static quality, Aesthetic remains comparable, while Imaging shows a consistent  
 427 and large advantage, directly reflecting fewer blurry frames and cleaner visuals.

## 428 5.2 COMPOSITIONAL PLANNING BENCHMARK

429 We present the robot manipulation success rates of 4 different scenes in Table 2. We separately  
 430 report the success rates of IND and OOD tasks, where IND represents the  $N$  tasks seen in the training

Scene	Type	Dynamic Quality $\uparrow$				Static Quality $\uparrow$			
		Motion Smoothness		Background Consistency		Aesthetic		Imaging	
		DiffCollage	Ours	DiffCollage	Ours	DiffCollage	Ours	DiffCollage	Ours
Tool-Use	IND	0.45 $\pm$ 0.06	<b>0.98</b> $\pm$ 0.03	0.48 $\pm$ 0.05	<b>0.94</b> $\pm$ 0.03	0.47 $\pm$ 0.02	<b>0.46</b> $\pm$ 0.02	0.42 $\pm$ 0.03	<b>0.70</b> $\pm$ 0.02
	OOD	0.41 $\pm$ 0.06	<b>0.96</b> $\pm$ 0.05	0.38 $\pm$ 0.07	<b>0.89</b> $\pm$ 0.06	0.42 $\pm$ 0.03	<b>0.48</b> $\pm$ 0.03	0.40 $\pm$ 0.05	<b>0.70</b> $\pm$ 0.04
Drawer	IND	0.43 $\pm$ 0.03	<b>0.85</b> $\pm$ 0.03	0.50 $\pm$ 0.04	<b>0.86</b> $\pm$ 0.03	0.50 $\pm$ 0.02	<b>0.54</b> $\pm$ 0.02	0.39 $\pm$ 0.02	<b>0.73</b> $\pm$ 0.02
	OOD	0.44 $\pm$ 0.05	<b>0.96</b> $\pm$ 0.05	0.41 $\pm$ 0.06	<b>0.89</b> $\pm$ 0.05	0.45 $\pm$ 0.03	<b>0.48</b> $\pm$ 0.03	0.42 $\pm$ 0.04	<b>0.70</b> $\pm$ 0.04
Cube	IND	0.40 $\pm$ 0.04	<b>0.96</b> $\pm$ 0.03	0.52 $\pm$ 0.04	<b>0.90</b> $\pm$ 0.03	0.49 $\pm$ 0.02	<b>0.51</b> $\pm$ 0.02	0.33 $\pm$ 0.03	<b>0.65</b> $\pm$ 0.02
	OOD	0.42 $\pm$ 0.06	<b>0.97</b> $\pm$ 0.05	0.39 $\pm$ 0.06	<b>0.91</b> $\pm$ 0.05	0.51 $\pm$ 0.03	<b>0.52</b> $\pm$ 0.03	0.41 $\pm$ 0.05	<b>0.63</b> $\pm$ 0.04
Puzzle	IND	0.40 $\pm$ 0.03	<b>0.96</b> $\pm$ 0.03	0.47 $\pm$ 0.04	<b>0.90</b> $\pm$ 0.03	0.49 $\pm$ 0.04	<b>0.48</b> $\pm$ 0.02	0.44 $\pm$ 0.03	<b>0.70</b> $\pm$ 0.02
	OOD	0.39 $\pm$ 0.06	<b>0.97</b> $\pm$ 0.05	0.48 $\pm$ 0.06	<b>0.90</b> $\pm$ 0.05	0.46 $\pm$ 0.03	<b>0.47</b> $\pm$ 0.03	0.38 $\pm$ 0.05	<b>0.70</b> $\pm$ 0.04
Overall	IND	0.41 $\pm$ 0.04	<b>0.94</b> $\pm$ 0.06	0.49 $\pm$ 0.04	<b>0.90</b> $\pm$ 0.04	0.49 $\pm$ 0.03	<b>0.50</b> $\pm$ 0.03	0.40 $\pm$ 0.05	<b>0.70</b> $\pm$ 0.03
	OOD	0.40 $\pm$ 0.06	<b>0.97</b> $\pm$ 0.05	0.45 $\pm$ 0.07	<b>0.90</b> $\pm$ 0.05	0.46 $\pm$ 0.04	<b>0.48</b> $\pm$ 0.03	0.39 $\pm$ 0.05	<b>0.69</b> $\pm$ 0.05

Table 1: **Comparison across four scenes on Dynamic/Static Quality.** Our results are averaged over 5 seeds and standard deviations are shown after the  $\pm$  sign.

data while OOD represents the  $N \cdot N - N$  unseen tasks. We observe that DiffCollage fails at almost all tasks. Qualitatively, we find that the synthesized images of DiffCollage tend to be blurry or even unrealisitc, perhaps due to its score averaging sampling scheme. Such suboptimal images will further confuses the inverse dynamic models, cause unstable behaviors and failures. In contrast, our method achieves significantly higher success rates, indicating that the generated visual plans are realistic and accurate for the inverse dynamic model to follow. We also include several multiple representative policy learning baselines, such as Goal-Conditioned Diffusion Policy (GCDP). We notice that though strong policy learning baseline is able to perform well on IND tasks, their performance suffers from a significant degradation on OOD tasks. In contrast, our method—enabled by the graphical chain formulation and message passing—maintains stable performance regardless of the task distribution.

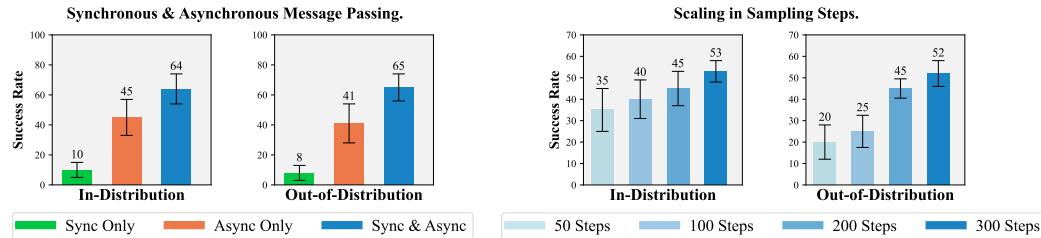
Scene	Type	LCBC	LCDP	GCBC	GCDP	DiffCollage	CompDiffuser	Ours
Tool-Use	IND	80 $\pm$ 7	95 $\pm$ 2	85 $\pm$ 5	96 $\pm$ 3	1 $\pm$ 2	<b>60<math>\pm</math>2</b>	<b>97<math>\pm</math>3</b>
	OOD	15 $\pm$ 3	37 $\pm$ 8	13 $\pm$ 6	42 $\pm$ 13	0 $\pm$ 0	<b>51<math>\pm</math>3</b>	<b>96<math>\pm</math>2</b>
Drawer	IND	35 $\pm$ 6	54 $\pm$ 6	30 $\pm$ 5	50 $\pm$ 6	0 $\pm$ 0	<b>20<math>\pm</math>5</b>	<b>53<math>\pm</math>5</b>
	OOD	6 $\pm$ 5	26 $\pm$ 14	7 $\pm$ 5	18 $\pm$ 16	0 $\pm$ 0	<b>18<math>\pm</math>3</b>	<b>52<math>\pm</math>6</b>
Cube	IND	28 $\pm$ 3	58 $\pm$ 5	26 $\pm$ 4	60 $\pm$ 3	0 $\pm$ 0	<b>32<math>\pm</math>8</b>	<b>64<math>\pm</math>10</b>
	OOD	8 $\pm$ 3	22 $\pm$ 12	5 $\pm$ 5	24 $\pm$ 13	0 $\pm$ 0	<b>34<math>\pm</math>6</b>	<b>65<math>\pm</math>9</b>
Puzzle	IND	23 $\pm$ 5	48 $\pm$ 5	19 $\pm$ 6	47 $\pm$ 3	0 $\pm$ 0	<b>10<math>\pm</math>3</b>	<b>50<math>\pm</math>11</b>
	OOD	0 $\pm$ 0	11 $\pm$ 9	0 $\pm$ 0	12 $\pm$ 11	0 $\pm$ 0	<b>9<math>\pm</math>3</b>	<b>50<math>\pm</math>13</b>
Overall	IND	33 $\pm$ 18	57 $\pm$ 15	30 $\pm$ 21	56 $\pm$ 16	0 $\pm$ 1	<b>17<math>\pm</math>2</b>	<b>59<math>\pm</math>17</b>
	OOD	2 $\pm$ 4	15 $\pm$ 12	15 $\pm$ 12	15 $\pm$ 13	0 $\pm$ 0	<b>16<math>\pm</math>2</b>	<b>54<math>\pm</math>14</b>

Table 2: **Quantitative Results on Compositional Planning Bench.** We benchmark our method on the 100 test-time tasks across 4 scenes with 30 episodes per task. Our results are averaged over 5 seeds and standard deviations are shown after the  $\pm$  sign.

### 5.3 ABLATION STUDIES

**Jointly Synchronous & Asynchronous Message Passing.** We compare the success rates of three variants of our test-time compositional sampling scheme in Cube Scene: Only Synchronous Loss (*Sync Only*), Only Asynchronous Loss (*Async Only*), and Joint Synchronous and Asynchronous Loss (*Sync & Async*), as shown in Figure 4. *Sync only* suffers from overly tight constraints that are difficult to optimize, leading to lower success rates. In contrast, the asynchronous variant performs better. Combining the two—*Sync & Async*—outperforms either alone, likely due to its more effective balance of constraint enforcement and flexibility.

486  
 487 **Scaling in Sampling Steps.** We study how the number of diffusion sampling steps affects the  
 488 planning performance on Drawer Scene (Figure 5). Success rates improve as the number of steps  
 489 increases, demonstrating that our method scales effectively with additional test-time compute. We  
 490 hypothesize that taking more steps enables deeper cross-factor message passing through repeated  
 491 denoising and guidance updates, which in turn reduces boundary inconsistencies and yields more  
 492 accurate, temporally coherent plans.



501 **Figure 4: Effect of synchronous and asyn-**  
 502 **chronous message passing.** Results are averaged  
 503 over 5 seeds on the Cube scene. Combining(Sync  
 504 & Async) achieves the best results for both in-  
 505 distribution and out-of-distribution tasks.

501 **Figure 5: Effect of sampling steps on planning perfor-**  
 502 **mance.** Results are averaged over 5 seeds on Drawer Scene.  
 503 Increasing the number of diffusion sampling steps improves  
 504 success rates for both in-distribution and out-of-distribution  
 505 tasks.

#### 506 5.4 REAL ROBOT EXPERIMENT

507 For the real-world experiments, we deploy our method on a Franka Emika Panda robotic manipulation,  
 508 controlled at 20Hz using joint impedance control. Visual image observations are captured using  
 509 an Intel RealSense D435 depth camera. For data collection, the robot is teleoperated using a  
 510 Meta Quest 3 headset, with tracked Cartesian poses converted to joint configurations through inverse  
 511 kinematics. Quantitative and qualitative results shows our compositional planner continues to produce  
 512 coherent long-horizon behaviors and maintains strong performance in the real world. We present the  
 513 corresponding results in table 3. More details are covered in appendix H.

Real Scene	IND:Task1	IND:Task2	OOD:Task3	OOD:Task4
DiffCollage	1/10	1/10	0/10	0/10
Ours	9/10	7/10	10/10	8/10

514 **Table 3: Real-robot success rates.** Our method substantially outperforms DiffCollage across both  
 515 in-distribution (IND) and out-of-distribution (OOD) tasks on real hardware.

## 522 6 DISCUSSION AND CONCLUSION

523 **Limitations.** Our method has several limitations. First, it relies on the accuracy of the estimated clean  
 524 data (Tweedie estimates) during denoising, since guidance losses are computed on these estimates.  
 525 This sensitivity could be mitigated by performing multi-step Tweedie estimation or by scaling up  
 526 training data and model capacity. Second, as in prior work, the number of test-time composed  
 527 segments,  $n$ , must be specified manually. Developing procedures that automatically infer  $n$  from  
 528 task structure and uncertainty would be an interesting future research direction. Lastly, our approach  
 529 can be more computationally demanding than direct averaging-based sampling, because test-time  
 530 guidance is implemented via gradient-based optimization(Table 6). Exploring lighter optimization  
 531 schedules could potentially reduce this overhead.

532 **Conclusion** We introduced Compositional Visual Planning, an inference-time method that composes  
 533 long-horizon plans by stitching overlapping video factors with message passing on Tweedie estimates.  
 534 A chain-structured factor graph imposes global consistency, enforced via joint synchronous and  
 535 asynchronous updates, while diffusion-sphere guidance balances alignment and diversity without  
 536 retraining. Compositional Visual Planning is plug-and-play with short-horizon diffusion video  
 537 prediction model, scales with test-time compute, and generalizes to unseen start–goal combinations.  
 538 Beyond robotics, the framework is applicable to broader domains, such as panorama image generation  
 539 and long-form text-to-video synthesis, which we leave for future exploration.

540 REPRODUCIBILITY STATEMENT  
541542 We promise to provide all the source code to reproduce the results in this paper, including the proposed  
543 algorithm and the evaluation benchmark.  
544545 LLM USAGE DISCLOSURE  
546547 We confirm that no large language models (LLMs) were used to generate, edit, or refine the scientific  
548 content, experimental design, results, or conclusions of this paper. All text, figures, algorithms, and  
549 analyses were produced entirely by the authors.  
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## APPENDIX

## A NOISY-BETHE GAP THEOREM

**Theorem 1** (Noisy-Bethe Gap Theorem). *Consider a linear chain  $z = [u^1, u^2, u^3]$  with pairwise factors  $[u^1, u^2]$  and  $[u^2, u^3]$ , where  $u^2$  is the transition boundary variable. Assume the forward noising processes are  $p(u_t^1, u_t^2 | u^1, u^2)$ ,  $p(u_t^2, u_t^3 | u^2, u^3)$ , and  $p(u_t^2 | u^2)$ . Let  $a(u^2) = \int p(u^1, u^2) p(u_t^1, u_t^2 | u^1, u^2) du^1$ ,  $b(u^2) = \int p(u^2, u^3) p(u_t^2, u_t^3 | u^2, u^3) du^3$ ,  $c(u^2) = p(u^2) p(u_t^2 | u^2)$ ,  $Z = \int c(u^2) du^2$ , and  $q(u^2) = c(u^2)/Z$ . Denote by  $p(u_t^1, u_t^2, u_t^3)$  the true noisy distribution and by  $\hat{p}(u_t^1, u_t^2, u_t^3)$  the estimator from Eq. 1. Then the gap between true distribution and estimated distribution is:*

$$\Delta = p(u_t^1, u_t^2, u_t^3) - \hat{p}(u_t^1, u_t^2, u_t^3) = Z \text{Cov}_{u^2 \sim q} \left[ \frac{a}{c}, \frac{b}{c} \right]. \quad (8)$$

*Proof.* The true noisy distribution is:

$$\begin{aligned} p(u_t^1, u_t^2, u_t^3) &= \int p(u_t^1, u_t^2, u_t^3, u^1, u^2, u^3) du^1 du^2 du^3 \\ &= \int p(u^1, u^2, u^3) p(u_t^1, u_t^2, u_t^3 | u^1, u^2, u^3) du^1 du^2 du^3 \\ &= \int \frac{p(u^1, u^2) p(u_t^1, u_t^2 | u^1, u^2) p(u^2, u^3) p(u_t^2, u_t^3 | u^2, u^3)}{p(u^2) p(u_t^2 | u^2)} du^1 du^2 du^3 \\ &= \int \frac{\int p(u^1, u^2) p(u_t^1, u_t^2 | u^1, u^2) du^1 \int p(u^2, u^3) p(u_t^2, u_t^3 | u^2, u^3) du^3}{p(u^2) p(u_t^2 | u^2)} du^2 \end{aligned} \quad (14)$$

The estimator used in (Zhang et al., 2023; Mishra et al., 2023; 2024) is:

$$\hat{p}(u_t^1, u_t^2, u_t^3) = \frac{\int p(u^1, u^2) p(u_t^1, u_t^2 | u^1, u^2) du^1 du^2 \int p(u^2, u^3) p(u_t^2, u_t^3 | u^2, u^3) du^2 du^3}{\int p(u^2) p(u_t^2 | u^2) du^2} \quad (15)$$

Define the left-factor message into  $u^2$  as  $a(u^2) = \int p(u^1, u^2) p(u_t^1, u_t^2 | u^1, u^2) du^1$ , the right-factor message into  $u^2$  as  $b(u^2) = \int p(u^2, u^3) p(u_t^2, u_t^3 | u^2, u^3) du^3$ , and the local boundary evidence  $c(u^2) = p(u^2) p(u_t^2 | u^2)$ . Then  $p(u_t^1, u_t^2, u_t^3) = \int \frac{ab}{c} du^2$  and  $\hat{p}(u_t^1, u_t^2, u_t^3) = \frac{\int a du^2 \int b du^2}{\int c du^2}$ .

Introduce a change of measure by setting  $q(u^2) = \frac{c}{Z}$  with  $Z = \int c du^2$ . For the true distribution, we have:

$$\begin{aligned} p(u_t^1, u_t^2, u_t^3) &= \int \frac{\int p(u^1, u^2) p(u_t^1, u_t^2 | u^1, u^2) du^1 \int p(u^2, u^3) p(u_t^2, u_t^3 | u^2, u^3) du^3}{p(u^2) p(u_t^2 | u^2)} \frac{p(u^2) p(u_t^2 | u^2)}{p(u^2) p(u_t^2 | u^2)} \frac{Z}{Z} du^2 \\ &= Z \int \frac{\int p(u^1, u^2) p(u_t^1, u_t^2 | u^1, u^2) du^1 \int p(u^2, u^3) p(u_t^2, u_t^3 | u^2, u^3) du^3}{p(u^2) p(u_t^2 | u^2)} \frac{p(u^2) p(u_t^2 | u^2)}{p(u^2) p(u_t^2 | u^2)} \frac{Z}{Z} du^2 \\ &= Z \mathbb{E}_{u \sim q(u^2)} \left[ \frac{a}{c} \frac{b}{c} \right] \end{aligned} \quad (16)$$

For the estimator, observe that:

$$\begin{aligned} \int a(u^2) du^2 &= \int \left( \int p(u^1, u^2) p(u_t^1, u_t^2 | u^1, u^2) du^1 \right) du^2 \\ &= \int \left( \int p(u^1, u^2) p(u_t^1, u_t^2 | u^1, u^2) du^1 \right) \frac{p(u_2) p(u_t | u_2)}{p(u_2) p(u_t | u_2)} \frac{Z}{Z} du^2 \\ &= Z \int \frac{\left( \int p(u^1, u^2) p(u_t^1, u_t^2 | u^1, u^2) du^1 \right) p(u_2) p(u_t | u_2)}{p(u_2) p(u_t | u_2)} \frac{Z}{Z} du^2 \\ &= Z \mathbb{E}_{u^2 \sim q(u^2)} \left[ \frac{a}{c} \right] \end{aligned} \quad (17)$$

864 By the same argument,  $\int b(u^2)du^2 = Z \mathbb{E}_{u^2 \sim q(u^2)} \left[ \frac{b}{c} \right]$ , therefore the estimated distribution:  
 865

$$866 \hat{p}(u_t^1, u_t^2, u_t^3) = Z \mathbb{E}_{u^2 \sim q(u^2)} \left[ \frac{a}{c} \right] \mathbb{E}_{u^2 \sim q(u^2)} \left[ \frac{b}{c} \right]. \quad (18)$$

867 Finally, the difference between the true and estimated distributions is:  
 868

$$\begin{aligned} 869 \Delta &= p(u_t^1, u_t^2, u_t^3) - \hat{p}(u_t^1, u_t^2, u_t^3) \\ 870 &= Z \mathbb{E}_{u \sim q(u^2)} \left[ \frac{a}{c} \frac{b}{c} \right] - Z \mathbb{E}_{u^2 \sim q(u^2)} \left[ \frac{a}{c} \right] \mathbb{E}_{u^2 \sim q(u^2)} \left[ \frac{b}{c} \right] \\ 871 &= Z \text{Cov} \left[ \frac{a}{c}, \frac{b}{c} \right]. \end{aligned} \quad (19)$$

873 This shows that the estimator departs from the true distribution by a covariance term under the  
 874 reweighted boundary measure  $q(u^2)$ , scaled by  $Z$ .  $\square$   
 875

## 876 B SYNCHRONOUS MESSAGE PASSING

879 **Theorem 2** (Synchronous Message Passing Constraint). *Let  $x^{1:n} \in \mathbb{R}^{n \times tchw}$  denote the concatenated  
 880 intermediate factors in a chain-structured factor graph with transition boundaries  $\psi_{i-1,i}$ . Given the start boundary  $\psi_{s,1}$  and the goal boundary  $\psi_{n,g}$ , the joint constraints distribution over all  
 881 intermediate factors is Gaussian:*

$$883 p_{\text{sync}}(x^{1:n} | s, g) \propto \exp \left( -\frac{1}{2} (x^{1:n})^T \Sigma^{-1} x^{1:n} + \eta^T x^{1:n} \right), \quad (10)$$

$$884 \text{where } \Sigma^{-1} = \begin{bmatrix} \frac{A_1^T A_1}{c_0} + \frac{B_1^T B_1}{c_1} & -\frac{B_1^T A_2}{c_1} & & \\ -\frac{A_2^T B_1}{c_1} & \frac{A_2^T A_2}{c_1} + \frac{B_2^T B_2}{c_2} & -\frac{B_2^T A_3}{c_2} & \\ & -\frac{A_3^T B_2}{c_2} & \frac{A_3^T A_3}{c_2} + \frac{B_3^T B_3}{c_3} & \\ & & \ddots & \end{bmatrix}, \quad \eta = \begin{bmatrix} \frac{A_1^T s}{c_0} \\ 0 \\ 0 \\ \vdots \\ \frac{B_n^T g}{c_n} \end{bmatrix}.$$

891 *Proof.*  $p_{\text{sync}}(x^{1:n} | s, g)$  can be represented as a product of dependencies over all boundary variables:  
 892

$$893 p(x^{1:n} | s, g)_{\text{sync}} = \psi_{s,1}(s, x_1) \psi_{1,2}(x_1, x_2) \cdots \psi_{n-1}(x_{n-1}, x_n) \psi_{n,g}(x_n, g). \quad (20)$$

894 The aim of this equation is to express the joint distribution over all intermediate states, given initial  
 895 and final state:  
 896

$$\begin{aligned} 897 p(x^{1:n} | s, g) &= \psi_0(s, x_1) \psi_1(x_1, x_2) \cdots \psi_{n-1}(x_{n-1}, x_n) \psi_n(x_n, g) \\ 898 &\propto \exp \left( -\frac{1}{c_0} \|s - A_1 x_1\|^2 - \frac{1}{c_1} \|B_1 x_1 - A_2 x_2\|^2 - \frac{1}{c_2} \|B_2 x_2 - A_3 x_3\|^2 \right. \\ 899 &\quad \left. - \cdots - \frac{1}{c_{n-1}} \|B_{n-1} x_{n-1} - A_n x_n\|^2 - \frac{1}{c_n} \|B_n x_n - g\|^2 \right) \\ 900 &= \exp \left( -\frac{1}{2} s^T \frac{I}{c_0} s + \frac{1}{2} s^T \frac{A_1}{c_0} x_1 + \frac{1}{2} x_1^T \frac{A_1}{c_0} s - \frac{1}{2} x_1^T \frac{A_1^T A_1}{c_0} x_1 \right. \\ 901 &\quad - \frac{1}{2} x_1^T \frac{B_1^T B_1}{c_1} x_1 + \frac{1}{2} x_1^T \frac{B_1^T A_2}{c_1} x_2 + \frac{1}{2} x_2^T \frac{A_2^T B_1}{c_1} x_1 - \frac{1}{2} x_2^T \frac{A_2^T A_2}{c_1} x_2 \\ 902 &\quad - \frac{1}{2} x_2^T \frac{B_2^T B_2}{c_2} x_2 + \frac{1}{2} x_2^T \frac{B_2^T A_3}{c_2} x_3 + \frac{1}{2} x_3^T \frac{A_3^T B_2}{c_2} x_2 - \frac{1}{2} x_3^T \frac{A_3^T A_3}{c_2} x_3 \\ 903 &\quad - \frac{1}{2} x_3^T \frac{B_3^T B_3}{c_3} x_3 + \frac{1}{2} x_3^T \frac{B_3^T A_4}{c_3} x_4 + \frac{1}{2} x_4^T \frac{A_4^T B_3}{c_3} x_3 - \frac{1}{2} x_4^T \frac{A_4^T A_4}{c_3} x_4 \\ 904 &\quad \vdots \\ 905 &\quad - \frac{1}{2} x_n^T \frac{B_n^T B_n}{c_n} x_n + \frac{1}{2} x_n^T \frac{B_n^T}{c_n} g + \frac{1}{2} g^T \frac{B_n}{c_n} x_n - \frac{1}{2} g^T \frac{I}{c_n} g \left. \right) \\ 906 &= \exp \left( -\frac{1}{2} x_{1:n}^T \Sigma^{-1} x_{1:n} + \eta^T x_{1:n} \right), \end{aligned} \quad (21)$$

$$\begin{aligned}
918 \quad & \Sigma^{-1} = \begin{bmatrix} \frac{A_1^T A_1}{c_0} + \frac{B_1^T B_1}{c_1} & -\frac{B_1^T A_2}{c_1} & -\frac{B_2^T A_3}{c_2} \\ -\frac{A_2^T B_1}{c_1} & \frac{A_2^T A_2}{c_1} + \frac{B_2^T B_2}{c_2} & -\frac{B_2^T A_3}{c_2} \\ -\frac{A_3^T B_2}{c_2} & -\frac{A_3^T A_3}{c_2} + \frac{B_3^T B_3}{c_3} & \ddots \end{bmatrix}, \eta = \begin{bmatrix} \frac{A_1^T s}{c_0} \\ 0 \\ 0 \\ \vdots \\ \frac{B_n^T g}{c_n} \end{bmatrix}.
\end{aligned}$$

## 926 C BASELINE IMPLEMENTATIONS

929 To ensure clarity about how the prior baselines are instantiated in our setting, we briefly summarize  
930 the exact formulations of all baselines below. We'd like to clarify DiffCollage / GSC/ CompDiffuser  
931 are joint denoising over the full long-horizon trajectory, starting from a single long-horizon Gaussian.  
932

933 **Language-Conditioned Behavioral Cloning (LCBC).** The policy uses a T5 text encoder to embed  
934 natural-language instructions. We concatenate the text embedding with image features extracted by a  
935 ResNet backbone, and feed the result into an MLP policy head. The model is trained in a supervised  
936 manner to predict a single action at each timestep, conditioned on both the language input and the  
937 current observation.

938 **Language-Conditioned Diffusion Policy (LCDP).** LCDP follows the same text encoding pipeline  
939 as LCBC with a T5 encoder, but replaces the MLP head with a Transformer-based policy head. The  
940 Transformer generates chunks of actions rather than single-step predictions, allowing multi-step  
941 reasoning conditioned on language and observations.

942 **Goal-Conditioned Behavioral Cloning (GCBC).** GCBC uses a ResNet backbone to encode both  
943 the current observation image and the goal image. The concatenated features are passed through  
944 an MLP policy head, which outputs a single action. This provides a goal-aware baseline without  
945 language conditioning.

946 **Goal-Conditioned Diffusion Policy (GCDP).** GCDP employs a ResNet backbone with a Transformer  
947 policy head, conditioned jointly on the current observation and the goal image. The model outputs  
948 action chunks, enabling multi-step planning toward the goal state.

949 **DiffCollage / GSC.** DiffCollage is a compositional test time generation method, and GSC refers to its  
950 adaptation for robotic planning. Given a start image and a goal image, the model directly generates  
951 an entire visual plan (a sequence of intermediate frames). It composes a long-horizon score following  
952 Eq. 2 and iteratively denoise the entire long trajectory. We then employ a separately trained inverse  
953 dynamics model to convert the visual plan into executable robot actions.

954 **CompDiffuser.** CompDiffuser is a trained joint-denoising baseline: it trains the short-horizon  
955 model to condition on its preceding and following noisy chunks. At test time, it subdivides the long  
956 trajectory into overlapping chunks, and denoises them jointly while conditioning each chunk on its  
957 neighbors. We then employ a separately trained inverse dynamics model to convert the visual plan  
958 into executable robot actions. CompDiffuser is SOTA joint-denoising baseline on OGBench 2D  
959 xy point maze navigation; however, its behavior on high-dimensional visual planning had not been  
960 evaluated prior to our experiments.

### 963 C.1 IMPLEMENTATION DETAILS

964 For DiffCollage/GSC, we use the official codebase with recommended hyperparameters and exactly  
965 the same pretrained checkpoints as our method, since all methods are training-free in the same setup  
966 as ours. For CompDiffuser, we use the official codebase and match the model capacity to ours. And  
967 we select the checkpoint with the best validation loss. We intentionally disable replanning in order to  
968 evaluate the benefit coming from the algorithm itself, rather than from repeated replanning.

### 971 C.2 SANITY TEST ON OGBENCH

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Env	DiffCollage	CompDiffuser	Ours
pointmaze-medium	100 $\pm$ 0	100 $\pm$ 0	100 $\pm$ 0
pointmaze-large	100 $\pm$ 0	100 $\pm$ 0	100 $\pm$ 0
pointmaze-giant	32 $\pm$ 4	67 $\pm$ 3	93 $\pm$ 4

977  
978 Table 4: **Results on OGBench.** Our reproduced results match the reported performance in CompDiffuser  
979 with only minor statistical variation.  
980

981 To faithfully verify that our implementations are correct and reasonably tuned, we first evaluate on  
982 OGBench tasks, where CompDiffuser reports results for both DiffCollage/GSC and CompDiffuser.  
983 Our reproduced results match the reported performance with only minor statistical variation.  
984

## 985 D COMPOSITIONAL PLANNING TASKS

### 986 D.1 DATASET AND SIMULATION SETUP

988 **Assets.** Our asset library combines 3D models and textures from ShapeNet (Chang et al., 2015)  
989 and RoboCasa (Nasiriany et al., 2024). We additionally apply simple high-quality texture(e.g.,  
990 wood, plastic, metal finishes) to increase visual fidelity. All simulations are conducted in the  
991 SAPIEN engine (Xiang et al., 2020), which provides high-fidelity physics and rendering for robotic  
992 manipulation.

993 **State and Action Space.** The observation space consists solely of RGB images with resolution  
994  $256 \times 256 \times 3$ , without access to privileged information such as depth or ground-truth states. The  
995 action space is parameterized by the end-effector position (3D Cartesian coordinates), orientation  
996 represented as a quaternion (4D), and a binary scalar controlling the gripper open/close state. During  
997 initialization, all object poses are randomized within a 0.2 m radius from their nominal positions,  
998 ensuring sufficient variability and out-of-distribution test cases.

999 **Demonstrations.** We provide 300 expert demonstrations for each of the  $N$  start–goal combinations  
1000 across all scenes, resulting in thousands of trajectories spanning tool-use, drawer manipulation,  
1001 cube rearrangement, and puzzle solving. Each demonstration is generated via scripted policies that  
1002 guarantee feasibility and success. For the LCBC and LCDP baselines, we further annotate each  
1003 demonstration with natural-language descriptions of the task. These annotations also help evaluate  
1004 how well language-conditioned models can generalize across start–goal variations.

1005 **Success Condition.** A rollout is considered successful if all objects reach their target configurations  
1006 within a predefined spatial threshold.  
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### 1008 D.2 TOOL

1009 The Tool scene (Figure 6) requires the robot to manipulate a tool in order to push the cube to the  
1010 target location. Success is achieved when the cube reaches the designated target area within a fixed  
1011 distance threshold. Direct manipulation is not possible, so the robot must use the provided tool to  
1012 accomplish the task.  
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### 1014 D.3 DRAWER

1015 The Drawer scene (Figure 7) requires the robot to manipulate drawers into closed states and use  
1016 the brush to draw on the canvas. Depending on the start state, the brush may be located in different  
1017 spaces, and the robot must go to the correct space and retrieve it. After grasping the brush, the drawer  
1018 must often be closed again before drawing, to avoid collision between the arm and the drawer.  
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### 1020 D.4 CUBE

1021 The Cube scene (Figure 8) requires the robot to manipulate multiple colored cubes, first arranging  
1022 them into a prescribed order and then placing each cube into its designated goal region. This task  
1023 evaluates the planner’s ability to identify and distinguish between object colors, maintain the correct  
1024 ordering, and execute precise placement into multiple targets.  
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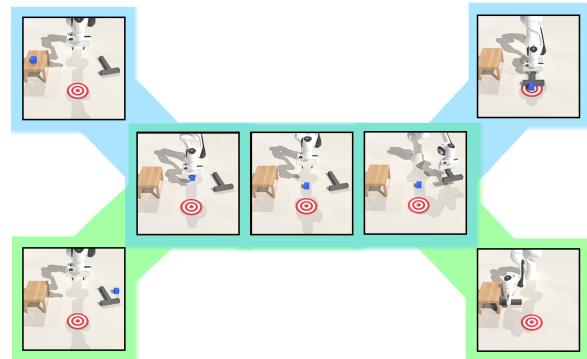
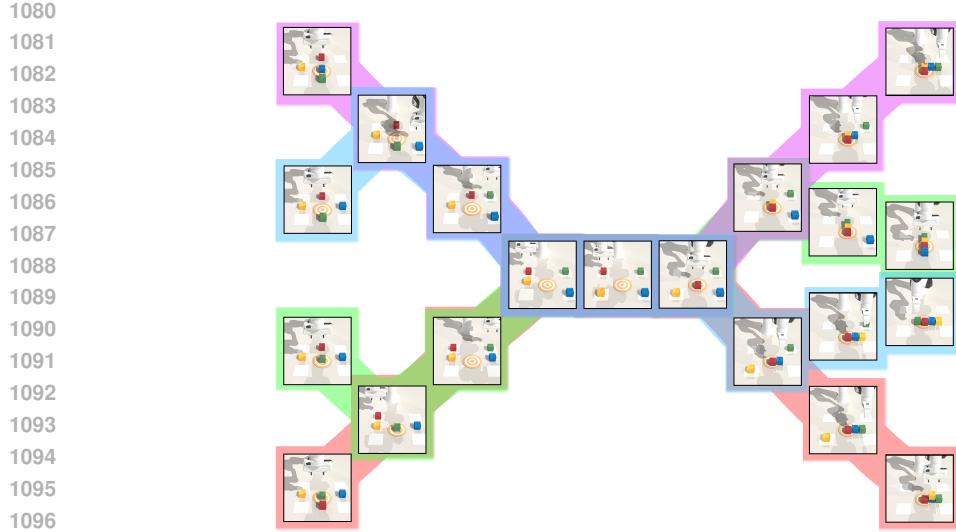


Figure 6: **Visualization of Tool Scene.**

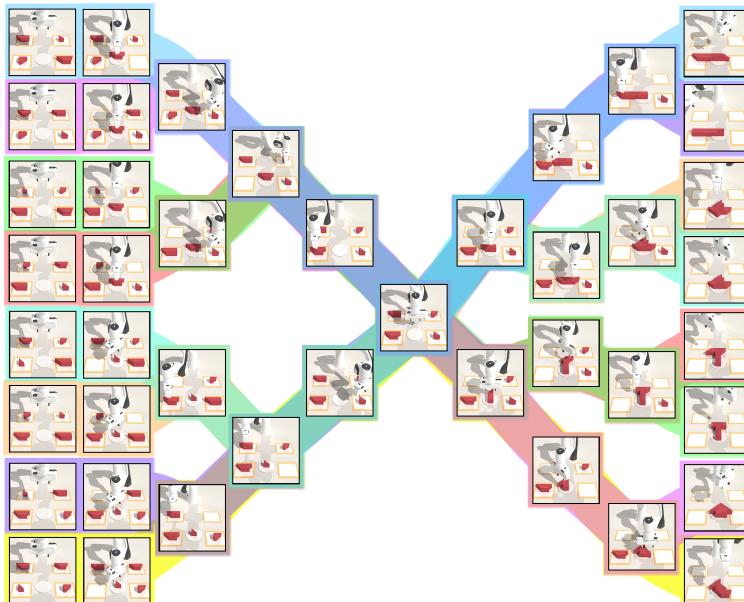


Figure 7: **Visualization of Drawer Scene.**

Figure 8: **Visualization of Cube Scene.**

### D.5 PUZZLE

The Puzzle scene (Figure 9) poses the most challenging test of compositional planning. The robot must first arrange multiple colored blocks into a specific intermediate configuration, and then place them into distinct goal slots. This requires not only accurate object manipulation and ordering, but also the ability to chain together multiple sub-tasks that were only observed in isolation during demonstrations.

Figure 9: **Visualization of Puzzle Scene.**

1134 **E IMPLEMENTATION DETAILS**  
11351136 **Software.** All experiments are conducted on Ubuntu 20.04.6 with Python 3.10 and PyTorch 2.2.1.  
11371138 **Training.** Models are trained on NVIDIA H200 GPUs.  
11391140 **Deployment Hardware.** For deployment, we use a single NVIDIA L40S GPU.  
11411142 **Model Inputs and Outputs.** All observations are first encoded into tokens using the Cosmos  
1143 tokenizer. We adopt DiT backbones for video generation, using DiT-L or DiT-XL (Peebles & Xie,  
1144 2023) depending on the scene. Video generation is performed entirely in the token space. For  
1145 control, we employ a simple MLP-based inverse dynamics model, which predicts low-level actions  
1146 conditioned on consecutive frames.  
11471148 **Hyperparameters.** We report all hyperparameters used during both training and inference for full  
1149 transparency. For fairness and reproducibility, we do not perform any hyperparameter search or use a  
1150 learning rate scheduler; all experiments are conducted with fixed values throughout. This ensures  
1151 that performance gains arise from the method itself rather than extensive hyperparameter tuning. We  
1152 also keep hyperparameters consistent across different tasks and scenes, unless otherwise specified, to  
1153 highlight the robustness of our approach.  
1154

Hyperparameter	Value
Diffusion Time Step	500
Batch Size	512
Optimizer	Adam
Learning Rate	$1 \times 10^{-4}$
Iterations	1M
Discount Factor $\gamma$	0.6
Sampling Time Steps	300
Guidance Weight $g_r$	0.6

1155 Table 5: **Relevant hyperparameters** used in our experiments.  
11561157 **F DEPLOYMENT TIME STUDY**  
11581159 **F.1 TIME STUDY & NFEs**  
11601161 All results in Table 6 are reported under the same setting of 300 DDIM steps. Compared to DiffCollage,  
1162 our method incurs higher wall-clock time because it requires test-time backpropagation through  
1163 the diffusion model in order to enforce consistency via message passing. This extra computation  
1164 accounts for the increase in sampling time, but is necessary to achieve the significant gains in success  
1165 rates reported in the main results. Our implementation is fully batched, so all factors require only  
1166 a single forward pass at each diffusion steps. We follow the standard convention of counting only  
1167 forward passes of the diffusion model as NFEs. At the same DDIM step count, our method requires  
1168 twice the NFEs because it performs two forward passes per step. The backward update does not  
1169 increase NFEs, but it does add a small amount of extra wall-clock time.  
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Scene	# Models Composed	Sampling Time (s) ↓	
		DiffCollage	Ours
Tool-Use	3	7.1	17.2
Drawer	5	18.9	30.4
Cube	5	21.1	31.7
Puzzle	6	30.4	61.8

1171 Table 6: **Sampling time during deployment.** This is measured as the mean wall-clock time across  
1172 all samples within a single scene.  
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1188	Tool-Scene	NFE 300	NFE 400	NFE 500
1189	IND	$1 \pm 2$	$0 \pm 0$	$3 \pm 2$
1190	OOD	$0 \pm 0$	$2 \pm 2$	$1 \pm 2$
(a) DiffCollage/GSC				
1194	Tool-Scene	NFE 3000	NFE 4000	NFE 5000
1195	IND	$1 \pm 0$	$2 \pm 1$	$5 \pm 3$
1196	OOD	$1 \pm 2$	$3 \pm 2$	$2 \pm 1$
(b) DiffCollage+Repainting (10 RePaint steps)				
1200	Tool-Scene	NFE 300	NFE 400	NFE 500
1201	IND	$60 \pm 2$	$62 \pm 3$	$64 \pm 3$
1202	OOD	$51 \pm 3$	$52 \pm 3$	$49 \pm 3$
(c) CompDiffuser				

Table 7: Success rates on tool-use scenes under different NFE budgets: (a) DiffCollage/GSC, (b) DiffCollage/GSC+Repainting with 10 RePaint steps, and (c) CompDiffuser. CompDiffuser consistently outperforms DiffCollage/GSC and DiffCollage+Repainting.

## F.2 HYPERPARAMETER SEARCH OVER NFEs

On our high-dimensional tool scenes, we perform a hyperparameter search for DiffCollage/GSC and DiffCollage+Repainting( Anonymous (2025)) over the number of NFEs, allowing many more NFEs than our method. Even with substantially more NFEs than our sampler, the noisy-Bethe gap still leads to blurry and inconsistent video generations, which in turn keeps the success rates low. In these high-dimensional settings, CompDiffuser consistently outperforms DiffCollage/GSC and DiffCollage+Repainting. We report the corresponding results in table 7.

## G EXTENSION OF OUR METHOD

A key advantage of our training-free compositional framework is its plug-and-play flexibility. We demonstrate this with three extensions. (1) **Language conditioning**: replacing the goal image with a text prompt and adding a CLIP-style guidance loss yields performance comparable to goal-image conditioning. (2) **Physical constraints**: environment knowledge can be injected through simple penalty terms (e.g., collision loss), improving performance in challenging settings. (3) **Classical planning**: the asynchronous objective naturally supports tree-structured planning, and we provide a bidirectional RRT-style stitching variant for reference.

### G.1 EXTENSION TO LANGUAGE CONDITIONING

Conceptually, the boundary condition is an important way to extend the planning horizon, and in our training-free setting the goal image can be directly replaced by a text description. Concretely, we define a CLIP-style guidance loss  $L_{\text{clip}}$  that measures the semantic match between the generated goal frame at each denoising step and the goal text prompt, and we drop the original goal-image term in  $L_{\text{sync}}$  and  $L_{\text{async}}$ . In a preliminary goal-text setting, this variant achieves performance comparable to the goal-image version.

We build the language conditioning setting based on the Tool scene while the goal image is replaced with a goal text condition. In this setting, the text prompts correspond to two goal configurations: “the robot arm pushes the cube under the table” and “the robot arm pushes the cube over a red target.” We present the corresponding results in table 8.

Tool-Scene	Goal Image	Goal Text
IND	97 $\pm$ 3	95 $\pm$ 3
OOD	96 $\pm$ 2	96 $\pm$ 3

Table 8: Success rates for goal-image vs. goal-text boundary conditions on tool-use scenes. Goal-text guidance uses  $L_{clip}$  to enforce semantic match between the generated goal frame and the text description. We observe that language conditioning achieves performance comparable to goal-image conditioning.

## G.2 EXTENSION TO PHYSICAL CONSTRAINTS

Our method does not forcibly stitch short plans together: boundary consistency is imposed through a *soft* guidance penalty rather than a hard constraint. Our DSG-based guidance preserves the underlying data manifold, so if the demonstrations never exhibit behaviors such as “passing through a wall,” the guided sampler has very little chance of generating such unrealistic trajectories.

Moreover, because our framework is fully training-free, we can incorporate additional physical constraints when available. For verifier design in the pointmaze domains, we use the ground-truth maze layout and compute a collision penalty from the position coordinates. If a point  $(x, y)$  is inside a maze wall box with center  $(c_x, c_y)$  and half-width  $d$ , we define the pointwise loss as the minimum distance to the four box walls:

$$L(x, y) = \min(x - (c_x - d), (c_x + d) - x, y - (c_y - d), (c_y + d) - y).$$

Summing this penalty along the trajectory yields  $L_{collision}$ , which we simply add to our guidance objective. As shown below, this explicit collision term further improves performance on the most challenging maze while leaving the easier settings unchanged. We present the corresponding results in table 9.

Env	Ours	Ours + $L_{collision}$
pointmaze-medium	100 $\pm$ 0	100 $\pm$ 0
pointmaze-large	100 $\pm$ 0	100 $\pm$ 0
pointmaze-giant	93 $\pm$ 4	97 $\pm$ 3

Table 9: Success rates on pointmaze environments with and without an explicit collision penalty  $L_{collision}$ . Incorporating  $L_{collision}$  can further improve performance on more challenging long-horizon tasks.

## G.3 EXTENSION TO CLASSICAL PLANNING

Extending our method to RRT-style, tree-structured planning is natural under the asynchronous objective. The objective decomposes into a forward term  $L_{fwd}$  that plans from the start and a backward term  $L_{bwd}$  that plans from the goal. We initialize a forward tree  $T_s$  rooted at  $s$  and a backward tree  $T_g$  rooted at  $g$ , and then iterate: (1) sample a batch of stitched paths using  $L_{fwd}$  to expand the forward tree, (2) sample a batch of stitched paths using  $L_{bwd}$  to expand the backward tree, and (3) connect  $T_s$  and  $T_g$  whenever newly added nodes fall below a distance threshold. On OGBench, this bidirectional stitching tree planner achieves a 100% success rate on pointmaze-large, and extending it to more complex settings will require additional exploration, which is beyond the scope of this work. Beyond classical sampling-based planners, our stitching framework naturally extends to Monte Carlo Tree Search as well, and recent work (Yoon et al. (2025a)) has already explored this direction.

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**Algorithm 2** Bidirectional RRT Compositional Tree Planner

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1: Initialize forward tree  $T_s \leftarrow \{s\}$ , backward tree  $T_g \leftarrow \{g\}$ 
2: while not connected do
3:   Sample a batch of stitched paths with  $L_{\text{fwd}}$  to expand  $T_s$  and collect new nodes  $\mathcal{N}_s$ 
4:   Sample a batch of stitched paths with  $L_{\text{bwd}}$  to expand  $T_g$  and collect new nodes  $\mathcal{N}_g$ 
5:   for each pair  $(x, y)$  with  $x \in \mathcal{N}_s, y \in T_g$  or  $x \in T_s, y \in \mathcal{N}_g$  do
6:     if  $\|x - y\| \leq \varepsilon$  then
7:       Connect  $T_s$  and  $T_g$  through  $(x, y)$  and return the stitched path
8:     end if
9:   end for
10: end while

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**H REAL ROBOT EXPERIMENT**

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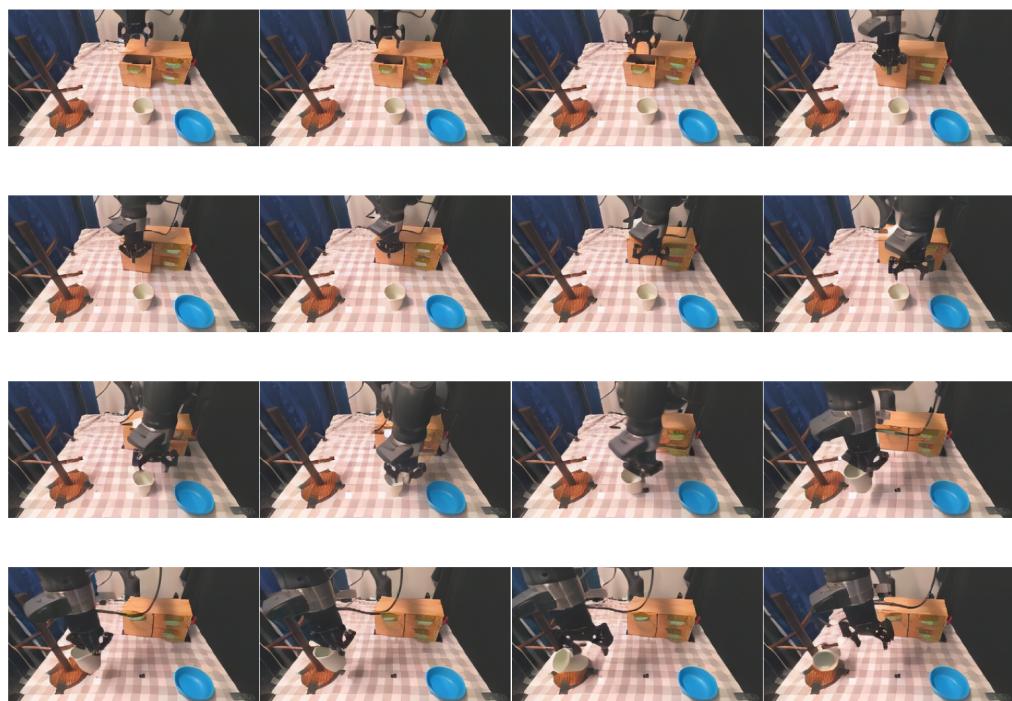
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Figure 10: **Real World Robot Setup Overview.**

1350 **H.2 QUALITATIVE VIDEO SYNTHESIS RESULTS**  
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1352 **H.2.1 IN DISTRIBUTION EVALUATION**  
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1377 **Figure 11: Task1 Synthetic Plan.**  
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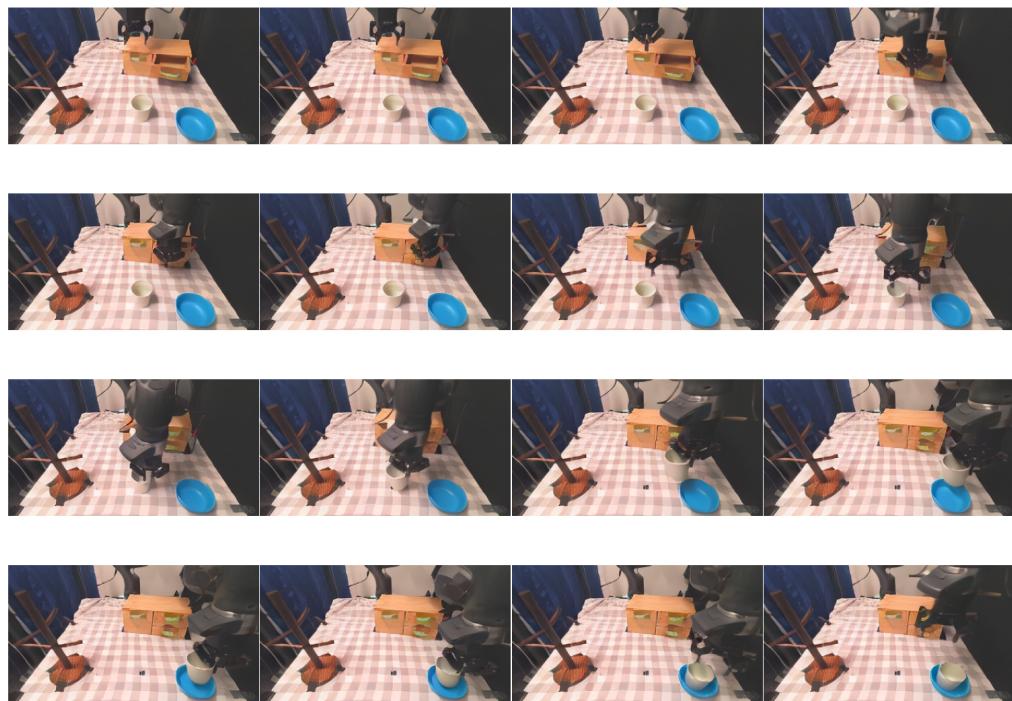
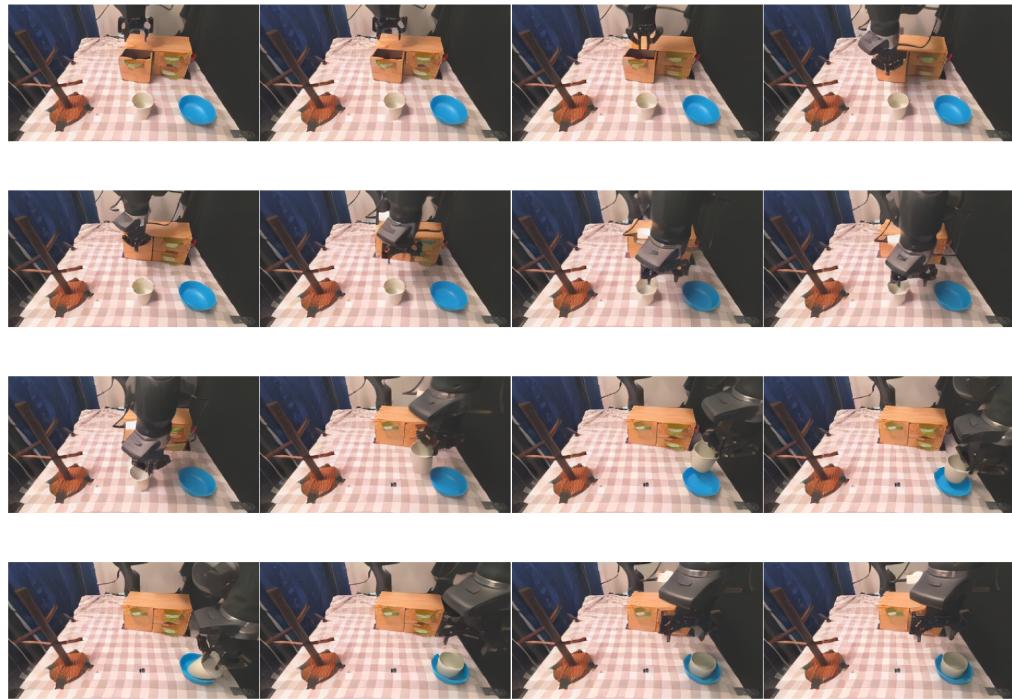


Figure 12: **Task2 Synthetic Plan.**

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1405 **H.2.2 OUT OF DISTRIBUTION EVALUATION**  
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Figure 13: **Task3 Synthetic Plan.**

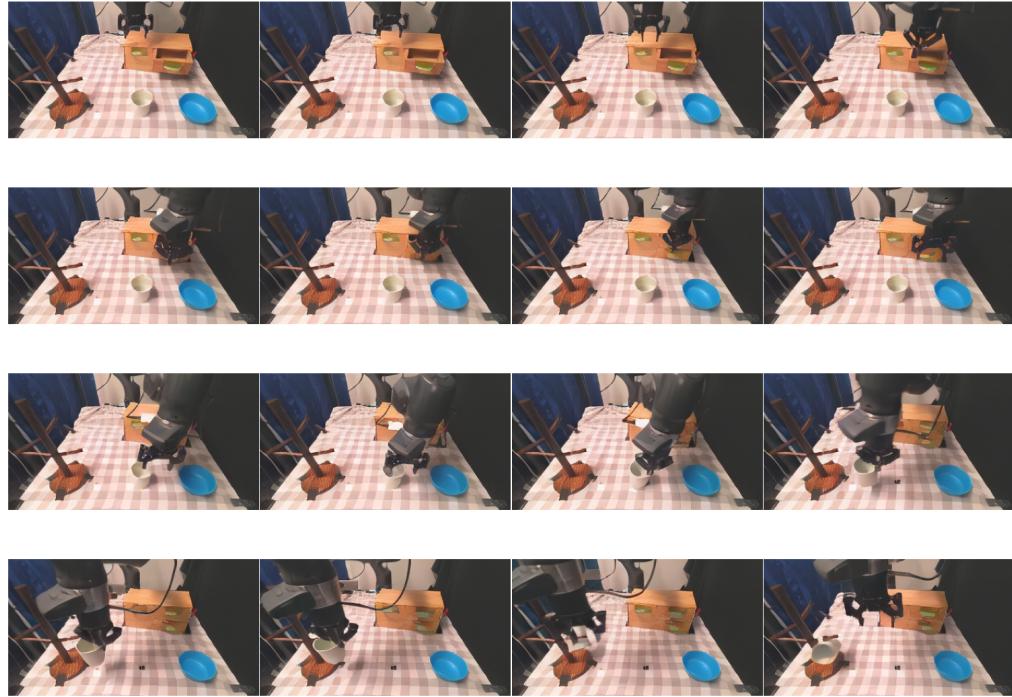
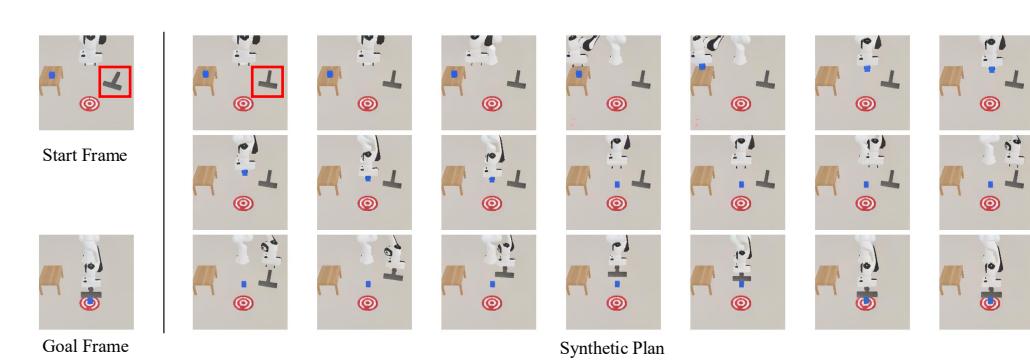


Figure 14: **Task4 Synthetic Plan.**

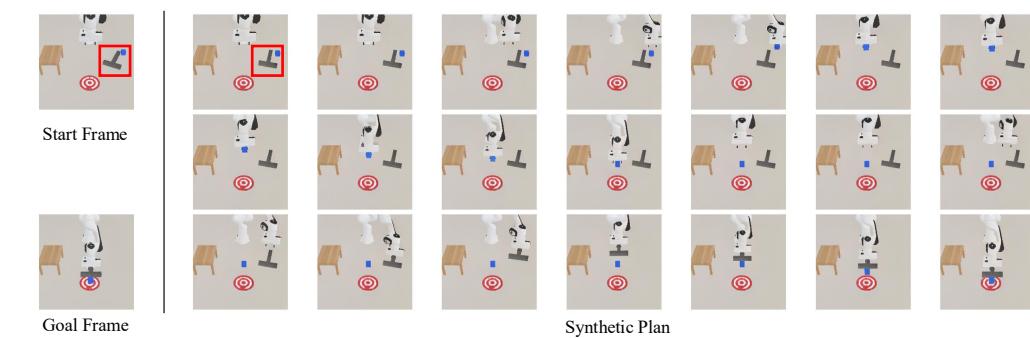
## 1458 I FAILURE CASE OF SYNCHRONOUS MESSAGE PASSING

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 1460 In our experiments, removing the discount factor does not cause plans to become globally incoherent,  
 1461 the generated videos remain smooth, and the intermediate states are generally consistent. The main  
 1462 failure mode is more subtle: the generated start and goal frames remain semantically correct but  
 1463 exhibit small spatial misalignments relative to the precise test-time requirement. This misalignment  
 1464 leads to plans that are coherent in motion but anchored to an incorrect spatial configuration.

1465 For example, in our failure cases 1 and 2, the robot performs a reasonable picking motion for the hook,  
 1466 yet the entire sequence is centered around the wrong object location: the gray tool is consistently  
 1467 generated with a small but noticeable orientation offset from its true pose required by start. We  
 1468 hypothesize that without the discount factor, the guidance signals from the start and goal conditions  
 1469 are not sufficiently emphasized, allowing this spatial drift to persist throughout the whole plan.



1482 **Figure 15: Failure Mode of Synchronous Message Passing: Case 1.**



1497 **Figure 16: Failure Mode of Synchronous Message Passing: Case 2.**