Elastic Aggregation for Federated Optimization

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Paper under double-blind review

Abstract

Federated learning enables the privacy-preserving training of neural network models using real-world data across distributed clients. FedAvg has become the preferred optimizer for federated learning because of its simplicity and effectiveness. FedAvg uses naïve aggregation to update the server model, interpolating client models based on the number of instances used in their training. However, naïve aggregation suffers from client-drift when the data is heterogenous (non-IID), leading to unstable and slow convergence. In this work, we propose a novel aggregation approach, elastic aggregation, to overcome these issues. Elastic aggregation interpolates client models adaptively according to parameter sensitivity, which is measured by computing how much the overall prediction function output changes when each parameter is changed. This measurement is performed in an unsupervised and online manner. Elastic aggregation reduces the magnitudes of updates to the more sensitive parameters so as to prevent the server model from drifting to any one client distribution, and conversely boosts updates to the less sensitive parameters to better explore different client distributions. Empirical results on real and synthetic data as well as analytical results show that elastic aggregation leads to efficient training in both convex and non-convex settings, while being fully agnostic to client heterogeneity and robust to large numbers of clients, partial participation, and imbalanced data. Finally, elastic aggregation works well with other federated optimizers and achieves significant improvements across the board.

1 Introduction

Unlike traditional centralized learning in which models are trained using large datasets stored in a central server (Goyal *et al.*, 2017), federated learning - first proposed in (McMahan *et al.*, 2017) - leverages data spread across many clients to learn classification tasks distributively without explicitly sharing data (Konečnỳ *et al.*, 2016a;b; Mohri *et al.*, 2019; Kairouz *et al.*, 2019), thereby ensuring a basic level of privacy.

There are four principal characteristics of federated learning (McMahan *et al.*, 2017):

- *Unreliable links*: Communication links between the server and clients are unreliable and there may only be a small subset of clients that are active at any given time.
- *Massive distribution*: The number of clients is large, while the amount of data per client is small.
- *Substantial heterogeneity*: Client data is heterogeneous (non-IID) (Konečnỳ *et al.*, 2016a), i.e., data in different clients are sampled from different regions of the sampling space.
- *Imbalanced data*: The amount of data per client is highly variable.

The most popular algorithm for federated learning is FedAvg (McMahan *et al.*, 2017), which tackles the communication bottleneck by performing multiple local updates on the available clients before communicating the overall change to the server. FedAvg uses naïve aggregation to interpolate client models, and has shown success in certain applications. However, its performance on heterogeneous data is still an active area of research (Li *et al.*, 2019b; Haddadpour and Mahdavi, 2019; Khaled *et al.*, 2020). According to (Khaled *et al.*, 2020), *training models on local data that minimize local empirical loss appears to be meaningful, but yet, doing so, is fundamentally inconsistent with*

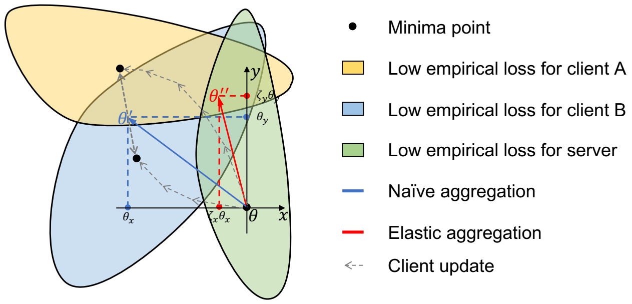

Figure 1: Illustration of naïve aggregation and elastic aggregation. The local updates of client A and client B drive the server model θ towards their individual minima (black dots in plot). Naïve aggregation simply averages the received model from clients A and B, yielding θ' as the new server model. Although θ' minimizes the local empirical loss of clients A and B, θ' drifts from ideal distribution for the server model. Elastic aggregation adjusts gradient with respect to parameter sensitivity. Parameter θ_x is more sensitive (has a larger gradient norm), and is restricted with $\zeta_x < 1$ to reduce the magnitude of its update. Parameter θ_y is less sensitive (has a smaller gradient norm), and is correspondingly boosted with $\zeta_y > 1$ to better explore the parameter space. This minimizes the loss for clients A and B, while not causing the server model to drift from its ideal distribution. Hence, elastic aggregation results in a better update θ''.

minimizing the global empirical loss. Client updates drive the server model away from the ideal distribution, a phenomenon known as 'client-drift'. Naïve aggregation (McMahan *et al.*, 2017) is efficient in aggregating client models but does not account for distribution inconsistencies across client data or the consequent objective inconsistency. In other words, with naïve aggregation, the server model risks converging to a stationary point of a mismatched objective function which can be arbitrarily different from the true objective (Wang *et al.*, 2020).

Prior works (McMahan *et al.*, 2017; Karimireddy *et al.*, 2019; Reddi *et al.*, 2016) attempt to overcome this issue by running fewer epochs or iterations of SGD on the devices or by stabilizing server-side updates so that the resulting models correspond to inexact minimizations and keep globally desirable properties. In this work, we propose a novel aggregation approach, elastic aggregation, to overcome client-drift. We measure parameter sensitivity using unlabeled samples of client data, by computing the changes to the overall function output for a given change to the parameter in question, without relying on the loss. This allows our method to not only avoid requiring labelled data, but importantly also pre-empts complications that could otherwise arise from the loss being at a local minimum with gradients close to zero. During the aggregation of client models, updates to the more sensitive parameters can then be reduced in magnitude, preventing the server model from drifting to any client distribution. Conversely, updates to the less sensitive parameters can be boosted to better explore different client distributions.

Contributions. Elastic aggregation tackles distribution inconsistency across client data using the concept of parameter sensitivity, and is simple to implement, requiring little hyper-parameter tuning. Furthermore, parameter sensitivity is computed in an online and unsupervised manner, and thus better utilizes the unlabeled data generated by the client during runtime. Elastic aggregation is easily integrated into different federated optimizers, achieving substantial improvements over naïve aggregation. The empirical results on real and synthetic data and analytical results show that elastic aggregation leads to efficient training in both convex and non-convex settings, across all four federated learning scenarios (unreliable links, massive distribution, substantial heterogeneity and imbalanced data).

2 Related work

Federated learning is a fast-evolving topic. The general setup involves server and client updates; each of these updates is associated with minimizing some local loss function. The server model then benefits from all client data and achieves superior performance, for tasks such as next word prediction (Hard *et al.*, 2018; Yang *et al.*, 2018), emoji prediction (Ramaswamy *et al.*, 2019), decoder models (Chen *et al.*, 2019b), vocabulary estimation (Chen *et al.*, 2019a), low latency vehicle-to-vehicle communication (Samarakoon *et al.*, 2018) and predictive models in health (Brisimi *et al.*, 2018). Nevertheless, federated learning raises several issues and has been the topic of much research effort, focusing on the issues of generalization and fairness (Mohri *et al.*, 2019; Li *et al.*, 2019b), the design of more efficient communication strategies (Konečný *et al.*, 2016b;a; Suresh *et al.*, 2017; Stich *et al.*, 2018; Karimireddy *et al.*, 2019; Basu *et al.*, 2019), the study of lower bounds (Woodworth *et al.*, 2018), differential privacy guarantees (Agarwal *et al.*, 2018), security (Bonawitz *et al.*, 2017), etc (Kairouz *et al.*, 2019). We focus here on relevant work that specifically address the four federated learning characteristics noted above - massive distribution, heterogeneity, unreliable links and imbalanced data.

Much of earlier work in this context propose optimizing for the local risk objective with SGD (Stich *et al.*, 2018) over mini-batches of client data, analogous to the centralized scenario, with the server then averaging the received models. FedAvg (McMahan *et al.*, 2017) is a generalization of local SGD, proposing a larger number of local SGD steps per round. In the case of identical clients, it reduces to parallel SGD for which asymptotic convergence has been proven (Zinkevich *et al.*, 2010; Stich *et al.*, 2018); more recently, (Patel and Dieuleveut, 2019; Khaled *et al.*, 2020) analyzed the same method under the name of local SGD, also for identical functions. FedAvg inexactly solves client-side optimization, requiring the tuning of the number of epochs and the learning rate hyper-parameters in order to achieve a good accuracy-communication trade-off (McMahan *et al.*, 2017; Li *et al.*, 2020a).

Despite the strong empirical performance of FedAvg in IID settings, performance degrades in non-IID scenarios (Zhao *et al.*, 2018). The analysis of FedAvg for heterogeneous clients is more delicate, due to the aforementioned client-drift, first empirically observed by (Zhao *et al.*, 2018). Several analyses bound this drift by assuming bounded gradients (Wang *et al.*, 2019; Yu *et al.*, 2019), viewing it as additional noise (Khaled *et al.*, 2020), or assuming that the client optima are ϵ-close (Li *et al.*, 2019b; Haddadpour and Mahdavi, 2019). (Liang *et al.*, 2019) proposes using variance reduction to deal with client heterogeneity but achieved slower convergence rates than SGD. Other variants include using a decreasing learning rate (Li *et al.*, 2020a), modifying client empirical loss dynamically (Li *et al.*, 2018; Acar *et al.*, 2021), adding regularizers on local updates (Karimireddy *et al.*, 2019; Shamir *et al.*, 2014; Karimireddy *et al.*, 2020), or modifying the server side updates (Hsu *et al.*, 2019; Reddi *et al.*, 2016). While these works do recognize the incompatibility of local and global stationary points, their proposed fixes are based on inexact minimization. Additionally, in order to establish convergence for non-IID situations, these works impose additional constraints.

Another class of work that extend the analysis of SGD type methods to federated learning settings specifically addresses the special case of full client participation (Konečný *et al.*, 2016a; Makhdoumi and Ozdaglar, 2017; Shamir *et al.*, 2014; Yuan and Ma, 2020; Pathak and Wainwright, 2020; Liang *et al.*, 2019; Li *et al.*, 2020b; Condat *et al.*, 2020; Gorbunov *et al.*, 2020; Li and Richtárik, 2020). For example, FedSVRG (Konečný *et al.*, 2016a) and DANE (Shamir *et al.*, 2014) need gradient information from all clients each round, and are as such not directly applicable to partial federated learning settings. Furthermore, it may not be trivial to extend these works to overcome the deficiency. For example, FedDANE (Li *et al.*, 2019a) is a version of DANE that works with partial participation. However, with partial participation, FedDANE empirically performs worse than FedAvg (Li *et al.*, 2019b). Similar to these works, FedPD (Zhang *et al.*, 2020) proposes distributed optimization with a different notion of 'participation'. FedPD activates either all clients or no clients per round, which again fails to satisfy the partial participation condition.

Finally, there have been earlier works aiming to decrease communication costs by compressing the transmitted models (Dutta *et al.*, 2020; Mishchenko *et al.*, 2019; Alistarh *et al.*, 2017), decreasing the bit-rate of the transmission. These ideas are complementary to our work, and can work together with our approach.

3 METHOD

In this section, we focus on the introduction of parameter sensitivity, which is our main innovations. *The proof of convergence is provided in the technical part of supplementary materials.*

Algorithm 1: Elastic aggregation within a single layer

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A variable with a superscript $i$ indicates the $i^{th}$ element of the variable. A variable with a
  subscript $k$ indicates the variable from $k^{th}$ client. $\eta, \eta'$ are learning rates of server and clients
  respectively. $\mu, \tau$ are the hyper-parameters. $\theta, \theta_k \in \mathbb{R}^n$ are the server's and the $k^{th}$ client's
  parameters respectively. $\Omega \in \mathbb{R}^n$ is the aggregated parameter sensitivity. $\Omega_k \in \mathbb{R}^n$ is the
  parameter sensitivity on the $k^{th}$ client.
Initialize $\theta$
$B_k \leftarrow$ Sample a subset of training data $D_k$.
$D_k \leftarrow$ Drop the samples of $B_k$ from $D_k$.
for each round do
    for each activated client $k$ do
        Initialize $\Omega_k$ as zeros.
        for each batch data $x \in B_k$ do
            $g = \nabla||F(\theta; x)||_2^2$
            for $i \in [1, \cdots, n]$ do
                $\Omega_k^i \leftarrow \mu\Omega_k^i + (1 - \mu)|g^i|$
        $\theta_k \leftarrow \theta$
        for each epoch do
            for each batch data $x \in D_k$ do
                $\theta_k \leftarrow \theta_k - \eta' \nabla \ell_k(F(\theta_k; x))$
        $\Delta_k = \theta_k - \theta$
    $w_k \leftarrow |D_k| / \sum_k |D_k|$; $\Omega = \sum_k (w_k \cdot \Omega_k)$; $\Omega' = \max(\Omega)$
    for $i \in [1, \cdots, n]$ do
        $\zeta^i = 1 + \tau - \Omega^i / \Omega'$
        $\Delta^i = \zeta^i \cdot \sum_k (w_k \cdot \Delta_k^i)$
        $\theta^i \leftarrow \theta^i - \eta \cdot \Delta^i$
```

3.1 Federated Learning with Naïve Aggregation

In federated learning, we solve an optimization problem of the form:

$$arg \min_{\theta \in \mathbb{R}^n} [\ell(\theta) \triangleq \frac{1}{m} \sum_{k=1}^{m} L_k(\theta)], \tag{1}$$

where $L_k(\theta) \triangleq \mathbb{E}_{(x,y) \backsim \mathcal{P}_k} [\ell_k(\theta; (x, y))]$ is the empirical loss of the k^{th} client, $\mathcal{P}_k$ is the data distribution for the k^{th} client across K clients.

FedAvg A common approach to solving Eq. 1 in federated settings is FedAvg (McMahan *et al.*, 2017). At each round, a subset of clients is selected (typically randomly) and the server broadcasts its model to each client. In parallel, the clients run SGD on their own loss function ℓ_k and then send their updated models to the server. The server then update its model to be the average of these client models.

Suppose that at the r^{th} round, the server has model θ and samples a client set $\mathcal{S}$. Here we use a standard gradient descent form to update parameters:

$$\theta \leftarrow \theta - \eta\Delta, \tag{2}$$

where Δ is the aggregated clients' gradients and η is the learning rate of server, which is typically 1.0. FedAvg uses naïve aggregation to compute Δ.

Naïve aggregation We can write naïve aggregation as:

$$\Delta = \sum_{k \in \mathcal{S}} (w_k \cdot \Delta_k), \tag{3}$$

where $\Delta_k = \theta - \theta_k$ is the accumulated gradients within a training round of the k^{th} client, and $w_k = |D_k| / \sum_{k \in \mathcal{S}} |D_k|$ is the aggregated weight of the k^{th} client across the activated clients $\mathcal{S}$. D_k, θ_k are the training dataset and trained parameters on the k^{th} client, respectively.

3.2 Exploring parameter sensitivity

Due to the non-IID-ness (heterogeneity) across different clients, $\mathcal{P}$ can vary substantially across different clients. Thus, the simple and efficient way to aggregate client models described in Eq. 3 cannot solve client drift and cause oscillation as well as slow convergence. We explore parameter sensitivity as an approach to overcome these issues.

The main idea we are proposing is that *less sensitive parameters can be freely updated to minimize the loss for individual clients in $\mathcal{S}$ without causing the server to drift; by the same reasoning, parameters that are more sensitive should not be updated as much.* This is illustrated in Fig. 1.

At the r^{th} round, the active clients receive the server model θ_r, which parameterizes the approximation function F of the true function $\bar{F}$. F maps the input x to the output y, and our goal is to largely maintain the same output y for the observed x while learning additional training instances $(x, y) \backsim P_k$. To this end, we measure how sensitive the output of function F is to changes in the network parameters. To reduce confusion, we ignore the symbols denoting the round r and the client k for all variables and only discuss parameter sensitivity locally.

For a given data point x, the output of the network is $F(\theta; x)$. A small perturbation δ in the parameters θ results in a change in the function output that can be approximated by:

$$F(\theta + \delta; x) - F(\theta; x) \approx g(\theta; x)\delta, \tag{4}$$

where $g(\theta; x) = \frac{\partial F(\theta; x)}{\partial \theta}$ is the gradient of the learned function with respect to the parameter θ evaluated at the data point x. Our goal is to maintain the output of the network (the learned function F) for each observed data point and thus we reduce the magnitudes of the updates to the parameters that are more sensitive for the data point. Conversely, we boost the updates to the less sensitive parameters to better fit client distributions.

Based on equation 4, and assuming a small constant change δ, we can measure the sensitivity of a parameter by the magnitude of the gradient $g(\theta; x)$, i.e., how much does a small perturbation to that parameter change the output of the learned function for data point x. There are many ways to measure the sensitivity of a parameter by the magnitude of the gradient, such as fishing matrix. However, in federated learning, we have to take the computation ability of each client into account, which is only with limited computation ability. Therefore, we use the exponentially-decayed moving average[1] to obtain sensitivity Ω^i for i^{th} parameter θ^i:

$$\Omega^i \leftarrow \mu \Omega^i + (1 - \mu)|g(\theta^i; x)|, \tag{5}$$

where μ is momentum and we empirical set to 0.9.

When the output function F is multi-dimensional, as is the case for most neural networks, equation 5 involves computing the gradients for each output, which requires as many backward passes as the dimensionality of the output. As a more efficient alternative, we propose to use the gradients of the squared L^2 norm of the learned function output, i.e. $g(\theta; x) = \nabla ||F(\theta; x)||_2^2$. Hence, we only need to compute one backward pass in order to estimate the parameters' sensitivity.

3.3 Elastic aggregation with respect to parameter sensitivity

The elastic aggregation for i^{th} parameter can be written as:

$$\Delta^i = \zeta^i \cdot \sum_{k \in \mathcal{S}} (w_k \cdot \Delta_k^i), \tag{6}$$

[1] While a Polyak-style average has more theoretical guarantees for accumulating Ω over batches, our results match the claim that “an exponentially-decayed moving average typically works much better in practice" (Martens, 2014). Thus, we use the exponentially-decayed moving average to accumulate Ω over batches. Refer to supplementary materials for more discussion.

τ	Naïve	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Train Acc.(%)	58.36	57.21	58.33	58.76	60.66	60.47	**61.44**	60.66	60.55	61.01	59.27	58.96
Test Acc.(%)	60.74	58.30	59.76	61.17	61.91	**63.39**	62.74	**63.39**	63.23	63.22	60.83	61.37
Boosted(%)	-	0.0	11.45	27.24	43.75	59.74	74.43	84.57	89.93	93.53	96.26	97.38

Table 1: The effect of τ. Using an unsuitable τ for elastic aggregation with cause it to even underperform naïve aggregation. The last row shows the percentage of parameters that are boosted for different τ.

where ζ^i is the adaptive coefficient with respect to the parameter sensitivity. The definition of ζ^i is:

$$\zeta^i = 1 + \tau - \Omega^i/\Omega', \tag{7}$$

where $\Omega' = \max(\Omega)$ is the maximum of parameter sensitivities in each layer, and $\Omega^i = \sum_{k\in\mathcal{S}}(w_k \cdot \Omega^i_k)$ is the aggregated sensitivity of i^{th} parameter across activated clients. The server is able to boost updates ($\zeta^i > 1.0$) to parameters which have low sensitivity (low Ω^i). Meanwhile, sensitive parameters (high Ω^i) have their updates restricted with a penalty ($\zeta^i < 1.0$). τ is a hyper-parameter determining the ratio of boosted or restricted parameters. FedAvg with elastic aggregation is formalized in Algorithm 1.

Discussion on τ In Eq. 6, $\zeta^i \in (1, 1+\tau]$ is applied to the less sensitive parameters, and $\zeta^i \in [\tau, 1)$ is applied to the more sensitive parameters. Thus, τ determines the portion of parameter to be restricted or boosted. Generally, a larger τ indicates that a larger proportion of parameters will be boosted, as shown in Tab. 1, and vice versa.

If a large proportion of parameters is restricted (τ is small, such as 0.1), the server model will not be updated much and exiting local minima becomes difficult. However, with a large τ such as 0.9, optimization of the server model becomes unstable, and this also causes performance degradation. It is therefore critical to maintain a balance between these extremes with a suitable τ and we find that $\tau = 0.5$ generalizes well to various experiments. At this value of τ, roughly three quarters of parameters are boosted.

4 Experiments Settings

Inconsistent usage of models, datasets and non-IID partition methods make it difficult to compare the performances of federated learning algorithms fairly. To enforce fair comparison, we explicitly specify the combinations of datasets, models, and non-IID partition methods to be used for experiments, which mostly consists of those used in existing work published at top tier machine learning venues (He *et al.*, 2020). Note that although the reported precisions are state-of-the-art for the models we have chosen, higher-capacity models can achieve higher performances on these datasets. Therefore, we compare the relative performance of elastic aggregation against naïve aggregation for various federated learning scenarios.

Implementation We measure our algorithm against the most comprehensive and representative suite of federated datasets and modeling tasks to date. For this purpose, we carry out simulations on five diverse and representative experiments on several datasets. In order to facilitate the experiments, we use the cosine annealing scheduler (Loshchilov and Hutter, 2016) for the client learning rate η'_r at the r^{th} round, which is given by $\eta'_r = (1 + \cos(\frac{r}{R}\pi))\eta'$. For all tasks, we measure the performance on the entire test set throughout training. Clients are sampled uniformly at random, without replacement within a given round, but with replacement across rounds. There are a variety of federated optimizers that improve the performance of federated learning. Here, we use the most basic and widely used federated optimization algorithm, i.e., FedAvg (McMahan *et al.*, 2017), as the default baseline in our experiments. With reference to (Reddi *et al.*, 2020), we implement all experiments with two important characteristics. First, instead of doing K training steps per client, we do E epochs of training over each client's data. Second, to account for varying numbers of gradient steps per client, we weight the average of the client outputs Δ^k_t by the number of training samples. This also follows the approach of (McMahan *et al.*, 2017) and often outperforms uniform weighting (Reddi *et al.*, 2018). We set $\mu = 0.95$ for the accumulation of the parameter sensitivity over batches in each client. $\tau = 0.5, \eta = 1.0$ is used for all experiments. While computing Eq. 7, we normalize the parameter sensitivity layer by layer, which means that the parameter sensitivity is divided by the maximum value within the same layer. Aside from experiments 5.1, we set $E = 1$ for each client for all experiments.

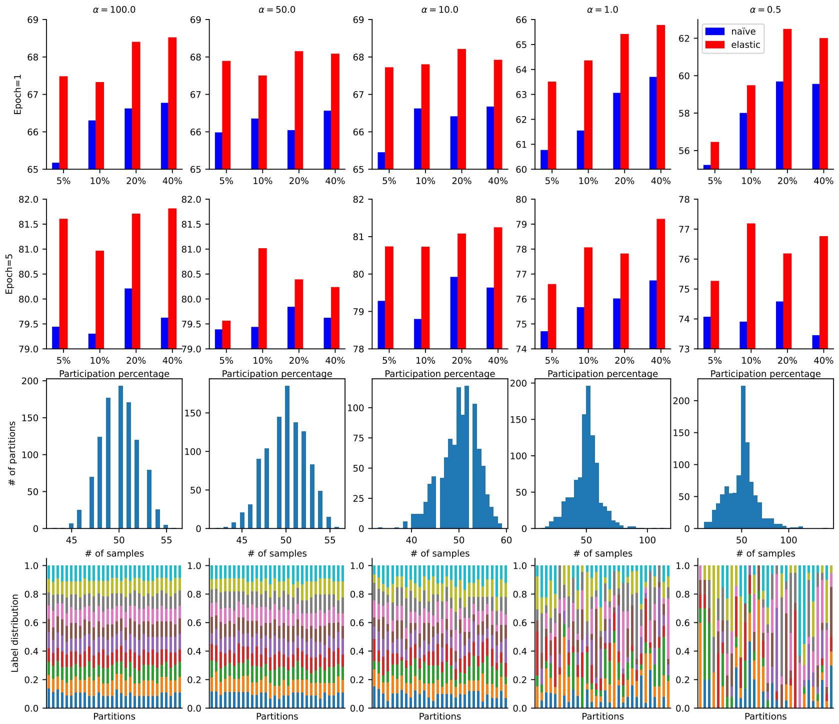

Figure 2: Performances across different partitioning distributions, participation rates and numbers of local epochs. Compared with naïve aggregation, elastic aggregation achieves significant improvements across these settings.

Generate synthetic federated datasets Differences in distribution have an enormous influence on the performance of federated optimization. The Dirichlet distribution is used on the label ratios to ensure uneven label distributions among clients for non-IID splits, as in (Yurochkin *et al.*, 2019). To generate imbalanced data, we sample the number of data points from a log-normal distribution. The degree of imbalance depends on the variance of the log-normal distribution, which is controlled by the hyper-parameter α. A smaller α indicates a stronger non-IID-ness in partitions, as shown in Fig. 2.

5 Experiments

Under all experiments, the samples used for computing parameter sensitivities at each client will be randomly sampled training data, and these sampled data will not be used for supervised training any more. For fair comparison, these sampled data will not be used in the baseline methods either.

5.1 Comparison with naive aggregation

MNIST As our baseline, we conduct a convex optimization experiment on MNIST (LeCun *et al.*, 1998) with logistic regression (Li *et al.*, 2018; 2019b; McMahan *et al.*, 2017). In this experiment, we generate a synthetic federated dataset of MNIST with $\alpha = 100.0$ and split the dataset into 1000 partitions. At each round of training, we randomly choose 100 clients without repeat in the partition. As shown in Tab. 3, elastic aggregation has a large advantage over naïve aggregation even with a large number of participating clients.

Federated EMNIST Due to resource constraints of edge devices, existing works commonly use shallow neural networks for experiments. Similarly, we include Federated EMNIST (Reddi *et al.*, 2020) with a shallow neural network consisting of two convolution layers followed by two linear layers (Reddi *et al.*, 2020). Notably, Fed-EMNIST (Reddi *et al.*, 2020) has a natural client partitioning that is highly representative of real-world federated learning problems. It contains 3,400 users (clients)

Optimizer	FedAvg	FedAvgM	FedProx	SCAFFOLD	AdaOpt	PFNM	Per-FedAvg	pFedMe
w/o Elastic.	49.34	62.52	51.52	72.10	78.00[3]	71.23	50.44	83.20
w/ Elastic.	52.37	64.48	53.64	75.42	80.34	73.64	53.24	85.63
Improvement.	**3.03**	**1.96**	**2.12**	**3.32**	**2.34**	**2.41**	**2.80**	**2.43**

Table 2: Elastic aggregation can be easily integrated with different federated optimizers, achieving performance improvements. (Notes that the comparison across different federated optimizers is meaningless since they are under different experiment settings.)

and 62 label classes. The dataset is split into 671,585 training examples and 77,483 testing examples. Rather than holding out specific users, we split each user's examples across train and test so that all users have at least one example in train and one example in test. Users with fewer than 2 examples are excluded from the data set. As shown in Tab. 3, elastic aggregation generalizes well to real-world federated learning problems.

CIFAR-100 In this experiment, we conduct a balanced but non-IID partition on Fed-CIFAR100 [2], which was introduced by TensorFlow Federated (Hsieh *et al.*, 2020) with ResNet-20 (He *et al.*, 2016) as the baseline model. As shown in Tab. 3, elastic aggregation significantly speeds up training. Furthermore, the final accuracy is improved by over 20% compared with the naïve baseline, which demonstrates the effectiveness of elastic aggregation.

CIFAR-10 and CINIC-10 To further demonstrate the performance improvement resulting from elastic aggregation, we generate two synthetic federated datasets for CIFAR-10 (Hsieh *et al.*, 2020) and CINIC-10 (Darlow *et al.*, 2018) with $\alpha = 100.0$. CINIC-10 extends CIFAR-10 with the addition of down-sampled ImageNet images, making for a more challenging dataset. This also enables measuring the performance of models trained on CIFAR images on ImageNet images for the same classes. As shown in Tab. 3, elastic aggregation achieves superior performances on both the CIFAR-10 and the more difficult CINIC-10 datasets, confirming that our method generalizes well to large scale datasets.

Natural language processing on Shakespeare dataset We maintain the same experimental setup as (McMahan *et al.*, 2017) on the most commonly used natural language processing tasks: Shakespeare LSTM. The epoch in each round is 1, the batch size is 10 and both elastic and naive aggregation are trained for 2k rounds. We get the final accuracy as 62.14% for naive aggregation and 64.73% for elastic aggregation. The results show that elastic aggregation not only has a significant improvement in CV related tasks, but also has an positive effect on NLP related tasks.

Imbalanced and non-IID We are also interested in understanding how parameter sensitivity can help improve convergence, particularly in cross-device and non-IID scenarios. Given the resource constraints of edge devices, large DNN models are usually trained under the cross-organization (also known as cross-silo) federated learning settings. Across the following experiments, we use CIFAR-10 (Krizhevsky *et al.*, 2009) with ResNet-20 (He *et al.*, 2016). We study three different parameters here: non-IID-ness, participation rate, and number of epochs. We explore the effects of varying non-IID-ness with $\alpha = 100.0, 50.0, 10.0, 1.0, 0.5$ across 1000 partitions. For participation rate, we test sampling $5\%, 10\%, 20\%, 40\%$ of the clients at each round. Finally, we test having $1, 5$ epochs of local updates, to better show the robustness of elastic aggregation. As shown in Fig. 2, elastic aggregation is especially superior for situations with massive distribution, substantial heterogeneity, and imbalanced data.

5.2 Integration with modern optimizer

Federated optimizer with elastic aggregation To illustrate the generalizability of elastic aggregation, we integrate elastic aggregation into FedAvg (McMahan *et al.*, 2017), FedAvgM (Reddi *et al.*, 2020) and FedProx (Li *et al.*, 2018). FedAvgM is an enhancement of FedAvg on the server-side, making use of the momentum strategy to update the server model. FedProx is an enhancement of FedAvg on the client-side, adding a penalty to the client updates. We have presented the details of FedAvg with elastic aggregation in Algorithm 1. Please refer to supplementary materials for more

[2] https://github.com/tensorflow/federated

details about the other federated optimizers with elastic aggregation. As shown in the left part of Tab. 2, elastic aggregation works well with other complementary approaches whether designed for client-side or server-side. In order to further verify the generality of elastic aggregation, we did further experiments on the stat-of-the-art methods in FL, i.e. SCAFFOLD (Karimireddy *et al.*, 2020), AdaOpt (Reddi *et al.*, 2020) and PFNM (Yurochkin *et al.*, 2019). Since these methods target on different aspect of federated learning, we conducted experiments based on their original experimental settings. As shown in middle part of Tab. 2, elastic aggregation plays a positive effect in most federated optimizers.

Influence on personalization Personalization in federated learning is also a topic of great concern. Specifically, we equipped Per-FedAvg (Fallah *et al.*, 2020), pFedMe (Dinh *et al.*, 2020) – both focus on the model personalization (local model performance) in federated learning with elastic aggregation and found that elastic aggregation is also do a favour in learning a better personalized model, as shown in the right part of Tab. 2.

Dataset	Rounds(Epochs)	Total	Sampled	Batch	Init. LR	Model	Naïve(%)	Elastic(%)
Balanced data across clients								
CIFAR-100	4000(∼80)	500	10	10	0.05	ResNet-20	32.31	**56.64**
Unbalanced data across clients								
MNIST	20(∼2)	1000	100	100	0.1	Logistic Regression	70.14	**73.64**
EMNIST	1000(∼2.5)	3400	10	100	0.1	2Conv+2Linear	88.71	**89.82**
CIFAR-10	4000(∼40)	1000	10	10	0.05	ResNet-20	66.84	**68.74**
CINIC-10	200(∼20)	1000	100	10	0.05	ResNet-20	35.81	**36.29**
CINIC-10	4000(∼40)	1000	10	10	0.05	ResNet-20	68.68	**69.25**

Table 3: Test accuracies for various datasets. Along with rounds, we list a more intuitive measure, i.e., epochs, to better reflect the total training computation.

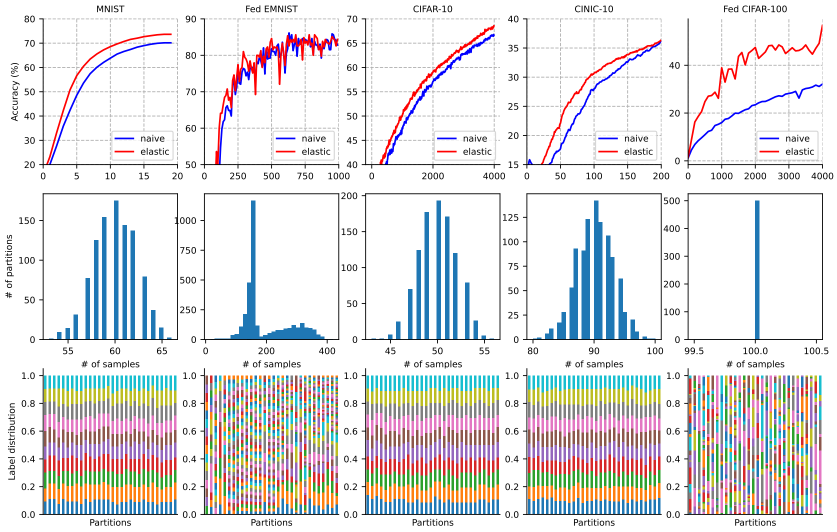

Figure 3: Convergence speeds on different datasets.

6 Conclusion

Our work studied the impact of heterogeneity on optimization methods for federated learning. We then proposed a new aggregation method, elastic aggregation, to overcome gradient dissimilarity by making use of parameter sensitivity. To the best of our knowledge, we are the first to use unlabeled client data to improve federated learning performances. Our empirical evaluations across a suite of federated datasets have validated our theoretical analysis and showed that the elastic aggregation approach significantly improves the convergence behavior of federated learning in realistic heterogeneous scenarios. Future directions include extending the elastic aggregation approach to adaptive optimization methods or gossip-based training methods.

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