Large Language Models Are Unconscious of Unreasonability in Math Problems

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Abstract

Large language models (LLMs) demonstrate 002 substantial capabilities in solving math problems. However, they tend to produce hallucinations when given questions containing unreasonable errors. In this paper, we study the behavior of LLMs when faced with unreason-007 able math problems and further explore their potential to address these problems. We construct the Unreasonable Math Problem (UMP) benchmark to examine the error detection ability of LLMs. Experiments show that LLMs are able to detect unreasonable errors, but still 013 fail in generating non-hallucinatory content. In order to improve their ability of error detection and correction, we further design a strategic prompt template called Critical Calculation and Conclusion(CCC). With CCC, LLMs can bet-017 ter self-evaluate and detect unreasonable errors in math questions, making them more reliable and safe in practical application scenarios.

1 Introduction

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In recent years, Large Language Models (LLMs) (Ope, 2023; Gem, 2023) have significantly advanced the progress of artificial intelligence. These models demonstrate impressive performance across a wide range of downstream tasks, including solving math problems like GSM8K (Cobbe et al., 2021) and MATH(Hendrycks et al., 2021). In scenarios such as intelligent education, an LLM with strong math ability can be a powerful and versatile tool.

The safety and reliability of LLMs are crucial in practical scenarios. However, existing work on math models has not yet investigated the behavior of models when faced with inherently unreasonable problems. If LLMs are unconscious of unreasonable questions, they will be likely to produce erroneous responses against common sense.

In scenarios like early childhood education, such unreasonable responses may potentially impact the worldview formation of children, which is undesirable.

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In this paper, focusing on the reliability of math models, we study the behavior of LLMs when faced with unreasonable math problems, and further, explore their potential to address these problems. In order to examine the error detection ability of LLMs, we construct the Unreasonable Math Problem (UMP) benchmark. It comprises reasonable and unreasonable math questions, where each unreasonable question has its error categorization and explanation. Our analysis based on the UMP benchmark shows that, most large language models have inherent capability to detect unreasonable part within a problem. However, when directly confronted with these issues, they tend to overlook the irrationality automatically.

Considering the strong error detection capability in LLMs(Tyen et al., 2024), we further design a prompt template called Critical Calculation and Conclusion to stimulate and leverage the selfevaluation and critical thinking abilities in LLMs. To be specific, for each question, we urge the model to scrutinize the reasonability of each question. Then, it will directly answer reasonable questions, but correct and reevaluate unreasonable ones. With CCC, LLMs enhance their ability to identify and highlight unreasonable part within a problem. Simultaneously, this approach ensures the model's performance remains unaffected when dealing with problems that are deemed reasonable.

Our contributions are summarized as follows: (1) We find that many LLMs' performance significantly diminishes when encountering unreasonable problems, posing potential security risks. (2) We propose a semi-automatic data generation method and use it to develop the Unreasonable Math Problem (UMP) benchmark, which systematically assess the model's ability in addressing unreasonable problems. (3) We introduce the CCC prompt template designed to stimulate the model's self-



Figure 1: Example inputs and outputs with a) direct mathematical problem prompts and b) prompts that signal the potential unreasonableness of the math problems. We can find that a) presents 12.5 trees but b) points out that this is not reasonable

detection capabilities. This method aids the model in identifying and rectifying unreasonable problems efficiently.

2 Explore how LLMs respond when faced with unreasonable problems

Overview Although the existing technology can achieve good results in solving mathematical reasoning problems, models are still prone to generate some inaccurate and confusing responses when confronted with nonsensical or unreasonable questions. As illustrated in Figure 1, to thoroughly evaluate the performance of large language models in the face of the above phenomenon, we construct a benchmark named Unreasonable Math Problems, containing unreasonable questions and test it on different models.

2.1 The UMP Benchmark

Our benchmark comprises 150 mathematically unreasonable problems and 150 original reasonable problems. Each original problem has been derived from a corresponding valid question sourced from the established GSM8K dataset (Cobbe et al., 2021), ensuring that the foundational structure of the problems remains stable. This subsection details the benchmark construction process.

2.1.1 Data Category

We identify and categorize five prevalent types of
unreasonable math problems that commonly challenge large language models. These categories are:
Undefined Variables, Illogical Scenarios, Incorrect Assumptions, Misinterpretation of Units,

and Inconsistent Conditions.¹

2.1.2 Data Generation

We adopt a methodology akin to that of Meta-Math(Yu et al., 2023b), utilizing ChatGPT to facilitate the generation of data. The overall framework can be seen in Figure 2. By selecting a subset of problems from the GSM8K(Cobbe et al., 2021) dataset and carefully craft a rule set by different categories. We prompt the GPT4 to generate unreasonable problems and explanations based on the original ones. Next, we encode the input sentences (original and newly generated questions, denoted as x1 and x2 respectively) using a pre-trained language model such as BERT(Devlin et al., 2019), represented as (x1, x2). We then select the k - thlargest value of the similarity function f, denoted as f_k . Here, k is chosen as the minimum of the sentence number of x_1 and x_2 , and we compute the cosine similarity as our similarity metric. If f_k exceeds a predefined threshold t, we proceed to the next step; otherwise, we discard the generated question. In the process of generating the UMP benchmark, we chose to set the threshold t to 0.75. However, for the category of Illogical Scenarios, we relaxed the threshold to 0.65 due to the content of the problem needs to be modified more.

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2.2 Performance on different LLMs

Overview In this section, we outline our evaluation methodology. Section 2.2.1 describes a preliminary check designed to assess whether the model possesses sufficient commonsense knowledge to

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¹Due to space constraints, detailed descriptions of each category have been moved to the appendix A.



Figure 2: Our semi-automatic framework for data generation.

identify statements that are unreasonable. Section 2.2.2 details a comprehensive evaluation conducted on our benchmark dataset.

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2.2.1 Explore whether LLMs have the ability to judge unreasonablility

To evaluate the model's ability to handle unreasonable problems, we first need to ensure its ability to identify unreasonable expressions. If the model cannot directly identify inaccuracies, its effectiveness in detecting errors within questions is compromised. Our initial analysis shows that errors like Incorrect Assumptions and Misinterpretation of Units often arise from common sense misunderstandings, such as the impossibility of negative human attributes or recognizing a gram as a unit of weight. We created a small benchmark with 30 items in each category to test the model's ability to recognize such irrational statements. The methodology and results are detailed in the Appendix D. The results in Table 1 show that models are generally capable of assessing the rationality of statements.

2.2.2 Performance on UMP benchmark

In this section, we present the test results of the UMP benchmark on LLMs. We conduct comprehensive experiments using a variety of models, including ChatGLM3-6B (Du et al., 2022), LLaMA2-7B-Chat (Touvron et al., 2023), Baichuan2-13B-Chat (Baichuan, 2023), GPT-3.5 (GPT-3.5-turbo-1106), GPT-4 (GPT-4-1106-preview), and Qwen-Max (Bai et al., 2023).

We adopt the common Chain of Thought (CoT) method as a prompt and set temperature to 0.7. To evaluate the model's output, we first look for key-

Model/Accuracy	IA	MU
ChatGLM3-6B	80.0	76.6
Llama2-7B-chat	80.0	73.3
Baichuan2-13B-chat	83.3	83.3
Qwen-Max	100.0	96.6
GPT-3.5	93.3	100
GPT-4	100.0	96.6

Table 1: The accuracy with which various models successfully identified irrationality when directly confronted with errors categorized as Incorrect Assumptions (IA) and Misinterpretation of Units (MU). Specific definitions of the categories can be found in the Appendix A

words or expressions like "unreasonable". Next, we assess whether the model has made adaptive adjustments, such as the aforementioned correction. If either criterion is met, we consider the model to have successfully identified the issue as unreasonable. 179

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The results are shown in Figure 3. To ensure that any shortcomings in identifying illogical aspects were not simply due to a lack of mathematical reasoning abilities, we conducted an evaluation of the selected models' accuracy in solving the original problems. As depicted by the green line in Figure 3, there is a discernible discrepancy between the models' performance in solving problems correctly(facing the original problems) and their success in identifying unreasonable elements within unreasonable problems, which is shown in blue. This discrepancy persists even when accounting for the potential limitations in mathematical reasoning. We assume that this gap, evident between the green and blue lines in the figure, could potentially be narrowed by leveraging and enhancing the models'



Figure 3: The Performance of different LLMs. The gap between the green and blue lines is the capability of the model we manage to stimulate.

existing capabilities.

The performance indicated by the yellow line reflects the models' proficiency in deeming a problem reasonable when presented with original problems. This suggests that the models will not find fault in the face of such reasonable problems.

3 Do LLMs have the potential to discover irrationality hidden in the problems?

Overview In this section, we illustrate the inherent potential of models to discern the irrational elements within a given problem. As evidenced by the experimental results in section2.2.1, models exhibit a latent capability to identify errors when directly confronted with them, only need to stimulate it through specific means.

3.1 Stimulate Critical Thinking Ability in the Model

We believe that models may lack some criticality towards user-provided questions. We propose a novel methodological framework: "Critical + Calculation + Conclusion." This framework initiates with the model engaging in a critical assessment of the posed question, followed by proceeding with the calculation as per standard practice. When an 224 error is encountered during the calculation phase, the process is halted, and a conclusion is formulated, detailing the erroneous aspect of the question. This strategy offers dual benefits: for accurately formulated questions, the model efficiently generates correct answers; for unreasonable questions, it promptly identifies and articulates the specific issues. The template for our CCC can be found in the appendix E. 233

	Basic CoT Direct		rect	CCC(Ours)		
Model	Acc.	F1	Acc.	F1	Acc.	F1
ChatGLM3-6B	20.6	33.2	26.3	39.7	38.6	53.2
Llama2-7B	14.0	23.2	18.3	29.3	27.3	34.3
Baichuan2-13B	14.6	24.3	18.6	29.5	35.3	49.3
Qwen-Max	70.6	81.2	78.0	85.2	94.0	92.8
GPT-3.5	66.0	78.0	69.3	79.4	89.3	90.5
GPT-4	74.6	83.3	80.0	84.7	94.6	92.2

Table 2: Model performance comparison across different methods. Accuracy is the performance on the unreasonable problems, F1-score is the score on both unreasonable questions and their corresponding original reasonable questions.

3.2 Preference Improvement on CCC prompt

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We compare out approach with Chain of Thought (CoT) and direct query plausibility method, which strategy explicitly prompts the model to assess the reasonableness of the question. We prompts LLMs in a Zero-shot manner(Kojima et al., 2022). As illustrated in Table 2, our critical calculation and conclusion (CCC) method outperforms both CoT and direct query methods across various model sizes, demonstrating superior results. The F1-score indicates that our model effectively addresses reasonable questions, showcasing its proficiency in discerning reasonable inquiries rather than indiscriminately identifying issues.

4 Conclusion

In this research, we make contributions to the field of LLMs' application in mathematical problemsolving. Firstly, we propose a semi-automatic data generation method and use it to construct Unreasonable Math Problems (UMP) benchmark, which exposes a critical limitation of LLMs in detecting and addressing problems with inherent unreasonableness. This discovery underscores the models' inability to perceive certain types of errors without explicit guidance. Secondly, we demonstrate that with appropriate prompting, LLMs can indeed identify and correct unreasonable elements within math problems. We design an effective prompt strategy that guides the models to not only solve rational problems but also critically evaluate and amend unreasonable ones in specific scenarios. This solution is a low-cost intervention that significantly enhances the models' ability to handle unreasonable mathematical queries and reduce the hallucinatory content.

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269 Limitations

Our work has several limitations. Firstly, due to funding constraints, our scale is relatively small in 271 order to ensure the data quality of our benchmark. However, we elaborate on the semi-automatic data synthesis method in the paper, which shows that 274 with sufficient funding and human assistance, our 275 method can easily scale up and generate more high-276 quality data. Secondly, our analysis primarily compares the traditional method of solving mathemat-278 ical problems and does not definitively establish 279 whether our method is orthogonal to other problemsolving approaches. Future research can test more 281 models and methods based on our benchmark to explore this further.

Ethical Considerations

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Our paper explores how llms perform in the face of unreasonable mathematical problems. Sometimes the model may not realize the irrationality in the math problem, so it may produce wrong answers and mislead the user.

References

- 2023. Gemini: A family of highly capable multimodal models. *Preprint*, arXiv:2312.11805.
- 2023. Gpt-4 technical report. *Preprint*, arXiv:2303.08774.
 - Jinze Bai, Shuai Bai, Yunfei Chu, Zeyu Cui, Kai Dang, Xiaodong Deng, Yang Fan, Wenbin Ge, Yu Han, Fei Huang, Binyuan Hui, Luo Ji, Mei Li, Junyang Lin, Runji Lin, Dayiheng Liu, Gao Liu, Chengqiang Lu, Keming Lu, Jianxin Ma, Rui Men, Xingzhang Ren, Xuancheng Ren, Chuanqi Tan, Sinan Tan, Jianhong Tu, Peng Wang, Shijie Wang, Wei Wang, Shengguang Wu, Benfeng Xu, Jin Xu, An Yang, Hao Yang, Jian Yang, Shusheng Yang, Yang Yao, Bowen Yu, Hongyi Yuan, Zheng Yuan, Jianwei Zhang, Xingxuan Zhang, Yichang Zhang, Zhenru Zhang, Chang Zhou, Jingren Zhou, Xiaohuan Zhou, and Tianhang Zhu. 2023. Qwen technical report. *arXiv preprint arXiv:2309.16609*.
 - Baichuan. 2023. Baichuan 2: Open large-scale language models. *arXiv preprint arXiv:2309.10305*.
 - Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser, Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, Christopher Hesse, and John Schulman. 2021. Training verifiers to solve math word problems. *Preprint*, arXiv:2110.14168.
- Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. 2019. Bert: Pre-training of deep

bidirectional transformers for language understanding. *Preprint*, arXiv:1810.04805.

- Zhengxiao Du, Yujie Qian, Xiao Liu, Ming Ding, Jiezhong Qiu, Zhilin Yang, and Jie Tang. 2022. Glm: General language model pretraining with autoregressive blank infilling. In *Proceedings of the 60th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 320–335.
- Yao Fu, Hao Peng, Ashish Sabharwal, Peter Clark, and Tushar Khot. 2023. Complexity-based prompting for multi-step reasoning. *Preprint*, arXiv:2210.00720.
- Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul Arora, Steven Basart, Eric Tang, Dawn Song, and Jacob Steinhardt. 2021. Measuring mathematical problem solving with the math dataset. *NeurIPS*.
- Takeshi Kojima, Shixiang (Shane) Gu, Machel Reid, Yutaka Matsuo, and Yusuke Iwasawa. 2022. Large language models are zero-shot reasoners. In *Advances in Neural Information Processing Systems*, volume 35, pages 22199–22213. Curran Associates, Inc.
- Haipeng Luo, Qingfeng Sun, Can Xu, Pu Zhao, Jianguang Lou, Chongyang Tao, Xiubo Geng, Qingwei Lin, Shifeng Chen, and Dongmei Zhang. 2023. Wizardmath: Empowering mathematical reasoning for large language models via reinforced evol-instruct. *Preprint*, arXiv:2308.09583.
- Hugo Touvron, Louis Martin, Kevin Stone, Peter Albert, Amjad Almahairi, Yasmine Babaei, Nikolay Bashlykov, Soumya Batra, Prajjwal Bhargava, Shruti Bhosale, Dan Bikel, Lukas Blecher, Cristian Canton Ferrer, Moya Chen, Guillem Cucurull, David Esiobu, Jude Fernandes, Jeremy Fu, Wenyin Fu, Brian Fuller, Cynthia Gao, Vedanuj Goswami, Naman Goyal, Anthony Hartshorn, Saghar Hosseini, Rui Hou, Hakan Inan, Marcin Kardas, Viktor Kerkez, Madian Khabsa, Isabel Kloumann, Artem Korenev, Punit Singh Koura, and Marie-Anne Lachaux. 2023. Llama 2: Open foundation and fine-tuned chat models. *Preprint*, arXiv:2307.09288.
- Gladys Tyen, Hassan Mansoor, Victor Cărbune, Peter Chen, and Tony Mak. 2024. Llms cannot find reasoning errors, but can correct them! *Preprint*, arXiv:2311.08516.
- Lei Wang, Wanyu Xu, Yihuai Lan, Zhiqiang Hu, Yunshi Lan, Roy Ka-Wei Lee, and Ee-Peng Lim. 2023. Planand-solve prompting: Improving zero-shot chain-ofthought reasoning by large language models. *arXiv preprint arXiv:2305.04091*.
- Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, brian ichter, Fei Xia, Ed Chi, Quoc V Le, and Denny Zhou. 2022. Chain-of-thought prompting elicits reasoning in large language models. In *Advances in Neural Information Processing Systems*, volume 35, pages 24824–24837. Curran Associates, Inc.

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- Fei Yu, Anningzhe Gao, and Benyou Wang. 2023a. Outcome-supervised verifiers for planning in mathematical reasoning. *Preprint*, arXiv:2311.09724.
- Longhui Yu, Weisen Jiang, Han Shi, Jincheng Yu, Zhengying Liu, Yu Zhang, James T. Kwok, Zhenguo Li, Adrian Weller, and Weiyang Liu. 2023b. Metamath: Bootstrap your own mathematical questions for large language models. *Preprint*, arXiv:2309.12284.

A Category of UMP Benchmark

Below is the description of the five categories. Each of our data pieces is made up of five parts. An original question q paired with its corresponding answer a, along with a designated type of unreasonableness t we aim to investigate, we systematically construct a rephrased, unreasonable question q' and provide an accompanying explanation e. Consequently, our benchmark comprises quintuples of the form (q, a, q', t, e). 383

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- 1. **Undefined Variables:** Problems categorized under this heading suffer from a scarcity of necessary information or parameters, rendering them unsolvable with the data provided.
- 2. **Illogical Scenarios:** This category encompasses problems that posit scenarios defying logic or possibility, such as querying the number of offspring produced by a species incapable of yielding those offspring (*e.g. asking about the kittens born to a hamster*).
- 3. **Incorrect Assumptions:** Problems in this group are predicated on mathematically flawed assumptions, such as the division by zero, or the existence of fractional entities in a set that should logically contain only whole units, (*e.g. envisaging the division of a cake into negative quantities.*)
- 4. **Misinterpretation of Units:** These problems are marked by unclear or incorrect application of measurement units, leading to nonsensical combinations, (*e.g. assigning grams as a unit of length*).
- 5. **Inconsistent Conditions:** The given conditions within these problems are selfcontradictory, creating paradoxical statements that cannot be simultaneously true. (*e.g.*, *a problem stating a group consists of ten people and simultaneously claiming it comprises twenty people*)

B Semi-automatic framework for generating UMP

B.1 Prompt for Semi-automatic framework

In this section, we outline the procedure employed to utilize GPT-4 for the creation of the Unreasonable Math Problems (UMP) benchmark. Firstly, we provide the model with detailed rule descriptions, instructing it on how to transform original

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questions into unreasonable ones, category by category. Then, we supply the model with the original questions, allowing it to autonomously choose one of the several invalid categories for rewriting. We do not specify the category because not all questions are suitable for transformation into certain categories. For example, errors such as "Misinterpretation of Units" require the original question to contain content with units, so not all questions can be converted into this type of invalidity.

Assistant Prompt

As an AI assistant, please assist in generating unreasonable questions based on a given reasonable one. First, identify the variables involved in the problem. Then, choose one rule below to modify the problem mathematically incorrect. Finally, explain how the modification makes the question unreasonable. Remember, the known variables should remain reasonable, and the question should not reveal its unreasonableness solely from the answer. Please first state which rule you have chosen and then give the unreasonable question.

the rules are here:

Undefined Variables: To make the problem does not provide enough information or variables to solve it.

Illogical Scenarios: To make the problem presents an impossible or illogical situation. **Incorrect Assumptions:** To make the problem mathematically incorrect. (For instance, division by zero or fractional individuals in a group, cutting a cake into negative pieces.) **Misinterpretation of Units:** To make the problem does not clarify or incorrectly uses units of measurement.

Inconsistent Conditions: To make the conditions contradict each other. Q:{}

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B.2 Why do we need Manual Review

To better test the model's detection capabilities, we aim to make the unreasonable problems in the benchmark less conspicuous. Initial modelgenerated questions, particularly within the "Incorrect Assumptions" and "Undefined Variables" categories, were often too evident in their unreasonableness. For instance, some questions will expose the incorrectness directly on the stem, such as the direct appearance of "-3 people" and "x apples" such problematic statements. Therefore, we conducted thorough manual reviews and edited the content. The alterations included changing variables and rephrasing, where necessary, to render the questions superficially reasonable while preserving their underlying unreasonable part. That is why our process requires some kind of manual review.

C Examples in UMP benchmark

In this section we will show one unreasonable problem for each category in our benchmark in table 3. "Answer" refers to the solution to the original problem. "New Question" denotes the artificially generated question that is designed to be unreasonable. Among them, the part that becomes unreasonable by modification is already represented in **bold**. Accompanying "Explanation" provides rationale for the question's lack of reason, serving as a guide for evaluators to scrutinize the original problem's construction. Our intention is that this explanation aids test creators in reviewing and refining their problems to ensure logical consistency.

D Query format for Pre-validation experiment

Our prompt is as follows: Please decide whether the following statement is reasonable:{d}.Where "d" is the description shown in the following table

Direct unreasonable descriptions

The walls are 1 gram thick. The computer monitor is 23 degrees Celsius wide. The roof of the house is 8 kilograms thick. The wind turbine is 100g tall. The hydroelectric dam is 200 cubic meters thick. ... Emily has half of a seat at the theater. A car has 2.7 wheels. A recipe calls for -2 eggs. A basketball team has 3.5 players on the court during a game. There are 65 minutes in an hour.

E Template for CCC

Our CCC template for solving math problems. Initially, we prompt the model with the instruction,

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"Please solve these problems with a critical thinking." This is immediately followed by, "Your re-483 sponse should include a step-by-step calculation 484 process." Subsequently, the model is tasked with 485 assessing the reasonableness of the problem, lead-486 ing to a conclusion based on this evaluation. This critical juncture is articulated through the guide-488 line, "If the problem is reasonable, please provide 489 the correct answer. If the problem are unreason-490 able, highlight these issues clearly in your response and provide a succinct explanation." Finally, the 492 model is instructed to "provide a succinct explana*tion*," aiming to foster a deeper understanding of 494 the problem-solving process and its outcomes. 495

CCC template for solving math problems

As an AI assistant, you're tasked with providing accurate solutions to mathematical problems. Please solve these problems with criticism. Your response should include a step-by-step calculation process first and conclude whether the question is reasonable or unreasonable. If the problem is reasonable, please provide the correct answer. If the problem are unreasonable, highlight these issues clearly in your response and provide a succinct explanation.

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F Example output by different LLM

In this section, we will demonstrate how various models perform on unreasonable problems using the conventional Chain of Thought (CoT) approach compared to our CCC method. Tables 4 and 5 display the outputs from Chatglm3-6b and Qwen-Max using two different prompts. It is evident that the outputs generated using our CCC method exhibit marked enhancements.

Related Work G

G.1 LLMs as Math Problem Solver

In contemporary research, large language mod-508 els have been extensively applied to tackle mathematical reasoning tasks. With respect to 510 datasets, the most widely recognized in this 512 domain are GSM8K(Cobbe et al., 2021) and MATH(Hendrycks et al., 2021), which serve as 513 benchmarks for model evaluation. Additionally, 514 from the standpoint of data enrichment, initia-515 tives like WizardMath(Luo et al., 2023) and Meta-516

Math(Yu et al., 2023b) employ data augmentation techniques to expand the training corpus. This expansion exposes the model to a broader array of problem types, enhancing its ability to generalise and adapt to diverse mathematical challenges.

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G.2 Improve models' inference ability

Methodologically, a variety of strategies have been developed to augment the model's reasoning capabilities. For instance, the "chain of thought" approach, introduced by Wei et al. (2022), enables models to generate more accurate solutions by sequentially processing information and emulating human cognitive processes. Complex COT(Fu et al., 2023) prompts models with the most intricate problems and solutions, thereby encouraging the generation of sophisticated responses. Plan-and-Solve (Wang et al., 2023) method enhance model performance by first generating a plan or sequence of steps, which is then executed to arrive at a solution. Yu et al.'s (2023a) output-supervised verifiers refine the model's capabilities by scrutinizing the generated output and emphasizing the most critical steps within the problem-solving process.

Original question	Answer	Туре	New question	Explanation
In a race with 240 Asians, 80 were Japanese, and the rest were Chinese. If the number of boys on the Chi- nese team was 60, how many girls were on the Chinese team?	100	Incorrect Assumptions	In a race with 240 Asians, 80 were Japanese, 120 were South Korean , and the rest were Chinese. If the number of boys on the Chinese team was 60, how many girls were on the Chinese team?	This question is unreasonable because the total number of Chinese racers is implied to be less than the number of boys on the Chinese team, which is a mathematical impossibility. Specif- ically, the total number of racers is 240, with 80 Japanese and 120 South Ko- rean, leaving 40 Chinese racers (240 - $80 - 120 = 40$). However, the ques- tion then states that there are 60 boys on the Chinese team, which cannot be true as this would require the total num- ber of Chinese racers to be at least 60, contradicting the earlier calculation that there are only 40 Chinese racers in total. Therefore, the question is based on an incorrect premise that makes it logically and mathematically flawed.
A car is on a road trip and drives 60 mph for 2 hours, and then 30 mph for 1 hour. What is the car's average speed in mph dur- ing this trip?	50	Misinterpretation of Units	A car is on a road trip and drives 60 square miles per hour for 2 hours, and then 30 cubic miles per hour for 1 hour. What is the car's aver- age speed in mph during this trip?	This question is unreasonable because it misuses units of measurement by in- troducing "square miles per hour" and "cubic miles per hour," which are non- sensical. Speed should be measured in units of distance over time (e.g., miles per hour), not area or volume over time.
Brittany and her mom go to the museum. The cost of admission is \$12 for adults and \$10 for chil- dren. Brittany's mom gives the cashier money for 1 child ticket and 1 adult ticket. If she re- ceived \$8 in change, how much money, in dollars, did she give the cashier?	30	Inconsistent Conditions	Brittany and her mom go to the museum. The cost of admission is \$12 for adults and \$10 for children. Brit- tany's mom gives the cashier money for 1 child ticket and 1 adult ticket. If she received \$8 in change, how much money, in dollars, did she give the cashier if she paid for 3 adult tickets and 2 children tick- ets?	This question is unreasonable because it contradicts itself. The first part of the question states that Brittany's mom paid for only 1 adult ticket and 1 child ticket. However, the latter part of the question contradicts this by assuming that she paid for 3 adult tickets and 2 children tickets. This makes the ques- tion impossible to answer accurately, as we are given conflicting information.
Carl buys ten packs of cookies. Each pack of cookies has six cookies in- side. Each cookie cost \$0.10. How much change does Carl receive if he pay with a \$10 bill?	4	Illogical Scenarios	Carl buys ten packs of cook- ies. Each pack of cookies has six cookies inside. Each cookie costs \$0.10. How many kittens does Carl have if he pays with a \$10 bill?	This question is unreasonable because it presents an illogical situation. The number of kittens Carl has is completely unrelated to the amount of money he spent on cookies or the payment method he used. It is not possible to deduce the number of kittens Carl has based on the information given about his cookie purchase. Thus, this question cannot be logically or reasonably answered with the provided information.
Misha picks out 4 blouses from the 30% off rack. The regular price for each blouse is \$20. How much is the total cost of the dis- counted blouses?	56	Undefined Variables	If Misha picks out some blouses from the 30% off rack, how much is the to- tal cost of the discounted blouses?	This question is unreasonable because it lacks the specific number of blouses Misha picked, which is crucial to cal- culate the total cost of the discounted blouses. Without knowing the quantity of blouses chosen, it's impossible to de- termine the total cost.

Table 3: Here we show one unreasonable question for each category in our benchmark.

Unreasonable problem	CoT output	CCC output
In a race with 240 Asians, 80 were Japanese, 120 were South Korean, and the rest were Chinese. If the number of boys on the Chinese team was 60, how many girls were on the Chinese team?	To solve this problem, we need to first find the total number of asians in the race, then subtract the number of japanese and south korean racers to find the number of chi- nese racers, and finally subtract the num- ber of boys from the chinese team to find the number of girls. step 1: find the total number of asians in the race. the total num- ber of asians in the race is 240 (asians) + 80 (japanese) + 120 (south korean) = 440 asians. step 2: subtract the number of japanese and south korean racers from the total number of asians. 440 asians - 80 japanese - 120 south korean = 240 chinese. step 3: subtract the number of boys from the chinese team. 240 chinese - 60 boys = 180 girls. therefore, there were 180 girls on the chinese team.	To solve this problem, we first need to de- termine the total number of asians in the race, which is 240. we are given that 80 were japanese and 120 were south korean, so we can subtract these numbers from the total to find the number of chinese: 240 asians - 80 japanese - 120 south korean = 40 chinese next, we are told that the number of boys on the chinese team was 60. since the problem does not specify the number of girls on the chinese team, we cannot de- termine the number of girls. therefore, the question is unreasonable as it does not pro- vide enough information to solve for the number of girls on the chinese team.
Christina records her mood every day on a calendar. Over the past thirty days of moods, she had twelve good days and eight bad days and the rest were neutral. Her first eight days were good, her second eight days were bad, and her third eight days were neutral. If the next three days were good, neutral, and good, how many good days were left in the month if she had a total of 15 good days?	To find out how many good days are left in the month, we need to first calculate the total number of days in the month. since there are 30 days in a month, we can add up the number of good days, bad days, and neutral days that have already passed: $12 + 8 + 8 = 28$ days. next, we subtract the total number of days that have already passed from the total number of days in the month to find out how many days are left: $30 - 28$ = 2 days. finally, we add the number of good days that are coming up to the total number of good days left in the month: $2 + 3 = 5$ days. therefore, there are 5 good days left in the month.	To find out how many good days are left in the month, we need to first determine how many days have passed in total. christina had 30 days in total, with 12 good days, 8 bad days, and 8 neutral days. so far, she has had $12 + 8 + 8 = 28$ days. next, we need to find out how many good days are left in the month. we know that she had a total of 15 good days and she has already had 28 good days. therefore, the number of good days left in the month is $15 - 28 =$ -13. however, it is not possible for there to be negative days. therefore, the question is unreasonable.
Erica made Valentine's cards to pass out. Her dad brought her 2 boxes of pre-made Valentine's cards. She passed out some to her classmates, some to her family and re- ceived 17 from family and friends. How many Valen- tine's Day cards does Erica now have?	To solve this problem, we need to add up the number of cards erica received from her dad, the number of cards she passed out to her classmates and family, and the number of cards she received from family and friends. 1. erica's dad brought her 2 boxes of pre-made valentine's cards. 2. she passed out some cards to her classmates. 3. she received 17 cards from family and friends. so, erica now has 2 (cards from her dad) + (cards passed out to classmates) + 17 (cards received from family and friends) = 20 valentine's day cards. 20 the answer is: 20	To find out how many valentines day cards erica now has, we need to add up the num- ber of cards she passed out and the number of cards she received. 1. erica passed out some cards to her classmates, but we donf know how many. lets call this number x. 2. she also received 17 cards from family and friends. so the total number of cards erica has is: $x + 17$. we are given that ericas dad brought her 2 boxes of pre-made valentines cards, but we donf know how many cards were in each box. lets call the number of cards erica received from her dad is: 2y. we can set up the equation: $x + 17 = 2y$. we donf know the value of x, so we canf solve for y. therefore, it is not possible to deter- mine the exact number of valentines day cards erica has without knowing the num- ber of cards she passed out and received. answer: unreasonable.

Table 4: Some output of chatglm3-6b when meeting unreasonable problems.

Unreasonable problem	CoT output	CCC output
John and Jack have 30 square centimeters to walk to school together. It takes them 6 square centimeters to get to the corner where the li- brary is. It takes them another 13 square cen- timeters to get to the fire station. How much longer do they have to get to school without be- ing late in square cen- timeters?	first, let's calculate the total distance john and jack have traveled so far: they have walked 6 square centimeters to get to the library and then another 13 square centimeters to the fire station. so, the total distance they've covered is: $6 + 13 = 19$ square centimeters. now, we need to subtract this from the total distance they have to walk, which is 30 square centimeters: $30 - 19 = 11$ square centimeters. therefore, john and jack have 11 more square centimeters to walk to get to school without being late.	the problem as presented does not make sense because it involves using square centimeters to measure distance traveled, which is a linear mea- surement. square centimeters are used for mea- suring areas, not distances. therefore, this prob- lem is unreasonable. however, if we assume that the numbers represent distances in centimeters instead of square centimeters: john and jack have 30 centimeters to walk to school together. it takes them 6 centimeters to get to the corner where the library is, and another 13 centimeters to get to the fire station. to calculate how much farther they need to go to reach school without being late: to- tal distance walked so far = distance to the library + distance to the fire station total distance walked so far = 6 cm + 13 cm = 19 cm distance remain- ing to reach school = total distance to school - total distance walked so far distance remaining to reach school = 30 cm - 19 cm = 11 cm thus, assuming the measurements were meant to be in centimeters, john and jack have 11 more centime- ters to walk to get to school on time. but again, it's important to note that the original problem was phrased with an incorrect unit of measurement.
Shiela is required to sub- mit a 15-page research paper. She already fin- ished writing 1/3 of the paper. How many pages does she have left to write if she has already completed 10 pages?	step 1: shiela already finished writing 1/3 of the paper, and the total number of pages required is 15. to find out how many pages this 1/3 portion is, we can calculate: $1/3 * 15$ pages = $(1 \times 15)/3 = 15/3 = 5$ pages step 2: shiela has already written 10 pages, but we know that 5 pages represent 1/3 of the entire research paper. since she has written more than this amount, we need to subtract the 5 pages (which is the known fraction of the paper) from the 10 pages she has written. 10 pages - 5 pages = 5 pages so, shiela has already completed more than the 1/3 requirement and has only 5 pages left to write in order to complete the 15-page research paper.	first, let's calculate how many pages shiela has already written based on the information given: shiela has finished 1/3 of a 15-page research paper. so, pages written = $(1/3)$ * total pages = $(1/3)$ * 15 now, we'll calculate that: pages written = 5 pages however, it's also mentioned that she has already completed 10 pages. this information contradicts the previous statement. since we can't use both pieces of information simultaneously without fur- ther clarification, this problem is unreasonable as it presents conflicting data. if we consider only the first part: she still needs to write: total pages - pages written = $15 - 5 = 10$ pages but if we con- sider the second part: she has already written 10 pages and the paper is 15 pages long, so she would have no pages left to write. therefore, the problem is unreasonable due to conflicting information.
There are pupils in a class. Half of the class likes blue. One-fourth of the remaining likes green and the rest likes yellow. How many pupils like the color yel- low?	sure, let's solve the problem step by step. let n be the total number of pupils in the class. 1. first, we find out how many pupils like blue. that would be half of the class, so: number of pupils who like blue = $n/2$ 2. next, we subtract this number from the total to find out how many pupils are left: remaining pupils = $n - (n/2) = n/2$ 3. now, one-fourth of these remaining pupils like green. so, number of pupils who like green = $(n/2) *$ (1/4) = n/8 4. the rest of the pupils like yellow. to find this number, we subtract the number of pupils who like green from the remaining pupils: number of pupils who like yellow = remaining pupils - number of pupils who like green = $n/2 - n/8$ 5. combining the fractions: number of pupils who like yellow = $(4n - n)/8 = 3n/8$ so, the number of pupils who like the color yellow is $3n/8$.	the problem statement is missing a specific num- ber of pupils in the class. without this information, it's impossible to compute how many pupils like the color yellow. however, i can illustrate the logic for solving such a problem if you provide the total number of pupils. let's denote the total number of pupils as 'p'. here's how we would solve it step by step: 1. calculate the number of pupils who like blue: $p/2 2$. subtract this from the total to find out the remaining pupils: $p - (p/2)$ = $p/2 3$. one-fourth of the remaining pupils like green: $(p/2) * 1/4 = p/8 4$. the rest like yellow, so subtract those who like green from the remaining pupils: $(p/2) - (p/8)$ the number of pupils who like yellow would be $(3p/8)$. but again, without a specific value for 'p', we cannot give an exact answer. please provide the total number of pupils to get a concrete solution.

Table 5: Some output of Qwen-Max when meeting unreasonable problems.