COIN: COUNTING THE INVISIBLE REASONING TOKENS IN COMMERCIAL OPAQUE LLM APIS

Anonymous authors
Paper under double-blind review

ABSTRACT

As post-training techniques evolve, large language models (LLMs) are increasingly augmented with structured multi-step reasoning abilities, often optimized through reinforcement learning. These reasoning-enhanced models outperform standard LLMs on complex tasks and now underpin many commercial LLM APIs. However, to protect proprietary behavior and reduce verbosity, providers typically conceal the reasoning traces while returning only the final answer. This opacity introduces a critical **transparency gap**: users are billed for invisible reasoning tokens, which often account for the majority of the cost, yet have no means to verify their authenticity. This opens the door to *token count manipulation*, where providers may overreport token usage or inject synthetic, low-effort tokens to inflate charges, a threat that can be carried out at *near-zero cost*. To address this issue, we propose `CoIn`, a verification framework that audits both the *quantity* and *semantic validity* of hidden tokens. `CoIn` constructs a verifiable hash tree from token embedding fingerprints to check token counts, and uses embedding-based relevance matching to detect fabricated reasoning content. Experiments demonstrate that `CoIn`, when deployed as a trusted third-party auditor, can effectively detect token count manipulation with a success rate reaching up to 94.7%, showing the strong ability to restore billing transparency in opaque LLM services. The dataset and code are available at `https://anonymous.4open.science/r/LLM-Auditing-CoIn-20F0`.

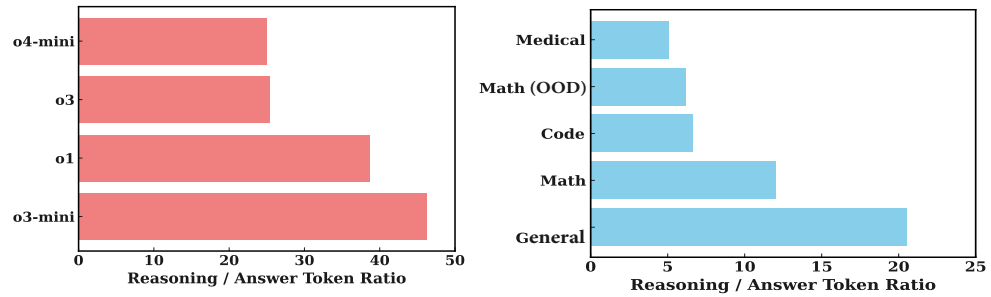
1 INTRODUCTION

Large language models (LLMs) have achieved significant advances in recent years. Yet, as pretraining begins to saturate available data resources (Zoph et al., 2020), the research community has increasingly turned to inference-time innovations (Hu et al., 2023; Kumar et al., 2025). Among these, reinforcement learning (RL)-optimized reasoning models have shown promise by generating longer, structured reasoning traces that improve performance, particularly in tasks involving mathematics and code (Guo et al., 2025; Muennighoff et al., 2025). Such models, exemplified by DeepSeek-R1 (Guo et al., 2025) and ChatGPT-O1 (Jaech et al., 2024), demonstrate that scaling at inference time can yield new capabilities without further pretraining.

With this shift, providers like OpenAI increasingly adopt new service models. Reasoning traces, while critical for quality, are often verbose, sometimes speculative (Jin et al., 2024; Zhang et al., 2025), and may reveal internal behaviors vulnerable to distillation (Gou et al., 2021; Sreenivas et al., 2024). To protect proprietary methods and streamline outputs, commercial APIs typically suppress these intermediate steps, exposing only the final answer. However, users are still charged for all generated tokens, including those hidden from view. We refer to such services as **Commercial Opaque LLM APIs (COLA)**—proprietary, pay-per-token APIs that conceal intermediate reasoning text.

A natural consequence of this design is a **verification gap**: users have no means to verify token usage or detect overbilling. Because reasoning tokens often outnumber answer tokens by more than an order of magnitude (Figure 1), this invisibility allows providers to **misreport token counts** or **inject low-cost, fabricated reasoning tokens to artificially inflate token counts**. We refer to this practice as **token count manipulation**. For instance, a single ARC-AGI run by OpenAI's o3 model consumed 111 million tokens, costing $66,772.[1] Given the scale of reasoning-heavy workloads, even

[1] `https://arcprize.org/blog/oai-o3-pub-breakthrough`



(a) (b)

Figure 1: Ratio of reasoning tokens to answer tokens across datasets and deployed APIs. (a) Token ratios on the OpenR1-Math dataset across different OpenAI reasoning models. (b) Token ratios of the DeepSeek-R1 (Guo et al., 2025) across various reasoning datasets. In both cases, the number of reasoning tokens often exceeds answer tokens by an order of magnitude or more.

small inaccuracies in billing could translate into substantial financial consequences. Although there is no evidence of deliberate misconduct, the asymmetry of information between providers and users underscores the importance of transparent billing mechanisms to safeguard user interests.

To tackle this problem, we design **`CoIn` (Counting the Invisible)**, a verification framework that enables third-party auditing of invisible reasoning tokens in COLA. ***Importantly, our aim is not to suggest that such practices are occurring in today's systems, but rather to highlight a structural vulnerability inherent to the COLA design.*** On the contrary, we acknowledge the provider's motivations for concealing reasoning traces, as well as the community's concerns about opacity. **`CoIn`** ***seeks to bridge this gap by providing a neutral auditing mechanism that ensures billing accountability while preserving the confidentiality of hidden content.***

`CoIn` consists of two key components: **(1) Token Quantity Verification**, which leverages a verifiable hash tree (Merkle, 1987) to store fingerprint embeddings of reasoning tokens. Upon an audit request, `CoIn` allows users to query a small subset of the token fingerprints in the hash tree to verify the number of invisible tokens, avoiding accessing the actual reasoning tokens; and **(2) Semantic Validity Verification**, which detects fabricated, irrelevant, or low-effort token injection via a semantic relevance matching head. This matching head takes the embeddings of both the reasoning tokens and the answer tokens as input, and outputs a relevance score indicating their semantic consistency. Users can assess this score to identify token count manipulation with low-effort token injection. Together, these components enable `CoIn` to identify misreported token counts and fabricated reasoning traces, enabling transparent billing without exposing proprietary data. In practice, `CoIn` can be deployed as a trusted third-party auditing service that ensures billing transparency while preserving the integrity and confidentiality requirements of COLA providers.

Our main contributions are as follows:

- We define the COLA architecture and formalize the emerging risk of *token count manipulation*, categorizing it into misreporting, naive inflation, and adaptive inflation strategies.
- We design `CoIn`, a verification framework combining *token quantity verification* via verifiable hashing and *semantic validity verification* via embedding relevance, to audit invisible tokens without exposing proprietary content.
- Our experiments demonstrate that `CoIn` can achieve a 94.7% detection success rate against various adaptive attacks with less than 40% embedding exposure and less than 4% token visibility. Moreover, even when 10% of tokens are maliciously forged by COLA, `CoIn` still maintains a 40.1% probability of successful detection.

2 Related Work

Reasoning Model. LLMs have shown strong performance on complex reasoning tasks by generating intermediate steps, a technique known as chain-of-thought prompting (Wei et al., 2022). This paradigm has been further enhanced by methods such as self-consistency (Wang et al., 2022) and program-aided reasoning (Gao et al., 2023). Recent research reveals that generating more reasoning steps at inference time can lead to higher answer accuracy, a phenomenon referred to as the test-

time scaling law, which has become a guiding principle for optimizing LLMs (Snell et al., 2024). Reasoning models are typically LLMs fine-tuned via RL (Rafailov et al., 2023; Wu et al., 2023; Ramesh et al., 2024) to produce structured reasoning traces before generating final answers, thereby improving answer quality. These reasoning traces are often longer, more indirect, and may include failed attempts, but are nonetheless closely tied to the final answer (Hao et al., 2024; Yang et al., 2025b). Since these reasoning tokens are generated in the same autoregressive manner as answer tokens, COLAs charge for them based on token count. However, the indirect and verbose nature of reasoning makes it challenging to audit their legitimacy without direct access to the reasoning traces themselves.

COLA Auditing. Several works have emerged to address the lack of transparency in COLA. Sun et al. (2025) systematically define the opacity problem of commercial LLM services and further extend it to multi-agent settings. Cai et al. (2025) propose a watermark-based method to audit whether a COLA uses the required LLM rather than a cheaper LLM. Similarly, Yuan et al. (2025) develop a user-verifiable protocol to detect nodes that run unauthorized or incorrect LLM in a multi-agent system. Another series of works (Zheng et al., 2025; Marks et al., 2025) proposes auditing some bad behaviors of LLMs, e.g., cheating and offensive outputs. These techniques mainly focus on the model auditing and lack attention to the token count auditing of COLA.

3 PRELIMINARY

Participants and Problem Formulation. The `CoIn` framework involves three roles: (1) COLA — a commercial LLM service provider (e.g., OpenAI) that performs multi-step reasoning and returns only the final output to the user; (2) User — an end-user who submits prompts and receives answers along with the billing summary; and (3) `CoIn` auditor — a trusted third party responsible for verifying the invisible reasoning tokens on behalf of the user.

In each service interaction, the user sends a prompt P to COLA. The LLM generates reasoning tokens $R = \{r_1, r_2, \ldots, r_m\}$, followed by answer tokens $A = \{a_1, a_2, \ldots, a_n\}$. Only the final answer A is returned to the user, while the reasoning trace R remains hidden. Billing is based on the total number of tokens $m + n$, including the invisible reasoning tokens. As Figure 1 shows, reasoning tokens often dominate the total count, i.e., $m \gg n$, resulting in a significant transparency gap.

Potential Token Count Manipulation. For a malicious COLA, we consider two strategies for token count manipulation:

- **Token Count Misreporting.** COLA reports a falsified token count $m_f > m$, leading to direct overbilling without modifying the output.
- **Token Count Inflation.** Anticipating user-side defenses (e.g., hash matching, spot-checking), COLA may append low-effort fabricated tokens to the original reasoning trace. These fabricated tokens can be generated via random sampling, retrieval from related documents, or repetition of existing tokens, and then indistinguishably mixed with genuine reasoning tokens. The inflated sequence is then used for billing, bypassing naive verification methods and still overcharging the user. Due to the trade-off between risk, benefit, and cost, we only consider ***near-zero cost token inflation***, while fabrications requiring LLM participation are beyond our scope.

To address these threats, `CoIn` employs two components: (1) **Token Quantity Verification**, which audits the reported token count using verifiable commitments and exposes embeddings; and (2) **Semantic Validity Verification**, which evaluates the relevance between reasoning and answer tokens to detect low-quality injections.

Threat Model. COLA has access to the user prompt P, the full reasoning trace R, and the answer A, and controls the billing report (m, n), where m is the claimed number of reasoning tokens and n is the number of answer tokens. It can manipulate the reported count without user visibility. The `CoIn` auditor operates as a trusted third party. It can access P, A, and (m, n), but cannot observe R directly or directly query the LLM used by COLA. However, it can request COLA to return the embeddings of R, computed using an embedding model fixed by the auditor to prevent tampering.

4 `CoIn`: COUNTING THE INVISIBLE REASONING TOKENS

`CoIn` comprises two complementary components: **token quantity verification** and **semantic validity verification**. The token quantity verification module treats embeddings of invisible reasoning tokens

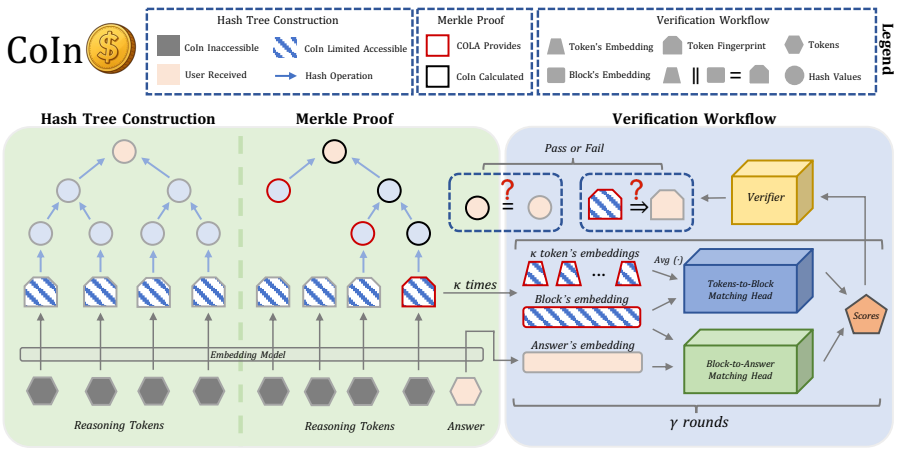


Figure 2: CoIn Framework.

as cryptographic fingerprints and organizes them into a verifiable hash tree. By querying a small subset of these fingerprints, users can audit the claimed number of invisible tokens without accessing their contents, thereby mitigating token count misreporting. The semantic validity verification module trains a lightweight neural network, referred to as a *matching head*, to evaluate the relevance between embeddings. During auditing, CoIn retrieves token embeddings from the hash tree and uses the matching head to compute relevance scores both among reasoning tokens and between reasoning and answer tokens. These scores help detect token count inflation through the injection of fabricated or irrelevant reasoning tokens. An overview of the CoIn framework is illustrated in Figure 2.

4.1 Token Quantity Verification

Token Fingerprint Generation. In CoIn, COLA is required to generate embeddings of its reasoning tokens using a third-party embedding model $\mathsf{Embd}(\cdot)$ designated by the CoIn auditor. These embeddings serve as token fingerprints used to construct a verifiable hash tree for auditing. This verifiable hash tree enables CoIn to audit the total number of invisible tokens without accessing them, while the per-token hash commitments preclude token count misreporting.

Specifically, given a reasoning sequence R, COLA first partitions R into α blocks. For each token r_i in block B_j, COLA computes: (1) the block embedding $\mathsf{Embd}(B_j)$, which embeds all the tokens inside the block; and (2) the token embedding $\mathsf{Embd}(r_i)$, which embeds the single token itself. Each reasoning token therefore acquires both the block embedding and the token embedding. For each reasoning token $\mathsf{Embd}(r_i)$, CoIn concatenates its block embedding and token embedding to form the token fingerprint: $\mathsf{Embd}(B_j) \,\|\, \mathsf{Embd}(t_i)$.

Fingerprint Hash Tree Construction. COLA applies a cryptographic hash function (e.g., SHA-256), agreed upon with CoIn, to each token fingerprint to construct the leaf nodes of a Merkle Hash Tree (Merkle, 1987). The number of leaf nodes is padded to the nearest power of two, and parent nodes are built recursively by hashing concatenated sibling nodes up to the Merkle Root. This root serves as a commitment to the full set of reasoning tokens and is submitted to CoIn. After constructing the hash tree, COLA gives the Merkle Root to CoIn for Merkle Proofs upon user's auditing request.

Merkle Proof. Upon receiving the answer A and the token counts m and n, a user may suspect token inflation. To verify the count of invisible reasoning tokens, the user selects a block B_j and randomly chooses token indices to audit. Upon receiving the request, CoIn auditor requests the following information from COLA: (1) the fingerprints of the selected tokens; and (2) the corresponding Merkle Path, which is a sequence of sibling hashes needed to reconstruct the Merkle Root from the corresponding token. CoIn recomputes the Merkle root from the provided data and checks for consistency with the original commitment by COLA. A successful match confirms the integrity of the selected token; a mismatch indicates possible fabrication and inflated token reporting. The construction and Merkle Proof procedure is illustrated in Figure 2-(a) and detailed further in Appendix F.1 , F.2.

The Merkle proof in token quantity verification ensures both the structural integrity and the correctness of the reported token count, effectively defending against token count misreporting. However, a dishonest COLA may still conduct token count inflation by injecting irrelevant or low-effort fabricated tokens that pass count verification. To address this limitation, we introduce semantic validity verification.

4.2 Semantic Validity Verification

To defend against token count inflation, we introduce the semantic validity verification component, as illustrated in Figure 2-(b). This component ensures that reasoning tokens are semantically meaningful and contribute to the final answer, preventing low-effort or fabricated token insertion. Based on this principle, `CoIn` verifies the semantic validity of invisible tokens from two perspectives:

- **Tokens-to-Block verification** checks whether each reasoning token r_i is semantically coherent within its enclosing block B_j. This defends against randomly injected or meaningless tokens.
- **Block-to-Answer verification** evaluates whether a reasoning block B_j is semantically aligned with the final answer A, thus identifying the insertion of low-cost content that is insufficiently relevant to the task.

To support both tasks, `CoIn` trains two lightweight neural modules called the *matching heads*, which are binary classifiers that determine whether two embeddings are semantically associated. Given two token embeddings a and b, the matching head first computes the cosine similarity: $\text{cos\_sim} = \frac{a \cdot b}{\|a\|\|b\|}$, and constructs the feature vector: $h = [a; b; a - b; a \odot b; \text{cos\_sim}] \in \mathbb{R}^{4d+1}$, where d is the embedding dimension, $[\,;\,]$ denotes concatenation, and $\odot$ denotes element-wise multiplication. The feature h is then passed through a two-layer feedforward network to produce a scalar match score $S \in [0, 1]$, representing the likelihood that a and b are semantically aligned. This process can be viewed as a regression function $S = \mathsf{MH}(a, b)$.

In `CoIn`, the matching heads $\mathsf{MH}_{\mathsf{tb}}(\cdot), \mathsf{MH}_{\mathsf{ba}}(\cdot)$ are trained offline for tokens-to-block and block-to-answer verification respectively. `CoIn` uses open-source corpora and the same embedding model in token fingerprinting to build the datasets for matching heads training.

Verification Protocol. In each verification round, the user randomly selects some reasoning tokens r_i (by default, 10% of the tokens within a selected block) from the hash tree. Since the token fingerprint consists of both the token embedding $\mathsf{Embd}(r_i)$ and the corresponding block embedding $\mathsf{Embd}(B_j)$, it can be directly used for Tokens-to-Block verification. For the Block-to-Answer verification, we use $\mathsf{Embd}(B_j)$ and the embedding of the whole answer to compute the score:

$$S_{tb} = \mathsf{MH}_{\mathsf{tb}}(\mathsf{AVG}(\mathsf{Embd}(r_i)), \mathsf{Embd}(B_j)), \qquad S_{ba} = \mathsf{MH}_{\mathsf{ba}}(\mathsf{Embd}(B_j), \mathsf{Embd}(A)). \tag{1}$$

Here, S_{tb} and S_{ba} represent the relevance scores for the two respective verification tasks. Each score reflects the estimated likelihood that the two input embeddings are semantically relevant.

4.3 Workflow of `CoIn`

Enforcing Billing Integrity with `CoIn`. When a user suspects token count manipulation in a specific response, they can initiate an audit request to `CoIn`. The audit begins with the user selecting a fraction γ of the total reasoning blocks for verification. `CoIn` then performs two Semantic Validity Verifications and multiple Merkle Proofs on these selected blocks. The resulting match scores are passed to a verifier, which issues a final decision. If the verifier accepts, the audit concludes successfully. If the verifier rejects, the user continues by randomly selecting another unverified block for auditing. This process repeats until either a successful judgment is reached or all blocks are exhausted. If no verification passes, the audit concludes with COLA being flagged for token inflation. The user may then request COLA to justify the charges by disclosing the original reasoning content. The complete procedure is outlined in Algorithm 4.

Verifier Design. Each audit round produces a variable-length sequence of match scores, as the number of verified blocks depends on verifier decisions. To handle this, we implement two types of verifiers: (1) **Rule-based**: Averages the scores from two semantic verifications. The audit passes if both averages exceed a threshold τ. (2) **Learning-based**: Uses a lightweight DeepSets model (Zaheer et al., 2017) to process the unordered set of match scores and audit will succeed if the confidence exceeds τ. Auditing outcomes enable users to assess the trustworthiness of a COLA provider.

Frequent failures in CoIn audits may erode user trust and damage provider reputation. By introducing verifiable accountability, the CoIn framework serves as a deterrent against token count manipulation in commercial LLM services.

Hyperparameter and Verification Cost. CoIn is governed by a few hyperparameters that control auditing granularity and cost. Specifically, α is the number of blocks, β the block size, γ the initial sampling ratio (default: 0.3), and k the number of tokens sampled per block (default: $0.1 * \beta$). A smaller β reduces exposure but increases overhead. The protocol begins with $\gamma \cdot \alpha$ rounds and may proceed up to α rounds under early stopping, so the number of verification rounds satisfies $\ell \in [\gamma \cdot \alpha,\ \alpha]$. As a result, the total number of Merkle Proofs is $k \cdot \ell$, and the number of Semantic Judgments is $2 \cdot \ell$.

5 EXPERIMENTS

We systematically evaluate the robustness and reliability of CoIn and its submodules under various adaptive inflation attacks across multiple datasets. We further analyze the construction cost of the Hash Tree, as well as whether the partially exposed block embeddings and tokens can be exploited to recover the reasoning tokens of COLA. Finally, we assess the difficulty of the dataset we constructed.

5.1 EXPERIMENT SETUP

Token Inflation Implementations. We study both *naive* and *adaptive* token count inflation strategies. To enable fine-grained evaluation and systematic dataset construction, we design four variants of adaptive inflation. All inflation types used in our experiments are summarized in Table 1. These strategies are applied to generate inflated samples for both training and evaluation.

Table 1: Token inflation types used in our experiments.

Type	Description
Naive Inflation	Randomly select tokens from the vocabulary for injection.
Ada. Inflation 1	Inject tokens with embeddings similar to P, R, or A.
Ada. Inflation 2	Inject tokens directly sampled from P, R, or A.
Ada. Inflation 3	Inject reasoning sequences extracted from other inputs.
Ada. Inflation 4	Inject retrieved sequences semantically similar to P, R, or A.

Datasets and Training Setup. We conduct experiments on five datasets derived from DeepSeek-R1 (Guo et al., 2025), covering diverse reasoning domains: medical (Chen et al., 2024a), code (Team, 2025; Face, 2025), mathematics (Face, 2025), general reasoning[2], and out-of-domain (OOD) mathematics (Team, 2025; Face, 2025). For training, we randomly sample 20,000 examples from each dataset and combine them into a joint dataset. Another 1,000 samples per dataset are held out to form the evaluation set for CoIn. We use the tokenizer of DeepSeek-R1 in our experiments.

For the matching head, we use all-MiniLM-L6-v2 (Reimers & Gurevych, 2019) as model structure. In the **tokens-to-block** verification task, we treat original samples as normal instances and apply Naive Inflation as well as Adaptive Inflation 1 and 2 to construct inflated samples. Normal and inflated samples are labeled 0 and 1 respectively and mixed at a 1:1 ratio to form the training set. For the **block-to-answer** verification task, we adopt a similar setup, but use Ada. Inflation 1–4 to construct inflated samples. This setting enables a thorough evaluation of the model's ability to detect both shallow and semantically sophisticated inflation attacks. The details are explained in Appendix C, D.

Metrics. We define the *Detection Success Rate (DSR)* as the classification accuracy of our module, computed separately for malicious and benign samples. Unlike conventional metrics (e.g. AUC), DSR highlights performance differences across negative sample types and reveals asymmetric detection difficulty that aggregated metrics may obscure. *Inflation Rate (IR)* indicates the percentage of fabricated tokens injected by COLA relative to the number of original reasoning tokens. For benign samples, the *Average Exposure Rate (AER)* refers to the proportion of blocks exposed during the CoIn multi-step verification process out of the total number of blocks.

[2] https://huggingface.co/datasets/glaiveai/reasoning-v1-20m

5.2 Detection Performance of CoIn

We evaluate CoIn's ability to detect various token count inflation attacks. Figure 3 shows the relationship between IR and DSR across five datasets. DSR increases with IR, indicating that more aggressive inflation is easier to detect. For example, in the Naive Inflation setting, DSR approaches 100% when IR exceeds 0.5, suggesting near-perfect detection for heavily inflated queries. Figure 4 illustrates the impact of τ_h. As τ_h increases, DSR for malicious samples increases, while for benign samples decreases. This highlights a tunable trade-off between user utility and COLA's protection.

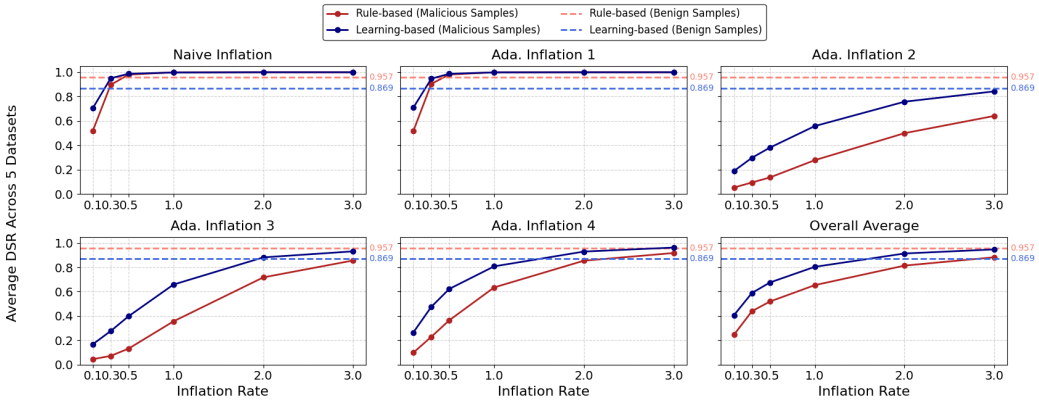


Figure 3: Performance of CoIn across different inflation methods and verifiers. The red lines and the blue lines represent the DSR of rule-based verifier and learning-based verifier, respectively.

Learning-based Verifier Excels at Detecting Malicious Samples. In Figure 3, for a fair comparison, we set the threshold $\tau = 0.5$ for learning-based verifier, and $\tau = 0.6$ for rule-based verifier. The initial sampling ratio γ is set to 0.3. Across all settings, the learning-based verifier consistently outperforms the rule-based variant in detecting inflated samples, achieving up to 94.7% average DSR at an inflation ratio (IR) of 3.0. Among the adaptive methods, Ada. Inflation 2 and Ada. Inflation 3 present greater detection challenges. Nevertheless, CoIn still achieves an average DSR of approximately 84.3% and 93.1% at IR = 3.0 for these cases. These results demonstrate that CoIn remains robust even under strong adaptive inflation, and that learning-based semantic verification offers significant advantages in practical detection scenarios.

Table 2: Influence of Block Size.

Metric	Block Size β		
	256	512	1024
Avg. α	16.8	8.6	4.5
Avg. ℓ	6.3	3.7	2.2
AER $\downarrow$	0.38	0.43	0.49

Rule-based Verifier Excels at Handling Benign Samples. Although it is less effective at detecting inflated samples, the rule-based verifier, as shown in Figure 3, performs better on benign samples when properly tuned, due to its lower false positive rate. Given our verification mechanism, a higher DSR on benign samples results in a lower Average Exposure Rate (AER), which makes COLA more favorable toward such mechanisms. Based on the rule-based verifier ($\tau = 0.6$), Table 2 reports the average number of blocks α, average verification rounds ℓ and AER across different block sizes β. With increasing β, ℓ decreases, indicating lower cost. However, AER increases, and the results in Appendix G show that increasing the block size reduces the DSR for adaptive inflation.

5.3 Performance of the Semantic Validity Verification

Block-to-Answer Verification. We separately evaluate the performance of the two types of matching heads introduced in Section 4.2. Table 3 shows the DSR of the matching heads on the *Block-to-Answer Verification* task. The model achieves an average DSR of 94.8% across attack types. Even for the Math (OOD) dataset, which was excluded from training, the model performs strongly, indicating good generalization. The DSR drops slightly on clean (non-inflated) samples due to the presence of reasoning blocks not directly contributing to the final answer (see Section 6). Additionally, Ada. Inflation 3 introduced hard negatives that resemble real data, making separation more difficult.

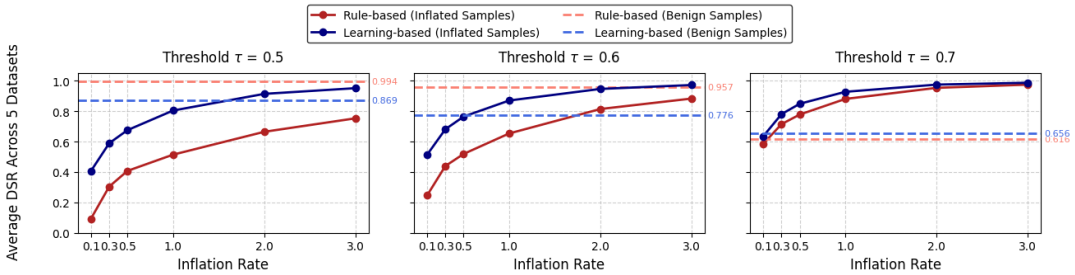


Figure 4: Impact of threshold τ on DSR.

Table 3: Block-to-Answer Verification Performance Across Attack Types and Domains.

Attack Type	Medical	Code	Math	General	Math (OOD)	Avg.
Naive Inflation	99.4	100.0	100.0	99.3	100.0	99.7
Ada. Inflation 1	95.3	98.7	98.6	96.8	98.2	97.5
Ada. Inflation 2	94.4	92.3	92.8	94.2	92.7	93.3
Ada. Inflation 3	89.2	81.5	84.3	92.9	84.6	86.5
Ada. Inflation 4	94.2	97.9	99.0	96.1	97.8	97.0
Avg. With Inflation	94.5	94.1	94.9	95.8	94.7	94.8
No Inflation	87.9	90.3	87.1	86.5	87.9	87.9

Tokens-to-Block Verification. Table 4 shows the results for tokens-to-block verification. The model performs well overall but struggles with Adaptive Inflation 2, where tokens reused from the same sample lead to significant lexical and semantic overlap. This overlap can blur the distinction between original and fabricated content, especially when reused tokens legitimately contribute to the block.

Table 4: Tokens-to-Block Verification Performance Across Attack Types and Domains.

Attack Type	Medical	Code	Math	General	Math (OOD)	Avg.
Naive Inflation	90.8	90.5	95.3	84.5	94.6	91.2
Ada. Inflation 1	95.1	96.1	95.8	95.5	95.8	95.6
Ada. Inflation 2	76.0	75.2	73.9	73.6	74.8	74.7
Avg. With Inflation	87.3	87.2	88.4	84.5	88.4	87.2
No Inflation	82.0	80.4	87.2	79.0	86.0	82.9

Cost of Building Hash Trees. We evaluate the computational overhead of constructing the Merkle Tree, with respect to input size and hidden dimension. Experiments were conducted on a dual-socket AMD EPYC 7763 system (128 cores, 256 threads). All constructions ran as single-threaded processes on one logical core. As shown in Figure 5, the construction time grows approximately linearly with the input length for a fixed hidden dimension, and increases more steeply with higher dimensions. Given that most LLM inference servers have underutilized CPUs, and the Merkle Tree construction process scales effectively with parallelism, the cost of building it is nearly negligible.

6 Discussion

Can the original text be recovered from the tokens and embeddings exposed by COLA? During the `CoIn` verification process, COLA may leak partial block embeddings and tokens to `CoIn`. To quantify the impact of such leakage, we design two experiments: (1) **Direct reconstruction from embeddings**. Our protection goal is to prevent large-scale reasoning data from being collected for reverse engineering or distillation, rather than avoiding leakage of individual reasoning fragments. We adopt Vec2Text (Morris et al., 2023), a "hypothesis–correction" iterative method for embedding inversion. As shown in Table 5, text can still be partially recovered when the block size is small (≤ 64), but recovery performance drops sharply as block size increases; at our framework's minimum block size of 256, the attack almost completely fails. Even when BLEU (Papineni et al., 2002) / F1 scores are relatively high, the reconstructed text often suffers from severe semantic distortion, making

it unsuitable for malicious data distillation. (2) **LLM-based reconstruction**. We further assume that a malicious CoIn may employ RAG to retrieve similar documents and use an LLM to reconstruct the original content (prompt design provided in Appendix H). Results on a math dataset (Table 7) show that the combination of high BERTScore/EmbedSim and low BLEU/ROUGE indicates that LLMs can preserve core semantics, but differ significantly in surface expression and syntactic structure. Further experimental details are provided in Appendix I.

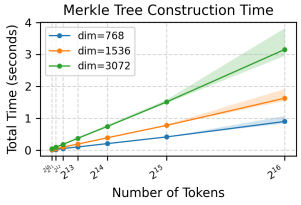


Figure 5: Merkle Tree Construction Time with Fluctuation Range.

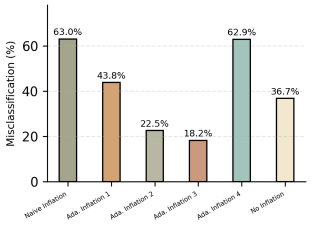


Figure 6: Misclassification Rates of LLMs on Constructed Datasets.

Table 5: Reconstruction performance on Math (see full results in Appendix I.1, Table 6).

Block Size	Math	
	BLEU	Token F1
16	43.83	0.7467
32	15.06	0.5123
64	12.62	0.4256
128	9.84	0.3441
256	4.07	0.2617
512	0.67	0.2016

How does CoIn defend against repetition-based token inflation? An important class of adversarial strategies we must consider involves dishonest providers artificially inflating token counts by repeating reasoning segments or appending permuted or LLM-rewritten variants. For simplicity, we did not incorporate such strategies into the core method presented in this paper, but CoIn can be naturally extended to mitigate them. For direct repetition, subtree equivalence checks in the Merkle tree can reveal duplicated segments. *If repeated content exists, identical hash values will appear at sibling nodes, implying structurally identical subtrees and thus duplicated token sequences.* For more complex manipulations, such as random permutations or LLM-based rewritings, we provide detailed experiments in Table 8. We find that even after rewriting by Qwen3-4B (Yang et al., 2025a), the rewritten text still exhibits high similarity to the original. Therefore, the proposed subtree similarity check retains strong potential to detect such inflation patterns. As a highly extensible framework, CoIn can incorporate these additional measures to defend against a broader range of attacks. Detailed analysis and experimental results are provided in Appendix J.

How difficult is the dataset we constructed? To investigate the dataset difficulty, we submitted the failed samples from the *Block-to-Answer Verification* task, along with their Answer, to a LLM. Based on the idea of LLM-as-a-Judge (Zheng et al., 2023; Li et al., 2024), we use a prompt to instruct the LLM to perform binary classification. The prompt used is provided in Appendix H. The relatively high misclassification rate suggests that the LLM, after reading the original text, tends to align with the matching head's judgment. The LLM shows high error rates on Naive Inflation, Ada. Inflation 1 and 4, indicating strong performance of the matching head in these cases. However, it still struggles with the remaining two adaptive inflations. Notably, 36.7% of real blocks were misclassified by the LLM, suggesting that some parts of the true reasoning steps may be unrelated to answer derivation.

7 Conclusion

This paper presents CoIn, a novel auditing framework designed to verify the token counts and semantic validity of hidden reasoning traces in COLA. We identify and formalize the problem of *token count manipulation*, in which service providers can overcharge users by injecting redundant or fabricated reasoning tokens that are not visible to the user, often at *near-zero computational cost*. To address this, CoIn integrates two complementary components: a hash tree-based token quantity verifier and a semantic relevance-based validity checker. Our extensive experiments demonstrate that CoIn can detect both naive and adaptive inflation strategies with high accuracy, even under limited exposure settings. By enabling transparent and auditable billing without revealing proprietary content, CoIn introduces a practical mechanism for accountability in commercial LLM services. We hope this work lays the foundation for future research on LLM API auditing, transparent reasoning, and verifiable inference services.

ETHICS STATEMENT

The central goal of this research is to enhance billing transparency and accountability in commercial opaque LLM APIs, thereby fostering greater trust between service providers and users. Our work identifies a potential vulnerability, *token count manipulation*, and proposes a defensive framework, `CoIn`, to address it. We stress that our position is entirely neutral: we are not suggesting, implying, or accusing any current commercial providers of engaging in such practices. Instead, we view this research as a proactive exploration of potential risks that could arise from information asymmetries between providers and users. Our objective is to contribute constructively to the ecosystem by identifying possible vulnerabilities early and proposing mitigations that help prevent the erosion of trust.

Ultimately, we believe this work contributes positively to the AI ecosystem by introducing a mechanism that balances the provider's need to protect intellectual property with the user's right to verifiable billing. We therefore believe this research raises no significant ethical concerns beyond the general considerations outlined in the ICLR Code of Ethics.

REPRODUCIBILITY STATEMENT

To ensure the reproducibility of our research, we have made our code and datasets publicly available under an anonymous license. The complete implementation of the `CoIn` framework, along with the scripts used for all experiments, can be found at the following anonymous repository:

`https://anonymous.4open.science/r/LLM-Auditing-CoIn-20F0`.

We provide extensive details of our experimental setup in the paper, specifically:

- **Dataset Construction:** The methodology for creating the evaluation and training datasets, including various token inflation strategies, is detailed in Section 5.1. Additional details, such as data sources with their corresponding HuggingFace links, dataset construction methods, and data-related experimental settings, are further discussed in Appendix C. Due to file size limitations, we provide partial data samples through an anonymous link; the complete dataset and the trained models involved will be released after the paper is accepted.
- **Model and Training:** The overall framework design, the architecture of the matching head, and its execution method are discussed in Section 4. Hyperparameters for training the matching heads and the verifier, as well as detailed model architecture descriptions, are provided in Appendix D.
- **Algorithms:** The core algorithms for Merkle Tree construction, Merkle Proof verification, and the complete multi-round `CoIn` workflow are formally described in Algorithms 2, 3, 4 in Appendix F.

Although we used LLMs to draft some data preprocessing scripts, all code has been manually reviewed and verified by the authors. Furthermore, algorithmic skeletons generated by LLMs were based on original Python code provided by the authors, and subsequently refined with additional details by hand. The final released codebase ensures complete reproducibility. We believe these resources provide a solid foundation for other researchers to verify our results and build upon our work.

REFERENCES

Will Cai, Tianneng Shi, Xuandong Zhao, and Dawn Song. Are you getting what you pay for? auditing model substitution in llm apis. *arXiv preprint arXiv:2504.04715*, 2025.

Charlie Chen, Sebastian Borgeaud, Geoffrey Irving, Jean-Baptiste Lespiau, Laurent Sifre, and John Jumper. Accelerating large language model decoding with speculative sampling. *arXiv preprint arXiv:2302.01318*, 2023.

Junying Chen, Zhenyang Cai, Ke Ji, Xidong Wang, Wanlong Liu, Rongsheng Wang, Jianye Hou, and Benyou Wang. Huatuogpt-o1, towards medical complex reasoning with llms, 2024a. URL https://arxiv.org/abs/2412.18925.

Lingjiao Chen, Matei Zaharia, and James Zou. How is chatgpt's behavior changing over time? *Harvard Data Science Review*, 6(2), 2024b.

Hugging Face. Open r1: A fully open reproduction of deepseek-r1, January 2025. URL https://github.com/huggingface/open-r1.

Xingcheng Gao, Swaroop Mishra, et al. Pal: Program-aided language models. *arXiv preprint arXiv:2211.10435*, 2023.

Jianping Gou, Baosheng Yu, Stephen J Maybank, and Dacheng Tao. Knowledge distillation: A survey. *International Journal of Computer Vision*, 129(6):1789–1819, 2021.

Daya Guo, Dejian Yang, Haowei Zhang, Junxiao Song, Ruoyu Zhang, Runxin Xu, Qihao Zhu, Shirong Ma, Peiyi Wang, Xiao Bi, et al. Deepseek-r1: Incentivizing reasoning capability in llms via reinforcement learning. *arXiv preprint arXiv:2501.12948*, 2025.

Shibo Hao, Yi Gu, Haotian Luo, Tianyang Liu, Xiyan Shao, Xinyuan Wang, Shuhua Xie, Haodi Ma, Adithya Samavedhi, Qiyue Gao, et al. Llm reasoners: New evaluation, library, and analysis of step-by-step reasoning with large language models. *arXiv preprint arXiv:2404.05221*, 2024.

Zhiqiang Hu, Lei Wang, Yihuai Lan, Wanyu Xu, Ee-Peng Lim, Lidong Bing, Xing Xu, Soujanya Poria, and Roy Ka-Wei Lee. Llm-adapters: An adapter family for parameter-efficient fine-tuning of large language models. *arXiv preprint arXiv:2304.01933*, 2023.

Aaron Jaech, Adam Kalai, Adam Lerer, Adam Richardson, Ahmed El-Kishky, Aiden Low, Alec Helyar, Aleksander Madry, Alex Beutel, Alex Carney, et al. Openai o1 system card. *arXiv preprint arXiv:2412.16720*, 2024.

Mingyu Jin, Qinkai Yu, Dong Shu, Haiyan Zhao, Wenyue Hua, Yanda Meng, Yongfeng Zhang, and Mengnan Du. The impact of reasoning step length on large language models. *arXiv preprint arXiv:2401.04925*, 2024.

Komal Kumar, Tajamul Ashraf, Omkar Thawakar, Rao Muhammad Anwer, Hisham Cholakkal, Mubarak Shah, Ming-Hsuan Yang, Phillip HS Torr, Fahad Shahbaz Khan, and Salman Khan. Llm post-training: A deep dive into reasoning large language models. *arXiv preprint arXiv:2502.21321*, 2025.

Haitao Li, Qian Dong, Junjie Chen, Huixue Su, Yujia Zhou, Qingyao Ai, Ziyi Ye, and Yiqun Liu. Llms-as-judges: a comprehensive survey on llm-based evaluation methods. *arXiv preprint arXiv:2412.05579*, 2024.

Baohao Liao, Yuhui Xu, Hanze Dong, Junnan Li, Christof Monz, Silvio Savarese, Doyen Sahoo, and Caiming Xiong. Reward-guided speculative decoding for efficient llm reasoning. *arXiv preprint arXiv:2501.19324*, 2025.

Chin-Yew Lin. Rouge: A package for automatic evaluation of summaries. In *Text summarization branches out*, pp. 74–81, 2004.

Samuel Marks, Johannes Treutlein, Trenton Bricken, Jack Lindsey, Jonathan Marcus, Siddharth Mishra-Sharma, Daniel Ziegler, Emmanuel Ameisen, Joshua Batson, Tim Belonax, et al. Auditing language models for hidden objectives. *arXiv preprint arXiv:2503.10965*, 2025.

Ralph C Merkle. A digital signature based on a conventional encryption function. In *Conference on the theory and application of cryptographic techniques*, pp. 369–378. Springer, 1987.

John X Morris, Volodymyr Kuleshov, Vitaly Shmatikov, and Alexander M Rush. Text embeddings reveal (almost) as much as text. *arXiv preprint arXiv:2310.06816*, 2023.

Niklas Muennighoff, Zitong Yang, Weijia Shi, Xiang Lisa Li, Li Fei-Fei, Hannaneh Hajishirzi, Luke Zettlemoyer, Percy Liang, Emmanuel Candès, and Tatsunori Hashimoto. s1: Simple test-time scaling. *arXiv preprint arXiv:2501.19393*, 2025.

Kishore Papineni, Salim Roukos, Todd Ward, and Wei-Jing Zhu. Bleu: a method for automatic evaluation of machine translation. In *Proceedings of the 40th annual meeting of the Association for Computational Linguistics*, pp. 311–318, 2002.

Rafael Rafailov, Archit Sharma, Eric Mitchell, Christopher D Manning, Stefano Ermon, and Chelsea Finn. Direct preference optimization: Your language model is secretly a reward model. *Advances in Neural Information Processing Systems*, 36:53728–53741, 2023.

Shyam Sundhar Ramesh, Yifan Hu, Iason Chaimalas, Viraj Mehta, Pier Giuseppe Sessa, Haitham Bou Ammar, and Ilija Bogunovic. Group robust preference optimization in reward-free rlhf. *Advances in Neural Information Processing Systems*, 37:37100–37137, 2024.

Nils Reimers and Iryna Gurevych. Sentence-bert: Sentence embeddings using siamese bert-networks. In *Proceedings of the 2019 Conference on Empirical Methods in Natural Language Processing*. Association for Computational Linguistics, 11 2019. URL https://arxiv.org/abs/1908.10084.

Charlie Snell, Jaehoon Lee, Kelvin Xu, and Aviral Kumar. Scaling llm test-time compute optimally can be more effective than scaling model parameters. *arXiv preprint arXiv:2408.03314*, 2024.

Sharath Turuvekere Sreenivas, Saurav Muralidharan, Raviraj Joshi, Marcin Chochowski, Ameya Sunil Mahabaleshwarkar, Gerald Shen, Jiaqi Zeng, Zijia Chen, Yoshi Suhara, Shizhe Diao, et al. Llm pruning and distillation in practice: The minitron approach. *arXiv preprint arXiv:2408.11796*, 2024.

Guoheng Sun, Ziyao Wang, Xuandong Zhao, Bowei Tian, Zheyu Shen, Yexiao He, Jinming Xing, and Ang Li. Invisible tokens, visible bills: The urgent need to audit hidden operations in opaque llm services. *arXiv preprint arXiv:2505.18471*, 2025.

Open Thoughts Team. Open Thoughts, January 2025.

Xuezhi Wang, Jason Wei, Dale Schuurmans, et al. Self-consistency improves chain of thought reasoning in language models. *arXiv preprint arXiv:2203.11171*, 2022.

Jason Wei, Xuezhi Wang, Dale Schuurmans, et al. Chain-of-thought prompting elicits reasoning in large language models. *arXiv preprint arXiv:2201.11903*, 2022.

Tianhao Wu, Banghua Zhu, Ruoyu Zhang, Zhaojin Wen, Kannan Ramchandran, and Jiantao Jiao. Pairwise proximal policy optimization: Harnessing relative feedback for llm alignment. *arXiv preprint arXiv:2310.00212*, 2023.

An Yang, Anfeng Li, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chang Gao, Chengen Huang, Chenxu Lv, et al. Qwen3 technical report. *arXiv preprint arXiv:2505.09388*, 2025a.

Shu Yang, Junchao Wu, Xin Chen, Yunze Xiao, Xinyi Yang, Derek F Wong, and Di Wang. Understanding aha moments: from external observations to internal mechanisms. *arXiv preprint arXiv:2504.02956*, 2025b.

Michael J Yuan, Carlos Campoy, Sydney Lai, James Snewin, and Ju Long. Trust, but verify. *arXiv preprint arXiv:2504.13443*, 2025.

Manzil Zaheer, Satwik Kottur, Siamak Ravanbakhsh, Barnabas Poczos, Russ R Salakhutdinov, and Alexander J Smola. Deep sets. *Advances in neural information processing systems*, 30, 2017.

Jintian Zhang, Yuqi Zhu, Mengshu Sun, Yujie Luo, Shuofei Qiao, Lun Du, Da Zheng, Huajun Chen, and Ningyu Zhang. Lightthinker: Thinking step-by-step compression. *arXiv preprint arXiv:2502.15589*, 2025.

Tianyi Zhang, Varsha Kishore, Felix Wu, Kilian Q Weinberger, and Yoav Artzi. Bertscore: Evaluating text generation with bert. *arXiv preprint arXiv:1904.09675*, 2019.

Lianmin Zheng, Wei-Lin Chiang, Ying Sheng, Siyuan Zhuang, Zhanghao Wu, Yonghao Zhuang, Zi Lin, Zhuohan Li, Dacheng Li, Eric Xing, et al. Judging llm-as-a-judge with mt-bench and chatbot arena. *Advances in Neural Information Processing Systems*, 36:46595–46623, 2023.

Xiang Zheng, Longxiang Wang, Yi Liu, Xingjun Ma, Chao Shen, and Cong Wang. Calm: Curiosity-driven auditing for large language models. *arXiv preprint arXiv:2501.02997*, 2025.

Barret Zoph, Golnaz Ghiasi, Tsung-Yi Lin, Yin Cui, Hanxiao Liu, Ekin Dogus Cubuk, and Quoc Le. Rethinking pre-training and self-training. *Advances in neural information processing systems*, 33: 3833–3845, 2020.

A LLM USAGE STATEMENT

In the development of this work, we employed several commercial LLMs at different stages to enhance the quality and robustness of our research. The specific applications are as follows:

Adversarial Brainstorming. After establishing the main framework of CoIn, we interacted with LLMs in an adversarial manner by prompting them to act as attackers and propose potential exploits and attack strategies against our design. This iterative process played an important role in identifying weaknesses in the framework and improving our defense mechanisms. For example, adversarial brainstorming inspired the idea of a multi-round verification mechanism: in cases of verification failure, the CoIn framework could require a COLA to provide the original text, which substantially increases the risk of fraudulent behavior being exposed. At the same time, we carefully considered and ultimately rejected certain potential directions, such as zero-knowledge-proof-based defenses, primarily due to concerns about practicality and scope.

Partial Code Generation and Code-to-Algorithm Conversion. We employed LLMs to assist with technical implementation. For instance, LLM was used to draft Python scripts for data preprocessing. For some of the more complex algorithms in the appendix, we provided original Python code as guidance and prompted LLMs to generate high-level LaTeX algorithmic skeletons based on this code. These outputs then served as blueprints, which were thoroughly reviewed, refined, and supplemented by the authors to ensure correctness and methodological consistency.

Manuscript Polishing. We used LLMs as writing assistants to improve sentence structure, check spelling errors, and enhance the clarity and readability of the manuscript.

All LLM-generated content (including conceptual challenges, code, and text) was critically reviewed by the authors, who take full responsibility for the scientific integrity and accuracy of the paper. We emphasize that LLMs are not authors and bear no responsibility for this work; all accountability lies with the human authors. This study did not involve any human subjects or sensitive data, and therefore raises no additional ethical concerns beyond those discussed in our Ethics Statement.

B LIMITATIONS

We acknowledge that CoIn, despite its merits, possesses certain limitations that warrant discussion.

- **Mechanistic limitations**: When the inflation rate is low, CoIn shows limited performance in detecting malicious samples; its probabilistic nature also inevitably leads to a non-zero misclassification rate. In cases where benign samples are misclassified as malicious, the protocol requires COLA to disclose the original text for verification. Furthermore, the auditing process of CoIn depends on COLA's active cooperation, which may constrain its applicability in practice.
- **Uncovered attack surfaces**: CoIn remains less effective against certain sophisticated attacks. For instance, LLM-based rewriting or expansion attacks may bypass detection, but they require substantial computational cost, far beyond the near-zero cost scenarios considered in this work, and are thus better categorized as model substitution Cai et al. (2025); Chen et al. (2024b), for which a separate line of research exists and could be combined with CoIn. Moreover, cascade-based inference Liao et al. (2025) or speculative sampling Chen et al. (2023) poses ambiguous cases of "inflation," where it is difficult to distinguish fraud from legitimate reasoning optimization, leaving this aspect unverified.
- **Potential privacy risks**: Although Section 6 shows that recovering original tokens from embeddings Morris et al. (2023) under CoIn's setting is difficult, future advances or attacks tailored specifically to our setting may still expose partial information. While such partial leakage is unlikely to enable the construction of high-quality distillation datasets, it may become a serious concern in sensitive domains such as healthcare.

C Dataset Construction and Experimental Details

C.1 Dataset Construction Details

We construct two verification datasets for Block-to-Answer and Tokens-to-Block verification, each dataset includes two types of inflated samples. The simple version consists entirely of artificially generated (inflated) tokens, while the hard version contains a mixture of real and inflated tokens. For Tokens-to-Block verification, we randomly sample between 3.125% and 12.5% of tokens from each block to create both training and test instances.

For both verification tasks, we generate 1,200,000 positive and negative samples respectively. The training set is uniformly distributed across four datasets. Since the difficulty levels of the samples vary, we adjust the composition using an adaptive inflation strategy (applied in Block-to-Answer) to ensure balanced learning.

For training the DeepSets model, we additionally sample 1,000 examples. To preserve generalization capability, the data used for training this model does not overlap with any samples seen by the matching heads.

C.2 Experimental Details

All evaluation results, unless stated otherwise, are reported on 1,000 examples. This applies to Block-to-Answer, Tokens-to-Block, and the test sets used within the `CoIn` framework. Each numeric result is computed over a minimum of 1,000 samples to ensure statistical significance. Please refer to the Algorithm 1 for our `CoIn` workflow test set construction process.

Algorithm 1 Streamlined Generation of Inflated Reasoning Sequences

Require: Original dataset D_{orig}, inflation ratios $\mathcal{K}$, strategies S_{list} with weights W_S, tokenizer $\mathcal{T}$, embedder $\mathcal{E}$, anchor source Src_{anchor}, segment length range $[L_{min}, L_{max}]$, insertion mode M_{ins}, and optional block range $[B_{min}, B_{max}]$ if using block mode.
Ensure: Inflated dataset $D_{inflated}$

Initialize $D_{inflated} \leftarrow \emptyset$
Build FAISS indexes for RAG-based strategies
for each data point $item_i = (P_i, R_i, A_i)$ in D_{orig} **do**
    $T_{orig} \leftarrow \mathcal{T}(R_i)$;
    if T_{orig} is empty **then continue**
    end if
    $T_{anchor} \leftarrow$ SelectAnchor$(item_i, Src_{anchor})$
    $N_{max} \leftarrow \lfloor |T_{orig}| \cdot \max(\mathcal{K}) \rfloor$
    $T_{pool} \leftarrow$ CollectTokens$(N_{max}, T_{anchor}, S_{list}, W_S)$
    for each $k \in \mathcal{K}$ **do**
        $N_k \leftarrow \lfloor |T_{orig}| \cdot k \rfloor$
        $T_k \leftarrow$ Subsample(T_{pool}, N_k)
        $T_{final} \leftarrow$ Insert$(T_{orig}, T_k, M_{ins}, [B_{min}, B_{max}])$
        Add $\mathcal{T}^{-1}(T_{final})$ to $D_{inflated}$ with metadata
    end for
end for
return $D_{inflated}$

C.3 Token Injection Strategy for Evaluation Dataset

To further clarify our dataset construction, we describe the token injection process used in generating evaluation samples. For each original reasoning sequence, we apply the following procedure:

- **Compute required malicious tokens.** The number of injected tokens is determined by a predefined inflation rate, ranging from 10% to 300% of the original reasoning length.
- **Sample malicious tokens.** Depending on the chosen inflation method, we collect sufficient malicious tokens. For continuous text segments (e.g., Ada. Inflation 4, sampled from Wikipedia), we ensure that each sampled document is between 256–512 tokens in length.

- **Partition into malicious blocks.** The malicious tokens are grouped into blocks of length between 32–256 tokens.
- **Random insertion.** Each malicious block is randomly inserted into the original reasoning content, leading to stochastic placement across different reasoning positions.
- **Final block partition.** After injection, the reasoning content (now containing malicious blocks) is split into fixed-size blocks according to the block size β.

This injection strategy creates more challenging and realistic evaluation cases. Because the placement of malicious blocks is random, some reasoning blocks may contain no malicious content, while others contain only 10–20% malicious tokens. Such variability increases the difficulty of detection and ensures robustness of the evaluation.

C.4 Source of Dataset

To evaluate `CoIn`'s performance across different domains, we constructed training and test sets based on five datasets distilled from `DeepSeek-R1` Guo et al. (2025), including Medical Chen et al. (2024a)[3], Code Team (2025); Face (2025)[4], Math Face (2025)[5], General[6], and Out-of-Domain data Math (OOD) Team (2025); Face (2025)[7]. Our final training set is a mixture of these five datasets.

D Training and Model Details

D.1 Hyperparameters

For the matching heads used in Tokens-to-Block verification and Block-to-Answer verification, we set the learning rate to 2×10^{-5}, the batch size to 128, and train for 3 epochs. We employ the Adam optimizer and use the focal loss function. The hidden dimension of the model follows that of the embedding model, set to 384.

For the DeepSets model in the verifier, we use a batch size of 128, a hidden dimension of 256, and train for 5 epochs. We adopt the Adam optimizer with a learning rate of 1×10^{-3} and use the binary cross-entropy (BCE) loss. All experiments are conducted with a fixed random seed of 42.

D.2 Training of Matching Heads

As described in the main text, `MH_tb` and `MH_ba` are two lightweight neural matching heads that take as input a pair of embeddings and output a score between 0 and 1, indicating their semantic relevance. Internally, these modules extract interaction features and feed them into two-layer feed-forward networks. Below we detail how we construct diverse training pairs for both modules.

Block-to-Answer Matching (MH_ba).

- **Benign samples:** Extract continuous segments from the original reasoning blocks.
- **Malicious samples:** Cover all inflation types discussed in the paper:
  1. *Naive Inflation:* Tokens randomly sampled from the vocabulary.
  2. *Adaptive Inflation 1:* Embed the prompt, reasoning, and answer, then inject semantically similar tokens based on cosine similarity.
  3. *Adaptive Inflation 2:* Tokens or phrases randomly sampled from the prompt/reasoning/answer and re-injected into the reasoning.
  4. *Adaptive Inflation 3:* For a given sample, find the most similar sample (via embedding) in the dataset and inject its reasoning into the current one.
  5. *Adaptive Inflation 4:* Similar to Inflation 1, but retrieves full paragraphs from an external corpus (e.g., Wikipedia) based on embedding similarity.

[3] `https://huggingface.co/datasets/FreedomIntelligence/Medical-R1-Distill-Data`
[4] `https://huggingface.co/datasets/open-r1/OpenThoughts-114k-Code_decontaminated`
[5] `https://huggingface.co/datasets/open-r1/OpenR1-Math-220k`
[6] `https://huggingface.co/datasets/glaiveai/reasoning-v1-20m`
[7] `https://huggingface.co/datasets/open-r1/OpenThoughts-114k-math`

To ensure difficulty, the ratio of real reasoning to injected tokens is uniformly controlled in the $[20\%, 80\%]$ range. We also construct fully malicious samples where all blocks are from inflation sources, maintaining a 1:1 ratio between partial and full malicious samples.

Tokens-to-Block Matching (MH_tb). This module is trained using the same dataset as block-to-answer. Instead of matching entire blocks, we randomly select 5–20 tokens from the original text, average their embeddings, and use this token-group embedding as input alongside the block embedding.

Dataset Size and Balance. For both tasks, we balance benign and malicious samples, with 1.2 million samples per class.

Due to space limitations, these details could not be included in the main text. They are now provided in Appendix D.2, alongside code in the supplementary material.

E Computational Resources

All experiments were conducted on a high-performance workstation running Ubuntu 20.04.6 LTS. The system is equipped with a dual-socket AMD EPYC 7763 processor, providing a total of 128 physical cores and 256 threads. For GPU acceleration, we utilized an NVIDIA RTX A6000 Ada graphics card.

F DETAILS OF CoIn

F.1 MERKLE TREE CONSTRUCTION

Algorithm 3 details the process COLA uses to construct the Merkle Hash Tree from a reasoning sequence R. This corresponds to the "Token Fingerprint Generation" and "Fingerprint Hash Tree Construction" paragraphs.

F.2 MERKLE PROOF VERIFICATION

Algorithm 2 describes how the CoIn auditor verifies the integrity of a token using its fingerprint and the Merkle path provided by COLA. This corresponds to the "Merkle Proof" paragraph.

Algorithm 2 Merkle Proof Verification

Require: Committed Merkle Root $MR_{committed}$ (from COLA).
Require: Token fingerprint fp_{token} of the audited token (from COLA).
Require: Merkle Path $P = [(h_1, pos_1), (h_2, pos_2), \ldots, (h_d, pos_d)]$ (from COLA), where h_k is a sibling hash and $pos_k \in \{\text{'left'}, \text{'right'}\}$ indicates h_k's position relative to the path node.
Require: Cryptographic hash function $H(\cdot)$.
Ensure: Boolean: **true** if verification succeeds, **false** otherwise.

$current\_computed\_hash \leftarrow H(fp_{token})$ ▷ Hash the provided token fingerprint
for each pair $(sibling\_hash, position) \in P$ **do**
  if $position =$ 'left' **then**
    $current\_computed\_hash \leftarrow H(sibling\_hash \,\|\, current\_computed\_hash)$
  else if $position =$ 'right' **then**
    $current\_computed\_hash \leftarrow H(current\_computed\_hash \,\|\, sibling\_hash)$
  else
    return false ▷ Error: Invalid position in Merkle Path
  end if
end for
$MR_{recomputed} \leftarrow current\_computed\_hash$
if $MR_{recomputed} = MR_{committed}$ **then**
  return true ▷ Verification successful: token integrity confirmed
else
  return false ▷ Verification failed: mismatch indicates potential issue
end if

Notes on Algorithms:

- **Padding (Algorithm 3):** The text states, "The number of leaf nodes is padded to the nearest power of two." Algorithm 3 implements this by duplicating the hash of the last actual leaf node if leaves exist. If the initial set of tokens (and thus fingerprints) is empty ($N = 0$), it assumes padding to $N_{pow2} = 1$ using a hash of a predefined value (e.g., an empty string). The exact nature of this padding for an empty set should be consistently defined between COLA and the auditor.
- **Merkle Path Representation (Algorithm 2):** The Merkle Path P is assumed to be a list of (hash, position) tuples. The 'position' indicates if the sibling hash is to the 'left' or 'right' of the node on the direct path from the audited leaf to the root.
- **Concatenation for Hashing:** The order of concatenation (e.g., $H(leftChild \,\|\, rightChild)$ vs. $H(rightChild \,\|\, leftChild)$) must be consistent throughout construction and verification. The algorithms assume a fixed order (left child first).

Algorithm 3 Merkle Tree Construction by COLA

Require: Reasoning tokens R; number of blocks α; embedding function $\mathsf{Embd}(\cdot)$; cryptographic hash function $H(\cdot)$.
Ensure: Merkle Root MR.

// Phase 1: Token Fingerprint Generation and Leaf Node Creation
$Blocks \leftarrow \text{Partition}(R, \alpha)$ ▷ Partition R into $B_1, \ldots, B_\alpha$
$Fingerprints \leftarrow \emptyset$ ▷ Initialize as an empty list
for each block $B_j \in Blocks$ **do**
  $e_{block_j} \leftarrow \mathsf{Embd}(B_j)$ ▷ Compute block embedding
  for each token $r_i \in B_j$ **do**
    $e_{token_i} \leftarrow \mathsf{Embd}(r_i)$ ▷ Compute token embedding
    $fp_i \leftarrow e_{block_j} \,\|\, e_{token_i}$ ▷ Form token fingerprint
    Add fp_i to $Fingerprints$
  end for
end for
$LeafNodes \leftarrow \emptyset$ ▷ Initialize as an empty list
for each fingerprint $fp \in Fingerprints$ **do**
  $leaf \leftarrow H(fp)$ ▷ Hash fingerprint to create leaf node
  Add $leaf$ to $LeafNodes$
end for
// Phase 2: Padding Leaf Nodes
$N \leftarrow \text{length}(LeafNodes)$
Let N_{pow2} be the smallest power of two such that $N_{pow2} \geq N$.
if $N < N_{pow2}$ **then**
  if $N = 0$ **and** $N_{pow2} > 0$ **then** ▷ e.g., $N = 0 \implies N_{pow2} = 1$
    $padding\_hash \leftarrow H(\text{""})$ ▷ Hash of empty string or other predefined padding value
    for $k \leftarrow 1$ **to** N_{pow2} **do**
      Add $padding\_hash$ to $LeafNodes$
    end for
  else if $N > 0$ **then**
    $last\_leaf\_hash \leftarrow LeafNodes[N-1]$ ▷ Get hash of the last actual leaf
    for $k \leftarrow 1$ **to** $N_{pow2} - N$ **do**
      Add $last\_leaf\_hash$ to $LeafNodes$ ▷ Pad by duplicating the last leaf's hash
    end for
  end if
end if
// Phase 3: Building the Tree Recursively
$CurrentLevelNodes \leftarrow LeafNodes$
while $\text{length}(CurrentLevelNodes) > 1$ **do**
  $NextLevelNodes \leftarrow \emptyset$
  for $k \leftarrow 0$ **to** $(\text{length}(CurrentLevelNodes)/2) - 1$ **do**
    $leftChild \leftarrow CurrentLevelNodes[2k]$
    $rightChild \leftarrow CurrentLevelNodes[2k+1]$
    $parentHash \leftarrow H(leftChild \,\|\, rightChild)$
    Add $parentHash$ to $NextLevelNodes$
  end for
  $CurrentLevelNodes \leftarrow NextLevelNodes$
end while
if $\text{length}(CurrentLevelNodes) = 1$ **then**
  $MR \leftarrow CurrentLevelNodes[0]$ ▷ The single remaining node is the Merkle Root
else ▷ Handles $N = 0$ and $N_{pow2} = 0$, resulting in an empty $CurrentLevelNodes$
  $MR \leftarrow H(\text{""})$ ▷ Define Merkle Root for an empty set of tokens, e.g., hash of empty string
end if
return MR

F.3 WORKFLOW OF CoIn

Algorithm 4 illustrates the complete verification procedure of CoIn.

Algorithm 4 Multi-Round Verification in CoIn

Require: COLA Response (containing reasoning blocks $\mathcal{B}_{\text{total}}$ and final answer A)
Require: Fraction γ of blocks for initial verification (e.g., 0.1)
Require: Pre-trained matching heads $\mathsf{MH}_{\mathsf{tb}}(\cdot,\cdot)$, $\mathsf{MH}_{\mathsf{ba}}(\cdot,\cdot)$
Require: Embedding function $\mathsf{Embd}(\cdot)$
Require: Verification threshold τ
Ensure: Audit decision: "Successful" or "COLA Flagged for Inflation"

// Initialization
$\mathcal{B}_{\text{unverified}} \leftarrow \mathcal{B}_{\text{total}}$
$\mathcal{B}_{\text{verified}} \leftarrow \emptyset$
audit_successful $\leftarrow$ **false**
all_blocks_audited $\leftarrow$ **false**
// Initial round of verification
Select an initial set of blocks $\mathcal{B}_{\text{current\_round}} \subseteq \mathcal{B}_{\text{unverified}}$ of size $\lceil \gamma \cdot |\mathcal{B}_{\text{total}}| \rceil$
if $\mathcal{B}_{\text{current\_round}}$ is empty **and** $|\mathcal{B}_{\text{total}}| > 0$ **then**
    $\mathcal{B}_{\text{current\_round}} \leftarrow$ one randomly selected block from $\mathcal{B}_{\text{unverified}}$
end if
while not *audit_successful* **and not** *all_blocks_audited* **do**
    if $\mathcal{B}_{\text{current\_round}}$ is empty **then**
        all_blocks_audited $\leftarrow$ **true** ▷ No more blocks to check
        goto *FinalDecision*
    end if
    round_scores $\leftarrow$ [] ▷ Initialize as an empty list/array
    for each block $B_j \in \mathcal{B}_{\text{current\_round}}$ **do**
        Randomly select a subset of reasoning tokens $\{r_i\}_{i=1}^{k}$ from B_j (e.g., 10
        $E_{\text{tokens}} \leftarrow \mathsf{AVG}(\{\mathsf{Embd}(r_i)\}_{i=1}^{k})$ ▷ Average embedding of selected tokens
        $E_{\text{block}} \leftarrow \mathsf{Embd}(B_j)$
        $E_{\text{answer}} \leftarrow \mathsf{Embd}(A)$
        $S_{tb} \leftarrow \mathsf{MH}_{\mathsf{tb}}(E_{\text{tokens}}, E_{\text{block}})$ ▷ Tokens-to-Block score
        $S_{ba} \leftarrow \mathsf{MH}_{\mathsf{ba}}(E_{\text{block}}, E_{\text{answer}})$ ▷ Block-to-Answer score
        Add pair (S_{tb}, S_{ba}) to *round_scores*
        CoIn performs Merkle Proofs on selected tokens in B_j (verification of token integrity)
    end for
    $\mathcal{B}_{\text{verified}} \leftarrow \mathcal{B}_{\text{verified}} \cup \mathcal{B}_{\text{current\_round}}$
    $\mathcal{B}_{\text{unverified}} \leftarrow \mathcal{B}_{\text{unverified}} \setminus \mathcal{B}_{\text{current\_round}}$
    verifier_decision $\leftarrow$ VERIFIER(*round_scores*, τ) ▷ Verifier can be rule-based or learning-based
    if *verifier_decision* $=$ Accept **then**
        audit_successful $\leftarrow$ **true**
    else
        if $\mathcal{B}_{\text{unverified}}$ is empty **then**
            all_blocks_audited $\leftarrow$ **true**
        else
            Select one new random block B_{next} from $\mathcal{B}_{\text{unverified}}$
            $\mathcal{B}_{\text{current\_round}} \leftarrow \{B_{\text{next}}\}$ ▷ Next round verifies this single block
        end if
    end if
end while
FinalDecision:
if *audit_successful* **then**
    return "Audit Successful"
else
    return "COLA Flagged for Token Inflation" ▷ User may request COLA to justify charges
end if

function VERIFIER(*scores_list*, τ) ▷ Example: Rule-based verifier
    if *scores_list* is empty **then return** "Reject"
    end if
    avg_S_tb $\leftarrow$ average of all S_{tb} in *scores_list*
    avg_S_ba $\leftarrow$ average of all S_{ba} in *scores_list*
    if *avg_S_tb* $> \tau$ **and** *avg_S_ba* $> \tau$ **then**
        return "Accept"
    else
        return "Reject"
    end if ▷ Alternatively, a learning-based verifier (e.g., DeepSets) could be used here.
end function

G DETECTION PERFORMANCE OF CoIn

We show the comparison of the two verifiers and the impact of τ under different block sizes, as shown in Figure 7,8,9,10

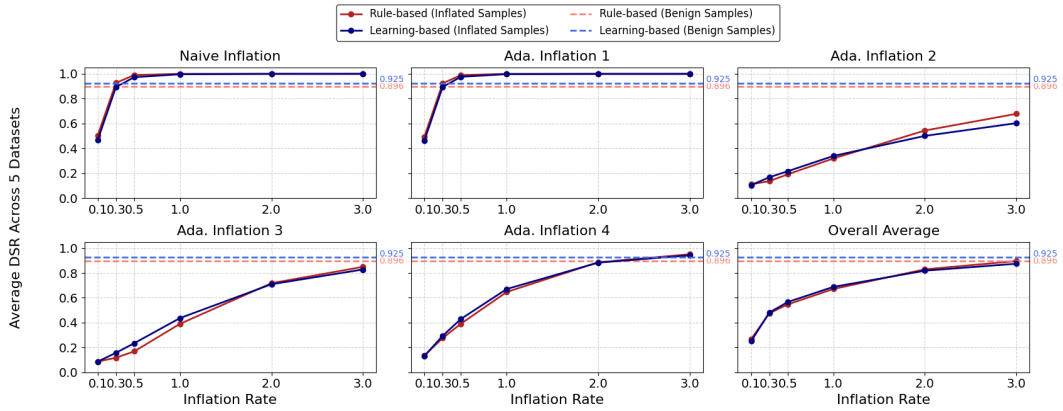


Figure 7: Performance of CoIn across different inflation methods and verifiers (Block Size = 512). The red lines and the blue lines represent the DSR of rule-based verifier and learning-based verifier, respectively. For a fair comparison, we set the threshold $\tau = 0.5$ for learning-based verifier, and $\tau = 0.6$ for rule-based verifier.

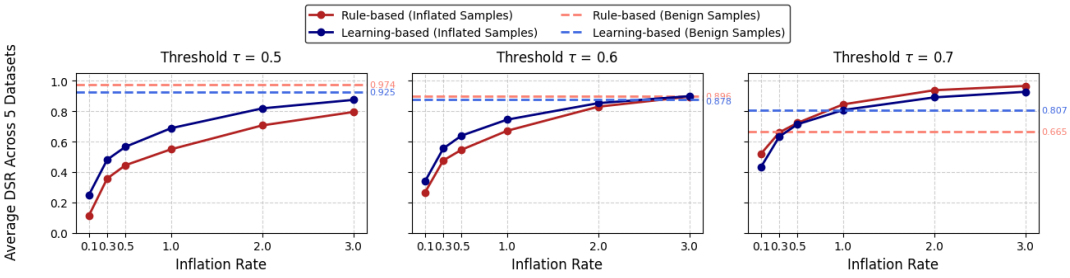


Figure 8: Impact of threshold τ on DSR (Block Size = 512).

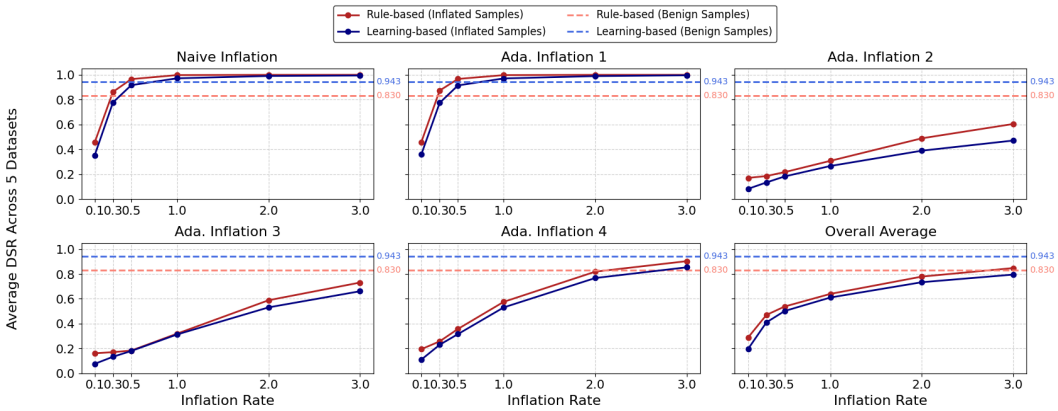


Figure 9: Performance of CoIn across different inflation methods and verifiers (Block Size = 1024). The red lines and the blue lines represent the DSR of rule-based verifier and learning-based verifier, respectively. For a fair comparison, we set the threshold $\tau = 0.5$ for learning-based verifier, and $\tau = 0.6$ for rule-based verifier.

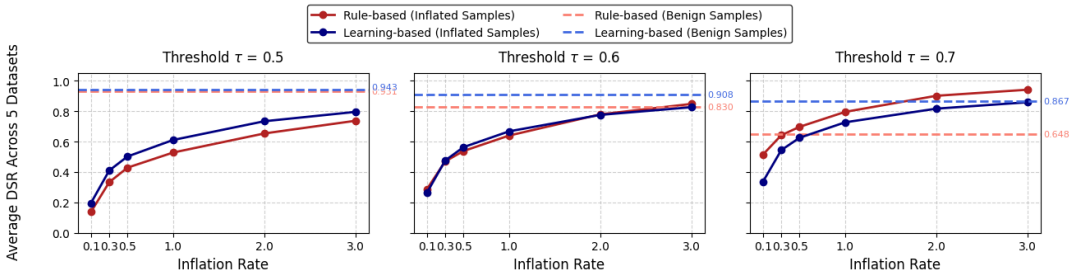


Figure 10: Impact of threshold τ on DSR (Block Size = 1024).

H PROMPTS USED IN DISCUSSION SECTION

Prompt 11 is used to explore the question "Can the original text be recovered from the tokens and embeddings exposed by COLA?", while Prompt 12 is used to explore "How difficult is the dataset we constructed?".

Prompt for Reconstructing Hidden Reasoning Passage

Reconstructing Hidden Reasoning Prompt
You are an expert in natural language reasoning and semantic retrieval.
Your task is to help recover a semantically meaningful and logically connected hidden passage that bridges a <Question> and an <Answer>.
This passage has been lost, but we know it is semantically related to both the <Question> and the <Answer>, and lies between them.

Given a – **<Question>**:
{question}

And the – **<Answer>**:
{answer}

We also know that some tokens from the original passage are still visible:
{sampled_token_text}

And we retrieved related documents from Wikipedia using the embedding of the original passage:
{retrieved_rag_docs}

Now, please help recover the most likely content of the hidden passage.
Return your answer strictly in the following JSON format:

```
\\recovered_json {
  "recovered_text": "<your reconstructed passage here>"
}
```

Figure 11: Prompt for Recovering a Hidden Reasoning Passage Using Question, Answer, Token Clues and Retrieved Wikipedia Documents.

Prompt for Evaluating Reasoning Passage Relevance

Evaluating Reasoning Process Prompt
You are a logical reasoning analyst.

Given a final answer and a randomly selected text passage, your task is to assess whether the text passage represents a reasoning process that leads to or supports the final answer.

The passage may or may not be relevant to the answer.
Your task is **not** to verify factual correctness, but to determine whether the passage semantically or logically connects to the answer and explains or justifies it in any meaningful way.

Random Text Passage:
{reason}

Final Answer:
{answer}

Please answer the following questions:
1. Is the text passage a plausible reasoning process that leads to the final answer?
2. Does it provide logical or semantic justification for the answer?

Respond in the following JSON format:

```
 \\reasoning_assessment
{
   "is_reasoning_process": true/false,
   "justification": "<your brief explanation of why the passage is or isn't
a reasoning process for the answer>"
}
```

Figure 12: Prompt for Judging Whether a Block Supports or Explains a Final Answer.

I CAN THE ORIGINAL TEXT BE RECOVERED FROM THE TOKENS AND EMBEDDINGS EXPOSED BY COLA?

I.1 DIRECT RECONSTRUCTION FROM EMBEDDINGS

Objective. Our framework aims to protect the provider's intellectual property by preventing large-scale harvesting of reasoning traces for distillation or reverse-engineering. We do *not* target the protection of isolated reasoning fragments, which are often trivially inferable from final answers or API summaries and are thus not considered sensitive. The real risk arises only when many reasoning steps are aggregated, enabling low-cost imitation of proprietary reasoning strategies.

Method. To assess direct reconstruction risk, we adopt the hypothesis–correction embedding inversion approach Vec2Text Morris et al. (2023) with the publicly released `inversion_model` and `corrector_model` trained for OpenAI's `text-embedding-ada-002`. We follow the default configuration (50 update steps; sequence beam width = 4).

Findings. As shown in Table 5, partial recovery is possible only when the block size is small (≤ 64). Consistent with prior observations Morris et al. (2023), reconstruction accuracy degrades sharply as length increases. In high-reasoning-load domains such as *math* and *code*, the reconstruction task becomes especially difficult. At our framework's minimum block size of 256, the attack completely fails. Even when BLEU/F1 appear non-trivial, the recovered text typically exhibits severe semantic drift, rendering it unsuitable for malicious distillation or dataset construction.

Block Size	Code		General		Math		Medical	
	BLEU	Token F1	BLEU	Token F1	BLEU	Token F1	BLEU	Token F1
16	31.08	0.6913	87.70	0.9292	43.83	0.7467	73.88	0.8845
32	22.98	0.5433	57.80	0.7828	15.06	0.5123	48.12	0.7195
64	12.31	0.4437	32.44	0.6412	12.62	0.4256	31.88	0.6291
128	7.98	0.3206	14.34	0.4816	9.84	0.3441	16.82	0.4928
256	3.24	0.2603	4.37	0.3740	4.07	0.2617	5.73	0.3801
512	0.46	0.2048	0.68	0.2989	0.67	0.2016	0.78	0.3101

Table 6: Direct reconstruction from embeddings using Vec2Text Morris et al. (2023) (50 update steps; beam width 4). Recovery drops rapidly with length; at block size 256 (our minimum), attacks effectively fail.

Example (Max Token Length = 64). Even with token-level overlap, the semantics are heavily distorted:

```
Original:
formula OH2 = 9R2 - (a2 + b2 + c2). If R=4, then 9R2=9*16=144.
The sum of squares of the sides: a2 + b2 + c2. From earlier, a=BC=55, so

Predicted:
formula: OH(a2+b2+c2) = R(a2+b2+c2) = R(a2+b2+c2) = R(a2+b2+c2) ...
= R(a2+b2+c2) = R(a2+b2+c2) = R(a2+b2+c2) = 55.
If a square has sides, then R(a2+b2+c2) = 9. If a square has sides, then
```

This illustrates that despite moderate BLEU/F1, the reconstructed content diverges in meaning and cannot serve as a basis for malicious distillation or dataset construction.

I.2 LM-BASED RECONSTRUCTION

During the verification process in `CoIn`, COLA leaks a certain number of block embeddings and tokens within the blocks to `CoIn`. To quantify the impact of such leakage, we assume a malicious `CoIn` leverages an RAG system to retrieve documents highly similar to the exposed embeddings and tokens, then feeds all retrieved information into an LLM to reconstruct the original content. The design and further details are provided in Appendix H. We randomly selected 100 samples from a mathematical dataset. We evaluated the similarity between the reconstructed blocks and the original

Table 7: Similarity Between Blocks Reconstructed by CoIn and Real Blocks.

Metric	Block Size β		
	256	512	1024
EmbedSim	0.65	0.66	0.75
BLEU	0.04	0.05	0.03
ROUGE-L	0.23	0.25	0.24
BERTScore	0.83	0.83	0.84

ones using embedding similarity, BLEU score Papineni et al. (2002), ROUGE-L Lin (2004) , and BERTScore Zhang et al. (2019). As shown in Table 7, we observe that a high BERTScore/EmbedSim combined with low BLEU/ROUGE indicates that the LLM successfully preserves the core semantics, while significantly differing from the real block in terms of surface expression and syntactic structure.

J Defending Against Repetition-Based Token Inflation Attacks

J.1 Threat Characterization

Beyond inflation attacks that CoIn primarily targets, an important class of adversarial strategies involves **repetition-based manipulation**. In such cases, a dishonest provider may artificially inflate the number of tokens by (1) duplicating subsequences of the reasoning text, or (2) appending variants of the original content generated through permutation or rewriting with small LLMs. These strategies differ from simple fabrication in that they preserve surface plausibility, but nonetheless undermine fair billing.

It is important to note that CoIn does not claim to defend against all advanced manipulations. Achieving complete security is difficult, and—as is the case across many trustworthy AI domains such as jailbreak defense, backdoor detection, and federated learning—robustness is typically achieved through iterative adversarial refinement. The value of CoIn lies in providing a balanced foundation that can be extended with additional defensive layers.

J.2 Detecting Direct Repetition

For the case of direct duplication, such as repeating hidden sequences or copying token fragments, CoIn can be extended with subtree equivalence checks on Merkle tree substructures. During an audit, the user may request all hash values at a specific level n of the Merkle tree. If repeated content exists, identical hash values will appear at sibling positions. This is because identical hashes at sibling nodes necessarily imply that the corresponding subtrees are structurally the same, which in turn means that the underlying token sequences are repeated. Since the provider must still produce valid Merkle proofs, these hash values necessarily correspond to real content; any attempt to falsify them would invalidate the proof.

In addition, as described in the CoIn design, the user retains the right to request the original reasoning text whenever suspicious billing arises. Once disclosed, repeated segments are trivially observable, making deception practically infeasible.

J.3 Addressing Permuted or Rewritten Variants

A more sophisticated attack involves appending **permuted or rewritten versions** of the original output, possibly produced by a small auxiliary LLM. While such strategies fall outside CoIn's initial threat model, since they require non-trivial computation rather than near-zero-cost inflation, they remain detectable through **redundancy analysis**.

To demonstrate this, we construct a controlled experiment:

- Let R_{ori} denote the original reasoning text.
- Generate an alternative sequence R_{llm} using a small LLM (Qwen/Qwen3-4B-Instruct-2507 Yang et al. (2025a)).
- Form the combined text $R_{\text{combined}} = R_{\text{ori}} \oplus R_{\text{llm}}$, which yields 100% inflation.

We then compute **normalized n-gram redundancy rates**. For a token sequence of length L and an n-gram window size n, let D_n denote the number of distinct n-grams appearing more than once. The redundancy rate is defined as:

$$\text{Redundancy}(n) = \frac{D_n}{L - n + 1}.$$

Intuitively, this metric captures how much repetition exists in a sequence: the higher the value, the more internally redundant the text is. Normalization by $(L - n + 1)$ eliminates the influence of text length, allowing fair comparisons across different samples.

J.4 Empirical Results

The redundancy rates for different domains and n-gram sizes are summarized in Table 8. The results show that R_{combined} consistently exhibits much higher redundancy than either R_{ori} or R_{llm}. Even when using LLM rewriting, substantial overlaps remain between the original and fabricated text. These redundancy patterns can be effectively detected by the Merkle subtree integrity check, thereby enabling `CoIn` to flag such manipulations.

Source	**n-gram size**	R_{ori}	R_{llm}	R_{combined}
Code	8	4.23	6.84	18.10
	16	1.00	4.40	17.23
	32	0.29	3.48	16.60
General	8	0.24	0.47	9.32
	16	0.04	0.14	8.95
	32	0.01	0.07	8.72
Math	8	5.21	7.28	12.18
	16	1.12	4.43	9.78
	32	0.13	3.45	8.90
Medical	8	1.40	3.45	9.27
	16	0.20	2.15	7.91
	32	0.03	1.65	7.14

Table 8: Normalized redundancy rates of original text (R_{ori}), LLM-generated variant (R_{llm}), and combined text (R_{combined}) across domains. Higher values indicate stronger repetition.

In summary, `CoIn`'s design can be naturally extended to resist repetition-based token inflation attacks:

- **Direct duplication** is detectable via subtree hash checks.
- **Permuted or rewritten variants** can be identified through redundancy analysis combined with Merkle tree verification.

Although such strategies are outside the strict scope of `CoIn`'s original threat model, these results illustrate the framework's extensibility. As with other trustworthy AI systems, we expect robustness to improve through iterative adversarial refinement and the integration of complementary defenses.