Crowdsourced Information Authentication: A Graph-based Model from the Science of Hadith

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Abstract

The authentication of prophetic traditions in Islam (Hadiths) is a cornerstone of Islamic jurisprudence, relying on meticulous examination of the chain of narrators and the transmitted content. This paper proposes a graph-based computational framework for the authentication of transmitted information, inspired by the principles of Hadith sciences. We jointly learn the authenticity score for each transmission and the reliability score for each narrator. The method explicitly accounts for the structure of transmission chains, narrator reliability metrics, content consistency, and the crucial aspect of corroboration (a form of collective verification) through multiple independent transmission paths, including a mechanism to discount overlapping paths. We explore several potential formulations and propose an iterative co-update algorithm with its convergence analysis. This work aims to exploit principles from over a millennium of extensive accumulated knowledge in the science of Hadith to inform the advancement of modern machine learning techniques for information verification.

Our formulation offers potential applications in digital information trustworthiness assessment and tools for Hadith scholars to leverage computational methods in analyzing the extensive Hadith corpora and its authentic collections.

1. Introduction

Hadith, the reported sayings and actions of the Prophet Muhammad (peace be upon him), form the second primary source of Islamic law after the Qur'an. The authenticity of a Hadith is paramount, and Islamic scholars developed a sophisticated critical methodology known as 'ilm al-Hadith' (the science of Hadith) over centuries. Key components include 'ilm al-Rijal' (knowledge of narrators), which assesses narrator reliability based on criteria like integrity ('Adalah') and accuracy ('Dabt'), and the analysis of the Isnad (chain of transmission) for connectivity ('Ittisal') and absence of hidden defects ('Illah'). The Matn (textual content) is also scrutinized for consistency with established principles.

A crucial aspect of Hadith authentication is cross-referencing, where multiple independent chains reporting the same or similar content significantly strengthen the authenticity claim. However, these chains might not be entirely independent, often sharing common narrators, a challenge that traditional scholarship addresses through nuanced analysis.

This paper introduces the *Iterative Co-Update Algorithm (ICUA)*, a method inspired by principles from Hadith scholarship for assessing information authenticity. ICUA utilizes a graph-based framework to represent transmission networks. It aims to formalize core Hadith authentication principles, including narrator reliability, Isnad integrity, and (optionally) Matn consistency. A key feature is a mechanism for aggregating evidence from multiple transmission paths while quantitatively discounting for overlaps due to shared narrators. The algorithm iteratively and jointly estimates information authenticity scores (S_H) and narrator (or more generally, transmitter) reliability scores (R_N). While rooted in Hadith methodology, we believe this framework contributes to the broader field of information trustworthiness assessment and offers a conceptual basis for potential new computational tools that Hadith scholars might choose to explore or adapt. Traditional Hadith evaluation methods, while rigorous, are manually intensive, and their comprehensive application to vast Hadith collections is challenging. Recent advances in graph-based machine learning, truth discovery, and information verification offer powerful tools that can help formalize and potentially scale complex reasoning processes analogous to those found in Hadith scholarship, applicable to the transmission of information in various domains, not limited to Hadiths.

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2. Related Work

The conceptual underpinnings of our proposed method are deeply rooted in the traditional Islamic science of Hadith ('ilm al-Hadith'). This field has a long history of sophisticated techniques for assessing the authenticity of reports. Central to this is 'ilm al-Rijal' (biographical evaluation of narrators), which involves assessing their integrity and accuracy, and 'Jarh wa Ta'dil' (the science of criticism and praise of narrators) (Brown, 2017). Meticulous analysis of the Isnad (chain of transmission) to ensure its connectivity and uncover any hidden defects is also foundational. There have been contemporary efforts to digitize Hadith collections and apply computational methods, often focusing on Isnad analysis, narrator network construction, or text mining (Najeeb, 2021). However, the joint iterative learning of report authenticity and narrator reliability, especially with sophisticated mechanisms for handling overlapping transmission paths that explicitly mirror the nuanced reasoning of traditional Hadith criticism, remains an area open for further exploration.

Our approach seeks to formalize some of these traditional principles within a computational framework, drawing parallels and inspiration from modern techniques in information trustworthiness. Truth discovery algorithms aim to identify true facts from multiple, often conflicting, sources of varying reliability (Li et al., 2015; Yin et al., 2008). Many of these methods, like TruthFinder (Yin et al., 2008), Hubs/Authorities (HITS) (Kleinberg, 1999), and PageRank (Page et al., 1999), employ iterative refinement where source reliability and claim believability are mutually dependent. Our proposed algorithm shares this iterative spirit of co-updating source (narrator) reliability and claim (Hadith) authenticity.

Graphs are natural representations for information propagation. Graph Neural Networks (GNNs) have shown success in detecting fake news and rumors by modeling propagation patterns, source characteristics, and content (Monti et al., 2019). These methods often learn representations of users and posts, and classify based on learned patterns. While our primary focus is on formalizing historical authentication methods rather than real-time fake news detection, the underlying principle of assessing information based on its propagation network and source reliability is shared. Shu et al. (2017) provide a comprehensive survey of fake news detection methods. Ruchansky et al. (2017) proposed CSI, a model that uses stance, user credibility, and content for fake news detection.

Iterative algorithms for belief propagation on graphs, like Loopy Belief Propagation (Murphy et al., 2013), have been used to infer states of nodes in a network. Trust and reputation systems on networks also often use iterative updates to assess node trustworthiness (Guha et al., 2004). Our model can be seen as a specialized iterative refinement mechanism, akin in spirit to belief propagation, tailored to the criteria derived from Hadith scholarship.

Our work distinguishes itself by specifically aiming to tailor the iterative updates and evidence aggregation (especially path overlap discounting) to the nuanced criteria found in Hadith authentication literature, while framing it within a formal, potentially ML-compatible general structure.

3. Problem Formulation

In the following sections, we primarily use vocabulary traditionally employed in Hadith sciences. However, the reader can easily substitute these terms for more general ones such as "information item" instead of Hadith, and "transmitter" or "source" instead of narrator, as alluded to in the introduction. This framework is presented with Hadith scholarship as its inspiration, but its mechanics can be generalized.

Let $\mathcal{H} = \{h_1, \ldots, h_M\}$ be the set of Hadith (information items) and $\mathcal{N} = \{n_1, \ldots, n_K\}$ be the set of narrators (transmitters). We aim to learn $S_H(h) \in [0, 1]$, the authenticity score for each Hadith $h \in \mathcal{H}$, and $R_N(n) \in [0, 1]$, the reliability score for each narrator $n \in \mathcal{N}$.

Hadith Transmission Graph: We model the transmission process as a directed graph where narrators are nodes. A specific Hadith h is associated with a set of Isnads (transmission paths) $\mathcal{P}_h = \{P_1, \ldots, P_{L_h}\}$. Each path $P_j \in \mathcal{P}_h$ is an ordered sequence of narrators $P_j = (n_{j,0} \to n_{j,1} \to \ldots \to n_{j,k_j})$. Here, $n_{j,0}$ is the ultimate source (e.g., in the context of Prophetic Hadith, this would be the Prophet Muhammad, peace be upon him (PBUH), whose statements are taken as the ground truth *for that specific report*, hence conceptually assigned an intrinsic reliability of 1 (for the purpose of evaluating the chain stemming from him). n_{j,k_j} is the final collector/reporter of the Hadith. Let $V(P_j)$ be the set of narrators in path P_j excluding the ultimate source $n_{j,0}$, as their reliabilities are what we assess within the path. Let $E(P_j)$ be the set of transmission edges (links between narrators) in P_j.

Path Score (S_{path}): The score of an individual path P for a Hadith h, given narrator reliabilities R_N, reflects the perceived strength of that chain of transmission. A principle often highlighted in Hadith science is the "weakest link" concept for Isnad evaluation:

$$S_{\text{path}}(P|\{R_N\}) = \min_{n \in V(P)} \{R_N(n)\} \tag{1}$$

While other formulations (e.g., product, average of reliabilities) are possible, the minimum is often most reflective of traditional Isnad criticism where a single unreliable narrator can compromise the entire chain.

Path Overlap (O): To account for the fact that multiple paths corroborating a Hadith might not be entirely independent (i.e., they share common narrators or segments), we measure the overlap between any two paths P_i, P_j for the same Hadith. The Edge Jaccard Index is a suitable measure for this:

$$O(P_i, P_j) = \frac{|E(P_i) \cap E(P_j)|}{|E(P_i) \cup E(P_j)|} \quad (2)$$

This quantifies the proportion of shared transmission links relative to the total unique links in both paths.

Matn Score ($M(h)$): This is an optional, potentially precomputed score $M(h) \in [0, 1]$ reflecting the textual consistency of a Hadith h with established, highly reliable sources (e.g., the Qur'an, Mutawatir Hadith, or undisputed historical facts) or internal textual coherence. This score could be derived from advanced Natural Language Processing (NLP) models or through expert scholarly input.

Priors: We can incorporate some prior knowledge into the model: $R_{\text{prior}}(n)$, reliability scores for some narrators, and $S_{H,\text{fixed}}(h)$, some fixed authenticity scores for benchmark Hadith. For instance, 'Mutawatir' Hadith (those transmitted by such a large number of distinct chains at each generation that their authenticity is considered definitive) could be assigned $S_{H,\text{fixed}}(h) = 1$.

4. Proposed Method

We propose an iterative algorithm, ICUA, that alternates between updating Hadith authenticity scores $S_H^{(t)}(h)$ and narrator reliability scores $R_N^{(t)}(n)$ at each iteration t, until the scores stabilize and converge.

4.1. Initialization (t=0)

- For each narrator $n \in \mathcal{N}$: If a prior reliability $R_{\text{prior}}(n)$ exists, initialize $R_N^{(0)}(n) = R_{\text{prior}}(n)$. Otherwise, initialize with a neutral value, e.g., $R_N^{(0)}(n) = 0.5$.
- For each Hadith $h \in \mathcal{H}$: If a fixed authenticity $S_{H,\text{fixed}}(h)$ exists, initialize $S_H^{(0)}(h) = S_{H,\text{fixed}}(h)$. Otherwise, initialize with a neutral value, e.g., $S_H^{(0)}(h) = 0.5$.

4.2. Iterative Updates

4.2.1. Step A: Update Hadith Authenticity

For each Hadith $h \in \mathcal{H}$: If a fixed authenticity score $S_{H,\text{fixed}}(h)$ exists for Hadith h, then $S_H^{(t)}(h) = S_{H,\text{fixed}}(h)$, and we proceed to the next Hadith. Otherwise:

1. **Calculate Individual Path Scores:** For each transmission path $P_j \in \mathcal{P}_h$ associated with Hadith h, compute its score $S_{\text{path}}(P_j | \{R_N^{(t-1)}\})$ using Eq. 1 with narrator reliabilities $R_N^{(t-1)}$ from the previous iteration $(t-1)$.
2. **Aggregate Path Scores with Overlap Discounting:** To combine evidence from multiple paths while accounting for their interdependencies, paths $P_{(1)}, \ldots, P_{(L_h)}$ in $\mathcal{P}_h$ are first sorted in descending order based on their individual scores $S_{\text{path}}(P_{(j)} | \{R_N^{(t-1)}\})$. Initialize the aggregated Isnad-based score for Hadith h, $S_{\text{isnad}}(h) = 0$. Maintain a set of paths already included in the aggregation, $P_{\text{included}} = \emptyset$. Iterate through the sorted paths $P_{(j)}$ for $j = 1, \ldots, L_h$:
   - Let $P_{\text{current}} = P_{(j)}$.
   - Calculate the maximum overlap of P_{current} with any path already in P_{included}: $O_{\max}(P_{\text{current}}) = \max_{P_k \in P_{\text{included}}} \{O(P_{\text{current}}, P_k)\}$. If P_{included} is empty, $O_{\max}(P_{\text{current}}) = 0$.
   - Determine the discount factor for P_{current}: $d_j = (1 - O_{\max}(P_{\text{current}}))$. This factor reduces the contribution of paths that are highly similar to already considered paths.
   - Update the aggregated score: $S_{\text{isnad}}(h) = S_{\text{isnad}}(h) + d_j \times S_{\text{path}}(P_{\text{current}} | \{R_N^{(t-1)}\})$.
   - Add the current path to the set of included paths: $P_{\text{included}} = P_{\text{included}} \cup \{P_{\text{current}}\}$.
3. **Incorporate Matn Score:** Let $S_{\text{interim}}(h) = S_{\text{isnad}}(h)$. If a Matn consistency score $M(h)$ is available and used for Hadith h: $S_{\text{interim}}(h) = S_{\text{interim}}(h) \times M(h)$.
4. **Normalize Authenticity Score:** The aggregated score $S_{\text{interim}}(h)$ can potentially exceed 1. To map this to the standard authenticity range $[0, 1]$, we apply a squashing function $\sigma(\cdot)$. For this paper, we use:

$$S_H^{(t)}(h) = \sigma(S_{\text{interim}}(h)) = \frac{S_{\text{interim}}(h)}{S_{\text{interim}}(h) + \alpha} \quad (3)$$

where α is a non-negative scaling constant (e.g., $\alpha = 1$). This function ensures $S_H^{(t)}(h) \in [0, 1]$ for non-negative $S_{\text{interim}}(h)$.

4.2.2. Step B: Update Narrator Reliability

For each narrator $n \in \mathcal{N}$: If a prior reliability $R_{\text{prior}}(n)$ exists and is designated as fixed, then $R_N^{(t)}(n) = R_{\text{prior}}(n)$, and we proceed to the next narrator. Otherwise: Let $\mathcal{H}_n$ be the set of all Hadith where narrator n appears in at least one of its transmission paths $P \in \mathcal{P}_h$. A narrator's reliability is derived from the authenticity of the Hadith they narrate, weighted by the significance of their role in transmitting those Hadith. In Step A, for each Hadith h and each path $P_j \in \mathcal{P}_h$, the discounted score contributed by that path to

$S_{\text{isnad}}(h)$ was $w(P_j, h) = d_j \times S_{\text{path}}(P_j | \{R_N^{(t-1)}\})$. To assess narrator n's reliability, we first calculate the normalized contribution of each path P_j (in which n appears) to its respective Hadith h's score: Let $\text{C}(P_j, h) = \frac{w(P_j,h)}{\sum_{P_k \in \mathcal{P}_h} w(P_k,h)}$, if $\sum_{P_k \in \mathcal{P}_h} w(P_k, h) > 0$; otherwise, $\text{C}(P_j, h) = 0$. This value represents the relative importance of path P_j among all paths for Hadith h.

The updated reliability $R_N^{(t)}(n)$ is then calculated as follows:

$$\text{Num}^{(t)}(n) = \sum_{h \in \mathcal{H}_n} \sum_{\substack{P_j \in \mathcal{P}_h \\ \text{s.t. } n \in V(P_j)}} S_H^{(t)}(h) \times \text{C}(P_j, h) \quad (4)$$

$$\text{Denom}^{(t)}(n) = \sum_{h \in \mathcal{H}_n} \sum_{\substack{P_j \in \mathcal{P}_h \\ \text{s.t. } n \in V(P_j)}} \text{C}(P_j, h) + \delta \quad (5)$$

where $\delta > 0$ is a small smoothing factor (e.g., $\delta = 1$) to prevent division by zero and to stabilize scores for narrators involved in few narrations. The reliability score is then:

$$R_N^{(t)}(n) = \frac{\text{Num}^{(t)}(n)}{\text{Denom}^{(t)}(n)} \quad (6)$$

This formulation, with $S_H^{(t)}(h) \in [0, 1]$ and normalized contributions, ensures $R_N^{(t)}(n)$ remains within or very close to $[0, 1]$; explicit clipping to $[0, 1]$ can be applied if necessary.

The iterative process continues until the changes in Hadith authenticity scores and narrator reliability scores between successive iterations fall below predefined small thresholds ϵ_S and ϵ_R, respectively. That is, when $\sum_{h \in \mathcal{H}} |S_H^{(t)}(h) - S_H^{(t-1)}(h)| < \epsilon_S$ and $\sum_{n \in \mathcal{N}} |R_N^{(t)}(n) - R_N^{(t-1)}(n)| < \epsilon_R$.

5. Convergence Analysis

Proving the convergence of the ICUA algorithm requires careful consideration of the update functions. Let $X^{(t)} = (\{S_H^{(t)}(h)\}_{h \in \mathcal{H}}, \{R_N^{(t)}(n)\}_{n \in \mathcal{N}})$ be the state vector encompassing all authenticity and reliability scores at iteration t. The ICUA effectively defines an operator $\mathcal{F}$ such that $X^{(t)} = \mathcal{F}(X^{(t-1)})$.

Boundedness and Compactness: The scores $S_H(h)$ and $R_N(n)$ are explicitly designed or constrained (e.g., by the squashing function σ and the formulation of Eq. 6) to lie within the interval $[0, 1]$. Thus, the state vector $X^{(t)}$ resides in a compact set, specifically the hypercube $[0, 1]^{|\mathcal{H}|+|\mathcal{N}|}$.

Continuity of the Operator $\mathcal{F}$: The update functions for $S_H^{(t)}(h)$ involve 'min' operations (Eq. 1), summations, products, divisions (for overlap calculation via Eq. 2 and for score normalization), and a squashing function σ (Eq. 3). The operations used in our iterations are continuous. The denominator in the overlap calculation (Eq. 2) is zero only if both paths are empty of edges, which is a trivial case not typically encountered. The δ in Eq. 5 ensures the denominator for the R_N update is strictly positive. Typical squashing functions σ, such as the one chosen in Eq. 3 are continuous (the same applies the update $R_N^{(t)}(n)$).

Given a finite set of Hadith and narrators, and fixed Matn scores and priors (if used), the overall operator $\mathcal{F}$ that maps $X^{(t-1)}$ to $X^{(t)}$ is continuous because it is a composition of continuous functions operating on these scores.

5.1. Existence of a Fixed Point:

Since $\mathcal{F}$ is a continuous function mapping a compact, convex set (the hypercube $[0, 1]^{|\mathcal{H}|+|\mathcal{N}|}$) into itself, by Brouwer's Fixed-Point Theorem, there exists at least one fixed point X^* such that $X^* = \mathcal{F}(X^*)$. The existence of a fixed point does not, by itself, guarantee that the iterative sequence $X^{(t)}$ generated by ICUA will converge to it. For the convergence of the sequence, stronger conditions are typically needed, for instance if $\mathcal{F}$ were a contraction mapping with respect to some norm $|| \cdot ||$ on the space of scores (i.e., $||\mathcal{F}(X) - \mathcal{F}(Y)|| \leq \lambda ||X - Y||$ for some constant $\lambda \in [0, 1)$ for all X, Y), then by the Banach Fixed-Point Theorem, $\mathcal{F}$ would have a unique fixed point, and the iteration $X^{(t+1)} = \mathcal{F}(X^{(t)})$ would converge to this fixed point from any initial state $X^{(0)}$. Proving that the complex operator $\mathcal{F}$ defined by ICUA is a contraction mapping is challenging, particularly due to the 'min' operation in path scoring and the coupled nature of the updates.

Many truth discovery algorithms that employ similar iterative, mutually reinforcing update structures have been observed to converge empirically (Yin et al., 2008; Li et al., 2015), and some possess theoretical convergence guarantees under specific assumptions. The averaging nature inherent in the narrator reliability update (Eq. 6) and the damping effect of the squashing function σ often contribute to the stability of such systems. The "weakest link" principle for path scores (Eq. 1), while crucial for reflecting Hadith authenticity criteria, can introduce non-smoothness that complicates formal proofs. Empirically, such iterative systems frequently converge in practice due to normalization, averaging effects, and the bounded nature of the scores. A formal proof of convergence for ICUA would likely require a more detailed analysis of the properties of $\mathcal{F}$, potentially under certain simplifying assumptions regarding the score functions or update rules.

For the purpose of this paper, we argue that the continuity of $\mathcal{F}$ on a compact set guarantees the existence of at least one fixed point. The empirical convergence observed in analogous systems (Li et al., 2015) suggests that ICUA is likely to converge to a meaningful equilibrium. However, a formal proof of convergence to a *unique* fixed point is

beyond the scope of this initial presentation and remains a direction for future work.

6. Conclusion and Future Work

We introduced ICUA, a graph-based iterative framework inspired by traditional Islamic Hadith scholarship for the joint assessment of information authenticity and source reliability. ICUA formalizes analogues of key Hadithic principles such as Isnad analysis, narrator evaluation, and corroboration with path overlap discounting. While a full convergence proof is future work, we sketched an argument for the existence of a fixed point for its iterative updates. Key directions for future research include:

- **Rigorous Convergence Proof:** Developing a full mathematical proof of convergence for ICUA, potentially identifying conditions under which convergence to a unique fixed point is guaranteed.
- **Learnable Components:** Exploring the possibility of making parameters of the ICUA model (e.g., the squashing function σ's parameters like α, the smoothing factor δ, or even weights within the path score or overlap calculations) learnable from data, should suitable labeled datasets become available. This could involve framing ICUA updates as layers in a Graph Neural Network.
- **Advanced Defect Detection ('Illah'):** Incorporating more sophisticated mechanisms to model, detect, or account for potential hidden defects ('Illah') in transmission chains (Isnads) or textual content (Matn), which is a nuanced aspect of Hadith criticism.
- **Enhanced Matn Analysis:** Deeper integration of advanced Natural Language Processing (NLP) techniques for Matn consistency checking, source comparison, and anomaly detection.
- **Generalization and Application**: Exploring the adaptation and application of the ICUA framework to other domains requiring information trustworthiness assessment in networked or multi-source environments.

Ultimately, ICUA serves as a conceptual model. We hope it stimulates further research into formalizing traditional epistemological principles for contemporary computational systems and encourages broader engagement with computational tools within Islamic studies for knowledge discovery.

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A. Algorithm Pseudo-code

Algorithm 1 Iterative Co-Update Algorithm (ICUA)

Initialize $S_H^{(0)}(h)$ for all $h \in \mathcal{H}$, $R_N^{(0)}(n)$ for all $n \in \mathcal{N}$.
$t \leftarrow 0$
repeat
    $t \leftarrow t + 1$
    // Step A: Update Hadith (Information Item) Authenticity Scores
    for all $h \in \mathcal{H}$ **do**
        if $S_{H,\text{fixed}}(h)$ exists **then**
            $S_H^{(t)}(h) \leftarrow S_{H,\text{fixed}}(h)$
        else
            Calculate $S_{\text{isnad}}(h)$ based on $\{R_N^{(t-1)}(n')\}_{n' \in \mathcal{N}}$ with overlap discount (as described in Section 4.2.1).
            $S_{\text{interim}}(h) \leftarrow S_{\text{isnad}}(h)$
            if $M(h)$ is used **then**
                $S_{\text{interim}}(h) \leftarrow S_{\text{interim}}(h) \times M(h)$
            end if
            $S_H^{(t)}(h) \leftarrow \sigma(S_{\text{interim}}(h))$ (using squashing function, e.g., Eq. 3)
        end if
    end for
    // Step B: Update Narrator (Transmitter) Reliability Scores
    for all $n \in \mathcal{N}$ **do**
        if $R_{\text{prior}}(n)$ exists and is fixed **then**
            $R_N^{(t)}(n) \leftarrow R_{\text{prior}}(n)$
        else
            Calculate Numerator $\text{Num}^{(t)}(n)$ using Eq. 4 (based on $\{S_H^{(t)}(h')\}_{h' \in \mathcal{H}}$).
            Calculate Denominator $\text{Den}^{(t)}(n)$ using Eq. 5.
            $R_N^{(t)}(n) \leftarrow \text{Num}^{(t)}(n)/\text{Den}^{(t)}(n)$ (ensuring result is in $[0, 1]$).
        end if
    end for
until convergence of $S_H^{(t)}$ and $R_N^{(t)}$ (e.g., below a threshold ϵ) =0