
Efficient and High-quality Ellipse Detection via Implicitly Excluding Most Useless Arc Groups and Enhancing Arc detection

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Abstract

1 Detecting ellipses from images is an fundamental problem in computer vision and
2 pattern recognition, and plays an important role in many applications. This paper
3 presents a new edge-link method for efficient and high-quality ellipse detection,
4 where the two steps of edge-link methods are improved by our two presented
5 novel measures respectively. The first is to adaptively adjust the search direction in
6 linking edge pixels to generate arcs as consistently as possible. The second is to
7 develop a novel measure for grouping arcs to check whether these arcs are from
8 a same ellipse, which is by employing a grid to manage the arcs and designing a
9 traversal path to visit grid cells continuously, through which most useless arc groups
10 can be implicitly excluded for efficiency. This is different from existing methods
11 that need explicitly check all possible arc groups. Based on these measures, we
12 design an algorithm to detect ellipses as many as possible. Experimental results
13 show that we can significantly improve both the accuracy and efficiency of ellipse
14 detection, much superior to existing methods. Thus, we can significantly improve
15 many applications.

16 1 Introduction

17 Ellipse detection is an important task in image processing, and required in many applications such as
18 industrial inspection [1], medical image analysis [2], autonomous driving [3], and robot vision [4].
19 With regard to this, the edge-link methods [5, 6, 7, 8, 9, 10, 11] are prominent due to their efficiency
20 and effectiveness, to be discussed in Sec. 2. These methods work by first extracting arcs with
21 continuous edge pixels and then checking arcs in groups whether they are co-elliptical, called *ellipse*
22 *checks*. The arcs from a same ellipse are called *co-elliptical* ones, and they are used to generate an
23 ellipse.

24 Arc groups are always in a large number, so that arc grouping for ellipse checks dominates the
25 efficiency. Considering that most arc groups are composed of arcs from different ellipses, which
26 cannot be used to generate ellipses, called *useless groups*, many methods have been proposed to
27 employ cheap calculations to exclude useless groups as soon as possible for efficiency, such as
28 constraining the search region for arc grouping [5], leveraging convex hulls to group [9], building an
29 adjacent matrix to represent the grouping relationships between arcs [8], and excluding many useless
30 groups by constraints from characteristic mapping [12] or the Candy’s theorem [13]. Even so, any
31 arc group should be checked once and this still wastes much time on useless groups.

32 In this paper, we address the challenge of implicitly excluding most useless groups for efficiency. It
33 is by using a grid to manage arcs and then visiting grid cells orderly from the center outward. For
34 a visited grid cell, each arc contained in the grid cell tries to find other arcs for arc grouping in its



	[8]	[7]	[14]	[10]	[15]	[12]	Ours
#TP	4	4	4	3	0	6	7
#FP	2	3	0	1	1	4	0
#FN	3	3	3	4	7	1	0
Time(ms)	7.86	8.12	15.28	11.24	52.23	7.07	5.94

Figure 1: Our method can obtain more accurate results than the state-of-the-art methods and cost less time, as illustrated for the example here. The detected ellipses are marked in red for the True Positive and green for the False Positive.

constrained search region [5]. This corresponds to have grid cells paired for arc grouping. As the constrained search region of an arc does not cover all grid cells in general, and a visited grid would not be processed again after it is visited, this would have many grid cells not paired for arc grouping, meaning their related useless arc groups are implicitly excluded. This will be discussed in detail in Sec. 3.

We also present a measure to generate arcs as consistently as possible, by which ellipses can be more effectively detected. This is by adaptively adjusting the search direction to link continuous edge pixels to generate contours, to be discussed in Sec. 4.1.

Based on our two novel measures, we develop an algorithm to detect ellipses as many as possible, where all formed arc groups are further checked by existing methods [9, 12] to exclude many more useless groups and finally co-elliptical arc groups are used to generate their corresponding ellipses with existing methods. As a result, we can detect many more ellipses and in a higher efficiency and a higher quality than existing methods, as illustrated in Fig. 1 and demonstrated in Sec. 5. Benefited from our improvements, many applications can be significantly promoted, as illustrated in Appendix C.

2 Related work

Ellipse detection methods can be coarsely classified into three categories: Hough transform based methods, edge-link based methods, and learning methods. Hough transform based methods [16, 17, 18, 19] take the ellipse detection task as a peak-finding process in a parametric voting space and use the Hough transform on pixels for a solution. Unfortunately, they are expensive and prone to incur incorrect results due to the complicated backgrounds and the lack of effective verification [20].

Recently, some learning methods [21, 22, 23, 24, 15, 25] have been proposed for ellipse detection. However, their potentials are limited by the difficulty of collecting high quality data for training, and they are always inefficient as they need learn a lot of features for a complex model, as shown in Fig. 1 and Table 3 for the result of [15].

Till now, edge-link based methods are prominent for ellipse detection [5, 6, 7, 8, 9, 10, 26, 11]. They link discrete edge pixels into arcs for ellipse detection, where local continuity information of contours can be well exploited to suppress interference from outliers and noise, and therefore increasing the detection accuracy. In the following, we have edge-link methods discussed briefly by their three sub-tasks, arc generation, arc grouping and ellipse checks.

For arc generation, Kim *et al.* [27] extract short straight line segments to approximate arcs, Prasad *et al.* [5] use curvature and convexity to extract smooth elliptic arcs, and there are two methods proposed for better corner detection to promote arc extraction, the adaptive Ramer Douglas Peucker (RDP) algorithm [28, 29] and a curvature-based method [30]. In implementing our method, we use the adaptive RDP algorithm [28] to divide contours into arcs because it need not frequent parameter adjustments. Based on the method of [8], Wang *et al.* [31] proposed a contrast-guided measure to enhance the extraction of arcs, but the improvement in detection capability is limited.

For arc grouping, some constraints are proposed to quickly exclude useless groups using simple computation, including arc-aware search regions [5], quadrant constraints [6], projection invariant

Table 1: Statistics about the ablation tests. The number of checked groups and time cost per image are the average results for all images in a dataset, where time refers to the total time cost on detecting ellipses in an image, including arc extraction, arc grouping and ellipse checks.

Datasets	Grouping via only arc-search regions		Our arc grouping measure		Implicit excluding rate
	Time(ms)	Checked groups	Time(ms)	Checked groups	
Prasad	8.44	18	4.09	4	$(18-4)/18=77.8\%$
Prasad+	21.52	54	6.61	14	$(54-14)/54=74.1\%$
Random	24.42	62	7.76	16	$(62-16)/62=74.2\%$
Smartphone	58.82	231	11.81	26	$(231-26)/231=88.7\%$

74 pruning [7], arc-support regions [14], characteristic mapping [12], the Candy’s theorem [13] and
 75 coherent chord computation [11]. As useless groups always take a very large portion of all possible
 76 groups, these measures still take much time and prevent efficiency promotion. There are also some
 77 data structures studied for improving ellipse detection by collecting the arcs that are very possibly
 78 co-elliptical, including undirected graphs [9] and disjoint-set forests [10]. Even so, they need to
 79 enumerate possible groups, and this still need check a large amount of useless groups.

80 To check whether an ellipse is valid, a commonly used criterion is the ratio of inliers, defined as
 81 the proportion of arc points that fit the ellipse well [6, 7, 10]. When the ratio is high, it means the
 82 estimated ellipse is consistent with arcs. Other criteria include gradient consistency [10] and the
 83 completeness of ellipse [5, 7], which can filter out bad ellipses, but may prevent detection of imperfect
 84 ellipses in images. In our implementation, we use the measure of [9] for valid check of ellipses.

85 Different from existing edge-link methods, we present an arc grouping method to implicitly exclude
 86 most useless groups, where arc-aware search regions [5] are used for grouping arcs that are possibly
 87 co-elliptical. To our knowledge, this is the first method that can implicitly exclude useless arc groups.
 88 Our method is orthogonal to existing methods and so easy to be integrated with them for improving
 89 ellipse detection. For example, the useless groups that are not implicitly excluded by our method
 90 can be further quickly excluded by characteristic mapping constrains [12]. As for arc generation, we
 91 will mainly use the measures of [9] but replace its strategy for contour extraction, where an adaptive
 92 strategy is developed to extract contours as smooth as possible for generating arcs consistently.

93 3 Grid-based arc grouping

94 Our measure for arc grouping is by using a grid to manage the arcs and then visiting grid cells by
 95 a traversing path, through which arcs are grouped for ellipse detection. In the following, we first
 96 introduce the steps of our measure and then discuss their implementation and the effectiveness on
 97 implicitly excluding useless groups. With an ablation study by four data sets, it is known that we can
 98 greatly reduce the arc groups to be checked in comparison with only using arc-search regions [5] for
 99 arc grouping, as listed in Table 1. This shows we can implicitly exclude most useless arc groups.

100 The steps of our measure are as follows. Firstly, a grid is generated by the bounding box for all
 101 extracted arcs. Then, arcs are recorded in the grid cells that contain or intersect with them. Finally, a
 102 traversing path is designed to visit cells sequentially from the center outwards gradually, by which
 103 each arc in the currently visited cell is taken as an active one to search for possible co-elliptical arcs
 104 (called *inactive arcs*) in its improved arc-search region (to be discussed in Sec. 3.2) for arc grouping.
 105 As illustrated in Fig. 2, the active arc R_3 finds its inactive arc R_4 in its arc-search region in red to
 106 form a group. In this way, all possible co-elliptical arcs can be grouped. In summary, the algorithm
 107 for our arc grouping method is given in Alg. 1.

108 3.1 Grid resolutions

109 Clearly, the grid resolutions have much influence on the detection efficiency. A lower grid resolution
 110 means fewer cells, so that a cell would be larger to contain more arcs and prevent efficiency. In
 111 contrast, a higher grid resolution will lead to smaller cells containing fewer arcs, but this will generate
 112 more cells, also preventing efficiency. As an ideal expectation, if the arcs are evenly distributed in the
 113 grid cells for each grid cell to contain only one arc, the number of grid cells would not be large and a
 114 grid cell contains the fewest arcs, which would have ellipse detection in a high efficiency.

115

Algorithm 1 Arc grouping for ellipse detection**Input:** Arcs: $R = \{r_i\}_{i=1}^n$, search regions: $\{\Omega_i\}_{i=1}^n$ **Output:** Arc groups: F

```

1: Define  $GC$  as the cell being processed;
2: Define  $IC$  as the set of VISITED cells;
3: Initialize  $GC$  as the central cell;
4: Mark all arcs as NOT_Active_USED;
5: while  $GC$  is not  $\emptyset$  do
6:   for arc  $r_i \in GC$  that is NOT_Active_USED do
7:     for arc  $r_j \in \Omega_i \setminus IC$  that is NOT_Active_USED do
8:       if  $r_i \in \Omega_j$  then
9:         Append arc group  $\langle r_i, r_j \rangle$  to  $F$ ;
10:      end if
11:    end for
12:    Mark  $r_i$  as USED;
13:  end for
14:  Add  $GC$  to  $IC$ ;
15:  Let  $GC$  be the next cell by the search order;
16: end while

```

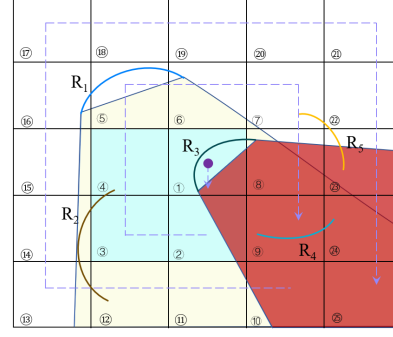


Figure 2: Our measure for arc grouping by orderly traversing the grid cells from the center outward, as marked by purple polylines with arrows.

116 Thus, we determine the grid resolution, NC_x and NC_y along the two axes, by Eq. 1,

$$\begin{aligned} NC_x &= \left\lfloor r_a \cdot \sqrt{N_{arcs}} \right\rfloor \\ NC_y &= \left\lfloor \frac{\sqrt{N_{arcs}}}{r_a} \right\rfloor \end{aligned} \quad (1)$$

117 where N_{arcs} is the number of arcs, and $r_a = \frac{\text{image_height}}{\text{image_width}}$ is the aspect ratio of the image.

118 With an investigation by many tests, such a grid resolution can always obtain good results and they
 119 are used in our implementation. Of course, arcs are generally in various lengths and distributed
 120 unevenly, which may influence the grid resolution in achieving high efficiency. As a future issue, we
 121 will further study these influences to optimize the grid resolution for high efficiency.

122 3.2 Improved arc-search regions

123 As discussed by Prasad *et al.* [5], an arc can only find its co-elliptical inactive arcs in a region, called
 124 an *arc-search region*. The arc-search region of an arc is bounded by the line connecting the two
 125 endpoints of the arc and two ray lines that are from its two endpoints and tangent to the arc, as
 126 illustrated by the red region for R_3 in Fig. 2. As we take the arcs of the visited grid cells each as
 127 active ones to find all their respective co-elliptical arcs, the visited grid cells would not be investigated
 128 in the following checks. Thus, our arc-search region for an arc should exclude the grid cells that have
 129 been visited. As illustrated in Fig. 2, the arc-search region of R_1 in the cell ⑤ should exclude the grid
 130 cells ①, ②, ③ and ④, as the cell ⑤ containing R_1 is visited after these cells. Clearly, this reduces
 131 the arc-search region of R_1 and implicitly exclude the arc group of R_1 and R_3 . Such a reduced
 132 arc-search is called an *improved arc-search region*, as illustrated by the yellow region for R_1 , which
 133 excludes the light green cells ①, ②, ③ and ④.

134 3.3 Traversing paths

135 For implicitly excluding useless groups as many as possible, we design a traversing path to visit grid
 136 cells from center outwards gradually. This is based on the following considerations:

- 137 • In general, active arcs nearer to the center of the grid often have smaller arc-search regions
 138 than those farther away from the center, e.g., the arc-search region of R_3 is smaller than
 139 that of R_1 in Fig. 2. Thus, first checking the arcs nearer the center of the grid can more
 140 effectively avoid checking useless groups, as a smaller arc-search region is less possible to
 141 have useless groups. Of course, it is also possible for an arc near the boundary of the grid to
 142 have a smaller arc-search region when it is towards the outside. However, such cases seldom
 143 occur, and so would not interfere with our efficiency.

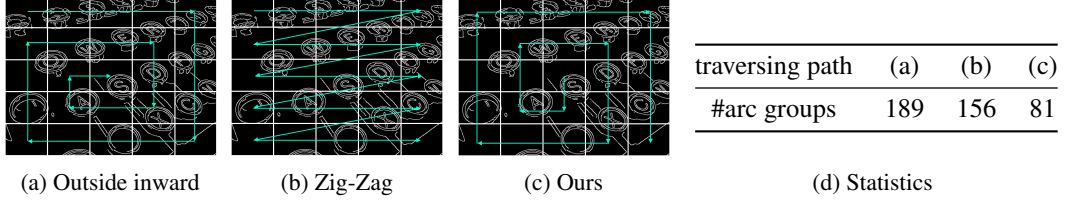


Figure 3: Comparison of the generated arc groups between using different traversing paths for the extracted arcs of the image in Fig. 10(c). The table shows the number of collected arc groups for different traversing paths.

- With such a traversing path, the grid cells far away from the center would have their arc-search regions improved, as discussed in Sec. 3.2. This is helpful for efficiency promotion. Otherwise, when the grid cells far away from the center are visited first, their arc-search region would be less improved, causing many useless groups generated. As illustrated in Fig. 2, if the grid cell containing R_1 is visited first, its arc-search region would be larger to include the light green cells, and so generating more useless groups.

As an investigation, we tested other traversing paths like the path from the outside inward and a zig-zag path, as illustrated in Fig. 3. The results show that using our path can generate much fewer arc groups than using the other paths. This shows the advantages of our designed traversing path for implicit exclusion of useless groups.

4 Improved ellipse detection

With our arc grouping measure, we present a new edge-link method for ellipses detection, where we mainly use the corresponding measures of [9] for arc generation and ellipse checks, and then take a new strategy for extracting ellipses as many as possible. The pipeline of our method is still by the steps for edge-link methods, extracting arcs, grouping arcs for ellipse checks and generating ellipses for co-elliptical arcs, as illustrated in Appendix A. For a complete introduction of our method for ellipse detection, we will first introduce the corresponding measures of [9] for arc generation, arc grouping and ellipse checks, and then discuss our improvements and our final algorithm for ellipse detection. Our improvements are as follows:

- In arc extraction, we present a novel measure to improve contour extraction for generating arcs more consistently than using the corresponding measure of [9].
- In arc grouping, our developed method in Sec. 3 is used, by which most useless groups can be implicitly excluded. Then, the collected arc groups could be further filtered by the characteristic mapping constraints [12] to more effectively obtain useful arc groups for ellipse checks.
- In ellipse generation, we take another strategy that first generates ellipse candidates as many as possible and then removes the redundant ones. Thus, ellipses can be detected many more than existing methods.

4.1 Arc extraction

The arc extraction measure of [9] includes the following five steps.

Edge detection. The Canny’s algorithm [32] is used to detect the edge pixels. For obtaining high quality edges, Gaussian filtering with small kernels is applied to smooth out noise and bilateral filtering is applied to smooth out textures.

Contour extraction. Continuous edge pixels are collected to obtain contour curves. Here, we develop an adaptive measure to extract contours as smoothly as possible for generating arcs as consistently as possible for helping ellipse detection, to be discussed later in this section.

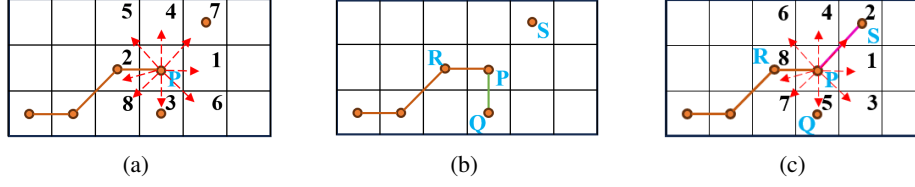


Figure 4: (a) The measure of [9] for contour generation is by starting from a seed edge pixel to extend gradually in a depth first search, and in a fixed search order of the left, right, down, up, up left, down right, up right, down left, as represented by the dashed arrow lines with numbers. (b) Shen *et al.* [9] may likely generate a very curved contour by extending from P to Q, not to S. (c) Our changed search order in extending a contour is by the angle difference from the last search order, as illustrated by the dashed arrow lines with numbers. Thus, we have P extended to S, not Q by the direction from R to P.

180 **Contour segmentation.** A contour may be composed of arcs from different ellipses. Thus, a
 181 contour should be segmented for arc extraction, which is by finding corner points, whose curvatures
 182 change abruptly in comparison with their respective neighboring points. More details are given in
 183 Appendix A.

184 **Arc determination.** The very short or very flat contour segments cannot be arcs of ellipses. They
 185 should be removed, and so the remained contour segments are the extracted arcs. Shen *et al.* [9] treat
 186 an arc as valid only when its length L satisfies $L > L_{arc}$, and the aspect ratio $B < B_{arc}$, where
 187 L_{arc} and B_{arc} are thresholds. Aspect ratio $B = \frac{box_width}{box_height}$ is used to describe the degree of flatness
 188 of the arc, where box_width and box_height are the longer side and the shorter side of the rotated
 189 rectangle with the minimum area bounding the arc.

190 In the above steps of [9] for arc extraction, there are some parameters. For the thresholds of these
 191 parameters to achieve good results, we set them by investigating the tested data sets, as done in
 192 existing methods [5, 7, 8, 30]. In our tests, we set $\theta_{arc} = 49^\circ$, $L_{arc} = 52$ and $B_{arc} = 29$.

193 With the above steps for arc extraction, we can obtain many more arcs for detecting ellipses as many
 194 as possible, especially those overlapped ones. This is superior to many methods like the learning
 195 based method [25], which mainly extracts the arcs on the outer contours of objects and so would miss
 196 many overlapped ellipses, as shown in Appendix B.

197 4.1.1 Improvement on contour extraction.

198 Contour extraction is to connect the edge pixels by the neighboring relationships between them to
 199 generate edges. Shen *et al.* [9] extracts an contour by randomly selecting an unused edge pixel as a
 200 seed to search for neighboring edge pixels iteratively until the contour cannot be extended, where
 201 the depth first search is used. After all edge pixels are used, it means all contours are extracted. As
 202 illustrated in Fig. 4(a), starting from the left most yellow pixel, a contour is generated. In the depth
 203 first search of [9], the search order is fixed as shown by the dashed lines with numbers in Fig. 4(a).
 204 With such a search order, from pixel P, the contour will be next connected to pixel Q, not to pixel S,
 205 as illustrated in Fig. 4(b). Thus, pixel P will be taken as a corner point in contour segmentation to
 206 generate shorter arcs.

207 For generating arcs as long as possible for improving ellipse detection, we change the search order for
 208 extending a contour as smooth as possible, which is by the angle difference from the search direction
 209 of the last extension. The neighboring pixels with the smaller angel difference will be searched more
 210 preferentially. As illustrated in Fig. 4(c), according to the last search direction from pixel R to P, our
 211 search order for extending the contour from P is determined by the changed ordered numbers. Thus,
 212 the contour is extended from P to S.

213 The measure of [9] for contour generation is by starting from a seed edge pixel with a depth first
 214 search. When the seed edge pixel is at the middle of an arc, this is unsuitable for extracting the
 215 arc completely. Considering this, we have two directions searched from each seed edge pixel for
 216 generating arcs as long as possible. For example, if pixel R is selected as a seed, the contour can be
 217 generated by search along two directions from R, as shown in Fig. 4(c).

4.2 Arc grouping with valid checks

With the obtained arcs, we first use our method in Sec. 3 to collect arc groups. Afterwards, for the collected arc groups, we could use the characteristic mapping constrains [12] for a further exclusion of useless groups. At last, the remained arc groups are used for ellipse generation.

For a collected arc group, its arcs are used for generating an ellipse, where we mainly use the corresponding measures of [9]. At first, an ellipse is estimated for them by the Least-Squares fitting method. Then, it is checked whether the estimated ellipse is valid. The valid estimated ellipses are our detected ellipses.

For valid checks, it is by the measure of [9] using the ratios of inliers, which are computed by the following equation:

$$S(e) = \frac{1}{|g^*|} \sum_{p \in g^*} Ind(dist(p, e) < \varepsilon) \quad (2)$$

where g^* is the set of arcs in a group, p is an arc point, e is the estimated ellipse, $dist(p, e)$ is the algebraic distance from point p to the estimated ellipse e , and $Ind(\cdot)$ refers to the indicator function.

When $S(e)$ has a high value, it means the estimated ellipse is valid. For this, a threshold S_{arc} is used for such a determination. By the suggestion of Shen *et al.* [9], we set $S_{arc} = 0.73$ in our tests, and always obtain good results.

4.3 Our algorithm for ellipse detection

With the measures discussed in the above subsections, we design an algorithm to detect ellipses as many as possible. Here, we generate candidates as many as possible and then remove the redundant ones, as discussed in the following.

Generating ellipse candidates. Our ellipse candidate generation is by the following steps. Firstly, an ellipse is estimated for each arc, as an arc may form an ellipse itself. Here, the estimated ellipse with its $S(e)$ less than S_{arc} , is discarded. Secondly, an ellipse is estimated for any a pair of arcs, where one arc is active and the other is one of its inactive arcs. It is by investigating the possibly estimated ellipses for pairs $(arc_{i,active}, arc_{j,inactive}), j = 1, 2, 3, \dots$, where $arc_{j,inactive}$ are the inactive arcs in the improved arc-search region of the active arc $arc_{i,active}$, and remaining the ones whose $S(e)$ value is greater than S_{arc} . In this way, if many arcs are co-elliptical, any two of them are used for ellipse estimation respectively. Clearly, this may cause redundant ellipses, but would not miss ellipses.

Removing redundant ellipses. We use two measures to remove redundant ellipses. Firstly, we guarantee that an arc can be used only once for ellipse detection. We queue up ellipse candidates by their $S(e)$ values from the highest to the lowest, and iteratively select the ellipse with the highest $S(e)$ values from the candidates which contains only unused arcs. Secondly, we merge similar ellipse candidates with the corresponding measure by [9], which computes a weighted L_2 difference between the ellipse parameters.

5 Results and discussion

To verify the effectiveness and efficiency of our method, we conducted extensive experimental studies and collected results on a personal computer installed with an Intel(R) Core i7-8700 CPU@3.2GHz and 48GB RAM, where we have a comparison with the state-of-the-art methods [5, 6, 7, 14, 8, 9, 10, 15, 12, 31]. Their source codes can be obtained from the internet except for the code of [31]. For the method of [31], we implemented it by ourselves. Prasad *et al.*[5] have their codes implemented in Matlab, Lu *et al.*[14] implemented in Matlab and C++, Wang *et al.*[15] implemented in Python, and the other methods implemented in C++. All methods run on the CPU except for [15], which runs on GPU GTX1080Ti.

Datasets. In our tests, we used four synthetic datasets for testing our effectiveness on ellipse detection and four real-world datasets for comparing with existing methods. The used synthetic datasets

Table 2: The test results of the compared methods on the four synthetic datasets. P, R and F represent for precision, recall and F-measure, respectively. Here, the values for the metrics are the averaged ones for an image in a dataset, and the **best** results and the **second best** results are marked in red and yellow respectively.

Method	Occlusion [5]			Overlapping [5]			Concentric [8]			Concurrent [8]		
	P↑	R↑	F↑	P↑	R↑	F↑	P↑	R↑	F↑	P↑	R↑	F↑
[14]	0.4889	0.4559	0.4685	0.6024	0.5287	0.5231	0.6627	0.8546	0.7465	0.6635	0.8392	0.7411
[8]	0.5558	0.1774	0.2492	0.4910	0.2680	0.3462	0.7428	0.6692	0.7041	0.7727	0.7340	0.7528
[9]	0.5955	0.4587	0.5174	0.6048	0.4267	0.4686	0.8742	0.8435	0.8586	0.8193	0.9135	0.8638
[10]	0.4441	0.1350	0.2009	0.7238	0.3874	0.4498	0.8095	0.8446	0.8267	0.6996	0.9337	0.7999
[15]	0.0863	0.0280	0.0422	0.0934	0.0249	0.0366	0.0310	0.0096	0.0147	0.1386	0.0622	0.0859
Ours	0.7074	0.5558	0.6191	0.6773	0.4827	0.5282	0.9117	0.8860	0.8987	0.8737	0.9430	0.9070

Table 3: The average F-measure and time cost of the compared methods on the four real-world datasets. The **best** and the **second best** results are marked in red and yellow respectively. *Ours refers to using our method with relaxing constraints on arc generation. Ours+CM refers to checking our selected arc groups by characteristic mapping [12] before they are sent for ellipse checks.

Method	F-measure↑				Time(ms)↓			
	Prasad	Prasad+	Random	Smartphone	Prasad	Prasad+	Random	Smartphone
[14]	0.5092	0.6540	0.6009	0.6403	162.70	550.49	640.23	1118.08
[8]	0.4293	0.5539	0.4997	0.5510	3.75	7.78	9.71	14.66
[9]	0.4265	0.5713	0.5838	0.6424	7.96	14.18	17.48	25.20
[10]	0.3552	0.4851	0.6022	0.6825	6.60	10.15	15.97	24.53
[15]	0.3866	0.4648	0.5559	0.5246	56.47	55.65	54.48	55.35
[12]	0.3425	0.5198	0.5144	0.5000	3.95	6.94	9.32	12.41
[31]	0.4332	0.5618	0.5104	0.5629	4.07	7.96	10.75	17.73
Ours	0.4632	0.6012	0.6106	0.7006	4.09	6.61	7.76	11.81
*Ours	0.5126	0.6589	0.5898	0.6108	5.95	10.01	13.49	24.10
Ours+CM	0.4381	0.5742	0.5815	0.6689	3.82	6.11	7.37	11.12

include the Occlusion/Overlapping dataset [5] and Concentric/Concurrent dataset [8]. The tested four real-world datasets are Prasad/Prasad+ dataset [5], and the Random/Smartphone dataset [6].

Evaluation metrics. Here, we use Precision, Recall and F-measure to evaluate the performance of an ellipse detector over a specific dataset. IoU is used to evaluate the similarity between a detected ellipse with an ellipse of ground truth.

5.1 Accuracy

We made tests on synthetic and real-world datasets, as discussed below. We also made tests on generated ellipses with various shapes, orientations and sizes in the supplementary materials, showing our superiority over existing methods.

Tests on the synthetic datasets. The results of the compared methods for the four synthetic datasets are shown in Table 2. Clearly, our method can always achieve the best results except the one for Precision metric on the Overlapping dataset, where Jiang *et al.*[10] has the best Precision result. This is because it is very rigorous in selecting ellipse candidates and reduces the ellipses to be generated in the cases when there are many overlapped ellipses, by which its Precision value is high. However, it would miss many true ellipses so that its Recall value is low. Some visualization are provided in the appendices.

Tests on the real-world datasets. The statistics on the real-world datasets are given in Table 3, where we use $\text{IoU} = 0.8$ as the threshold to validate ellipses, as suggested by [7]. For the details about comparison on detection performance with various setting of IoU, please see Appendix B in the supplementary materials. From the statistics in Table 3, it is known that our method always achieve the highest F-measure values than existing methods except for that Lu *et al.*[14] achieves better F-measure values than ours on Prasad and Prasad+. This is because the images of these two datasets are of low pixel resolutions, so that our arc generation with Gaussian filtering may have some elliptical arcs missed. When the images are of high pixel resolutions, as those in the datasets Random

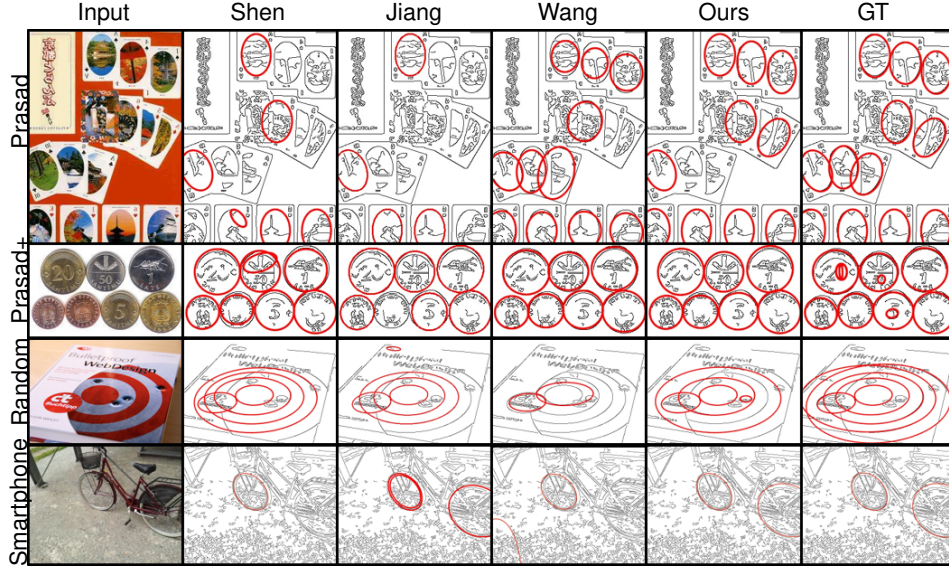


Figure 5: Some detection results of the methods in comparison on real-world datasets. We can detect more ellipses than the others, and in a higher quality.

and Smartphone, our method can obtain better results than Lu’s method. For further verification, we made tests with relaxing constraints on arc generation (referred to as *Ours in Table 3), where we have better F-measure values than [14] on Prasad and Prasad+ datasets. Overall, we can always obtain many more accurate ellipses while producing fewer wrong ellipses than existing methods, as illustrated in Fig. 1 and Fig. 5.

5.2 Efficiency

We made tests on the four real-world datasets to check our efficiency on ellipse detection. By the statistic data in Table 3, it is known that ours can be faster than existing methods except in handling the Prasad dataset, in which each image has very few ellipses, leading the generated matrices for the method of [8] to detect ellipses very small, so that [8] is the fastest in handling this dataset. This also makes [12] faster than ours. As for the other cases that have many ellipses in the image, ours is faster than them, especially ours+CM, which is by combining ours with the characteristics mapping constraints [12]. This shows our higher performance than existing methods.

6 Conclusions

Edge-link methods are prominent for ellipse detection. In this paper, we presented two novel measures to improve edge-linking methods, one for generating arcs more consistently and the other for saving a large amount of computation by implicitly excluding most useless arc groups. Meanwhile, we develop an algorithm to detect ellipses as many as possible by checking whether an arc or any a group of arcs can form an ellipse. Experimental results show that we can more efficiently detect ellipses, while obtaining many more ellipses and in a higher quality, than existing methods.

Limitation. Our method is dependent on arc extraction. When arcs are sufficiently detected, their corresponding ellipses can be almost detected by our method. Though we improve arc extraction, some arcs may be still missed to prevent ellipse detection, especially in handling the overlapped ellipses. As a future issue, arc extraction would be seriously investigated.

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397 A Some details for implementation of our method

398 **Pipeline of ellipse detection.** The pipeline of our method for ellipse detection follows the standard
 399 workflow of all edge-linking methods, as illustrated in Fig. 6. For an input image, we first extract the
 400 elliptic arcs, which involves edge detection, contour extraction and splitting. Then, we group the arcs
 401 that are likely from a same ellipse, where we use our grid-base method to implicitly eliminate most
 402 useless groups. Finally, we generate a candidate ellipse for each group, and apply further checks to
 remove redundant ones.

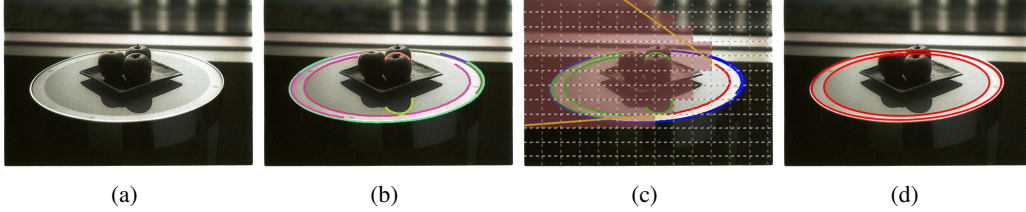


Figure 6: The pipeline for edge-link based ellipse detection. (a) The input image. (b) Extracting arcs. (c) Grouping arcs for ellipse checks. (d) Generating ellipses for co-elliptical arcs.

403

404 **Contour segmentation.** After extraction of curves, we subdivide the curves into elliptic arcs using
 405 corner points. The corner point can have an abrupt change in the magnitude of curvature, or be related
 406 to direction bending. Here, we provide an explicit example of this procedure. As shown in Fig. 7, we
 407 calculate the angle θ_i between the connected straight lines, and when θ_i is greater than a threshold
 408 θ_{arc} or its sign is different from θ_{i-1} , we mark the point as a corner point. We have $\theta_{arc} = 46^\circ$ in
 409 our experiments as suggested by [9].

410 **Removing redundant ellipses.** For the evaluation of redundant ellipses, we use the measure by [9]
 411 to compute the difference between two candidate ellipses, say e_i and e_j , $Diff(e_i, e_j)$, in the following
 412 formula,

$$Diff(e_i, e_j) = |x_i - x_j|^2 + |y_i - y_j|^2 + |a_i - a_j|^2 + |b_i - b_j|^2 + k_\theta \cdot \delta\theta \quad (3)$$

413 where (x_*, y_*) are the center coordinates of these two ellipses, a_* and b_* are the lengths of the
 414 semi-major axis and semi-minor axis, respectively, $\delta\theta$ is the angle between the two semi-major
 415 axes of these two ellipses, and $k_\theta = \min \left\{ \frac{a_i - b_i}{a_i + b_i}, \frac{a_j - b_j}{a_j + b_j} \right\}$ is used to attenuate the effect of $\delta\theta$ on
 416 $Diff(e_i, e_j)$ when one of the two ellipses is close to the circle. When two ellipses have a very low
 417 $Diff(e_i, e_j)$, meaning they are very similar and regarded as redundant ones. Here, we use a threshold
 418 $Th_d = 9.8$ for determining redundant ellipses, as suggested by [9].

419 B Additional experimental results

420 **Details of used datasets.** We test on four synthetic datasets, the Occlusion dataset and the Over-
 421 lapping dataset proposed in [5], and the Concentric dataset and the Concurrent dataset constructed
 422 by Meng *et al.*[8]. The Occlusion dataset and the Overlapping dataset each contain 300 images,
 423 containing many incomplete ellipses and broken arcs respectively. The Concentric dataset and the
 424 Concurrent dataset have 720 images and 1200 images respectively, whose contained concentric
 425 arcs and concurrent arcs are generally difficult to handle. The tested four real-world datasets are
 426 Prasad Dataset, Prasad+ Dataset, Random Dataset, and Smartphone Dataset. Prasad Dataset and
 427 Prasad+ Dataset [5] totally contain 400 images from Caltech256 dataset [33] with low resolutions,
 428 whose ellipses and occlusion cases are fewer than the other datasets. Random Dataset [6] has 400
 429 images, whose quality is better than Prasad Dataset, containing overlapping ellipses and complicated
 430 backgrounds. Smartphone Dataset has 629 images from 6 video shots [6], which are mainly from
 431 traffic signs and bicycle wheels of various perspectives, generally used to test the performance of the
 432 methods in practical application scenarios.

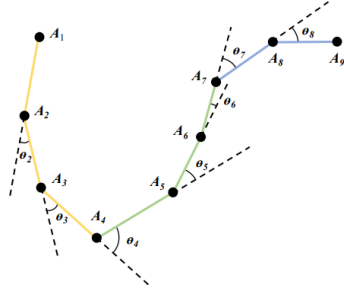


Figure 7: Contour segmentation by corner points. Here, A_4 is a corner point with an abrupt change in the magnitude of curvature, A_7 is a corner related to direction bending along the contour, and these two corner points segment the contour into three parts, marked in different colors.

Table 4: Recall rates for the methods on detecting various shape of ellipses.

Method	Recall(%)	
	First set	Second set
[6]	57.11	79.14
[7]	54.78	78.81
[8]	70.72	89.56
[10]	60.34	71.51
Ours	66.43	90.38

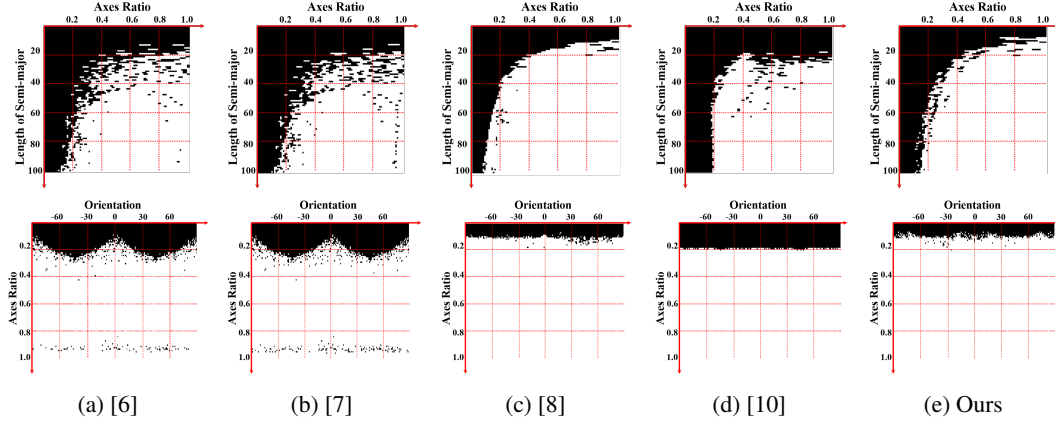


Figure 8: Performance comparison for detecting ellipses in various shapes, orientations, and sizes. The left shows the relationship between the ratio of the minor axis to the major axis and the length of the major axis. The right shows the relationship between the ratio of the minor axis to the major axis and the orientation. Here, the white area indicates the set of ellipses that can be correctly detected, while the black area indicates the failed ones. Wang’s method [15] fails to work in these tests.

Evaluation on variant axes ratio and orientation of ellipse. We generated two sets of ellipses in various shapes, orientations, and sizes to investigate the potential of our method. In the first set, 10,000 ellipses are each generated in images respectively, whose centers and orientations are fixed, but their semi-majors have lengths varied from 1 pixel to 100 pixels with an interval of 1 pixel, and their semi-minors have the lengths by the ratios of the minor axis to the major axis that vary from 0.01 to 1.0 with an interval of 0.01. In the second set, 18,000 ellipses are each generated in images respectively, whose orientations vary from -90° to 89° in a step of 1° . Here, with a direction, the semi-majors for the ellipses are fixed in the length of 100 pixels, and their semi-minors have the lengths varied by the ratios of the minor axis to the major axis varying from 0.01 to 1 in a step of 1. The results for the methods in comparison to detect the generated ellipses are shown in Fig. 8. The effectiveness of these methods to detect various ellipses are measured by the recall rates. The statistics in Table 4 show that our method can effectively detect a larger range of ellipses than the compared methods in general, except that in handling the first set, where ours is a little inferior to Meng *et al.*[8] in detecting the ellipses that are too small or too flat.

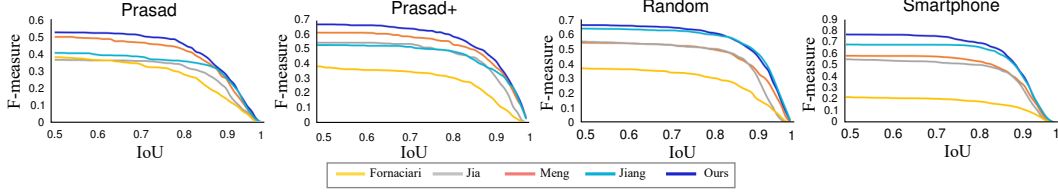


Figure 9: Ellipse detection performance of our method in comparison with state-of-the-art methods by the threshold for IoU varying from 0.5 to 0.99 with the interval of 0.01 on four real-world datasets, Prasad dataset, Prasad+ dataset, Random dataset and Smartphone dataset, respectively.

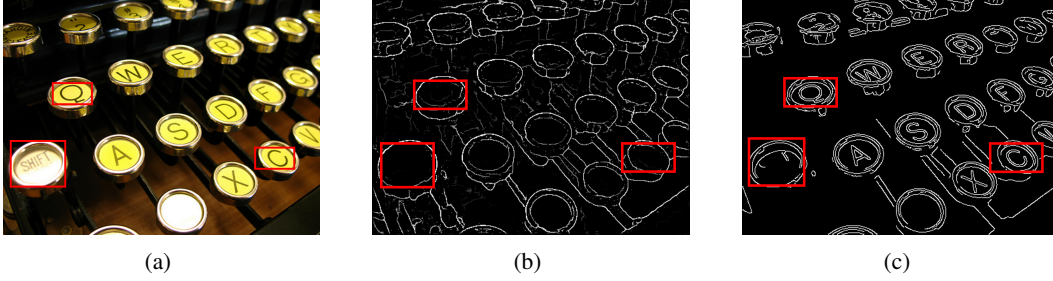


Figure 10: Comparison on edge extraction. (a) Input image. (b) Results of the learning method [25]. (c) Ours. Clearly, we can obtain many more arcs than using the learning method, as marked in the red boxes.

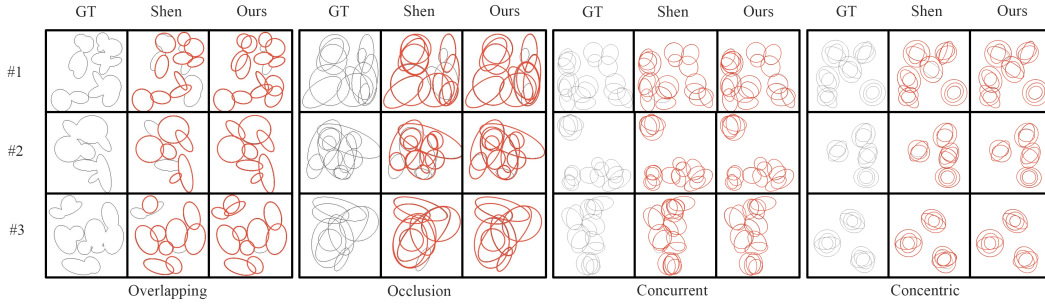


Figure 11: Some visualizations for the detection results of our method on the four synthetic datasets.

Performance over different IoU In general, when a detected ellipse has its IoU bigger than a threshold, it is regarded as correctly detected. In the paper, we use the results by setting the threshold as 0.8 for real-world datasets and 0.9 for the synthetic datasets. To further demonstrate the performance of our method, we performed experiments with the threshold ranging from 0.5 to 0.99. The results for the four real-world datasets are shown in Fig. 9, showing that our method achieves the best performance. When the threshold is lower than 0.75, our F-measure values no longer change significantly, showing our potentials for high-quality ellipse detection.

Comparison on edge extraction. We compare our edge extraction results with that of the learning based method [25]. As shown in Fig. 10, we can obtain many more arcs for detecting ellipses as many as possible, especially those overlapped ones. The method of [25] mainly extracts the arcs on the outer contours of objects, missing many overlapped ellipses, as shown in the red boxes in Fig. 10.

More results on synthetic datasets. We provide some visual comparison between the results of our method and that of [9] in Fig. 11 and Table 5. On the whole, we can always obtain many more correct ellipses for some complicated cases. In this experiment, we have $IoU = 0.9$ for validating ellipses because these images dose not contains noise or texture that may disturb arc extraction, as suggested by [7, 8, 9]. We also list the statistics of comparison on precision, recall and F-measure in Table 6.

Table 5: Statistics of the detection results in Fig. 11. We can detect more ellipses and in higher quality than [9], which can better extract ellipses than the other existing methods on synthetic datasets according to Table 6. TP, FP and FN stand for True Positive, False Positive and False Negative, respectively.

Method	Type	Occlusion#1	Occlusion#2	Occlusion#3	Overlapping#1	Overlapping#2	Overlapping#3
Shen	TP	9	6	9	8	7	7
	FP	1	0	0	3	4	1
	FN	7	2	3	4	5	1
Ours	TP	14	8	11	10	11	8
	FP	0	0	0	1	0	0
	FN	2	0	1	2	1	0

Method	Type	Concentric#1	Concentric#2	Concentric#3	Concurrent#1	Concurrent#2	Concurrent#3
Shen	TP	19	16	15	15	11	11
	FP	0	2	5	1	2	1
	FN	1	0	1	1	1	1
Ours	TP	20	16	16	16	12	12
	FP	2	1	0	0	0	0
	FN	1	0	0	0	0	0

Table 6: The test results of the compared methods on the four synthetic datasets. Here, the values for the metrics are the averaged ones for an image in a dataset, and the **best** results and the **second best** results are marked in red and yellow respectively.

Method	Occlusion			Overlapping			Concentric			Concurrent		
	Precision↑	Recall↑	F-measure↑	Precision↑	Recall↑	F-measure↑	Precision↑	Recall↑	F-measure↑	Precision↑	Recall↑	F-measure↑
[6]	0.0904	0.3624	0.1353	0.0881	0.2216	0.1260	0.0542	0.7881	0.1015	0.0684	0.8926	0.1271
[7]	0.4674	0.2688	0.2944	0.3197	0.1659	0.2155	0.4587	0.6426	0.5353	0.4370	0.8079	0.5672
[14]	0.4889	0.4559	0.4685	0.6024	0.5287	0.5231	0.6627	0.8546	0.7465	0.6635	0.8392	0.7411
[8]	0.5558	0.1774	0.2492	0.4910	0.2680	0.3462	0.7428	0.6692	0.7041	0.7727	0.7340	0.7528
[9]	0.5955	0.4587	0.5174	0.6048	0.4267	0.4686	0.8742	0.8435	0.8586	0.8193	0.9135	0.8638
[10]	0.4441	0.1350	0.2009	0.7238	0.3874	0.4498	0.8095	0.8446	0.8267	0.6996	0.9337	0.7999
[15]	0.0863	0.0280	0.0422	0.0934	0.0249	0.0366	0.0310	0.0096	0.0147	0.1386	0.0622	0.0859
Ours	0.7074	0.5558	0.6191	0.6773	0.4827	0.5282	0.9117	0.8860	0.8987	0.8737	0.9430	0.9070

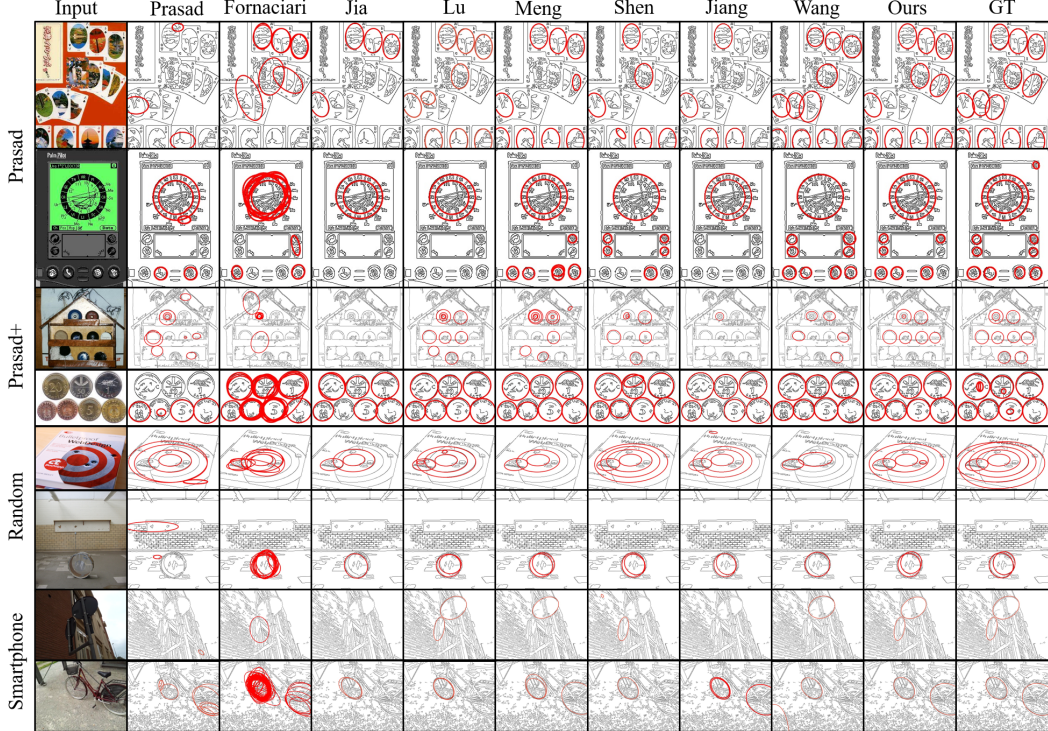


Figure 12: Some detection results of the methods in comparison on real-world datasets.

Table 7: The test results of the compared methods on the four real-world datasets. Here, the values for the metrics are the averaged ones for an image in a dataset, and the **best** results and the **second best** results are marked in red and yellow respectively.

Method	Prasad		Prasad+		Random		Smartphone	
	F-measure $\uparrow$	Time $\downarrow$	F-measure $\uparrow$	Time $\downarrow$	F-measure $\uparrow$	Time $\downarrow$	F-measure $\uparrow$	Time $\downarrow$
[5]	0.2874	2253.82	0.2108	5697.04	0.3112	6185.56	0.2226	13721.00
[6]	0.2888	4.48	0.2072	12.18	0.3063	13.58	0.1919	18.63
[7]	0.3343	4.10	0.4896	8.32	0.5016	10.79	0.5222	14.58
[14]	0.5092	162.70	0.6540	550.49	0.6009	640.23	0.6403	1118.08
[8]	0.4293	3.75	0.5539	7.78	0.4997	9.71	0.5510	14.66
[9]	0.4265	7.96	0.5713	14.18	0.5838	17.48	0.6424	25.20
[10]	0.3552	6.60	0.4851	10.15	0.6022	15.97	0.6825	24.53
[15]	0.3866	56.47	0.4648	55.65	0.5559	54.48	0.5246	55.35
[12]	0.3425	3.95	0.5198	6.94	0.5144	9.32	0.5000	12.41
[31]	0.4332	4.07	0.5618	7.96	0.5104	10.75	0.5629	17.73
Ours	0.4632	4.09	0.6012	6.61	0.6106	7.76	0.7006	11.81
*Ours	0.5126	5.95	0.6589	10.01	0.5898	13.49	0.6108	24.10
Ours+CM	0.4381	3.82	0.5742	6.11	0.5815	7.37	0.6689	11.12

Notes: 1) [15] runs on GPU, and the other methods run on CPU. Time is in millisecond.

2) “*Ours” refers to using our method without filtering in edge detection and with constraints relaxed in arc determination.

3) [12] replace characteristic number with characteristic mapping(CM) for arc grouping of [7].

4) “Ours+CM” refers to that our arc groups are further filtered by the CM constraints [12] before ellipse generation.

More results on real-world datasets. In the main paper, we only provide the statistics of some recent methods and limited visual results. Here, we provide more quantitative comparison in Table 7, and more visualization of detected ellipses in Fig. 12.

C Promotion to application of autonomous driving

Traffic sign detection is a crucial problem in autonomous driving, where it is very important to detect traffic signs as early and thoroughly as possible. Among all traffic signs, circular signs account for a large proportion, and provide key information about traffic rules and restrictions. Thus, ellipse detection in the captured images of the cameras for autonomous driving are much required.

We made a test by comparing our method and Jia *et al.* [12] on ellipses detection. Here, the used images are from the dataset collected from video frames captured by a mobile phone [12] and a set of complex real images containing circular traffic signs from the public Traffic Sign Detection Dataset (TSDD)¹.

As illustrated by some results in Fig. 13, we can detect more traffic signs than the method of Jia *et al.* [12]. Thus, we can promote the safety of autonomous driving, as discussed in the following. Firstly, we can more effectively detect small-sized ellipses, as shown in Fig. 13(a)(f), which means that traffic signs can be recognized from a greater distance, improving the timely response of autonomous driving systems. Secondly, our method detects more traffic signs, as shown in Fig. 13(c)(e), thereby avoiding the risk of missing critical information. Thirdly, we can effectively identify incomplete signs, as shown in Fig. 13(b), which are quite common in real-world scenarios due to limitations such as camera field of view or obstructions. Clearly, with our method, autonomous driving can be promoted a lot.

¹It is part of the Chinese Traffic Sign Database (<https://nlpr.ia.ac.cn/pal/trafficdata/index.html>) collected by Huang *et al.*.



Figure 13: Visualization results of our method and Jia-CM [12] on real-world scenes of traffic sign detection.

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