

# DETERMINE-THEN-ENSEMBLE: NECESSITY OF TOP-K UNION FOR LARGE LANGUAGE MODEL ENSEMBLING

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## ABSTRACT

Large language models (LLMs) exhibit varying strengths and weaknesses across different tasks, prompting recent studies to explore the benefits of ensembling models to leverage their complementary advantages. However, existing LLM ensembling methods often overlook model compatibility and struggle with inefficient alignment of probabilities across the entire vocabulary. In this study, we empirically investigate the factors influencing ensemble performance, identifying model performance, vocabulary size, and response style as key determinants, revealing that compatibility among models is essential for effective ensembling. This analysis leads to the development of a simple yet effective model selection strategy that identifies compatible models. Additionally, we introduce the UNION Top- $k$  Ensembling (UNITE), a novel approach that efficiently combines models by focusing on the union of the top- $k$  tokens from each model, thereby avoiding the need for full vocabulary alignment and reducing computational overhead. Extensive evaluations across multiple benchmarks demonstrate that UNITE significantly enhances performance compared to existing methods, offering a more efficient framework for LLM ensembling.

## 1 INTRODUCTION

Large language models (LLMs) have demonstrated remarkable performance across a wide range of tasks and have shown promising results in real-world applications (OpenAI, 2023; Yang et al., 2024; Dubey et al., 2024). Given the diversity in data sources, model architectures, and training methods, LLMs exhibit varying strengths and weaknesses depending on the task at hand. Consequently, rather than relying solely on training an LLM from scratch, an alternative approach is to create an ensemble of LLMs. This method allows for leveraging the complementary advantages of different LLMs (Jiang et al., 2023b; Lu et al., 2024; Yu et al., 2024b).

Existing model ensembling methods can be broadly categorized into three types: output-level, probability-level, and training-level approaches. Output-level methods (Jiang et al., 2023b; Lu et al., 2024; Shnitzer et al., 2023) aggregate the complete outputs of multiple candidate models. Probability-level methods (Huang et al., 2024; Yu et al., 2024b), integrate outputs based on probability distributions at each generation step through the intersection or union of the vocabulary. Training-level methods (Wan et al., 2024a; Xu et al., 2024) utilize output probability vectors as labels for richer information extraction during training. While output-level methods are constrained by the limitations of existing outputs, and training-level methods introduce additional computational overhead, probability-level methods have garnered particular attention.

Existing methods for ensembling LLMs, such as DEEPEN (Huang et al., 2024) and GAC (Yu et al., 2024b), grapple with two significant challenges. First, these approaches concentrate solely on the ensembling technique, sidestepping the crucial discussion of which types of models can be effectively combined. This oversight is critical because LLMs with substantial differences in architecture, size, and tokenizer may be inherently incompatible, leading to potential incompatibilities that undermine the benefits of ensembling (Dong et al., 2023; Lee et al., 2023). Second, these methods tend to align the probabilities across the entire vocabulary at each generation step. Such a strategy introduces substantial computational overhead during inference, which hinders performance and efficiency.

To address these challenges, we conduct a thorough investigation into the factors that truly influence ensembling performance. Our empirical analysis identifies three key factors: model performance,

054 vocabulary size, and response process. We evaluate multiple widely recognized LLMs across various  
055 benchmarks to explore ensembling from these perspectives. Our findings reveal several important  
056 insights: 1) Variations in performance levels among base LLMs significantly affect their compati-  
057 bility for ensembling; 2) The impact of vocabulary size is marginal; and 3) Even when performance  
058 and vocabulary size are aligned across LLMs, substantial differences in the reasoning process in the  
059 response can hinder successful ensembling.

060 Building on this analysis, we adopt a determine-then-ensemble strategy by starting with the best-  
061 performing LLM for target tasks, then iteratively selecting the next best-performing LLMs that  
062 meets ensembling criteria until the maximum number of models is reached or no further suitable can-  
063 didates exist. Additionally, we seek to improve the efficiency and performance of ensembling. In-  
064 stead of aligning the entire vocabulary of different LLMs, we propose the **Union Top- $k$  Ensembling**  
065 (**UNITE**). UNITE constructs a union of the top- $k$  tokens from each model and expands this set us-  
066 ing each model’s tokenizer. This is followed by probability aggregation to determine the next token.  
067 UNITE avoids the need for auxiliary mapping matrices and full vocabulary alignment, respecting  
068 the unique tokenization of each base LLM. Extensive experiments demonstrate the effectiveness of  
069 UNITE across various tasks, consistently outperforming state-of-the-art LLM ensemble methods.

070 Overall, our key contributions are: 1) We conduct an extensive analysis of existing model ensem-  
071 bling approaches and derive three significant insights. Building on these findings, we introduce  
072 a general model selection strategy for model ensembling; 2) We propose a well-motivated model  
073 ensembling method that efficiently operates on the top- $k$  candidate tokens rather than the entire vo-  
074 cabulary. 3) Experiments demonstrate significant performance improvements, reduced operational  
075 tokens to less than **0.04%** of current methods, and minimized latency to only *10 ms* longer than the  
076 individual model, validating the effectiveness of our proposed approach.

## 077 078 2 RELATED WORKS 079 080

081 Based on the sequence of inference and fusion processes, multiple model collaboration methods can  
082 be broadly classified into two categories: model ensembling (Jiang et al., 2023b; Yu et al., 2024b)  
083 and model merging (Yu et al., 2024a; Akiba et al., 2024). Model ensembling follows an inference-  
084 then-fusion approach, aiming to integrate the outputs of various models to achieve a more refined  
085 response. Conversely, model merging adopts a fusion-then-inference strategy, wherein different  
086 models are combined into a single model before inference. While model merging is typically appli-  
087 cable only to homologous models, our focus in this study is on the more general approach, namely  
088 model ensembling. We discuss different model ensembling methods as follows:

089 **Output-Level Model Ensembling** methods involve selecting multiple candidate models and uti-  
090 lizing their complete outputs for aggregation. For instance, Jiang et al. (2023b) developed PAIR-  
091 RANKER, an additional ranking model, to evaluate and select the best candidate output. Similarly,  
092 Lu et al. (2024) and Shnitzer et al. (2023) designed a router that determines the most appropriate  
093 candidate model based on the given question. However, these approaches are restricted by the exist-  
094 ing outputs and become ineffective if all options are incorrect. Some studies have addressed this by  
095 training fusion models to combine outputs (Jiang et al., 2023b; Wang et al., 2024b), which alleviates  
096 the limitation of relying solely on available candidates and often yields improved results. Neverthe-  
097 less, achieving generalization with fusion models remains a significant challenge, as they may not  
098 fully utilize the probability information generated at each step.

099 **Probability-Level Model Ensembling** methods focus on integrating outputs from different mod-  
100 els by utilizing the probability distribution at each generation step. COOL-FUSION (Liu et al.,  
101 2024) let each source LLM generate tokens until reaching common word boundaries, then jointly  
102 reranks the segments. However, this method relies on common word boundaries, limiting generation  
103 flexibility and diversity. Additionally, managing multiple source models can increase complexity.  
104 DEEPEN (Huang et al., 2024), based on the principles of relative representation theory, transforms  
105 each model’s probability distribution from its individual space into a common relative space for ag-  
106 gregation, which is determined through the intersection of the models’ vocabularies. On the other  
107 hand, GAC (Yu et al., 2024b) constructs a mapping matrix that projects the probability vectors of  
multiple LLMs into the dimensional space of their union vocabulary, aggregating outputs to select  
the next token at each generation step. It is worth noting that all of these existing approaches operate

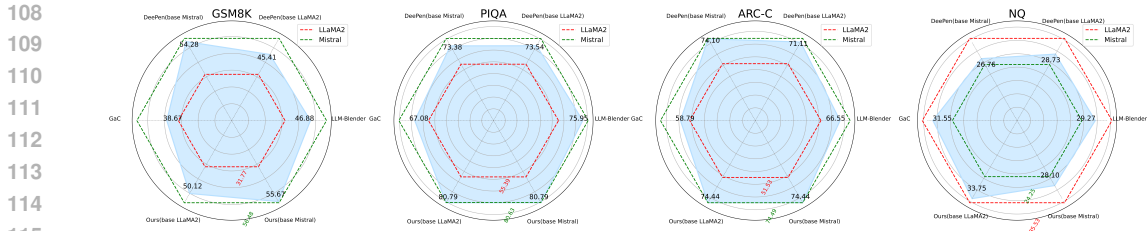


Figure 1: The impact of performance disparity among models on ensemble effectiveness across different datasets and methods is examined. We compare these methods to the individual performances of LLaMA2 and Mistral, indicated by dashed lines.

on the entire vocabulary at every single step, resulting in significant computational overhead. In this work, we focus on the probability-level model ensembling without the need for additional training.

**Training-Level Model Ensembling** methods exemplified by FUSELLM (Wan et al., 2024a), leverage output probability vectors from various models during the training process, using these vectors as labels instead of one-hot representations. This technique serves as a unique form of distillation, enabling the trained model to extract richer information from the probability outputs of the ensemble of teacher models. FUSECHAT (Wan et al., 2024b) extends the framework of FUSELLM to fuse multiple chat LLMs with diverse architectures and scales. However, distillation is predominantly aimed at enhancing smaller models, which limits its effectiveness in further advancing larger models. On the other hand, EVA (Xu et al., 2024) addresses vocabulary discrepancies by learning token alignment across different vocabularies, utilizing overlapping tokens for assistance. However, vocabulary alignment should involve both output embedding alignment and weight alignment, EVA achieves vocabulary alignment solely by solving the mapping matrix of output embeddings.

### 3 UNDERSTANDING MODEL ENSEMBLING FROM MODEL CAPACITY, VOCABULARY SIZE AND TASK

Existing model ensembling approaches (Huang et al., 2024; Yu et al., 2024b) only focus on designing the model ensembling method, offering limited discussion on the selection of base models for ensembling. However, insights from prior research (Dong et al., 2023; Lee et al., 2023), indicate that not all model pairs are compatible to be combined, particularly when there are significant differences in size, vocabulary, and performance. In our preliminary experiments, we explored various factors that might affect the performance of model ensembling, including model size (e.g., 3B/7B/8B/13B/70B), model architecture (dense/sparse), performance discrepancies, tokenization strategies (BPE/WordPiece), vocabulary size (e.g., 102K/64K/32K) and tasks variations (e.g., text generation/QA/multiple choices). Finally, we identified three representative factors for further analyses, including performance discrepancy, vocabulary size, and tasks variations.

#### 3.1 IMPACT OF MODEL PERFORMANCE DISCREPANCY

To investigate the impact of performance disparity on model ensembling, we select LLaMA2-13B-Chat and Mistral-7B-Instruct-v0.3 as base models, which show a significant performance gap across various tasks. We evaluated the GSM8K (Cobbe et al., 2021), PIQA (Bisk et al., 2020), ARC-C (Clark et al., 2018), and NQ (Kwiatkowski et al., 2019) datasets with three comparative methods.

As shown in Fig. 1, we found that when there is a significant performance disparity between models, LLM-BLENDER, which selects the optimal response from model candidates, is not suitable for model ensembling. Similarly, for methods like DEEPEN and GAC, which are achieved through token probability averaging, a substantial performance gap also leads to certain performance drops. This observation is straightforward and easily understood, as lower-performing models may introduce noise and bias, adversely affecting overall effectiveness. Nonetheless, it has been observed that DEEPEN and GAC consistently enhance the performance of less effective models when combined with a superior model. Although the resultant performance does not surpass that of the superior model, these methods can also serve as an alternative way for training-free knowledge distillation.

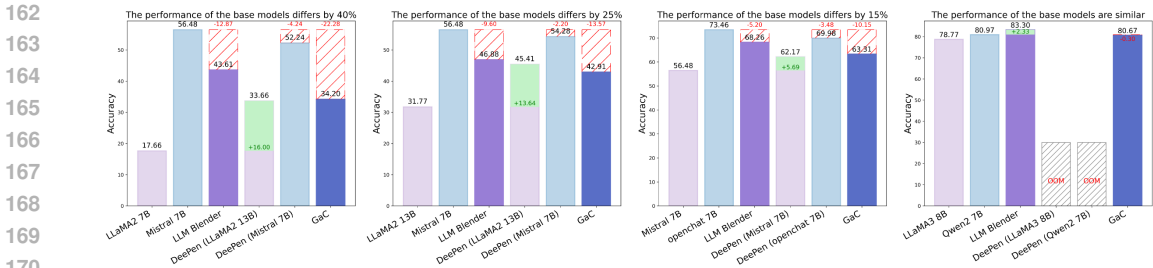


Figure 2: The impact of performance differences on model ensembling effectiveness on GSM8K dataset. OOM represents out of memory issue.

Methods	Dataset		Methods	Dataset		Methods	Dataset	
	GSM	PIQA		MMLU	ARC-C		NQ	ARC-C
DeepSeek-7B(102K)	59.67	72.66	DeepSeek-7B(102K)	46.97	58.73	Mistral-7B(32K)	24.25	74.49
Mistral-7B(32K)	56.48	80.63	LLaMA2-13B(32K)	49.61	51.53	Yi-6B(64K)	22.55	73.21
LLM-BLENDER	61.16	76.54	LLM-BLENDER	48.86	57.84	LLM-BLENDER	22.97	72.48
	(+1.49)	(-4.09)		(-0.75)	(-0.89)		(-1.28)	(-2.01)
DEEPEN(DeepSeek-7B)	55.00	77.65	DEEPEN(DeepSeek-7B)	52.81	60.00	DEEPEN(Mistral-7B)	25.26	76.50
	(-4.67)	(+4.99)		(+5.84)	(+1.27)		(+1.01)	(+1.01)
DEEPEN(Mistral-7B)	61.28	76.32	DEEPEN(LLaMA2-13B)	54.09	62.39	DEEPEN(Yi-6B)	22.80	77.86
	(+4.80)	(-4.31)		(+4.48)	(+10.86)		(+0.25)	(+4.65)
Ours(DeepSeek-7B)	62.77	81.81	Ours(DeepSeek-7B)	48.90	60.16	Ours(Mistral-7B)	24.76	76.54
	(+3.10)	(+9.15)		(+1.93)	(+1.43)		(+0.51)	(+2.05)
Ours(Mistral-7B)	58.88	81.81	Ours(Mistral-7B)	48.90	60.16	Ours(Yi-6B)	23.28	76.54
	(+2.40)	(+1.18)		(-0.71)	(+8.63)		(+0.73)	(+3.33)

Table 1: The influence of vocabulary size on model ensembling effectiveness

To further figure out how performance discrepancy impacts the ensembling, we then identify model pairs based on the GSM8K dataset with performance gaps of approximately 40% (LLaMA2-7B-Chat and Mistral-7B-Instruct-v0.3), 25% (LLaMA2-13b-Chat and Mistral-7B-Instruct-v0.3), 15% (OpenChat-3.5 and Mistral-7B-Instruct-v0.3), and less than 10% (LLaMA3-8B-Instruct and Qwen2-7B-Instruct). Fig. 2 shows that as the performance gap increases, the improvement from ensembling inferior models becomes more pronounced. In contrast, for superior models, the ensembling effect consistently falls below the baseline performance. When the performance difference is within 10%, ensembling may yield better results.

Moreover, it was observed that with the increase in vocabulary size, as exemplified by models such as LLaMA3 and Qwen2, whose vocabulary size exceeds 120K, approaches like DEEPEN, which depend on high-dimensional matrix operations, encounter heavy memory limitations that diminish their effectiveness.

**TAKEAWAY I: Smaller performance gaps lead to greater gains from model ensembling.**

3.2 INFLUENCE OF VOCABULARY SIZE

Yu et al. (2024b) found that large models employing different tokenization methods, such as BPE and BBPE, exhibit over 90% overlap in tokenizing Oxford 5000 common words. This indicates that tokenization methods have a marginal impact on model ensembling, leading us to focus on the effect of vocabulary size on ensemble performance. To this end, we selected four models with varying vocabulary sizes that exhibit similar performance on certain datasets, namely LLaMA2-13B-Chat (vocabulary size: 32,000), Mistral-7B-Instruct-v0.3 (vocabulary size: 32,768), Yi-6B (vocabulary size: 64,000), and DeepSeek-LLM-7B-Chat (vocabulary size: 102,400).

As presented in Table 1, existing methods demonstrate consistent performance irrespective of the gap in vocabulary size, thereby indicating that vocabulary size does not significantly influence the efficacy of model ensembling. To elucidate the underlying causes, we conducted a manual analysis of 100 responses generated by various LLMs. Our investigation revealed that approximately 80% of the words in the responses were common words while the tokenization of these common words exhibited no discernible differences across the different LLMs. Therefore, the vocabulary size, as well as the tokenization strategy, exerts minimal influence on the performance of model ensembling.

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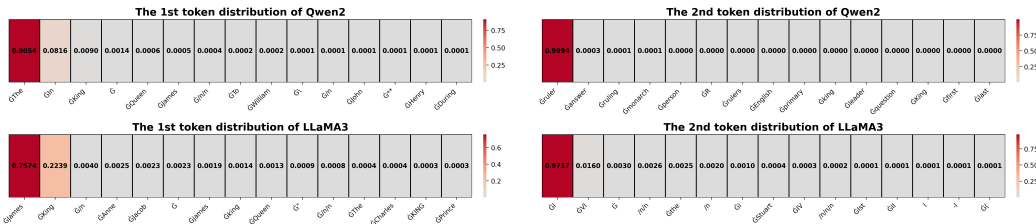


Figure 3: Token distribution of different models. We extract the top 15 words from the vocabulary of each model and apply a softmax processing to their corresponding logits.

Question	who wrote he ain't heavy he's my brother lyrics who was the ruler of england in 1616
Gold answer	Bob Russell James I
LLaMA3 response	Bob Russell ✓ James I ✓
Qwen2 response	He Ain't Heavy, He's My Brother was written by Mike D'Abo. ✗ The song was originally recorded by the British singer-songwriter and released as a single in 1969. It became a hit and has since been covered by numerous artists. ... ----- The ruler of England in 1616 was King James I. ✓ He was the first monarch of the Stuart dynasty and reigned from March 24, 1603, until his death on July 19, 1625. ...
LLM-Blender response	He Ain't Heavy, He's My Brother was written by Mike D'Abo. ✗ The song was originally recorded by the British singer-songwriter and released as a single in 1969. It became a hit and has since been covered by numerous artists. ... ----- The ruler of England in 1616 was King James I. ✓ He was the first monarch of the Stuart dynasty and reigned from March 24, 1603, until his death on July 19, 1625. ...

Table 2: Responses of different methods on NQ dataset.

Notably, despite significant variations in vocabulary size, the ensembling results of models with similar performance suggest that when the performance gap between base models is within 10%, model ensembling may achieve better results than the baseline model, which is aligned with findings in Section 3.1. More interestingly, results of evaluating DEEPEN on the ARC-C dataset indicate that when model performances are comparable, ensembling inferior models can even outperform superior ones. This might imply that when performance differences are minimal, the selection of main base models does not necessarily need to prioritize those with superior performance. Besides, it is important to note that these findings related to the vocabulary and tokenization consistently apply to LLM-BLENDER, DEEPEN and our method, with the exception of GAC. We will detail the reason in Section 4.2.

**TAKEAWAY II: The influence of vocabulary size for model ensembling is marginal.**

3.3 TASK-SPECIFIC CHALLENGES IN MODEL ENSEMBLING

In the aforementioned experiments, we identified an interesting phenomenon: Even when performance and vocabulary size are aligned across models, substantial differences in the response style can also hinder successful ensembling on specific tasks. For example, LLaMA3-8B and Qwen2-7B exhibit comparable performance across various tasks. Existing ensembling approaches such as LLM-BLENDER have been shown to enhance outcomes on GSM8K and PIQA tasks when utilizing these two base models. However, ensembling these models to address the questions in TriviaQA and NQ datasets proves to be impractical due to the significant discrepancies in their response styles.

As shown in Table 2, under identical prompting conditions, LLaMA3 tends to directly return the answers, whereas Qwen2 is inclined to conduct analysis. In voting-based LLM-BLENDER, response length significantly influences judgments, with PAIRRANKER consistently favoring longer responses. For instance, in our analysis of 100 randomly selected NQ and TriviaQA responses, the

PAIRRANKER favored the longer response as the answer in approximately 80% and 70% of cases accordingly, possibly due to its training data characteristics. More detailed analysis of TriviaQA and more experimental results are in appendix A. Similarly, probability-based ensembling methods like DEEPEN and GAC are also affected by the differences in the reasoning process, leading to significant deviations in vocabulary probability distributions. Figure 3 reveals a significant disparity in top token distributions between LLaMA3 and Qwen2 for the first two tokens. Averaging the probabilities directly could result in an excessively smoothed distribution, which is inappropriate. The underlying reason for this phenomenon may be the differences in LLMs’ training data. Therefore, caution is warranted in specific tasks to avoid substantial disparities, despite effective ensembling in most other tasks.

**TAKEAWAY III: Even though the performance and vocabulary size are aligned across models, substantial differences in response style could also hinder successful ensembling.**

## 4 METHODOLOGY

### 4.1 MODEL SELECTION STRATEGY

Given the aforementioned insights, we present a strategy to assess the compatibility of two models for ensembling and offer guidance for selecting base models from the candidate model pool.

When ensembling two models, we begin by comparing their performance on the target tasks. An overall performance gap within 10% is desirable, as it indicates the potential compatibility for successful ensembling. If the performance gap falls within this range, it is crucial to compare the response styles of different models. This comparison is relatively subjective, with differences often identifiable through superficial features such as text length and BLEU score.

In this work, we use text length as a guiding metric by constraining the length of longer responses to not exceed twice the length of the shorter ones. When the response manner is also consistent across models, it is highly probable that these two models can be successfully ensembled to produce a superior response. We exclude discrepancies in vocabulary size from consideration, as prior analyses have demonstrated its insignificance in affecting ensembling outcomes.

To select base models from the pool, we recommend initially choosing the best-performing model for the target tasks. Subsequently, select the next best-performing model that satisfies the criteria for successful ensembling with the first chosen model, continuing this process iteratively until the maximum number of base models is reached or no further suitable models can be found.

### 4.2 UNION TOP- $k$ ENSEMBLING

Apart from the model selection strategy, we also endeavor to improve the efficiency and performance of model ensembling. Existing probability-level model ensembling methods, e.g. DEEPEN and GAC, attempt to combine the models through full vocabulary alignment. We argue this approach is sub-optimal since the candidate next token typically resides within the top- $k$  tokens. Conversely, incorporating tokens of lower probability may introduce unnecessary noise, thereby diminishing the overall accuracy and effectiveness of the ensemble. Motivated by this, we propose the UNION Top- $k$  Ensembling (UNITE), a novel and efficient ensembling approach that only aligns the top- $k$  tokens in each decoding step.

Given the base models for ensembling, denoted as  $LLM_i$  with their respective tokenizer  $T^i$  and vocabulary  $V^i$ , we feed the input text *prompt* into  $LLM_i$  and obtain the probability distribution. Instead of aligning the probability distributions across different models, we only focus on the subset of top- $k$  tokens since the candidate next token is highly likely to be within the subset. The selected top- $k$  tokens of  $LLM_i$  is presented as  $TopK_i = (t_1^i, t_2^i, \dots, t_k^i)$  with corresponding probability distribution  $P^i = (p_1^i, p_2^i, \dots, p_k^i)$ .

Next, we aim to construct the union set  $V^u$  of  $TopK_i$  across the base models. The most straightforward solution to obtain the union set is to directly remove the duplicate tokens and retain all other distinct tokens. For the token in  $V^u$  but not in  $V^i$ , the probability of this token in  $LLM_i$  is assigned to 0. This strategy is adopted by GAC (Yu et al., 2024b). However, we contend that this strategy is not entirely reasonable since it is heavily influenced by the tokenizer and vocabulary. For exam-

**Algorithm 1** Union top- $k$  ensembling

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**Require:** LLM $_i$ , Vocabulary  $V_i$ , Tokenizer  $T_i$ , Demonstration *prompt*, Stop condition *stop*(\*)

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1: while not stop(*) do                                     ▷ Stopping criteria
2:   TopK $_i$ ,  $P^i \leftarrow$  LLM $_i$ (prompt)                     ▷ Generate top- $k$  tokens and their probabilities
3:   for each model do
4:     if token  $w \in V^u$  and  $w \in$  TopK $_i$  then
5:        $P_{\{w\}}^i$  and TopK $_i$  remains unchanged.
6:     else if token  $w \in V^u$  and  $w \in V^i$  and  $w \notin$  TopK $_i$  then
7:       TopK $_i \leftarrow w$ ,  $\hat{P}^i \leftarrow p_w^i$ 
8:     else if token  $w \in V^u$  and  $w \notin V^i$  then
9:        $w^1, \dots, w^m \leftarrow T^i(w)$ 
10:      TopK $_i \leftarrow w^1$ ,  $\hat{P}^i \leftarrow p_{w^1}^i$ 
11:     end if
12:   end for
13:    $\hat{P}_{norm}^i = \text{softmax}(\hat{P}^i)$ ,  $\hat{P}_{avg} = \frac{1}{n} \sum_{i=1}^n \hat{P}_{norm}^i$ 
14:    $w = \text{argmax}_{w \in \text{TopK}_i}(\hat{P}_{avg})$                                ▷ Predict the next token
15:   prompt  $\leftarrow$  prompt +  $w$                                    ▷ Update input sequence
16: end while

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ple, the candidate token “James” is within the top- $k$  subset of LLM $_1$  but not in  $V^2$  while the token “Jam” appears in TopK $_2$ . The word “James” can be tokenized into “Jam” and “es” by LLM $_2$ . If the token probabilities  $p_{\{Jam\}}^1$  and  $p_{\{James\}}^2$  are set to 0, the overall probability of generating the word “James” should be inherently reduced, even if both base models exhibit a preference for this word. Therefore, we present a new token probability alignment strategy on the union set. Firstly, we obtain the top- $k$  union vocabulary  $V^u$  by directly merging the TopK $_i$  subsets. Then, the token distribution  $P^i$  and TopK $_i$  are updated according to following criteria:

1. If the token  $w$  appears in both  $V^u$  and TopK $_i$ ,  $P_{\{w\}}^i$  and TopK $_i$  remains unchanged.
2. If the token  $w$  in  $V^u$  is absent in TopK $_i$  but present in  $V^i$ , it will be appended to TopK $_i$ .  $P^i$  also updates accordingly.
3. If the token  $w$  in  $V^u$  does not exist in  $V^i$ , it should be tokenized by  $T^i$ . The first token of the result along with its token probability is then updated to TopK $_i$  and  $P^i$ .

Up to now, we can obtain the aligned top tokens TopK $_i$  and the token distributions  $\hat{P}^i$  of based models. Consequently, we normalize  $\hat{P}^i$  as:  $\hat{P}_{norm}^i = \text{softmax}(\hat{P}^i)$ .

Since our method eliminates the need for full vocabulary alignment, it is essential to designate one model as the primary base model. As discussed in Section 3.2, the selection of primary base model is flexible when the candidate models exhibit comparable performance. To simplify the process, we directly employ the best-performing model as the primary base model. Then, the primary base model employs the average token probability  $\hat{P}_{avg}$  to predict the next token, where  $\hat{P}_{avg} = \frac{1}{n} \sum_{i=1}^n \hat{P}_{norm}^i$ .

The next token is determined using the maximization-based greedy strategy (Li et al., 2016). The chosen token will be appended to the input text. This process is iteratively repeated until the pre-determined stopping criterion is met, such as generating an end-of-sentence token or reaching the maximum length.

## 5 EXPERIMENTS

### 5.1 SETUP

**Models** All of our experiments are conducted with the following commonly used models, including LLaMA2-7B-Chat (Touvron et al., 2023), LLaMA2-13B-Chat (Touvron et al., 2023), LLaMA3-8B-Instruct (Dubey et al., 2024), LLaMA3.1-8B-Instruct (Dubey et al., 2024), Mistral-7B-Instruct-v0.3 (Jiang et al., 2023a), DeepSeek-LLM-7B-Chat (Bi et al., 2024), Yi-6B (Young et al., 2024), OpenChat-3.5 (Wang et al., 2024a), Qwen2-7B-Instruct (Yang et al., 2024), Mixtral-8×7B (Jiang et al., 2024) and Qwen1.5-72B (Bai et al., 2023).



Method	Dataset						Avg.
	GSM8K	PIQA	MMLU	ARC-C	TriviaQA	NQ	
Mistral	56.48	80.63	59.28	74.49	64.30	24.25	59.91
DeepSeek	59.67	72.66	46.97	58.73	47.63	11.37	49.51
OpenChat	73.46	87.10	60.80	78.05	61.77	31.08	65.38
LLM-BLENDER	70.79(-2.67)	83.28(-3.82)	60.10(-0.70)	76.29(-1.76)	56.35(-5.42)	25.57(-5.51)	62.06(-3.32)
DEEPEN	73.06(-0.40)	76.04(-11.06)	61.91(+1.11)	72.14(-5.91)	67.24 (+5.47)	28.26(-2.82)	63.11(-2.27)
GAC	62.85(-10.61)	67.85(-19.25)	55.15(-5.65)	73.04(-5.01)	62.22(+0.45)	20.72(-10.36)	56.97(-8.41)
UNITE	73.31(-0.15)	87.50 (+0.40)	62.13 (+1.33)	78.70 (+0.65)	65.80(+4.03)	31.78 (+0.70)	66.54 (+1.16)

Table 3: Results of ensembling on Mistral, DeepSeek, OpenChat. OpenChat is chosen as primary model for these experiments.

Method	Dataset					Avg.
	GSM8K	PIQA	ARC-C	MMLU		
LLaMA3	78.77	79.08	79.01	64.58		75.36
LLaMA3.1	80.83	82.86	79.49	66.69		77.47
Qwen2	80.78	84.57	84.92	64.96		78.81
<i>Two-model ensembling (LLaMA3+Qwen2)</i>						
LLM-BLENDER	82.69 (+1.91)	82.53 (-2.04)	82.98 (-1.94)	62.07 (-2.89)		77.57 (-1.24)
DEEPEN	- OOM -					
GAC	80.67 (-0.11)	80.96 (-3.61)	84.93 (+0.01)	67.05 (+2.09)		78.40 (-0.41)
UNITE	84.17 (+3.39)	85.53 (+0.96)	85.07 (+0.15)	69.78 (+4.82)		81.14 (+2.33)
<i>Three-model ensembling</i>						
LLM-BLENDER	83.30(+2.52)	83.47(-1.10)	83.48(-1.44)	62.55(-2.41)		78.20(-0.61)
DEEPEN	- OOM -					
UNITE	84.99 (+4.21)	84.98 (+0.41)	85.39 (+0.47)	69.12 (+4.16)		81.12 (+2.31)

Table 4: Results of ensembling on LLaMA3, LLaMA3.1, Qwen2. Qwen2 is chosen as primary model for these experiments.

**Baselines** We selected three typical model ensembling methods for further analyses. 1) **LLM-BLENDER** (Jiang et al., 2023b) includes a reward model, PAIRRANKER, to evaluate LLM responses, and a fusion model, GENFUSER, to combine them. We only use PAIRRANKER due to significant over-generation issues with GENFUSER. 2) **DEEPEN** (Huang et al., 2024) employs relative representation theory to map each model’s probability distribution to a universal space for aggregation, computed through the intersection of model vocabularies. 3) **GAC** (Yu et al., 2024b) projects multiple LLMs’ probability vectors into a unified vocabulary dimension using a mapping matrix and aggregates outputs at each generation step to select the next token. Although GAC suggests the importance of identifying keywords to improve latency, we still exclude it owing to its hindrance on the ensembling performance.

**Benchmarks** We evaluate six benchmarks, which can be categorized into three main groups. 1) **Comprehensive Examination:** MMLU (5-shot) (Hendrycks et al., 2021), covering 57 subjects that humans typically learn; ARC-C (0-shot) (Clark et al., 2018), collected from standardized natural science tests. 2) **Reasoning Capabilities:** GSM8K (Cobbe et al., 2021) (4-shot), a dataset of high-quality problems at the grade school math level; PIQA (Bisk et al., 2020) (0-shot), a commonsense reasoning dataset. 3) **Knowledge Capacities:** TriviaQA (5-shot) (Joshi et al., 2017), compiled by Trivia enthusiasts; NaturalQuestion (NQ) (5-shot) (Kwiatkowski et al., 2019), a question-answering corpus consisting of queries issued to the Google search engine.

For a comprehensive evaluation, we conduct two- and multi-model ensembling experiments on the aforementioned benchmarks. We select base models following the strategy presented in Section 4.1. We choose two pairs from Mistral, Deepseek, and OpenChat for the two-model experiments. For the closely performing LLaMA3, LLaMA3.1, and Qwen2, we select LLaMA3 and Qwen2 for the two-model ensemble based on their release dates and subsequently employ all three models for the multi-model ensembling evaluations. The hyper-parameter  $k$  is set to 10 in this work. All experiments are conducted on 46G NVIDIA L40 GPUs.

## 5.2 MAIN RESULTS

As shown in Table 3 and Table 4, we have following observations:



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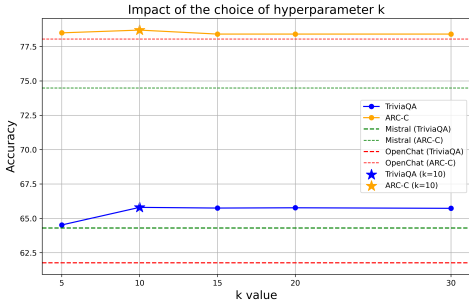


Figure 4: Impact of the choice of hyperparameter  $k$  on the ARC and TriviaQA datasets. Increasing  $k$  beyond a certain point leads to a slight decline or no improvement in performance.

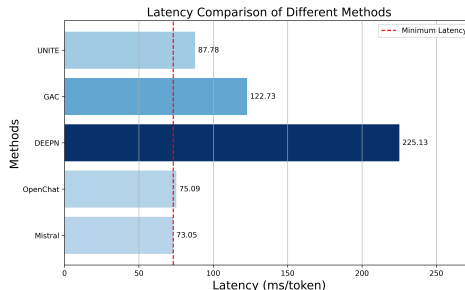


Figure 5: Latency comparison of different methods. The darker the color, the greater the latency.

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**(1) UNITE enhances individual model performance when the base models exhibit similar performance levels.** As demonstrated, the ensemble of models such as OpenChat achieved an average improvement of approximately 1.2% across five benchmark tasks, including ARC-C and MMLU. However, in the GSM8K task, a 15% performance gap between DeepSeek and OpenChat led to a slight decline in overall performance, reinforcing our observation in Section 3.1 that ensemble methods are most effective when model performances are closely aligned. Furthermore, experiments with LLaMA3 and Qwen2 validated our approach, resulting in performance increases of 3.39% and 4.82% on the GSM8K and MMLU tasks, respectively.

**(2) UNITE demonstrates greater robustness and generality.** Ensembling experiments with LLaMA3 and Qwen2 indicate that while LLM-BLENDER improves performance on the GSM8K dataset, it significantly underperforms compared to baseline models in the PIQA, ARC-C, and MMLU benchmarks. Specifically, BLENDER demonstrates a decline of 2.89% in the MMLU task, and this issue is exacerbated in ensembles with the baseline models OpenChat and Mistral, which exhibit relatively low baseline performance, leading to an average performance drop of 3.32%. Additionally, GAC fails to improve the performance of individual models across most tasks and exhibits a significant decline of over 10% when combined with Mistral and OpenChat. Furthermore, due to its intersection limitations, DEEPEN is ill-suited for ensembling with the LLaMA3 model, which requires handling a large vocabulary. In contrast, across tasks with comparable performance levels, UNITE achieves the highest improvements and overall performance across the board, with the exception of a slight under-performance relative to DEEPEN on TriviaQA. This outcome underscores the effectiveness and robustness of UNITE.

**(3) Collaborating with comparable LLMs does not necessarily yield better results.** The ensembling experiments conducted with three models, following the integration of LLaMA3.1, demonstrate improved performance on the GSM8K and ARC-C benchmarks compared to the ensemble of LLaMA3 and Qwen2. However, this approach results in suboptimal outcomes on PIQA and MMLU. This observation is justified, as the analysis in Section 3.2 indicates that while combining models with similar performance may enhance overall efficacy, such improvement is not guaranteed.

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5.3 ABLATION STUDY

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**Effect of hyperparameter  $k$  selection** To investigate the impact of the hyperparameter  $k$ , we conducted experiments using the Mistral and OpenChat models on the TriviaQA and ARC-C datasets. As illustrated in Fig. 4, increasing  $k$  from 5 to 10 enhances ensemble performance. However, further increasing  $k$  beyond 10 leads to either a slight decline or no change in performance. This finding supports our assertion that, in probability-level ensembling, aligning the entire vocabulary is unnecessary for predicting the next token.

**Effect of the next token selection strategy** We also explored the impact of deterministic decoding and top- $k$  sampling (Fan et al., 2018) on the next token emission. It is important to clarify that our UNITE focuses on the fact that, when predicting the next token, we only need to ensemble a subset of tokens rather than the entire vocabulary. In contrast, greedy decoding and top- $k$  sampling emphasize how to determine the next token’s ID after the ensemble process is complete. To investigate

Base Model	Method	Tokens Manipulated Each Step
Mistral(32678)	DEEPEN	32000
	GAC	32770
OpenChat(32002)	UNITE	14.46
LLaMA3(128256)	DEEPEN	109566
	GAC	170336
Qwen2(151646)	UNITE	14.43

Table 5: Tokens manipulated at each step. It’s noteworthy that DEEPEN encountered out-of-memory issues when conducting LLaMA3 and Qwen2 ensembling.

Model	ARC-C	PIQA
Qwen1.5-72B(Dense)	69.03	73.83
Mixtral-8x7B(Sparse)	73.98	74.59
UNITE	78.84 (+4.86)	81.88 (+7.29)

Table 6: Ensemble learning of the dense LLM Qwen1.5-72B and the sparse MOE model Mixtral-8x7B.

this, we conducted experiments using LLaMA3.1 and Mistral on the PIQA and ARC-C datasets correspondingly. As shown in Figure 6, for these deterministic tasks, the maximization-based greedy method outperforms the random sampling approach, which aligns with intuition.

#### 5.4 FURTHER ANALYSIS

**Latency analysis** Following the settings detailed in GAC (Yu et al., 2024b), we recorded the latency (*ms/token*) of different methods. The data presented in Fig. 4 reveals significant differences in latency among the various methods. Notably, the latency of UNITE is 87.78 *ms/token*, which is substantially lower than that of DEEPEN and GAC, and only about 10 *ms* longer than that of the individual models.

**Tokens manipulated each step** Table 5 presents tokens manipulated at each step. Compared to DEEPEN and GAC, UNITE significantly reduces the number of tokens manipulated at each step, indicating its efficiency. As the vocabulary of the base model expands, the number of manipulated tokens of DEEPEN and GAC increases significantly, which not only increases the computational burden but may also lead to performance bottlenecks. Contrastly, UNITE is minimally affected, maintaining its effectiveness.

**Ensemble of the dense model and the sparse model** We evaluate our method on the ensemble learning of the dense model and the sparse MoE model for reasoning and comprehensive examination tasks. Specifically, we utilize the widely-used large-scale dense model Qwen1.5-72B (Bai et al., 2023) alongside the popular sparse MoE model Mixtral-8x7B (Jiang et al., 2024) as our base models. As shown in Tabel 6, our UNITE achieves +4.86% and +7.29% performance on the ARC-C and PIQA datasets respectively, despite the base models already exhibiting high levels of performance.

## 6 CONCLUSION

In conclusion, our research highlights the effectiveness of ensemble methods in enhancing the performance of LLMs. By examining existing techniques, we identified key factors influencing ensembling success, such as model performance and response processes, while finding that vocabulary size has a minimal impact. In addition, the issue of vocabulary redundancy exposed by existing methods during ensembling lead us to propose UNITE, which efficiently aggregates a bunch of tokens from multiple LLMs without the computational overhead. Through extensive experimentation, UNITE consistently outperformed state-of-the-art ensemble methods, demonstrating its effectiveness in leveraging the strengths of diverse LLMs. Our contributions not only advance the understanding of model ensembling but also provide a practical framework for selecting and integrating LLMs to achieve superior performance.

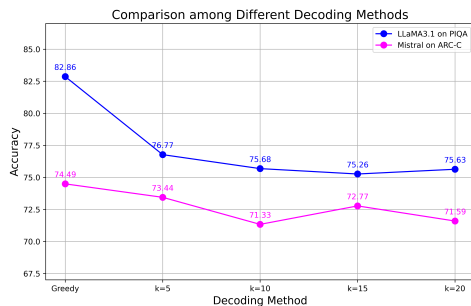


Figure 6: Comparison among different decoding methods. The greedy decoding strategy is more effective for eliciting the next token in deterministic tasks.

## REFERENCES

- 540  
541  
542 Takuya Akiba, Makoto Shing, Yujin Tang, Qi Sun, and David Ha. Evolutionary optimization of  
543 model merging recipes. *CoRR*, abs/2403.13187, 2024. doi: 10.48550/ARXIV.2403.13187. URL  
544 <https://doi.org/10.48550/arXiv.2403.13187>.
- 545 Jinze Bai, Shuai Bai, Yunfei Chu, Zeyu Cui, Kai Dang, Xiaodong Deng, Yang Fan, Wenbin Ge,  
546 Yu Han, Fei Huang, Binyuan Hui, Luo Ji, Mei Li, Junyang Lin, Runji Lin, Dayiheng Liu, Gao Liu,  
547 Chengqiang Lu, Keming Lu, Jianxin Ma, Rui Men, Xingzhang Ren, Xuancheng Ren, Chuanqi  
548 Tan, Sinan Tan, Jianhong Tu, Peng Wang, Shijie Wang, Wei Wang, Shengguang Wu, Benfeng  
549 Xu, Jin Xu, An Yang, Hao Yang, Jian Yang, Shusheng Yang, Yang Yao, Bowen Yu, Hongyi  
550 Yuan, Zheng Yuan, Jianwei Zhang, Xingxuan Zhang, Yichang Zhang, Zhenru Zhang, Chang  
551 Zhou, Jingren Zhou, Xiaohuan Zhou, and Tianhang Zhu. Qwen technical report. *arXiv preprint*  
552 *arXiv:2309.16609*, 2023.
- 553 Xiao Bi, Deli Chen, Guanting Chen, Shanhuang Chen, Damai Dai, Chengqi Deng, Honghui Ding,  
554 Kai Dong, Qiusi Du, Zhe Fu, Huazuo Gao, Kaige Gao, Wenjun Gao, Ruiqi Ge, Kang Guan,  
555 Daya Guo, Jianzhong Guo, Guangbo Hao, Zhewen Hao, Ying He, Wenjie Hu, Panpan Huang,  
556 Erhang Li, Guowei Li, Jiashi Li, Yao Li, Y. K. Li, Wenfeng Liang, Fangyun Lin, Alex X. Liu,  
557 Bo Liu, Wen Liu, Xiaodong Liu, Xin Liu, Yiyuan Liu, Haoyu Lu, Shanghao Lu, Fuli Luo, Shirong  
558 Ma, Xiaotao Nie, Tian Pei, Yishi Piao, Junjie Qiu, Hui Qu, Tongzheng Ren, Zehui Ren, Chong  
559 Ruan, Zhangli Sha, Zhihong Shao, Junxiao Song, Xuecheng Su, Jingxiang Sun, Yaofeng Sun,  
560 Minghui Tang, Bingxuan Wang, Peiyi Wang, Shiyu Wang, Yaohui Wang, Yongji Wang, Tong  
561 Wu, Y. Wu, Xin Xie, Zhenda Xie, Ziwei Xie, Yiliang Xiong, Hanwei Xu, R. X. Xu, Yanhong Xu,  
562 Dejian Yang, Yuxiang You, Shuiping Yu, Xingkai Yu, B. Zhang, Haowei Zhang, Lecong Zhang,  
563 Liyue Zhang, Mingchuan Zhang, Minghua Zhang, Wentao Zhang, Yichao Zhang, Chenggang  
564 Zhao, Yao Zhao, Shangyan Zhou, Shunfeng Zhou, Qihao Zhu, and Yuheng Zou. Deepseek LLM:  
565 scaling open-source language models with longtermism. *CoRR*, abs/2401.02954, 2024. doi: 10.  
566 48550/ARXIV.2401.02954. URL <https://doi.org/10.48550/arXiv.2401.02954>.
- 567 Yonatan Bisk, Rowan Zellers, Ronan Le Bras, Jianfeng Gao, and Yejin Choi. PIQA: reasoning  
568 about physical commonsense in natural language. In *The Thirty-Fourth AAAI Conference on*  
569 *Artificial Intelligence, AAAI 2020, The Thirty-Second Innovative Applications of Artificial Intel-*  
570 *ligence Conference, IAAI 2020, The Tenth AAAI Symposium on Educational Advances in Artifi-*  
571 *cial Intelligence, EAAI 2020, New York, NY, USA, February 7-12, 2020*, pp. 7432–7439. AAAI  
572 Press, 2020. doi: 10.1609/AAAI.V34I05.6239. URL <https://doi.org/10.1609/aaai.v34i05.6239>.
- 573  
574 Peter Clark, Isaac Cowhey, Oren Etzioni, Tushar Khot, Ashish Sabharwal, Carissa Schoenick, and  
575 Oyvind Tafjord. Think you have solved question answering? try arc, the AI2 reasoning challenge.  
576 *CoRR*, abs/1803.05457, 2018. URL <http://arxiv.org/abs/1803.05457>.
- 577  
578 Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser,  
579 Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, Christopher Hesse, and John  
580 Schulman. Training verifiers to solve math word problems. *CoRR*, abs/2110.14168, 2021. URL  
581 <https://arxiv.org/abs/2110.14168>.
- 582 Peijie Dong, Lujun Li, and Zimian Wei. Diswot: Student architecture search for distilla-  
583 tion without training. In *IEEE/CVF Conference on Computer Vision and Pattern Recogni-*  
584 *tion, CVPR 2023, Vancouver, BC, Canada, June 17-24, 2023*, pp. 11898–11908. IEEE, 2023.  
585 doi: 10.1109/CVPR52729.2023.01145. URL <https://doi.org/10.1109/CVPR52729.2023.01145>.
- 586  
587 Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, Aiesha  
588 Letman, Akhil Mathur, Alan Schelten, Amy Yang, Angela Fan, Anirudh Goyal, Anthony  
589 Hartshorn, Aobo Yang, Archi Mitra, Archie Sravankumar, Artem Korenev, Arthur Hinsvark,  
590 Arun Rao, Aston Zhang, Aurélien Rodriguez, Austen Gregerson, Ava Spataru, Baptiste Rozière,  
591 Bethany Biron, Binh Tang, Bobbie Chern, Charlotte Caucheteux, Chaya Nayak, Chloe Bi, Chris  
592 Marra, Chris McConnell, Christian Keller, Christophe Touret, Chunyang Wu, Corinne Wong,  
593 Cristian Canton Ferrer, Cyrus Nikolaidis, Damien Allonsius, Daniel Song, Danielle Pintz, Danny  
Livshits, David Esiobu, Dhruv Choudhary, Dhruv Mahajan, Diego Garcia-Olano, Diego Perino,

- 594 Dieuwke Hupkes, Egor Lakomkin, Ehab AlBadawy, Elina Lobanova, Emily Dinan, Eric Michael  
595 Smith, Filip Radenovic, Frank Zhang, Gabriel Synnaeve, Gabrielle Lee, Georgia Lewis Ander-  
596 son, Graeme Nail, Grégoire Mialon, Guan Pang, Guillem Cucurell, Hailey Nguyen, Hannah Ko-  
597 revaar, Hu Xu, Hugo Touvron, Iliyan Zarov, Imanol Arrieta Ibarra, Isabel M. Kloumann, Ishan  
598 Misra, Ivan Evtimov, Jade Copet, Jaewon Lee, Jan Geffert, Jana Vranes, Jason Park, Jay Ma-  
599 hadeokar, Jeet Shah, Jelmer van der Linde, Jennifer Billock, Jenny Hong, Jenya Lee, Jeremy  
600 Fu, Jianfeng Chi, Jianyu Huang, Jiawen Liu, Jie Wang, Jiecao Yu, Joanna Bitton, Joe Spisak,  
601 Jongsoo Park, Joseph Rocca, Joshua Johnstun, Joshua Saxe, Junteng Jia, Kalyan Vasuden Al-  
602 wala, Kartikeya Upasani, Kate Plawiak, Ke Li, Kenneth Heafield, Kevin Stone, and et al. The  
603 llama 3 herd of models. *CoRR*, abs/2407.21783, 2024. doi: 10.48550/ARXIV.2407.21783. URL  
604 <https://doi.org/10.48550/arXiv.2407.21783>.
- 605 Angela Fan, Mike Lewis, and Yann N. Dauphin. Hierarchical neural story generation. In Iryna  
606 Gurevych and Yusuke Miyao (eds.), *Proceedings of the 56th Annual Meeting of the Association  
607 for Computational Linguistics, ACL 2018, Melbourne, Australia, July 15-20, 2018, Volume 1:  
608 Long Papers*, pp. 889–898. Association for Computational Linguistics, 2018. doi: 10.18653/V1/  
609 P18-1082. URL <https://aclanthology.org/P18-1082/>.
- 610 Dan Hendrycks, Collin Burns, Steven Basart, Andy Zou, Mantas Mazeika, Dawn Song, and Jacob  
611 Steinhardt. Measuring massive multitask language understanding. In *9th International Confer-  
612 ence on Learning Representations, ICLR 2021, Virtual Event, Austria, May 3-7, 2021*. OpenRe-  
613 view.net, 2021. URL <https://openreview.net/forum?id=d7KBjmI3GmQ>.
- 614 Yichong Huang, Xiaocheng Feng, Baohang Li, Yang Xiang, Hui Wang, Bing Qin, and Ting  
615 Liu. Enabling ensemble learning for heterogeneous large language models with deep paral-  
616 lel collaboration. *CoRR*, abs/2404.12715, 2024. doi: 10.48550/ARXIV.2404.12715. URL  
617 <https://doi.org/10.48550/arXiv.2404.12715>.
- 618 Albert Q. Jiang, Alexandre Sablayrolles, Arthur Mensch, Chris Bamford, Devendra Singh Chaplot,  
619 Diego de Las Casas, Florian Bressand, Gianna Lengyel, Guillaume Lample, Lucile Saulnier,  
620 L  lio Renard Lavaud, Marie-Anne Lachaux, Pierre Stock, Teven Le Scao, Thibaut Lavril, Thomas  
621 Wang, Timoth  e Lacroix, and William El Sayed. Mistral 7b. *CoRR*, abs/2310.06825, 2023a.  
622 doi: 10.48550/ARXIV.2310.06825. URL [https://doi.org/10.48550/arXiv.2310.  
623 06825](https://doi.org/10.48550/arXiv.2310.06825).
- 624 Albert Q. Jiang, Alexandre Sablayrolles, Antoine Roux, Arthur Mensch, Blanche Savary, Chris  
625 Bamford, Devendra Singh Chaplot, Diego de Las Casas, Emma Bou Hanna, Florian Bressand, Gi-  
626 anna Lengyel, Guillaume Bour, Guillaume Lample, L  lio Renard Lavaud, Lucile Saulnier, Marie-  
627 Anne Lachaux, Pierre Stock, Sandeep Subramanian, Sophia Yang, Szymon Antoniak, Teven Le  
628 Scao, Th  ophile Gervet, Thibaut Lavril, Thomas Wang, Timoth  e Lacroix, and William El Sayed.  
629 Mixtral of experts. *CoRR*, abs/2401.04088, 2024. doi: 10.48550/ARXIV.2401.04088. URL  
630 <https://doi.org/10.48550/arXiv.2401.04088>.
- 631 Dongfu Jiang, Xiang Ren, and Bill Yuchen Lin. Llm-blender: Ensembling large language models  
632 with pairwise ranking and generative fusion. In Anna Rogers, Jordan L. Boyd-Graber, and Naoaki  
633 Okazaki (eds.), *Proceedings of the 61st Annual Meeting of the Association for Computational  
634 Linguistics (Volume 1: Long Papers), ACL 2023, Toronto, Canada, July 9-14, 2023*, pp. 14165–  
635 14178. Association for Computational Linguistics, 2023b. doi: 10.18653/V1/2023.ACL-LONG.  
636 792. URL <https://doi.org/10.18653/v1/2023.acl-long.792>.
- 637 Mandar Joshi, Eunsol Choi, Daniel S. Weld, and Luke Zettlemoyer. Triviaqa: A large scale distantly  
638 supervised challenge dataset for reading comprehension. In Regina Barzilay and Min-Yen Kan  
639 (eds.), *Proceedings of the 55th Annual Meeting of the Association for Computational Linguistics,  
640 ACL 2017, Vancouver, Canada, July 30 - August 4, Volume 1: Long Papers*, pp. 1601–1611.  
641 Association for Computational Linguistics, 2017. doi: 10.18653/V1/P17-1147. URL <https://doi.org/10.18653/v1/P17-1147>.
- 642 Tom Kwiatkowski, Jennimaria Palomaki, Olivia Redfield, Michael Collins, Ankur P. Parikh, Chris  
643 Alberti, Danielle Epstein, Illia Polosukhin, Jacob Devlin, Kenton Lee, Kristina Toutanova, Llion  
644 Jones, Matthew Kelcey, Ming-Wei Chang, Andrew M. Dai, Jakob Uszkoreit, Quoc Le, and Slav  
645 Petrov. Natural questions: a benchmark for question answering research. *Trans. Assoc. Comput.*

- 648 *Linguistics*, 7:452–466, 2019. doi: 10.1162/TACL\\_A\\_00276. URL [https://doi.org/](https://doi.org/10.1162/tacl_a_00276)  
649 [10.1162/tacl\\_a\\_00276](https://doi.org/10.1162/tacl_a_00276).
- 650
- 651 Hayeon Lee, Rui Hou, Jongpil Kim, Davis Liang, Sung Ju Hwang, and Alexander Min. A study  
652 on knowledge distillation from weak teacher for scaling up pre-trained language models. In  
653 Anna Rogers, Jordan L. Boyd-Graber, and Naoaki Okazaki (eds.), *Findings of the Association*  
654 *for Computational Linguistics: ACL 2023, Toronto, Canada, July 9-14, 2023*, pp. 11239–11246.  
655 Association for Computational Linguistics, 2023. doi: 10.18653/V1/2023.FINDINGS-ACL.714.  
656 URL <https://doi.org/10.18653/v1/2023.findings-acl.714>.
- 657 Haonan Li, Yixuan Zhang, Fajri Koto, Yifei Yang, Hai Zhao, Yeyun Gong, Nan Duan, and  
658 Timothy Baldwin. CMMLU: measuring massive multitask language understanding in chi-  
659 nese. In Lun-Wei Ku, Andre Martins, and Vivek Srikumar (eds.), *Findings of the Association*  
660 *for Computational Linguistics, ACL 2024, Bangkok, Thailand and virtual meeting, August*  
661 *11-16, 2024*, pp. 11260–11285. Association for Computational Linguistics, 2024. doi: 10.  
662 18653/V1/2024.FINDINGS-ACL.671. URL [https://doi.org/10.18653/v1/2024.](https://doi.org/10.18653/v1/2024.findings-acl.671)  
663 [findings-acl.671](https://doi.org/10.18653/v1/2024.findings-acl.671).
- 664 Jiwei Li, Will Monroe, Alan Ritter, Dan Jurafsky, Michel Galley, and Jianfeng Gao. Deep re-  
665 inforcement learning for dialogue generation. In Jian Su, Xavier Carreras, and Kevin Duh  
666 (eds.), *Proceedings of the 2016 Conference on Empirical Methods in Natural Language Pro-*  
667 *cessing, EMNLP 2016, Austin, Texas, USA, November 1-4, 2016*, pp. 1192–1202. The Asso-  
668 ciation for Computational Linguistics, 2016. doi: 10.18653/V1/D16-1127. URL [https://](https://doi.org/10.18653/v1/d16-1127)  
669 [doi.org/10.18653/v1/d16-1127](https://doi.org/10.18653/v1/d16-1127).
- 670 Cong Liu, Xiaojun Quan, Yan Pan, Liang Li, Weigang Wu, and Xu Chen. Cool-fusion: Fuse large  
671 language models without training. *CoRR*, abs/2407.19807, 2024. doi: 10.48550/ARXIV.2407.  
672 19807. URL <https://doi.org/10.48550/arXiv.2407.19807>.
- 673 Keming Lu, Hongyi Yuan, Runji Lin, Junyang Lin, Zheng Yuan, Chang Zhou, and Jingren  
674 Zhou. Routing to the expert: Efficient reward-guided ensemble of large language models.  
675 In Kevin Duh, Helena Gómez-Adorno, and Steven Bethard (eds.), *Proceedings of the 2024*  
676 *Conference of the North American Chapter of the Association for Computational Linguistics:*  
677 *Human Language Technologies (Volume 1: Long Papers), NAACL 2024, Mexico City, Mex-*  
678 *ico, June 16-21, 2024*, pp. 1964–1974. Association for Computational Linguistics, 2024. doi:  
679 10.18653/V1/2024.NAACL-LONG.109. URL [https://doi.org/10.18653/v1/2024.](https://doi.org/10.18653/v1/2024.naacl-long.109)  
680 [naacl-long.109](https://doi.org/10.18653/v1/2024.naacl-long.109).
- 681 OpenAI. GPT-4 technical report. *CoRR*, abs/2303.08774, 2023. doi: 10.48550/ARXIV.2303.08774.  
682 URL <https://doi.org/10.48550/arXiv.2303.08774>.
- 683
- 684 Tal Shnitzer, Anthony Ou, Mírian Silva, Kate Soule, Yuekai Sun, Justin Solomon, Neil Thomp-  
685 son, and Mikhail Yurochkin. Large language model routing with benchmark datasets. *CoRR*,  
686 abs/2309.15789, 2023. doi: 10.48550/ARXIV.2309.15789. URL [https://doi.org/10.](https://doi.org/10.48550/arXiv.2309.15789)  
687 [48550/arXiv.2309.15789](https://doi.org/10.48550/arXiv.2309.15789).
- 688 Mirac Suzgun, Nathan Scales, Nathanael Schärli, Sebastian Gehrmann, Yi Tay, Hyung Won Chung,  
689 Aakanksha Chowdhery, Quoc V Le, Ed H Chi, Denny Zhou, , and Jason Wei. Challenging big-  
690 bench tasks and whether chain-of-thought can solve them. *arXiv preprint arXiv:2210.09261*,  
691 2022.
- 692 Hugo Touvron, Louis Martin, Kevin Stone, Peter Albert, Amjad Almahairi, Yasmine Babaei, Niko-  
693 lay Bashlykov, Soumya Batra, Prajjwal Bhargava, Shruti Bhosale, et al. Llama 2: Open founda-  
694 tion and fine-tuned chat models. *arXiv preprint arXiv:2307.09288*, 2023.
- 695
- 696 Fanqi Wan, Xinting Huang, Deng Cai, Xiaojun Quan, Wei Bi, and Shuming Shi. Knowledge  
697 fusion of large language models. In *The Twelfth International Conference on Learning Rep-*  
698 *resentations, ICLR 2024, Vienna, Austria, May 7-11, 2024*. OpenReview.net, 2024a. URL  
699 <https://openreview.net/forum?id=jiDsk12qcz>.
- 700 Fanqi Wan, Ziyi Yang, Longguang Zhong, Xiaojun Quan, Xinting Huang, and Wei Bi. Fusechat:  
701 Knowledge fusion of chat models. *CoRR*, abs/2402.16107, 2024b. doi: 10.48550/ARXIV.2402.  
16107. URL <https://doi.org/10.48550/arXiv.2402.16107>.

- 702 Guan Wang, Sijie Cheng, Xianyuan Zhan, Xiangang Li, Sen Song, and Yang Liu. Openchat: Advancing open-source language models with mixed-quality data. In *The Twelfth International Conference on Learning Representations, ICLR 2024, Vienna, Austria, May 7-11, 2024*. OpenReview.net, 2024a. URL <https://openreview.net/forum?id=AOJyfhWYHf>.
- 703  
704  
705  
706
- 707 Hongyi Wang, Felipe Maia Polo, Yuekai Sun, Souvik Kundu, Eric P. Xing, and Mikhail Yurochkin. Fusing models with complementary expertise. In *The Twelfth International Conference on Learning Representations, ICLR 2024, Vienna, Austria, May 7-11, 2024*. OpenReview.net, 2024b. URL <https://openreview.net/forum?id=PhMrGCMIRL>.
- 708  
709  
710
- 711 Yangyifan Xu, Jinliang Lu, and Jiajun Zhang. Bridging the gap between different vocabularies for LLM ensemble. In Kevin Duh, Helena Gómez-Adorno, and Steven Bethard (eds.), *Proceedings of the 2024 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies (Volume 1: Long Papers), NAACL 2024, Mexico City, Mexico, June 16-21, 2024*, pp. 7140–7152. Association for Computational Linguistics, 2024. doi: 10.18653/V1/2024.NAACL-LONG.395. URL <https://doi.org/10.18653/v1/2024.naacl-long.395>.
- 712  
713  
714  
715  
716  
717
- 718 An Yang, Baosong Yang, Binyuan Hui, Bo Zheng, Bowen Yu, Chang Zhou, Chengpeng Li, Chengyuan Li, Dayiheng Liu, Fei Huang, Guanting Dong, Haoran Wei, Huan Lin, Jialong Tang, Jialin Wang, Jian Yang, Jianhong Tu, Jianwei Zhang, Jianxin Ma, Jianxin Yang, Jin Xu, Jingren Zhou, Jinze Bai, Jinzheng He, Junyang Lin, Kai Dang, Keming Lu, Keqin Chen, Kexin Yang, Mei Li, Mingfeng Xue, Na Ni, Pei Zhang, Peng Wang, Ru Peng, Rui Men, Ruize Gao, Runji Lin, Shijie Wang, Shuai Bai, Sinan Tan, Tianhang Zhu, Tianhao Li, Tianyu Liu, Wenbin Ge, Xiaodong Deng, Xiaohuan Zhou, Xingzhang Ren, Xinyu Zhang, Xipin Wei, Xuancheng Ren, Xuejing Liu, Yang Fan, Yang Yao, Yichang Zhang, Yu Wan, Yunfei Chu, Yuqiong Liu, Zeyu Cui, Zhenru Zhang, Zhifang Guo, and Zhihao Fan. Qwen2 technical report. *CoRR*, abs/2407.10671, 2024. doi: 10.48550/ARXIV.2407.10671. URL <https://doi.org/10.48550/arXiv.2407.10671>.
- 721  
722  
723  
724  
725  
726  
727  
728
- 729 Alex Young, Bei Chen, Chao Li, Chengen Huang, Ge Zhang, Guanwei Zhang, Heng Li, Jiangcheng Zhu, Jianqun Chen, Jing Chang, Kaidong Yu, Peng Liu, Qiang Liu, Shawn Yue, Senbin Yang, Shiming Yang, Tao Yu, Wen Xie, Wenhao Huang, Xiaohui Hu, Xiaoyi Ren, Xinyao Niu, Pengcheng Nie, Yuchi Xu, Yudong Liu, Yue Wang, Yuxuan Cai, Zhenyu Gu, Zhiyuan Liu, and Zonghong Dai. Yi: Open foundation models by 01.ai. *CoRR*, abs/2403.04652, 2024. doi: 10.48550/ARXIV.2403.04652. URL <https://doi.org/10.48550/arXiv.2403.04652>.
- 730  
731  
732  
733  
734
- 735 Le Yu, Bowen Yu, Haiyang Yu, Fei Huang, and Yongbin Li. Language models are super mario: Absorbing abilities from homologous models as a free lunch. In *Forty-first International Conference on Machine Learning, ICML 2024, Vienna, Austria, July 21-27, 2024*. OpenReview.net, 2024a. URL <https://openreview.net/forum?id=fq0NaiU8Ex>.
- 736  
737  
738
- 739 Yao-Ching Yu, Chun-Chih Kuo, Ziqi Ye, Yu-Cheng Chang, and Yueh-Se Li. Breaking the ceiling of the LLM community by treating token generation as a classification for ensembling. *CoRR*, abs/2406.12585, 2024b. doi: 10.48550/ARXIV.2406.12585. URL <https://doi.org/10.48550/arXiv.2406.12585>.
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757	Question	Who had a 70s No 1 hit with Kiss You All Over? The flag of Libya is a plain rectangle of which color?
758	Gold answer	Sir Henry Campbell Bannerman Green
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760	LLaMA3 response	Henry Campbell-Bannerman ✓ Green ✓
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762	Qwen2 Response	The hit song Kiss You All Over was performed by the band Arrows, featuring vocalist <b>Jim McCarty</b> . ✗ The song reached number one on the UK Singles Chart in 1976. The song was written by Arrows' lead singer Jim McCarty, along with the band's guitarist, Steve Pig Harrison. It was produced by Tony Visconti, who had previously worked with David Bowie and T. Rex. Kiss You All Over was released as a single from the band's debut album, Arrows, which was also released in 1976. The song's catchy chorus and upbeat tempo made it a popular hit of the 1970s, and it remains a well-known and beloved track from that era. The song's success helped to establish Arrows as a popular band of the time, and it continues to be played on classic rock and oldies radio stations today. ----- The flag of Libya is a plain rectangle of <b>green</b> color. ✓ It was adopted in 1977 and features a green color field with no other symbols or designs. The color green is significant in the country's history and culture, representing hope, fertility, and the country's rich agricultural heritage. The flag is rectangular in shape and is often flown vertically. It is an important symbol of the Libyan state and is used in various official and ceremonial contexts.
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774	LLM-Blender response	The hit song Kiss You All Over was performed by the band Arrows, featuring vocalist <b>Jim McCarty</b> . ✗ The song reached number one on the UK Singles Chart in 1976. The song was written by Arrows' lead singer Jim McCarty, along with the band's guitarist, Steve Pig Harrison. It was produced by Tony Visconti, who had previously worked with David Bowie and T. Rex. Kiss You All Over was released as a single from the band's debut album, Arrows, which was also released in 1976. The song's catchy chorus and upbeat tempo made it a popular hit of the 1970s, and it remains a well-known and beloved track from that era. The song's success helped to establish Arrows as a popular band of the time, and it continues to be played on classic rock and oldies radio stations today. ----- The flag of Libya is a plain rectangle of <b>green</b> color. ✓ It was adopted in 1977 and features a green color field with no other symbols or designs. The color green is significant in the country's history and culture, representing hope, fertility, and the country's rich agricultural heritage. The flag is rectangular in shape and is often flown vertically. It is an important symbol of the Libyan state and is used in various official and ceremonial contexts.
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Table 7: Responses of different methods on TriviaQA dataset

## 791 A APPENDIX

### 794 A.1 TASK CHALLENGES ANALYSIS ON TRIVIAQA

797 Table 7 provides a detailed comparison of the differences in response styles between LLaMA3  
798 and Qwen2 on the TriviaQA dataset. It is easily observable that, regardless of correctness, LLM-  
799 BLENDER consistently tends to choose longer responses as answers.

800 We try to address the response style issue via preprocessing steps. Specifically, we provide an  
801 alternative simple solution by using the few-shot examples to standardize the response format. We  
802 employed a new 5-shot prompt designed to elicit answers in the format: The answer is xxx. The  
803 responses are presented in the Table 8.

804 As shown in Table 9, the original prompts elicit accuracy in the brackets. Since the tedious response  
805 style illustrated in Table 7, Qwen incorporates answers into the analysis, we randomly sampled 100  
806 instances from the 1500-test set to manually extract the predictions. After adjusting the prompt, we  
807 can easily analyze the results for the entire test set. Consistent with our findings presented in the  
808 main text, when the base model's performance gap exceeds 10%, ensemble learning may yield little  
809 to no improvement. Additionally, it is important to note that our UNITE approach still outperforms  
its competitors.



810	TriviaQA Question	Which Lloyd Webber musical premiered in the US on 10th December 1993?
811		Question: In the 1971 Number One hit Ernie by Benny Hill,
812		what was the name of Ernie’s horse who was kicked by his rival,
813	Original prompt (Response style referring to Table 2)	Two-ton Ted from Teddington?
814		Answer: <b>Triggers.</b>
815		...
816		Question: In the 1971 Number One hit Ernie by Benny Hill,
817	New Prompt	what was the name of Ernie’s horse who was kicked by his rival,
818		Two-ton Ted from Teddington?
819		Answer: <b>The answer is Triggers.</b>
820		...
821	LLaMA3 response	The answer is Sunset Boulevard.
822	Qwen2.5 response	The answer is Sunset Boulevard.

Table 8: Case study of preprocessing on TriviaQA.

Method	TriviaQA
LLaMA3-8b-instruct	70.68(67)
Qwen2.5-7b-instruct	57.85(52)
LLM-BLENDER	64.77
UNITE	67.45

Table 9: Results of ensembling LLaMA3 and Qwen2.5 on TriviaQA after preprocessing. LLaMA3 is chosen as primary model for these experiments.

Method	BBH
LLaMA3-8b-instruct	73.00
Qwen2-7b-instruct	68.60
LLM-BLENDER	68.79
DEEPEN	-OOM-
GAC	69.86
UNITE	73.52

Table 10: Results of ensembling LLaMA3 and Qwen2 on BBH. LLaMA3 is chosen as primary model for these experiments.

## A.2 GENERALIZATION EVALUATION ON BIG-BENCH HARD BENCHMARK

We conduct additional experiments using the BBH (BIG-Bench Hard) benchmark (Suzgun et al., 2022), a diverse evaluation suite of 23 challenging tasks such as symbolic reasoning, to further validate UNITE’s generalizability. Due to computational constraints and limited rebuttal time, we randomly selected 10 subsets, and the results are presented below. Consistent with our main findings, UNITE demonstrates superior performance compared to other methods, highlighting the effectiveness and generalizability of our approach.

## A.3 MULTILINGUAL TOKENIZATION CONSIDERATIONS

We also conducted experiments on the CMMLU (Li et al., 2024), a Chinese Multitask Language Understanding Evaluation benchmark to validate our findings in section 3.2. We used Yi-6B (vocabulary size: 64,000) and Qwen2-7b-instruct (vocabulary size: 152,064) as our base models, both supporting the Chinese language. Qwen2-7b-instruct is the primary model, the results of various ensemble approaches are presented in Table 11. Similar to the results listed in Section 3.2, irrespective of the gap in vocabulary size, existing methods still demonstrate improvements, thereby indicating that vocabulary size for model ensembling is marginal.

Method	CMMLU
Yi-6B	75.21
Qwen2-7b-instruct	83.22
LLM-BLENDER	79.08
DEEPEN	-OOM-
GAC	75.88
UNITE	83.89

Table 11: Results of ensembling Yi and Qwen2 on CMMLU. Qwen2 is chosen as primary model for these experiments.

Setting	TriviaQA
Mistral	64.30
OpenChat	61.77
$k = 5$	64.52
$k = 10$	65.80
$k = 20$	65.77
$k = 100$	65.65
$k = 1000$	65.74
$k = 10000$	65.72

Table 12: Impact of the large hyperparameter  $k$  on TriviaQA dataset.

#### A.4 FURTHER ANALYSIS OF HYPERPARAMETER $k$

We clarify that the enhanced efficiency and effectiveness arise from reduced token options and our specialized union mapping method, as outlined in Section 4.2. UNITE constructs a union of the top- $k$  tokens from each model and expands this set using each model’s tokenizer, followed by probability aggregation to determine the next token. UNITE eliminates the need for auxiliary mapping matrices and full vocabulary alignment, respecting the unique tokenization of each base LLM.

We further test  $k$  with extremely large values to validate our effectiveness. As shown in Table 12, further increasing  $k$  leads to either a slight decline or no change in performance. This finding reinforces our assertion that, in probability-level ensembling, it is unnecessary to align the entire vocabulary to predict the next token.

#### A.5 CASE STUDY OF UNITE

Table 13 presents the specific cases of UNITE and other ensembling approaches, showcasing the validity and stability of our method.

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921	Question	Tom plants 10 trees a year. Every year he also chops down 2 trees a year. He starts with 50 trees. After 10 years 30% of the trees die. How many trees does he have left?	
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923	Gold Answer	He gets $10 - 2 = 8$ new trees a year. After 10 years he has $8 * 10 = 80$ new trees so he had $80 + 50 = 130$ trees He lost $130 * .3 = 39$ trees That means he had $130 - 39 = 91$ trees	
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928	LLaMA3 Response	Tom starts with 50 trees. He plants 10 trees a year, so after 10 years he has $50 + 10 * 10 = 150$ trees. However, he chops down 2 trees a year, so after 10 years he has $150 - 2 * 10 = 140$ trees. 30% of the trees die, which means $30 / 100 * 140 = 42$ trees die. Tom is left with $140 - 42 = 98$ trees. The answer is boxed{ 98 }	
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933	LLaMA3.1 Response	Tom starts with 50 trees and plants 10 trees a year, so after 10 years he will have $50 + 10 * 10 = 150$ trees. However, he also chops down 2 trees a year, so after 10 years he will have $150 - 2 * 10 = 140$ trees. After 10 years, 30% of the trees die, so 30% of 140 is $0.3 * 140 = 42$ trees die. The number of trees Tom has left is $140 - 42 = 98$ trees. The answer is boxed{ 98 }	
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937	Qwen2 Response	Tom starts with 50 trees and plants 10 trees a year, so after 10 years he will have $50 + 10 * 10 = 150$ trees. However, he also chops down 2 trees a year, so after 10 years he will have $150 - 2 * 10 = 140$ trees. After 10 years, 30% of the trees die, so 30% of 140 is $0.3 * 140 = 42$ trees die. The number of trees Tom has left is $140 - 42 = 98$ trees. The answer is boxed{ 98 }	
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941	2-models	LLM-Blender	
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944			Tom starts with 50 trees and plants 10 trees a year, so after 10 years he will have $50 + 10 * 10 = 150$ trees. However, he also chops down 2 trees a year, so after 10 years he will have $150 - 2 * 10 = 140$ trees. After 10 years, 30% of the trees die, so 30% of 140 is $0.3 * 140 = 42$ trees die. The number of trees Tom has left is $140 - 42 = 98$ trees. The answer is boxed{ 98 }
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Table 13: Case study of UNITE and other ensembling methods on GSM8K