

Online Two-Sided Markets: Many Buyers Enhance Learning

Facilitating trades between sellers and buyers is fundamental in several economic systems. We study a repeated trading problem in which a mechanism designer facilitates trade between a single seller and multiple buyers through the lens of online learning. Our model generalizes the classic bilateral trade setting to a multi-buyer environment. Specifically, the mechanism designer runs a second-price auction among the buyers—extending the fixed-price mechanism used in bilateral trade—before proposing a price to the seller. While this setting introduces new challenges compared to bilateral trade, it also provides an informational advantage. Indeed, the presence of multiple buyers enhances competition, inducing them to reveal their valuations in order to win the auction. However, as in bilateral trade, the seller faces a binary decision: whether to accept the proposed price or not. We show that this asymmetric feedback, which is more informative than in bilateral trade, allows us to break some lower bounds on regret minimization with a single buyer.

Our first result is an assumption-free $\tilde{O}(T^{2/3})$ regret upper bound for Strong Budget Balance mechanisms. We show that our asymmetric feedback can be used to design an adaptive grid to work on. Interestingly, we show that it is sufficient to use an adaptive grid only on the buyers' price, on which we have stronger feedback. Intuitively, we include in the grid the prices corresponding to valuations that appear with sufficiently high probability, while guaranteeing that valuations in between two points of the grid occurs with small probability. Such property mimics the bounded density assumption that in previous works has been essentially used to bound the probability that a valuation lies between two points of an uniform grid.

Our second main result establishes the first sublinear regret guarantees with respect to Global Budget Balance mechanisms. The main challenge is that the problem becomes two dimensional, as price offered to the sellers and the one asked to the buyers can differ from each other. This implies that, if each price lies on a grid of size K , we have to optimize over $K \times K$ couples, which makes impossible to learn without sharing the feedback. To provide the regret guarantees our proof relies on three main components: (i) an adaptive grid that works only when the buyers and seller distributions are independent or the joint distribution has bounded density, (ii) a multiplicative-additive revenue maximizing grid, and (iii) a tool to share feedback among arms in constrained MABs. By combining these results, we provide a $\tilde{O}(T^{2/3})$ regret bound when either the buyers and seller valuations are independent, or their joint distribution admits bounded density.