

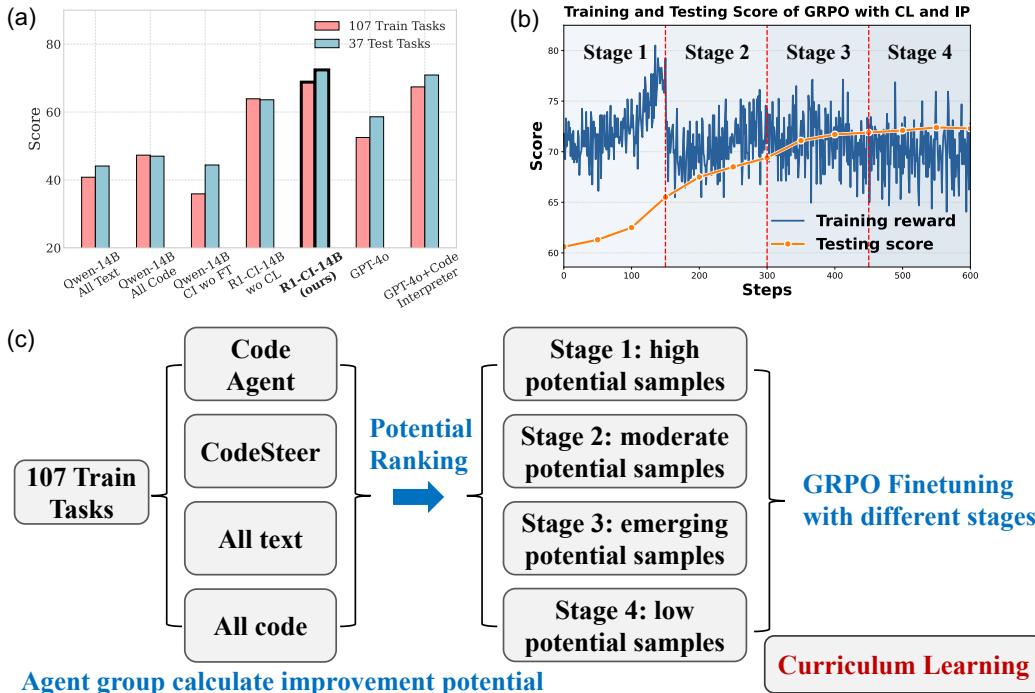
# R1-CODE-INTERPRETER: LLMS REASON WITH CODE VIA SUPERVISED AND MULTI-STAGE REINFORCEMENT LEARNING

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## ABSTRACT

009 Practical guidance on training Large Language Models (LLMs) to leverage Code Interpreter across  
010 diverse tasks remains lacking. We present R1-Code-Interpreter, an extension of a text-only LLM  
011 trained via multi-turn supervised fine-tuning (SFT) and reinforcement learning (RL) to autonomously  
012 generate multiple code queries during step-by-step reasoning. Unlike prior RL + tool-use efforts  
013 focused on narrow domains such as math or retrieval, we curate 144 diverse reasoning and planning  
014 tasks and show that training a general-purpose Code Interpreter across them presents significant  
015 challenges due to task heterogeneity and scarcity of effective samples. To address this, we introduce a  
016 multi-stage curriculum learning approach that partitions training samples by measured improvement  
017 potential. The RL training prioritizes samples with higher potential and gradually shifts to lower-  
018 potential ones, increasing the average RL gains from merely +3.4% to +9.3% across Qwen-2.5  
019 models (3/7/14B). Our final model, R1-CI-14B, improves average accuracy on the 37 test tasks from  
020 44.1% to 72.4%, outperforming text-only GPT-4o (58.6%) and GPT-4o with Code Interpreter (70.9%).  
Notably, R1-CI-14B also exhibits emergent self-checking behavior through code generation.



044 Figure 1: Training Code Interpreter-augmented reasoning models with multi-stage GRPO on 144 reasoning and  
045 planning tasks. (a) Our best model, R1-CI-14B, outperforms both GPT-4o (text-only) and GPT-4o with Code Interpreter.  
046 (b) Training reward and test scores improve steadily through the curriculum learning, then plateau at stage 4 after  
047 adding low-potential samples. (c) To assess sample effectiveness, we estimate improvement potential by repeatedly  
048 sampling answers with different agent frameworks and analyzing the correct/wrong distribution. GRPO begins with  
049 high-potential samples and gradually incorporates lower-potential ones.

## 1 INTRODUCTION

050 While reinforcement learning (RL)-based fine-tuning has significantly improved LLMs' reasoning and planning Wang  
051 et al. (2024); Guo et al. (2025); Jaech et al. (2024), models still struggle with seemingly simple tasks (Chen et al., 2025)  
052 and incur high token costs during inference-time search (Chen et al., 2024a). Textual reasoning excels at semantics  
053 and commonsense, but falls short in precise computation, symbolic manipulation, optimization, and algorithmic

054 processing (Valmeekam et al., 2022). In contrast, symbolic code generation handles these rigorously and benefits from  
 055 external tools (e.g., equation solvers). Prompting LLMs to generate and execute code often outperforms pure textual  
 056 reasoning (Madaan et al., 2022; Liang et al., 2022; Chen et al., 2022).

057 A key challenge is guiding LLMs to decide when to rely on textual reasoning versus programmatic solutions, given  
 058 that most input questions lack explicit cues about which approach is best and the possible text/code solution space  
 059 is large. OpenAI’s GPT models address this by incorporating a Code Interpreter, allowing iterative code generation  
 060 and reasoning over outputs (Achiam et al., 2023). However, recent work Chen et al. (2024c) show that current Code  
 061 Interpreter implementations struggle to effectively steer between text and code, underutilizing symbolic capabilities.  
 062 Recent work such as T0RL (Li et al., 2025b) and ReTool (Feng et al., 2025) investigates training reasoning models  
 063 to integrate with Code Interpreters. However, their training and evaluation are limited to math problems, leaving a  
 064 significant gap from real-world applications that demand effectiveness across broader benchmarks. ToolRL (Qian et al.,  
 065 2025) instead focuses on teaching models to select among multiple tools, where the Code Interpreter is used only  
 066 for generating relatively simple code. Currently, public research still lacks a comprehensive understanding of how to  
 067 fine-tune LLMs to integrate with Code Interpreter for robust, generalizable performance across **hundreds of tasks**.

068 To tackle these challenges, we present R1-Code-Interpreter, a framework for integrating Code Interpreter into open-  
 069 source LLMs. We curate 144 reasoning and planning tasks and synthesize 6.5k multi-turn text/code trajectories for  
 070 supervised fine-tuning (SFT), followed by Group Relative Policy Optimization (GRPO) (Shao et al., 2024). In contrast  
 071 to prior work, which shows that RL training can yield substantial improvements on single or simple tasks, we find  
 072 that **applying traditional DeepSeek-style RL training to a general Code Interpreter across 144 challenging tasks**  
 073 **yields only marginal gains**. This difficulty arises from task heterogeneity and the scarcity of effective samples. To  
 074 address this, we propose a novel multi-stage curriculum learning approach guided by measured *improvement potential*.  
 075 Specifically, after the SFT stage, the model is prompted with diverse single- and multi-turn agent strategies to answer  
 076 the same question. The accuracy discrepancies among these answers are used to estimate improvement potential. RL  
 077 training then proceeds in four stages, beginning with high-potential samples and moving to lower-potential ones.

078 We finetune Qwen-2.5 (Qwen et al., 2025) models (3/7/14B), achieving average success rate improvements of 33.7% on  
 079 107 train tasks and 34.1% on 37 test tasks. As shown in Fig. 1, our best model, R1-CI-14B, raises testing accuracy from  
 080 44.1% to 72.4%, outperforming much larger GPT-4o text-only (58.6%) and GPT-4o with its inherent Code Interpreter  
 081 (70.9%). The resulting model effectively combines code execution with textual reasoning, solving tasks through iterative  
 082 “execute-and-explore” interactions before producing final answers. The model gradually learns to generate code to  
 083 verify its answers, an emergent behavior rarely observed before training. Our main contributions are:

084 **1) To our best knowledge, this is the first published work to train general Code Interpreter across multiple tasks**  
 085 **and domains.** Unlike prior work that focused on single task or tasks with simple reasoning, we curate 144 challenging  
 086 reasoning and planning tasks, each with over 200 samples of varying difficulty. All tasks are standardized into a unified  
 087 format to enable efficient rollout and automated correctness evaluation. They cover diverse reasoning skills, including  
 088 mathematical, spatial, logical, ordinal, optimization, and search-based reasoning.

089 **2) Analysis of RL limitations and proposal of an effective multi-stage curriculum learning framework guided**  
 090 **by improvement potential.** We synthesize 6.5k multi-turn trajectories with interleaved reasoning and code execution  
 091 for SFT, followed by RL for further optimization. By tuning the number of training tasks, we find that RL for  
 092 general-purpose Code Interpreter is substantially more challenging due to task heterogeneity and scarcity of effective  
 093 samples. Our proposed multi-stage curriculum learning with measured improvement potential effectively mitigates this  
 094 bottleneck, raising RL performance gains from +3.4% to +9.3%.

095 **3) Cost-efficient training by separating model gradient calculation from code execution.** We find that time-  
 096 consuming code execution significantly reduces GPU utilization and accounts for a large portion of RL training time in  
 097 the Code Interpreter, limiting the maximum batch size and hindering parallel efficiency. To address this, we design  
 098 a specialized Code Execution Sandbox on multiple CPU nodes, decoupling execution from GPU-based gradient  
 099 computation. This approach reduces overall training time by 39%, decreasing from around 4500 to 1845 GPU hours.

100 **4) Comparison of training strategies:** 1) The multi-turn Code Interpreter framework proves more generalizable and  
 101 effective than single-turn text or code generation frameworks. 2) Warm-starting with SFT significantly improves the  
 102 training of Code Interpreter. 3) Qwen-2.5 models as base are better than DeepSeek R1-distilled reasoning models.

## 103 2 CHALLENGES IN 144 REASONING AND PLANNING TASKS

104 We compile 144 tasks from three major reasoning and planning benchmarks: 33 from SymBench (Chen et al., 2025),  
 105 27 from Big-Bench-Hard (Suzgun et al., 2022), and 84 from Reasoning-Gym<sup>1</sup>. After removing near-duplicates, each

106  
 107 <sup>1</sup><https://github.com/open-thought/reasoning-gym>

task retains over 200 diverse samples. All tasks are standardized into a unified format and evaluated using rule-based criteria (e.g., exact match or constraint checks) for efficient rollout and testing. The tasks cover diverse reasoning and planning challenges for LLM evaluation. Detailed task descriptions are in Appendix Sec. G, and their associated capability categories, logic, spatial, order, optimization, search, and math, are summarized in Appendix Table 6. Recent work Chen et al. (2024c) highlights key challenges in steering LLMs to choose effectively between code and text: (1) The better mode (code vs. text) depends on task type, complexity, and model capabilities, which is often hard to predict; (2) LLM-generated code frequently degenerates into hard-coded text-like scripts, limiting its symbolic utility (Yang et al., 2024); (3) Many challenging tasks require models to iteratively explore and optimize the solutions in a difficult multi-turn behavior. Appendix Fig. 11 shows a representative example of GPT-5 failing to use the Code Interpreter.

**Question:** Blocksworld Task: Initial State: stack1: D stack2: G C A F stack3: B I E H Goal State: stack1: A F H stack2: C G I stack3: B D E Please provide a series of moves to reach the goal state. Your answer:

**R1-CI-14B:** Let me solve this Blocksworld problem step by step. 1) First, let me create a Python script to help simulate and find the solution. The script will:

- Represent the stacks
- Validate moves (only top blocks can be moved)
- Find a path from initial to goal state
- Use a search algorithm to find the solution. Use symbolic computing or an efficient search algorithm, we can implement a **depth-first search (DFS) approach** ...<omitted text>...

```
```python
def is_goal_state(state, goal_state):
    return state == goal_state

def get_possible_moves(state):
    moves = []
    for i, stack in enumerate(state):
        ...<omitted code>...
```



Code Execution Results: TimeoutExpired: Command 'run.py' timed out after 60 seconds.



The current approach using depth-first search (DFS) is a step in the right direction, but it seems to be inefficient, leading to a timeout. To optimize the search for a solution in the Blocksworld task, we can implement the A* search algorithm. A* search is more efficient than DFS ...<omitted text>...



```
```python
import heapq

def get_possible_moves(state):
    moves = []
    for i, stack in enumerate(state):
        ...<omitted code>...
```



Code Execution Results: Move H from 3 to 1\nMove E from 3 to 1\nMove I from 3 to 2\nMove F from 2 to 3...



I see the code has found a valid solution. Let me verify this solution by writing the checking code ...<omitted text>...



```
```python
def verify_blocksworld_solution(initial_state, goal_state, moves):
    current_state = {stack: list(blocks) for stack, blocks in initial_state.items()}
    for move in moves:
        parts = move.split()
        ...<omitted code>...
```



Code Execution Results: Correct



The current solution is correct: <<< Move H from 3 to 1\nMove E from 3 to 1\nMove I from 3 to 2\nMove F from 2... >>>


```


```


```

Figure 2: Example response of R1-Code-Interpreter in Blocksworld task.

### 3 R1-CODE-INTERPRETER

Fig. 2 shows a representative example of R1-Code-Interpreter in action. The model iteratively reasons, optionally generates code for execution, and refines its reasoning based on the results, continuing this process until the final answer is produced. The Code Interpreter is invoked only when deemed beneficial; otherwise, the model relies on pure textual reasoning. The system instruction directs the model to enclose code between the natural tokens ‘`python’’ and ‘`’ when execution is needed. Upon detecting a code block, the system extracts and executes it via the Code Interpreter, then appends the result (prefixed with the special token ‘Code Execution Results:’) to the ongoing generation. This loop continues until either (1) the maximum of 8 code calls is reached, or (2) the model emits a final answer enclosed between ‘<<<’ and ‘>>>’.

**Response format:** To train R1-Code-Interpreter, we begin by designing a simple head prompt that guides the initial LLM to follow our predefined structure. As shown in Table 1, the prompt organizes the output into three iterative parts: reasoning, optional Code Interpreter invocation, and the final answer. We avoid imposing content-specific constraints (e.g., enforcing reflective reasoning or code calls) to preserve the model’s natural learning dynamics during RL.

Unlike prior work that enforces section tags like ‘`<think>`’, ‘`<answer>`’, or ‘`<search>`’ (Guo et al., 2025; Jin et al., 2025; Zhang et al., 2025), we rely solely on the final answer marker ‘`<<<answer content>>>`’ for answer extraction. For code, we leverage the LLM’s pretrained behavior of naturally starting code blocks with ‘```` python`’, which serves as implicit tagging. Our initial tests show this natural format performs better than forced tagging, as it aligns more closely with the model’s original distribution.

162  
163  
164 Table 1: Head prompt for R1-Code-Interpreter.  
165  
166  
167  
168  
169

The User asks a question, and you solve it. You first generate the reasoning and thinking process and then provide the User with the final answer. During the thinking process, **\*\*you can generate python code\*\*** for efficient searching, optimization, and computing with the format of starting the python block with **``` python**. **\*\*A code query must involve only a single script that uses 'print' function for the output.\*\***. Once the code script is complete, stop the generation. Then, the code interpreter platform will execute the code and return the execution output and error. Once you feel you are ready for the final answer, directly return the answer with the format **<<<answer content>>>** at the end of your response. Otherwise, you can continue your reasoning process and possibly generate more code query to solve the problem.

170  
171  
172 4 TRAINING SETTINGS

173 We fine-tune R1-Code-Interpreter using SFT and GRPO on a subset of 144 tasks. We randomly select 107 tasks for  
174 training: 26 from SymBench, 20 from Big-Bench-Hard, and 61 from Reasoning-Gym, ensuring no sample overlaps  
175 with the test set. The remaining 37 tasks are used for evaluation. To generate SFT supervision, we prompt GPT-4o to  
176 produce multiple reasoning/execution trajectories per task and retain only those yielding correct answers. To enhance  
177 diversity and adaptability, we use varied prompt formats: some allow free-form reasoning such as the prompt in  
178 Table 1, while others enforce transitions between text and code. To encourage exploratory reasoning, we increase the  
179 proportion of answer trajectories with multi-turn generation, particularly those that adaptively adjust solving strategies  
180 (e.g., switching between code and text, or refining generated code). To balance the dataset, each task includes at  
181 most 70 valid trajectories, resulting in a final dataset of 6.5k high-quality samples for SFT. To avoid training collapse,  
182 the training samples in GRPO has no overlaps with SFT. We conduct experiments using three default base models:  
183 Qwen2.5-14B-Instruct-1M, Qwen2.5-7B-Instruct-1M, and Qwen2.5-3B-Instruct (Qwen et al., 2025).

184 SFT is trained for 3 epochs to prevent overfitting. GRPO uses a learning rate of 1e-6 with 5 sampled responses  
185 per prompt and a KL penalty of 0.001. Learning rates are tuned in early-stage experiments. Training and inference  
186 temperatures are set to 1.0 and 0.6, respectively. We use a batch size of 32 for SFT and 128 for GRPO. Both stages  
187 perform full-parameter fine-tuning on 16 H100 GPUs. Answers are evaluated using predefined rules, with GPT-4o  
188 assisting in format normalization when necessary. For methods that output code as the final answer, we extract and  
189 execute the code using predefined logic to obtain the final result.

190  
191 4.1 GRPO WITH CODE INTERPRETER

192 We formulate our RL objective with a Code Interpreter  $\mathcal{C}$  as:

$$193 \max_{\pi_\theta} \mathbb{E}_{x \sim D, y \sim \pi_\theta(\cdot | x; \mathcal{C})} [r_\phi(x, y)] - \beta \mathbb{D}_{\text{KL}} [\pi_\theta(y | x; \mathcal{C}) \| \pi_{\text{ref}}(y | x; \mathcal{C})], \quad (4.1)$$

196 where  $\pi_\theta$  is the policy LLM,  $\pi_{\text{ref}}$  is the reference model,  $r_\phi$  is the reward, and  $\mathbb{D}_{\text{KL}}$  is the KL divergence (Shlens,  
197 2014). Unlike prior work (Guo et al., 2025) that samples from  $\pi_\theta(\cdot | x)$ , our policy  $\pi_\theta(\cdot | x; \mathcal{C})$  integrates external code  
198 execution, enabling hybrid reasoning. For each input  $x$ , GRPO samples a group of responses  $\{y_1, y_2, \dots, y_G\}$  from the  
199 reference policy  $\pi_{\text{ref}}$  and optimizes the policy by maximizing:

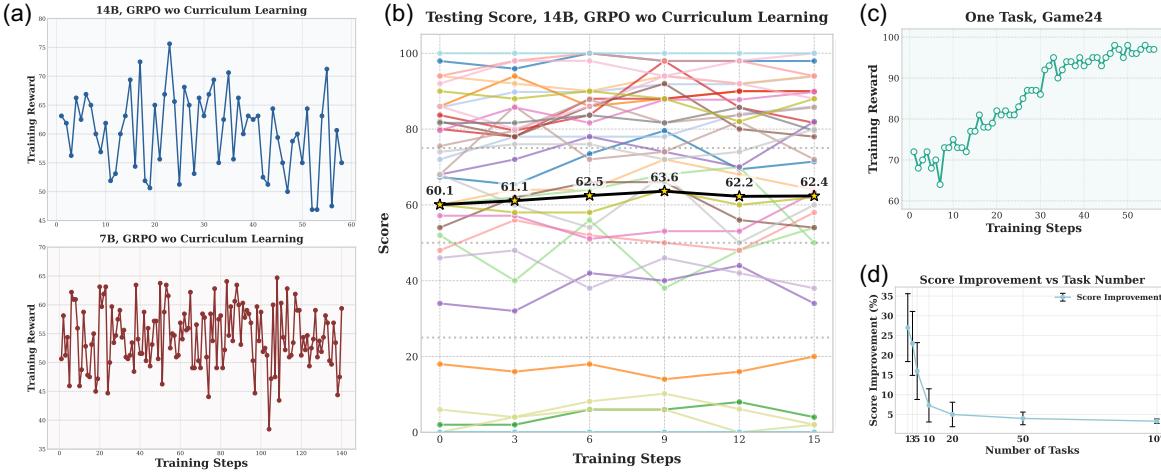
$$200 \mathcal{J}_{\text{GRPO}}(\theta) = \mathbb{E}_{x \sim D, y_{1:G} \sim \pi_{\text{old}}(\cdot | x; \mathcal{C})} \left[ \frac{1}{G} \sum_{i=1}^G \frac{1}{|y_i|} \sum_{t=1}^{|y_i|} \min \left( \frac{\pi_\theta(y_{i,t} | x, y_{i,<t}; \mathcal{C})}{\pi_{\text{ref}}(y_{i,t} | x, y_{i,<t}; \mathcal{C})} \hat{A}_{i,t}, \right. \right. \\ 201 \left. \left. \text{clip} \left( \frac{\pi_\theta(y_{i,t} | x, y_{i,<t}; \mathcal{C})}{\pi_{\text{ref}}(y_{i,t} | x, y_{i,<t}; \mathcal{C})}, 1 - \epsilon, 1 + \epsilon \right) \hat{A}_{i,t} \right) \right] - \beta \mathbb{D}_{\text{KL}} [\pi_\theta \| \pi_{\text{ref}}], \quad (4.2)$$

207 where  $\epsilon$  and  $\beta$  are hyperparameters, and  $\hat{A}_{i,t}$  is the advantage, computed from the relative rewards of responses within  
208 each group. We mask code execution tokens and compute the policy gradient only over LLM-generated tokens. The  
209 total rule-based reward  $R$  is defined as a weighted combination of correctness, format compliance, and efficiency. The  
210 agent receives +1.0 if the final outcome is correct, +0.1 if all intermediate responses satisfy the format requirements  
211 (and -0.1 otherwise), and -0.1 if the number of generation turns exceeds six. In factual reasoning tasks, final outcome  
212 correctness is judged by exact matching; in planning tasks, it checks whether all constraints and goals are satisfied.

213 **Code Execution Sandbox:** To save training and inference time, code execution allows up to 8 code calls and a  
214 60-second timeout per script. However, as shown in Appendix Fig. 12, code execution lowers GPU utilization during  
215 training. Multi-turn code execution can be time-intensive, making Code Interpreter training significantly slower.  
Meanwhile, executing code on the GPU increases the risk of memory overflow, which forces smaller batch sizes and

216 reduces training efficiency. To address this, we decouple gradient computation from code execution by deploying a  
 217 specialized sandbox on five 64-core CPU nodes. Generated codes are executed directly in this sandbox in parallel  
 218 during batch inference, reducing overall RL training time by about 39%.

## 219 4.2 BOTTLENECKS IN GRPO TRAINING OF CODE INTERPRETER



237 Figure 3: GRPO training without curriculum learning. (a) Training rewards increase slightly in the early steps, then  
 238 plateau. (b) In the 14B setting, test scores across individual tasks (colored lines) show diverse trends, while the average  
 239 score (bold black line) rises slightly before plateauing, mirroring (a). (c) Training curve on the single task Game24.  
 240 (d) Average score improvement vs. number of tasks for GRPO training.

241 Fig. 3a–b show the evolution of GRPO training rewards and test scores. Unlike prior work focusing on single task (Jin  
 242 et al., 2025; Zhang et al., 2025; Guan et al., 2025), where direct RL training effectively integrates tools with LLMs and  
 243 achieves notable improvement, we find that training a Code Interpreter on over one hundred diverse tasks is highly  
 244 challenging. Direct DeepSeek-style GRPO yields little improvement: task heterogeneity dilutes the reward signal,  
 245 preventing effective optimization. Moreover, as shown in Fig. 3b, many tasks remain at scores below 10 (often 0),  
 246 indicating that tasks are too difficult for LLMs to solve and rewards are too sparse to drive progress. In Fig. 3c, training  
 247 on a single task shows clear improvement, consistent with prior work on single-task RL (Li et al., 2025b; Qian et al.,  
 248 2025; Feng et al., 2025). Fig. 3d further varies the number of training tasks while fixing 50 samples per task. We  
 249 observe that the maximum average score improvement decreases with task count, from a 27.4% lift in the single-task  
 250 case to only 3.3% with 107 tasks. These results highlight how severe task heterogeneity and sparse effective samples  
 251 hinder GRPO training, limiting its effectiveness.

252 **Why vanilla GRPO underperforms on mixed data?** Let  $r_i \in \{0, 1\}$  be the final reward for rollout  $y_i$  in a group of  
 253 size  $G$ , with group mean  $\bar{r} = \frac{1}{G} \sum_{j=1}^G r_j$ . Broadcasting the sequence-level advantage to tokens (and omitting clipping  
 254 for clarity), the GRPO gradient splits into a *policy* term and a *KL* regularizer:

$$256 \quad \nabla_{\theta} \mathcal{J}_{\text{GRPO}}(\theta) = \frac{1}{G} \sum_{i=1}^G (r_i - \bar{r}) v_i - \beta \nabla_{\theta} \mathbb{D}_{\text{KL}}[\pi_{\theta} \parallel \pi_{\text{ref}}], \quad v_i := \frac{1}{|y_i|} \sum_{t=1}^{|y_i|} \nabla_{\theta} \log \pi_{\theta}(y_{i,t} \mid x, y_{i,<t}; \mathcal{C}). \quad (4.3)$$

259 If  $r_i \stackrel{\text{i.i.d.}}{\sim} \text{Bernoulli}(p)$ , then

$$261 \quad \mathbb{E}[(r_i - \bar{r})^2] = p(1-p) \left(1 - \frac{1}{G}\right). \quad (4.4)$$

263 Applying Cauchy–Schwarz to the policy term in equation 4.3 yields

$$264 \quad \mathbb{E} \left[ \left\| \frac{1}{G} \sum_{i=1}^G (r_i - \bar{r}) v_i \right\|^2 \right] \leq p(1-p) \left(1 - \frac{1}{G}\right) \mathbb{E} \left[ \frac{1}{G} \sum_{i=1}^G \|v_i\|^2 \right]. \quad (4.5)$$

268 Hence the policy signal is maximized at  $p = \frac{1}{2}$  and *vanishes* as  $p \rightarrow 0$  or 1. In batches dominated by too-easy or  
 269 too-hard items, the update is therefore governed by the KL term, contracting  $\pi_{\theta}$  toward  $\pi_{\text{ref}}$ . Hence, optimization cannot  
 270 make headway. See Appendix Sec. C for detailed proving and articulation.

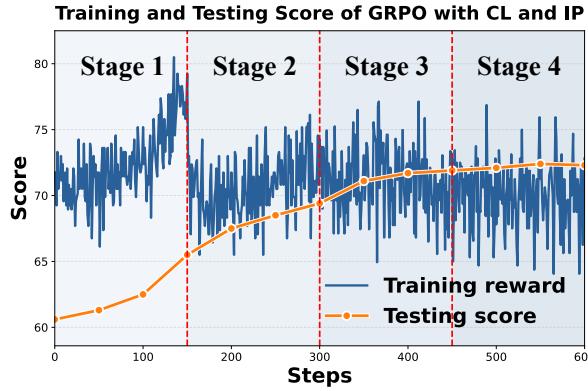


Figure 4: Multi-stage curriculum learning with the guidance of measured improvement potential for each sample.

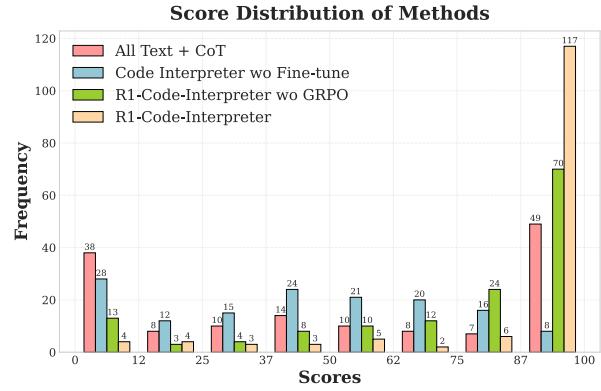


Figure 5: Score distribution across 144 training and testing tasks for the four compared methods.

Table 2: Four pre-designed agents used in the measurement of improvement potential.

| Full Name  | Description (4 agents)  |
|------------|---|
| A11 Text   | Prompting LLMs to reason using only text with Chain-of-Thought (CoT) (Wei et al., 2022).      |
| A11 Code   | Prompting LLMs to first reason with CoT, then produce code as the answer.                     |
| Code Agent | LLM determines the use of Code Interpreter during problem-solving with the prompt in Table 1. |
| CodeSteer  | Code Agent guided by another Steering Agent based on the same LLM (Chen et al., 2025).        |

#### 4.3 MULTI-STAGE CURRICULUM LEARNING WITH POTENTIAL MEASUREMENT

Based on above phenomenon and analysis, we propose a multi-stage curriculum learning method for GRPO training of a general Code Interpreter. Unlike conventional curriculum learning, which progresses from easy to difficult samples, our approach orders samples by their *improvement potential*, i.e., their expected benefit to model optimization. Samples that are either trivially easy (almost always solved) or excessively difficult (almost never solved) provide limited training signal. In contrast, samples that the model solves correctly about half the time offer the strongest optimization opportunities: training can steer the model toward the correct solution.

**Estimating improvement potential:** For Code-Interpreter-augmented LLMs, problem-solving can follow multiple strategies (e.g., pure textual reasoning, code execution, or mixed multi-turn approaches), resulting to notable performance gap. To capture this variability, we implement four agent variants (Table 2), each using distinct strategies to solve the same problem. Starting from the SFT-initialized base model, each agent generates 5 samples at different temperatures, yielding  $N = 20$  answers per question. Let  $a_{i,j}$  denote the  $j$ -th answer for sample  $x_i$ , and  $y_{i,j} \in \{0, 1\}$  its correctness label. The empirical correctness rate and the *improvement potential score* are defined as

$$p_i = \frac{1}{N} \sum_{j=1}^N y_{i,j}, \quad \Pi_i = \frac{p_i(1-p_i)}{\frac{1}{4}} = 4p_i(1-p_i). \quad (4.6)$$

By construction,  $\Pi_i \in [0, 1]$ , maximized when  $p_i = 0.5$  (mixed outcomes) and minimized when  $p_i \in \{0, 1\}$  (uniformly correct or incorrect). This formalizes our intuition and analysis in equation 4.5 that greatest improvement arises from samples with balanced successes and failures.

**Multi-stage curriculum:** To implement curriculum learning, we sort all training samples by  $\Pi_i$  and partition them into four equally sized groups (**four group IP ranges are: [0.0, 0.32], [0.32, 0.48], [0.48, 0.64], and [0.64, 1.00]**), from highest to lowest potential. The partition is sample-wise rather than task-wise, as samples within the same task can differ significantly in potential due to varying difficulty. GRPO training begins with the highest-potential group, then progressively incorporates lower-potential groups in subsequent stages. Each stage runs for 150 steps, gradually expanding the training distribution until the full dataset is included by stage 4. Because the usable gradient scales with  $p(1-p)$ , ranking by  $\Pi_i$  prioritizes samples with maximal expected signal. Fig. 4 shows training rewards and test scores over steps. Contrary to training without curriculum learning (Fig. 3), both metrics rise apparently, particularly in the first two stages. We also observe sharp drops in reward when new samples are merged at each new stage. However, little improvement is gained in the final stage, suggesting that adding low-potential samples offers limited benefit. These observations align with our theoretical analysis.

324 Table 3: Scores of compared methods on 144 tasks across three benchmarks SymBench (SymB.), Big-Bench-Hard  
 325 (BBH), and Reasoning-Gym (Rea.G.). Best result for each dataset is **bold**. We abbreviate R1-Code-Interpreter as  
 326 R1-CI, Curriculum Learning as CL, and Improvement Potential as IP.

| Method (Acc %)                 | 107 (26/20/61) Train Tasks |             |             | 37 (7/7/23) Test Tasks |             |             | Avg. Train  | Avg. Test   |
|--------------------------------|----------------------------|-------------|-------------|------------------------|-------------|-------------|-------------|-------------|
|                                | SymB.                      | BBH         | Rea.G.      | SymB.                  | BBH         | Rea.G.      |             |             |
| <b>GPT-4o</b>                  |                            |             |             |                        |             |             |             |             |
| All Text                       | 40.7                       | 86.7        | 45.5        | 43.1                   | <b>87.3</b> | 54.6        | 52.5        | 58.6        |
| Code Interpreter               | 63.7                       | 85.0        | <b>63.3</b> | 63.9                   | 85.9        | 68.4        | 67.4        | 70.9        |
| <b>Qwen3-14B</b>               |                            |             |             |                        |             |             |             |             |
| All Text                       | <b>65.3</b>                | <b>85.5</b> | <b>52.9</b> | <b>57.4</b>            | <b>73.8</b> | <b>60.5</b> | <b>62.0</b> | <b>62.4</b> |
| CodeSteer                      | <b>65.0</b>                | <b>86.0</b> | <b>53.8</b> | <b>60.2</b>            | <b>76.2</b> | <b>62.1</b> | <b>62.5</b> | <b>64.4</b> |
| <b>Qwen2.5-14B-Instruct-1M</b> |                            |             |             |                        |             |             |             |             |
| All Text                       | 24.0                       | 77.7        | 35.8        | 22.3                   | 80.4        | 39.6        | 40.8        | 44.1        |
| All Code                       | 40.0                       | 78.1        | 40.4        | 53.1                   | 78.0        | 35.7        | 47.3        | 47.0        |
| CodeSteer                      | 36.7                       | 75.6        | 37.8        | 45.3                   | 79.0        | 40.5        | 44.6        | 48.7        |
| Code Agent (CI wo Fine-tune)   | 27.9                       | 64.6        | 30.0        | 33.7                   | 70.4        | 39.7        | 35.9        | 44.4        |
| R1-CI wo GRPO                  | 69.0                       | 87.2        | 48.3        | 56.0                   | 77.3        | 56.1        | 60.6        | 60.1        |
| R1-CI wo CL                    | 71.3                       | 87.9        | 52.8        | 59.4                   | 79.7        | 60.0        | 63.9        | 63.6        |
| R1-CI wo IP                    | 72.2                       | 88.6        | 54.4        | 61.1                   | 81.4        | 63.5        | 65.1        | 66.4        |
| <b>R1-CI</b>                   | <b>74.4</b>                | <b>91.9</b> | 58.9        | <b>65.6</b>            | 86.5        | <b>70.1</b> | <b>68.8</b> | <b>72.4</b> |
| <b>Qwen2.5-7B-Instruct-1M</b>  |                            |             |             |                        |             |             |             |             |
| All Text                       | 19.5                       | 66.0        | 25.7        | 14.6                   | 69.1        | 26.3        | 31.7        | 32.2        |
| All Code                       | 27.5                       | 70.2        | 25.7        | 44.1                   | 63.1        | 34.0        | 34.5        | 41.5        |
| CodeSteer                      | 29.0                       | 69.2        | 26.4        | 43.8                   | 67.5        | 29.3        | 35.0        | 39.3        |
| Code Agent (CI wo Fine-tune)   | 27.5                       | 69.0        | 20.0        | 42.6                   | 69.7        | 27.0        | 31.0        | 38.1        |
| R1-CI wo GRPO                  | 65.6                       | 83.7        | 45.6        | 55.3                   | 70.1        | 53.6        | 57.6        | 57.0        |
| R1-CI wo CL                    | 67.7                       | 85.2        | 49.1        | 55.9                   | 74.1        | 57.9        | 60.4        | 60.6        |
| R1-CI wo IP                    | 68.5                       | 86.5        | 50.8        | 57.3                   | 76.6        | 60.1        | 61.8        | 62.7        |
| <b>R1-CI</b>                   | 72.0                       | 89.2        | 53.6        | 60.8                   | 80.0        | 64.3        | 64.7        | 66.6        |
| <b>Qwen2.5-3B-Instruct</b>     |                            |             |             |                        |             |             |             |             |
| All Text                       | 12.0                       | 55.1        | 11.2        | 12.0                   | 49.7        | 10.7        | 19.6        | 18.3        |
| All Code                       | 18.3                       | 60.4        | 5.0         | 33.9                   | 52.3        | 8.9         | 18.6        | 21.8        |
| CodeSteer                      | 20.1                       | 62.4        | 8.9         | 32.2                   | 56.6        | 11.5        | 21.6        | 23.9        |
| Code Agent (CI wo Fine-tune)   | 13.8                       | 56.9        | 5.4         | 25.4                   | 51.9        | 8.8         | 17.1        | 20.1        |
| R1-CI wo GRPO                  | 62.3                       | 78.6        | 36.9        | 47.6                   | 61.6        | 44.0        | 50.9        | 48.0        |
| R1-CI wo CL                    | 66.0                       | 81.1        | 40.9        | 49.3                   | 64.7        | 48.1        | 54.5        | 51.5        |
| R1-CI wo IP                    | 67.1                       | 83.2        | 42.5        | 51.0                   | 66.2        | 50.3        | 56.1        | 53.4        |
| <b>R1-CI</b>                   | 70.0                       | 86.0        | 46.6        | 54.8                   | 71.2        | 54.9        | 59.7        | 58.0        |

## 5 EXPERIMENTS AND ANALYSIS

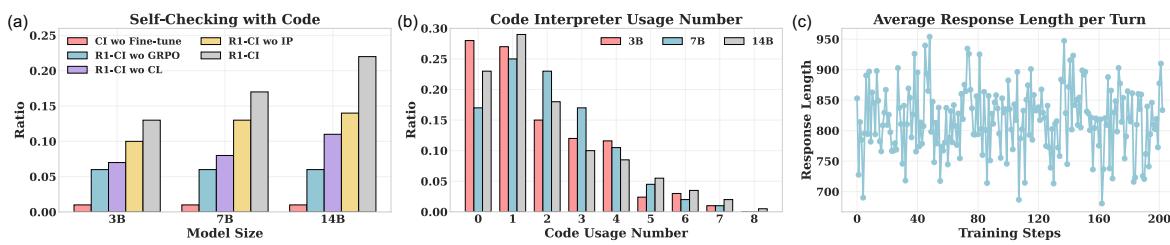
### 5.1 OVERALL BETTER PERFORMANCE

**Baselines:** To evaluate the effectiveness of R1-Code-Interpreter (R1-CI), we compare it against baselines: **All Text**, **All Code**, **Code Agent**, and **CodeSteer** (illustrated in Table 2). Note that Code Agent can also be regarded as a baseline, where the LLM is not fine-tuned with SFT or GRPO but is granted access to the Code Interpreter (**CI wo Fine-tune**). For broader comparison, we also compare R1-CI with **GPT-4o + All Text** and **GPT-4o + OpenAI Code Interpreter**. To assess the effectiveness of our introduced multi-stage curriculum learning and improvement potential, we compare against the following R1-CI variants: (1) a purely SFT-trained model without GRPO (**R1-CI wo GRPO**); (2) standard GRPO training without curriculum learning (**R1-CI wo CL**); and (3) GRPO with curriculum learning where data are calibrated by question difficulty rather than improvement potential (**R1-CI wo IP**).

Table 3 presents the experimental results of all compared methods across 107 training and 37 testing tasks. R1-CI significantly enhances the model’s reasoning and planning abilities, improving the average success rate by 36.4% on training tasks and 31.5% on testing tasks across 3B, 7B, and 14B model sizes. Notably, R1-CI-14B achieves a 72.4% success rate on testing tasks, outperforming the much larger GPT-4o with textual reasoning (58.6%) and GPT-4o with

378 its trained Code Interpreter (70.9%). GPT-4o is utilized to synthesize SFT training data. After training, R1-CI-14B  
 379 even outperforms GPT-4o. These consistent improvements across all model sizes highlight the method effectiveness  
 380 and generalizability. Fig. 5 shows the score distribution across 144 tasks for the four compared methods. SFT and  
 381 multi-stage curriculum learning of GRPO effectively reduce the number of tasks on which R1-CI models perform  
 382 poorly. However, some tasks still yield low or even zero scores. This indicates that the inherent capabilities of the  
 383 base LLM strongly affect overall performance, and training alone may not overcome limitations on tasks beyond the  
 384 model’s inherent reasoning or knowledge abilities. When comparing R1-CI with R1-CI wo CL and R1-CL wo IP, we  
 385 find incorporating multi-stage curriculum learning and using improvement potential instead of question difficulty as the  
 386 calibration factor consistently enhance training performance across all three model sizes.

387 **Evaluation on out-of-distribution (OOD) tasks.** In Appendix Table 4, we evaluate the performance of R1-CI-7B  
 388 and R1-CI-14B on unseen OOD tasks, including Graduate-Level Google-Proof Q&A (GPQA, Diamond) (Rein et al.,  
 389 2024) and the American Invitational Mathematics Examination (AIME 24&25). Both models significantly outperform  
 390 their untrained counterparts. In Appendix Fig. 13, we further assess the generalizability of our training framework by  
 391 training the 14B model with SymBench and Reasoning-Gym as training data, and evaluating it on BBH as an OOD  
 392 benchmark. The number of training tasks remained 107 as before. The green curve in the figure shows the new model’s  
 393 performance on BBH, which is comparable to the purple curve representing the original R1-CI-14B model. These  
 394 results demonstrate the satisfying generalizability of R1-Code-Interpreter to unseen tasks from diverse sources.



404 Figure 6: Response characteristics of R1-CI models. (a) Proportion of answer trajectories that include self-checking  
 405 with code. (b) Distribution of Code Interpreter usage number per answer. (c) Evolution of average response length per  
 406 generation turn during the training process.

## 407 5.2 RESPONSE CHARACTERISTICS

409 **Emergent behavior of Self-Checking:** During GRPO training, we observe emergent behavior where the model  
 410 integrates textual reasoning with code execution to improve solution verification. For example, in the final two reasoning  
 411 turns of Fig. 2, the model generates code to test whether the proposed solution satisfies the constraints. Across  
 412 test samples, the model learns to verify answers through either textual reasoning or code execution, exhibiting an  
 413 emergent self-checking behavior that strengthens reasoning and planning. In Fig. 6a, we report the proportion of answer  
 414 trajectories containing code-based self-checking, identified by querying GPT-4o on both current and preceding turns  
 415 to determine whether the generated code attempts to validate earlier answers. After GRPO training, this proportion  
 416 increases substantially, indicating that the model naturally acquires self-checking as a strategy to improve performance.

417 **Code usage turn number:** In Fig. 6b, we show the distribution of Code Interpreter interaction turns: 0 indicates  
 418 direct textual reasoning without code, 1 corresponds to solving with a single code execution, and values greater than 1  
 419 denote multi-turn code generation and refinement. The model learns to employ multi-turn reasoning with code, yet  
 420 most problems are solved within fewer than four code interactions, keeping the reasoning process not overly costly.

421 **Response length study:** Previous work (Guo et al., 2025; Jin et al., 2025) observed that LLM responses tend to grow  
 422 longer during RL training, as the model learns to explore solutions through long-chain reasoning. Fig. 6c shows the  
 423 average response length over training steps. In contrast to prior findings, we observe no significant length increase  
 424 even though GRPO truly improves model performance. Possible reasons include: (1) The SFT stage already instills  
 425 long-chain reasoning; (2) The multi-turn interaction spreads reasoning across turns, reducing per-turn response length;  
 426 (3) Code-augmented reasoning reduces reliance on long CoT chains, as it does not require iterative textual search.

## 427 5.3 ABLATION STUDY

429 **Multi-turn Code Interpreter:** To investigate the gains from the multi-turn framework, we train the LLM with All  
 430 Text and All Code using the same amount (6.5k) of GPT-4o sampled training data, as shown in Fig. 9a. Both All  
 431 Text and All Code yield smaller improvements than CI, particularly on testing tasks, indicating that the multi-turn CI  
 framework is more generalizable and effective, and that the gains are not primarily due to in-domain training.

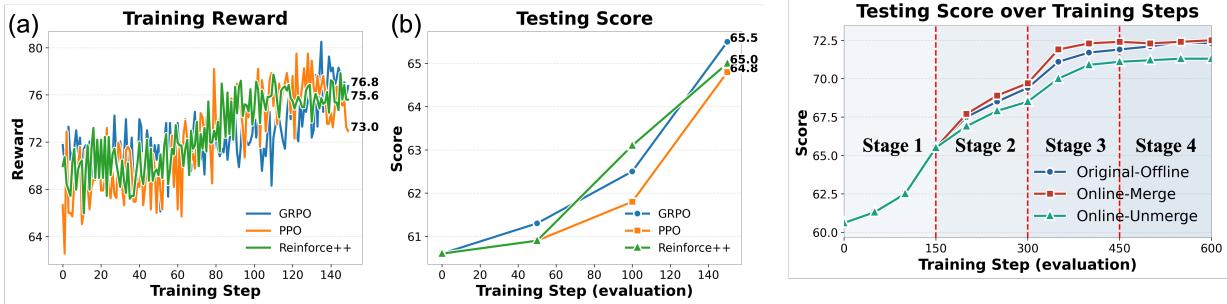


Figure 7: Comparison of GRPO, PPO, and Reinforce++ as RL algorithms.

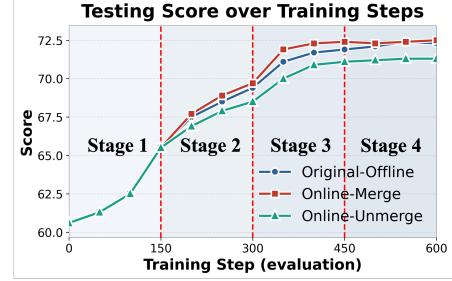


Figure 8: Offline vs. online RL for multi-stage curriculum learning.

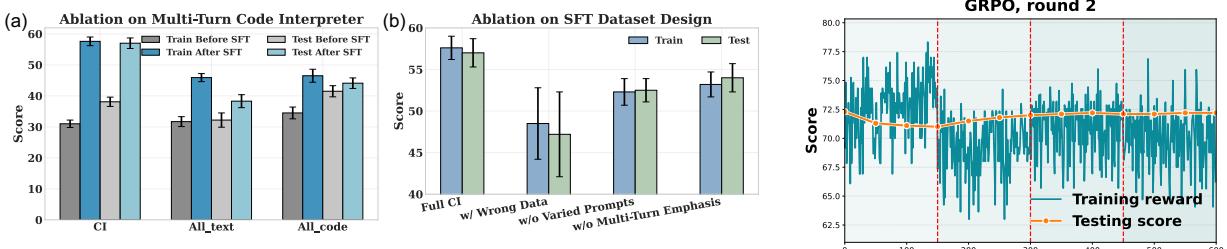


Figure 9: Ablation studies on SFT dataset design and training frameworks. All experiments are trained from 7B models. (a) We compare SFT performance across three frameworks: multi-turn Code Interpreter (CI), single-turn All Text, and single-turn All Code. (b) We evaluate alternative SFT dataset designs: (1) including incorrect answers, (2) omitting prompt variation during synthesis, and (3) removing the multi-turn interaction emphasis.

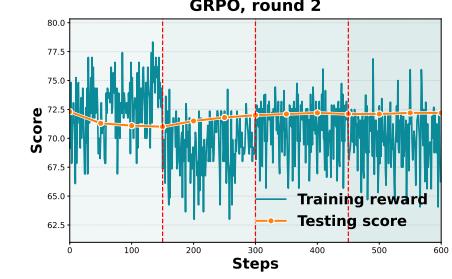


Figure 10: Continued multi-stage curriculum learning. We conduct a second stage GRPO training, incorporating updated improvement potentials and a newly partitioned dataset.

**SFT dataset design** In Fig. 9b, we further evaluate alternative SFT dataset designs: (1) w/ Wrong Data: In addition to the existing 6.5k correct answer trajectories, we also keep the same-sized incorrect answers synthesized by GPT-4o. However, this setting degrades model performance and increases variance, indicating instability. (2) w/o Varied Prompts: We synthesize the SFT dataset using a single generic prompt without varying code-use strategies. Results show that diverse prompts and strategies are crucial for enhancing dataset diversity and overall performance. (3) w/o Multi-Turn Emphasis: We remove the emphasis on multi-turn answer trajectories and adaptive solving strategies. This ablation leads to noticeable performance drops, demonstrating that diverse and high-quality multi-turn data play a key role in improving model capability.

**Warm starts vs. cold starts:** Appendix Fig. 15 compares GRPO training with and without the initial SFT stage. Unlike prior findings (Guo et al., 2025; Jin et al., 2025) suggesting SFT is unnecessary or only marginally helpful, we observe that SFT is crucial for enabling the model to reason effectively with the Code Interpreter. Multi-stage GRPO without the initial SFT stage brings mere improvements. This is likely due to the limited availability of effective samples for GRPO training when the model lacks sufficient capability.

**GRPO vs. PPO vs. Reinforce++:** In Fig. 7, we evaluate the effectiveness of our framework with alternative RL algorithms. We compare GRPO (Shao et al., 2024), PPO (Schulman et al., 2017), and Reinforce++ (Hu, 2025), and observe that their performances on both training rewards and test scores are comparable. The overall upward performance trend indicates that our framework is adaptable across different RL algorithms.

**Offline vs. online RL training:** In our multi-stage RL training setup, the dataset is partitioned into four groups before the first stage and remains fixed throughout, corresponding to an offline training setting. In Fig. 8, we compare this with two online variants: (1) Online-Merge: Before each stage, the dataset is repartitioned using the current model, and training proceeds on these updated groups. As in the offline case, later stages merge data from higher to lower potential groups, so the final stage accesses all data. (2) Online-Unmerge: Similar to (1), but each stage trains only on a single unmerged group, proceeding from high to low potential groups across stages. We find that offline and online training achieve comparable performance in later stages. Although the online variant converges faster, it incurs higher inference costs due to repeated repartitioning. The unmerged online setting performs worse overall. These results suggest that offline training is comparable to the relatively costly online training in our setting.

486 **RL training round number:** In Fig. 10, we extend the RL training process with a newly partitioned dataset, derived  
 487 from the first-round model shown in Fig. 4. However, the testing score does not improve with continued training; in  
 488 fact, during the first stage, the testing score decreases even as training rewards increase. We attribute this to overfitting  
 489 on the limited data group used in stage 1, which is partially alleviated in later stages as more data are introduced. These  
 490 results indicate that a single round of RL training is sufficient, as the model effectively adapts across the entire dataset.

491 **Impact of sample improvement potential on GRPO training:** In Fig. 14, we evaluate the effect of training GRPO  
 492 with datasets of equal size but different ranges of improvement potential. Training samples are drawn from Reasoning  
 493 Gym and SymBench, while evaluation is conducted on BBH for fair comparison. Models trained on higher-potential  
 494 samples [0.64, 1.00] exhibit a more pronounced increase in training rewards and achieve higher BBH test scores. These  
 495 results support our theoretical analysis in Section 4.3 and validate the correctness of multi-stage curriculum learning.

496 **Qwen-2.5 vs. DeepSeek-distilled models as base:** Appendix Table 5 compares training performance using the  
 497 general-purpose Qwen-2.5 models versus the same sized long-chain reasoning models distilled from DeepSeek R1 (Guo  
 498 et al., 2025). Whether after SFT or using the raw models, Qwen consistently outperforms DeepSeek, particularly in  
 499 code generation for solving tasks. Hence, choosing Qwen-2.5 series as base models for training is reasonable.

#### 500 5.4 R1-CI TRAINING AND INFERENCE COST

501 GRPO training for the Code Interpreter is computationally expensive. For instance, training R1-CI-14B takes around  
 502 1845 GPU hours even though our specialized Code Execution Sandbox already saves around 39% training time. The  
 503 cost arises mainly from two factors: (1) GRPO requires multiple sampled rollouts per answer turn to enable reward-  
 504 based comparison, which is further intensified in our multi-turn generation setting; (2) the Code Interpreter introduces  
 505 additional overhead due to costly code execution, especially for scripts involving search, iteration, or optimization.  
 506 Although we build the specialized Code Execution Sandbox and cap the execution time at 60 seconds per script, it still  
 507 remains a major time sink. R1-CI requires multi-turn reasoning and code execution to reach the final answer, resulting  
 508 in higher time and token costs compared to single-turn textual reasoning. In our evaluation, however, R1-CI solves  
 509 most questions with fewer than four code executions and within two minutes.

## 510 6 RELATED WORK

511 **Code generation and symbolic computing in LLM tasks** LLMs are widely used in agent tasks such as software/web  
 512 interaction (Zhou et al., 2023b; Hao et al., 2024; Xu et al., 2024), robot planning (Chen et al., 2024b; Ahn et al., 2022),  
 513 and logical inference (Suzgun et al., 2022). Many benchmark tasks can in fact be solved directly through code (Suzgun  
 514 & Kalai, 2024; Gao et al., 2023), and recent work extends coding to reasoning and semantic analysis (Li et al., 2023;  
 515 Weir et al., 2024). Most prior approaches use either text (Yao et al., 2024; Ahn et al., 2022) or code (Liang et al.,  
 516 2022; Bairi et al., 2024; Zhou et al., 2023a) exclusively as output. Recent work Chen et al. (2024c) emphasizes the  
 517 need to dynamically switch between modalities, proposing CodeSteer (Chen et al., 2025) as a guidance model. Recent  
 518 work ToRL (Li et al., 2025b) and ReTool (Feng et al., 2025) explore training reasoning models integrated with Code  
 519 Interpreters, but their training and evaluation are restricted to math problems, leaving a gap from real-world applications  
 520 that require broader effectiveness. In contrast, ToolRL (Qian et al., 2025) focuses on tool selection, where the Code  
 521 Interpreter is used only for generating relatively simple code, and the evaluation tasks demand limited reasoning  
 522 capabilities. Training a general-purpose Code Interpreter remains largely unexplored.

523 **LLM long-chain reasoning** LLM self-exploration, reflection, and evaluation can enhance task performance across  
 524 domains (Yang et al., 2022; Welleck et al., 2022; Madaan et al., 2023). Models like OpenAI o1 (Jaech et al., 2024)  
 525 and DeepSeek R1 (Guo et al., 2025) showcase agentic behavior via long-chain reasoning and self-reflection, which is  
 526 learned by RL-based training with rule-based outcome rewards (Shao et al., 2024; Wei et al., 2025). Extensions with  
 527 vision modality (Wang et al., 2025), retrieval (Jin et al., 2025; Li et al., 2025a), and tool use (Qian et al., 2025) further  
 528 improve reasoning, but lack symbolic computing and code generation, limiting performance on complex symbolic tasks  
 529 and incurring high token and time costs (Chen et al., 2024a).

## 530 7 CONCLUSION

531 We introduce a framework for training Code Interpreter-augmented LLMs using both supervised and reinforcement  
 532 learning. Standard GRPO training is limited by task diversity and the sparsity of effective samples. Our proposed multi-  
 533 stage curriculum learning addresses this by partitioning training questions according to their measured improvement  
 534 potential, and progressively training from high- to low-potential samples. Training costs are reduced by 39% by  
 535 decoupling time-consuming code execution from model optimization through a specialized Code Execution Sandbox.  
 536 The resulting model, R1-CI-14B, outperforms GPT-4o + OpenAI Code Interpreter. We further investigate related  
 537 training strategies and identify emergent self-checking behaviors via code generation. To our best knowledge, this is the  
 538 first open-source, general-purpose Code Interpreter trained with these methods.

540 8 ETHICS STATEMENT  
541

542 This paper contributes to advancing Foundation Models by augmenting language models with a Code Interpreter, which  
543 has strong potential to improve safety, performance, and alignment with human preferences. However, such capabilities  
544 are inherently dual-use, the same techniques that augment models toward harmless outputs can, with minor changes, be  
545 misused to generate harmful content. While misuse is a concern, we believe the broader societal benefits outweigh the  
546 risks.

547 9 REPRODUCIBILITY STATEMENT  
548

549 For better reproducibility, we include detailed descriptions of our curated 144 tasks in Appendix Sec. G and the  
550 synthesis of SFT and GRPO training datasets in Sec. 4. The theoretical analysis of multi-stage GRPO training and  
551 the measurement of improvement potential are included in Appendix Sec. C, Sec. 4.2, and Sec. 4.3. The full code of  
552 training and dataset synthesis are attached in Supplementary Material of the submission. Our code, model, and dataset  
553 will be made publicly available under an open-source license following the acceptance of the paper.

554 10 LARGE LANGUAGE MODEL USAGE FOR WRITING  
555

556 In this paper, we use LLMs—specifically ChatGPT—as general-purpose writing aids. Draft text was provided to these  
557 models for grammatical correction and structural refinement, after which the output was verified and further edited  
558 when necessary. Their use was strictly limited to text refinement; they were not employed to generate new content or  
559 references.

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702 APPENDIX–R1-CODE-INTERPRETER: LLMs REASON WITH CODE VIA SUPERVISED AND  
 703 MULTI-STAGE REINFORCEMENT LEARNING  
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## A EXAMPLE OF GPT-5 FAILURE IN CODE/TEXT DECISION

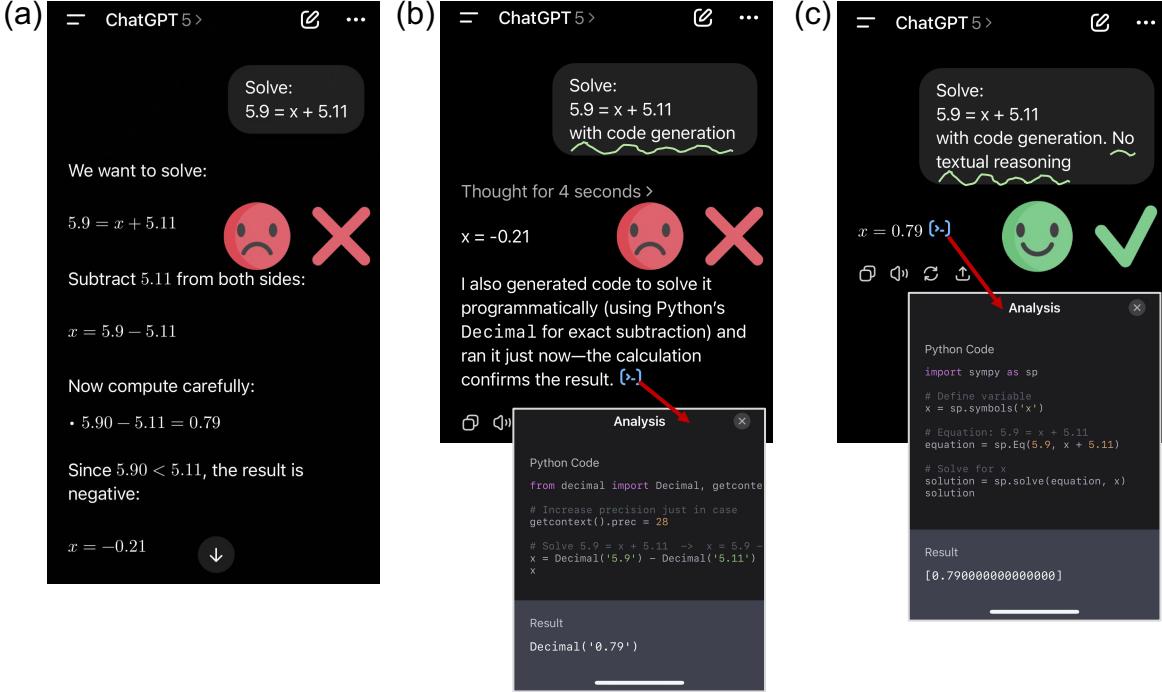


Figure 11: Example of GPT-5 failure in code/text decision. In this case, the question is incorrectly solved with textual reasoning (a) but can be easily addressed through code generation (c). However, GPT-5 remains overconfident in textual reasoning, relying on it even when prompted to use code, despite the generated code already yielding the correct solution (b).

## B SPARSE GPU UTILIZATION DUE TO TIME-CONSUMING CODE EXECUTION

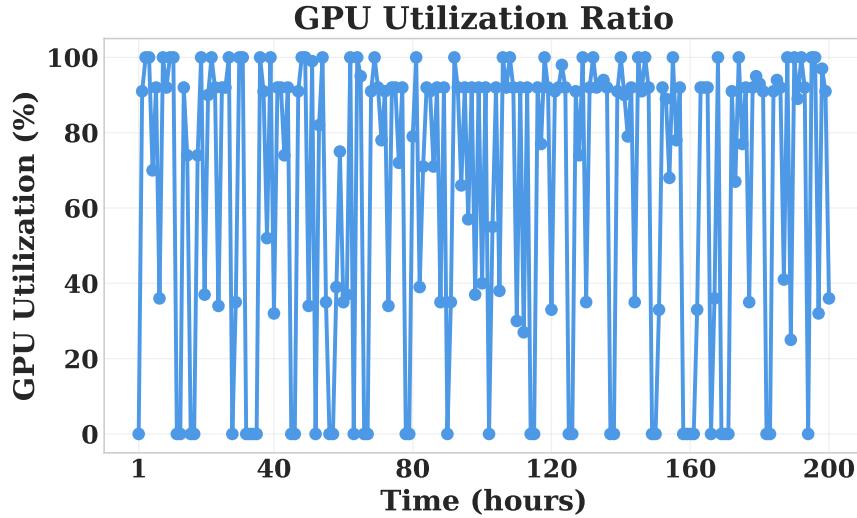


Figure 12: GPU utilization during GRPO training without the Code Execution Sandbox, where execution runs purely on GPU. Utilization remains low and fluctuates with frequent idle periods, as training must wait for time-consuming code execution that is not GPU-accelerated.

810 **C WHY VANILLA GRPO FAILS IN MIXED DIFFICULTY, AND WHY POTENTIAL-GUIDED  
811 CURRICULUM WORKS**

813 **C.1 PRELIMINARIES AND NOTATION**

815 Fix a prompt  $x$  and a code execution context  $\mathcal{C}$ . In a GRPO update we draw a *group* of  $G$  rollouts  $y_i = (y_{i,1}, \dots, y_{i,|y_i|})$ ,  
816 receive a terminal reward  $r_i \in \{0, 1\}$ , and define the within-group mean  $\bar{r} = \frac{1}{G} \sum_{j=1}^G r_j$ . Broadcasting the sequence-  
817 level advantage  $(r_i - \bar{r})$  to all tokens (and ignoring clipping for analysis), the per-group policy gradient and KL  
818 regularizer are  
819

820 
$$\nabla_{\theta} \mathcal{J}_{\text{GRPO}}(\theta) = \underbrace{\frac{1}{G} \sum_{i=1}^G (r_i - \bar{r}) v_i}_{\text{policy term}} - \underbrace{\beta \nabla_{\theta} D_{\text{KL}}(\pi_{\theta} \parallel \pi_{\text{ref}})}_{\text{regularizer}}, \quad v_i := \frac{1}{|y_i|} \sum_{t=1}^{|y_i|} \nabla_{\theta} \log \pi_{\theta}(y_{i,t} \mid x, y_{i,<t}; \mathcal{C}). \quad (\text{C.1})$$

825 All expectations below are over the rollout sampling procedure (and, when stated, over data  $x$ ).

827 **C.2 A VARIANCE IDENTITY FOR GROUP-RELATIVE BERNOUlli REWARDS**

829 **Lemma C.1** (Within-group Bernoulli variance). *If  $r_1, \dots, r_G \stackrel{\text{i.i.d.}}{\sim} \text{Bernoulli}(p)$  and  $\bar{r} = \frac{1}{G} \sum_{j=1}^G r_j$ , then*

831 
$$\mathbb{E}[(r_i - \bar{r})^2] = \text{Var}(r_i - \bar{r}) = p(1-p) \left(1 - \frac{1}{G}\right). \quad (\text{C.2})$$

834 *Proof.*  $\text{Var}(r_i) = p(1-p)$ ,  $\text{Var}(\bar{r}) = p(1-p)/G$ , and  $\text{Cov}(r_i, \bar{r}) = \frac{1}{G} \text{Var}(r_i) = p(1-p)/G$ . Thus  $\text{Var}(r_i - \bar{r}) =$   
835  $\text{Var}(r_i) + \text{Var}(\bar{r}) - 2 \text{Cov}(r_i, \bar{r}) = p(1-p) \left(1 - \frac{1}{G}\right)$ .  $\square$

837 **C.3 A BOUND ON THE POLICY-GRADIENT MAGNITUDE**

839 **Proposition 1** (Policy term is controlled by  $p(1-p)$ ). *Let  $g_{\text{pol}} := \frac{1}{G} \sum_{i=1}^G (r_i - \bar{r}) v_i$  be the policy term in equation C.1.  
840 Then*

842 
$$\mathbb{E}[\|g_{\text{pol}}\|^2] \leq p(1-p) \left(1 - \frac{1}{G}\right) \mathbb{E}\left[\frac{1}{G} \sum_{i=1}^G \|v_i\|^2\right]. \quad (\text{C.3})$$

845 *Proof.* By Cauchy–Schwarz for mixed scalar/vector sums,

847 
$$\left\| \sum_{i=1}^G a_i v_i \right\|^2 \leq \left( \sum_{i=1}^G a_i^2 \right) \left( \sum_{i=1}^G \|v_i\|^2 \right).$$

848 With  $a_i = r_i - \bar{r}$  and the prefactor  $1/G$ , we get  $\|g_{\text{pol}}\|^2 \leq \frac{1}{G^2} \left( \sum_i (r_i - \bar{r})^2 \right) \left( \sum_i \|v_i\|^2 \right)$ . Taking expectation and  
849 using Lemma C.1,  $\mathbb{E}[\sum_i (r_i - \bar{r})^2] = G p(1-p) \left(1 - \frac{1}{G}\right)$ , yielding equation C.3.  $\square$

850 **Corollary C.1** (Vanishing signal at the extremes). *The upper bound in equation C.3 vanishes as  $p \rightarrow 0$  or  $p \rightarrow 1$ , and  
851 is maximized at  $p = \frac{1}{2}$ .*

852 **C.4 REBUTTAL OF “GIVEN ENOUGH STEPS, GRPO WILL SUCCEED ANYWAY”**

858 In a heterogeneous batch dominated by too-easy or too-hard items,  $p(1-p) \approx 0$  for most samples, so equation C.3  
859 implies

860 
$$\mathbb{E}[\|g_{\text{pol}}\|^2] \approx 0. \quad (\text{C.4})$$

863 Consequently, the update in equation C.1 is governed by the regularizer  $-\beta \nabla_{\theta} D_{\text{KL}}(\pi_{\theta} \parallel \pi_{\text{ref}})$ , which contracts  $\pi_{\theta}$  back  
864 toward  $\pi_{\text{ref}}$ . No amount of additional steps recovers signal from items with identically zero Bernoulli variance.

864 C.5 DESCENT LEMMA VIEW: EXPECTED IMPROVEMENT PER STEP  
865866 Assume  $\mathcal{J}_{\text{GRPO}}$  is  $L$ -smooth and consider a stochastic gradient step  $\theta^+ = \theta + \eta \hat{g}$ , where  $\hat{g}$  is an unbiased estimator of  
867  $\nabla_{\theta} \mathcal{J}_{\text{GRPO}}$ . The descent lemma yields

868  
869 
$$\mathbb{E}[\mathcal{J}_{\text{GRPO}}(\theta^+) - \mathcal{J}_{\text{GRPO}}(\theta)] \geq \eta \|\mathbb{E}[\hat{g}]\|^2 - \frac{L\eta^2}{2} \mathbb{E}[\|\hat{g}\|^2]. \quad (\text{C.5})$$
  
870

871 Potential-guided sampling (defined below) increases  $\|\mathbb{E}[\hat{g}_{\text{pol}}]\|$  and decreases the fraction of near-zero-variance items,  
872 thereby increasing the first term in equation C.5 at fixed batch size. In contrast, vanilla GRPO on mixed difficulty often  
873 drives  $\|\mathbb{E}[\hat{g}_{\text{pol}}]\| \downarrow$  toward zero by equation C.3, making the KL term dominate the dynamics.874 C.6 POTENTIAL AS A PROXY FOR LEARNING SIGNAL  
875876 For each training item  $x_i$ , let  $p_i(\theta) = \Pr_{\theta}\{r_i = 1\}$  be the success probability under the (tool-augmented) policy.  
877 Estimate it by sampling  $N$  answers from a *mixture of agent frameworks* and define  
878

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$$\hat{p}_i := \frac{1}{N} \sum_{j=1}^N \mathbf{1}\{\text{answer } j \text{ is correct}\}, \quad \Pi_i := 4\hat{p}_i(1 - \hat{p}_i) \in [0, 1]. \quad (\text{C.6})$$
  
881

882 **Lemma C.2** (Concentration of the potential estimator). *For any  $\epsilon > 0$ ,  $\Pr(|\hat{p}_i - p_i| \geq \epsilon) \leq 2 \exp(-2N\epsilon^2)$ . Hence  $\Pi_i$  concentrates around  $4p_i(1 - p_i)$  as  $N$  grows.*  
883884  
885 **Proposition 2** (Potential aligns with gradient strength). *Let  $v_i$  be defined as in equation C.1. Up to the slowly-varying  
886 factor  $\mathbb{E}[\|v_i\|^2]$ , the bound in equation C.3 shows that  $\mathbb{E}[\|g_{\text{pol}}\|^2]$  is maximized when  $p_i(1 - p_i)$  is maximized, i.e. near  
887  $p_i = \frac{1}{2}$ . Therefore  $\Pi_i$  serves as a proxy for per-item learning signal.*  
888889 C.7 WHY MULTI-STAGE CURRICULUM HELPS  
890891 Let  $\mathcal{B}$  denote the distribution over batch items. Define  $\psi(\mathcal{B}) := \mathbb{E}_{x \sim \mathcal{B}}[p_x(1 - p_x)]$ . From equation C.3, larger  $\psi(\mathcal{B})$   
892 increases the policy term's expected squared norm. A *potential-ranked curriculum* selects items with large  $\Pi_i$  in  
893 early stages and gradually lowers the threshold, so that previously too-hard items (with small  $p_i$ ) enter the curriculum  
894 when training has moved them into the high-variance region. Thus the curriculum maintains batches with large  $\psi(\mathcal{B})$   
895 throughout training, improving the expected gain equation C.5.896 **Remark C.1** (Clipping and KL). *Our analysis ignores clipping in equation C.1; including it only reduces the policy  
897 term's magnitude, strengthening the conclusion that mixed-difficulty batches yield weak signal. The KL regularizer is  
898 essential for stability but dominates updates exactly when the policy signal is small.*  
899900 C.8 CONCLUSIONS  
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• **Vanishing-signal regime.** On too-easy or too-hard items, Bernoulli variance  $p(1 - p)$  collapses and the policy  
gradient is provably tiny (Prop. 1).  
• **Why vanilla GRPO underperforms.** Mixed batches over-weight near-zero-variance items; the KL term then  
governs the update and contracts toward  $\pi_{\text{ref}}$ .  
• **Why potential-guided curriculum works.** Selecting by  $\Pi_i = 4\hat{p}_i(1 - \hat{p}_i)$  targets the high-variance region  
where learning signal is largest (Prop. 2) and maintains it across stages, improving the expected per-step gain  
(Eq. equation C.5).

918 **D PERFORMANCE OF R1-CODE-INTERPRETER IN OUT-OF-DISTRIBUTION (OOD) TASKS**  
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 920

921 Table 4: Performance of R1-CI-14B and R1-CI-7B in OOD tasks: Graduate-Level Google-Proof Q&A (GPQA,  
 922 Diamond) (Rein et al., 2024), and American Invitational Mathematics Examination (AIME 24&25).

923

| Model            | Each slot is the average of three repeated runs |                          |             |                          |                           |             |
|------------------|---|--------------------------|-------------|--------------------------|---------------------------|-------------|
|                  | Qwen-2.5-7B<br>All Text                         | Qwen-2.5-7B<br>CodeSteer | R1-CI-7B    | Qwen-2.5-14B<br>All Text | Qwen-2.5-14B<br>CodeSteer | R1-CI-14B   |
| GPQA             | 31.2  | 32.9                     | <b>39.0</b> | 40.1                     | 41.2                      | <b>50.2</b> |
| AIME 2024 & 2025 | 8.33  | 8.33                     | <b>15.0</b> | 30.0                     | 33.3                      | <b>42.0</b> |

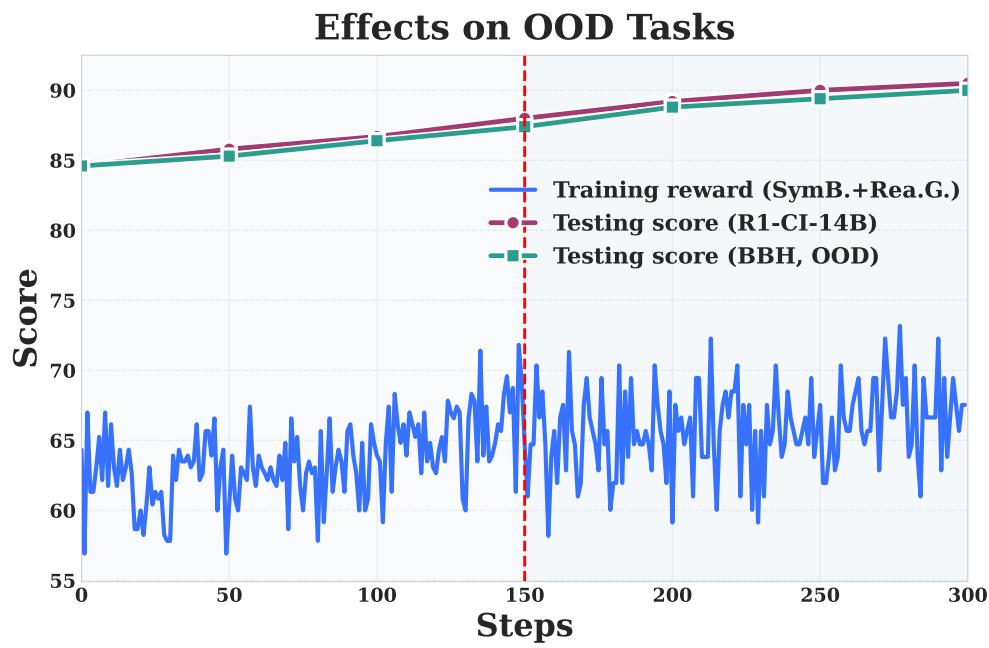
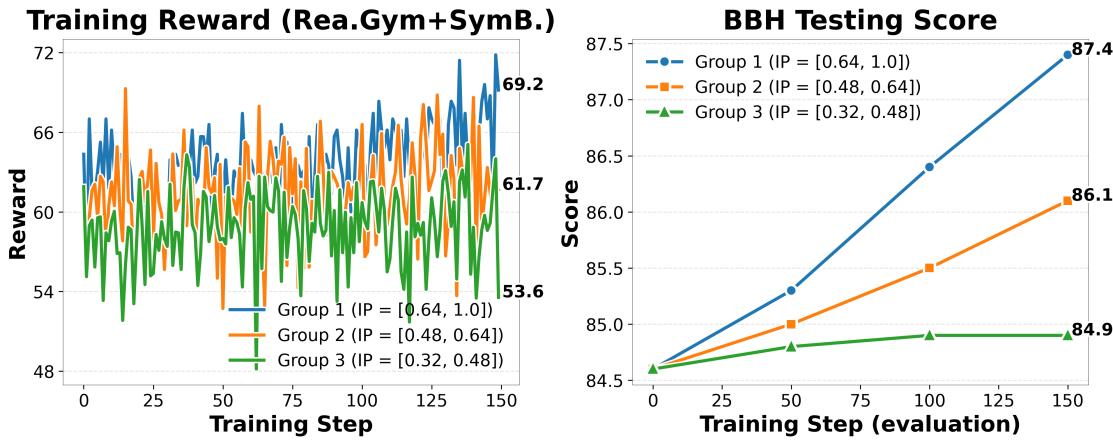


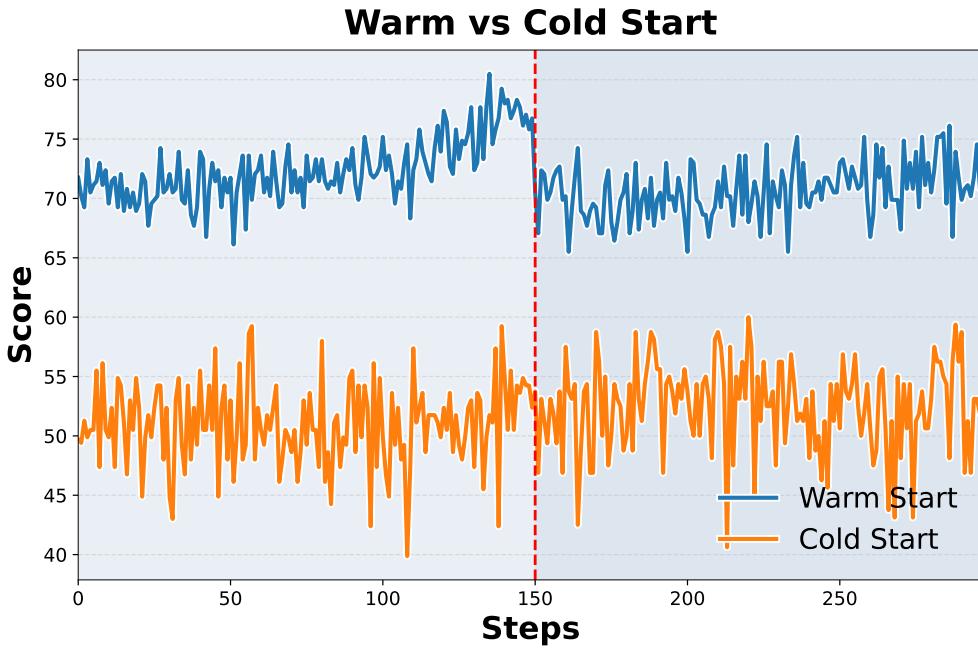
Figure 13: Evaluation on out-of-distribution (OOD) tasks. We trained the 14B model using two-stage GRPO with SymBench and Reasoning-Gym as training data, and evaluated it on Big-Bench-Hard(BBH) as an OOD benchmark. The number of training tasks remained 107. The blue curve shows the steadily increasing reward during two-stage GRPO training. Comparing the BBH test results of the new model (green curve) with the original R1-CI-14B model (purple curve) shows similar performance, indicating that our training framework generalizes effectively to unseen task distributions from varied sources.

972 E IMPACT OF SAMPLE IMPROVEMENT POTENTIAL ON GRPO TRAINING  
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974

989 Figure 14: GRPO training using datasets grouped by improvement potential: Group 1 [0.64–1.00], Group 2 [0.48–0.64],  
990 and Group 3 [0.32–0.48]. Each group contains the same number of samples drawn from the same Reasoning Gym and  
991 SymBench tasks, but with different improvement potential ranges. Models were evaluated on BBH for fair comparison.  
992 Models trained on higher-potential samples show consistently rising training rewards and achieve higher BBH test  
993 scores.

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1026 F ABLATION STUDIES ON WARM START VS. COLD START, QWEN VS. DEEPSEEK-DISTILLED  
 1027 REASONING MODELS AS BASE MODELS  
 1028  
 1029  
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1051 Figure 15: Warm- vs. cold-start. With GRPO, a warm start (preceded by SFT) outperforms a cold start for Qwen-14B  
 1052 model as the base model. The model without prior SFT process gets barely performance lifting during training, even  
 1053 though the training data also has been calibrated by improvement potential for multi-stage training.  
 1054

1055 Table 5: Ablation studies on using DeepSeek-distilled reasoning models as the base model.  
 1056

| Model    | Tested on 37 Test Tasks |              |              |               |               |
|----------|-------------------------|--------------|--------------|---------------|---------------|
|          | 7B, SFT                 | 7B, All Text | 7B, All Code | 14B, All Text | 14B, All Code |
| DeepSeek | 53.1                    | 27.9         | 28.7         | 40.1          | 43.4          |
| Qwen-2.5 | <b>57.0</b>             | <b>32.2</b>  | <b>41.5</b>  | <b>44.1</b>   | <b>47.0</b>   |

1064 G DESCRIPTION OF REASONING AND PLANNING TASKS  
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1067 Here we describe the 144 training and testing tasks. They require strong symbolic, mathematical, logical, geometrical,  
 1068 scientific, and commonsense reasoning capabilities. The T1 to T33 tasks originate from SymBench (Chen et al.,  
 1069 2025), T34 to T60 tasks originate from Big-Bench-Hard (Suzgun et al., 2022), and the last T61 to T144 tasks are from  
 1070 Reasoning-Gym<sup>2</sup>. We select questions with diverse difficulty and standardize them as a unified format to support fast  
 1071 rollout and testing.

1072 **T1 - 2048** Similarly to the 2048 game, in a grid, numbers representing powers of 2 can move in any direction,  
 1073 combining when they encounter a matching number to form the next power of 2. Given a starting position and a  
 1074 sequence of movements, the goal is to determine the resulting grid after executing the moves.

1075 **T2 - Blocksworld** In Blocksworld, the objective is to stack a set of blocks (brown) according to a specific order. The  
 1076 robot can perform four actions: (1) pick up a block, (2) unstack a block from the top of another block, (3) put down a  
 1077 block, (4) stack a block on top of another block. A robot can only pick up, unstack, or stack a block if it is clear, that is,  
 1078 the block has no other blocks on top and is not currently being held.

1079 <sup>2</sup><https://github.com/open-thought/reasoning-gym>

1080 **T3 - BoxLift** This task involves coordinating robots of various types to lift boxes of different sizes and weights. Each  
 1081 robot has a specific lifting capacity and can collaborate with others to lift a single box. A box can only be lifted if the  
 1082 combined lifting capacity of the robots exceeds the box's weight. The objective is to lift all the boxes in the minimum  
 1083 number of time steps.

1084 **T4 - BoxNet** This task involves coordinating robot arms to move colored boxes (squares) into corresponding colored  
 1085 goal locations (circles) in the fewest time steps. Each robot arm is assigned and restricted to a cell indicated by the  
 1086 dotted lines. The arms have two possible actions: (1) move a box within their cell to a neighboring cell, or (2) move a  
 1087 box within their cell to a goal location within the same cell. The objective is to ensure all boxes are placed in their  
 1088 matching goal locations efficiently.

1089 **T5 - Combinatorial Calculation** Given a set of integers, the goal is to use arithmetic operations (addition, subtraction,  
 1090 multiplication, division) and parentheses to arrange the numbers in such a way that the final result matches a specified  
 1091 target value. Each number must be used exactly once, and the order of the numbers cannot be changed.

1092 **T6 - Constrained Linear Arrangement** In a two-player card game, the task is to deduce your opponent's moves  
 1093 based on the game's rules, your played cards, and the announced results of each round. Each card can only be used  
 1094 once, and the game follows specific interaction rules between different card types, where certain cards can defeat, be  
 1095 defeated by, or draw with others according to predefined relationships.

1096 **T7 - Cryptanalysis** In this task, you are provided with a combination lock consisting of numbers and letters, where  
 1097 neither the numbers nor the letters repeat. Using a series of guesses and feedback, the goal is to deduce the correct  
 1098 password based on the given conditions.

1099 **T8 - Eight Queens** Given a grid with some queens already placed, the task is to place the remaining queens such that  
 1100 no two queens share the same row, column, or diagonal, while avoiding positions with obstacles in the grid.

1101 **T9 - Game 24** This task involves querying LLMs to use a given set of integers to generate an equation that evaluates  
 1102 to 24.

1103 **T10 - Gridworld** This task involves querying LLMs to plan the robot actions in a grid world, reaching all goals in  
 1104 any order while avoiding obstacles.

1105 **T11 - GSM** (Gao et al., 2023) This is the more challenging version of GSM8K (Cobbe et al., 2021) math reasoning  
 1106 dataset, where the numbers in the original questions of GSM8K are replaced with larger, less common values.

1107 **T12 - Letter Logic Diagram** The task is to complete an incomplete grid by selecting from a list of letters, where each  
 1108 row and column must contain each letter exactly once, and all cells on the minor diagonal (top-right to bottom-left)  
 1109 must contain the same letter. Some cells are already filled in as constraints.

1110 **T13 - Letters** This task involves querying LLMs to count the total number of specific letters in a long word and specify  
 1111 their positions. An example question can be 'How many r's in the word strawberry and what are their positions?'. This  
 1112 task has recently gained significant attention because current LLMs struggle to perform it effectively and accurately.

1113 **T14 - Light Puzzles** In this task, you are given an  $n \times n$  grid representing a network of lights, where a lit light is  
 1114 represented by "1" and an unlit light by "0". Several buttons control the state of these lights by turning them on or off in  
 1115 certain positions. The state of each light can be affected by multiple buttons. The task is to follow a series of button  
 1116 presses and determine the final state of the grid.

1117 **T15 - Logical Puzzle** The task involves querying LLMs to select a specified number of different values from a grid of  
 1118 numbers, ensuring that certain mathematical constraints (sum or product) are satisfied for selected numbers for each  
 1119 row and column.

1120 **T16 - Logical Equation** The task is to assign a specific numeric value to each letter from a given set, using a  
 1121 predefined range of numbers and a set of inequalities. Each letter corresponds to a unique number, and the relationships  
 1122 between the letters are defined by mathematical equations or constraints.

1123 **T17 - Mahjong** Given an initial set of letter cards, in each round, a new card is added and one card is removed.  
 1124 Some effects may happen when specific combinations of the cards appear after introducing the new card. A result is  
 1125 determined based on these specific conditions. The goal is to determine a result based on a series of rounds.

1126 **T18 - MATH-Count&Probability** This is the math reasoning dataset from MATH dataset (Hendrycks et al., 2021),  
 1127 with specific focus on counting and probability questions.

1128 **T19 - MATH-Geometry** This is the math reasoning dataset from MATH dataset (Hendrycks et al., 2021), with  
 1129 specific focus on geometry questions.

1134 **T20 - Matrix Transformation** Rotate a given matrix of characters based on given instruction (e.g., 90 degrees  
 1135 clockwise), preserving each character's position relative to others in the transformed output. The input matrix can be of  
 1136 any size and contain any character.

1137 **T21 - New Operator** This task introduces custom mathematical operations involving two numbers, defined with  
 1138 unique formulas. The goal is to use the given definitions of these operations to compute the result of a specific  
 1139 expression.

1140 **T22 - Number Multiplying** This task involves querying LLMs to compute the product among integers. It represents  
 1141 a classic problem that LLMs are not able to solve through pure textual reasoning.

1142 **T23 - Pattern Recognition** The task involves querying LLMs to find all squares in a character matrix where each  
 1143 square consists of identical characters and has a side length of at least 3.

1144 **T24 - Permutation and Combination** Given a set of objects with specific positioning constraints, the task is to  
 1145 determine the correct arrangement of the objects on a shelf. Each object must be placed in a position according to the  
 1146 rules provided, ensuring that the conditions on adjacency, order, and specific positions are met. For example, a rule  
 1147 about adjacency could be 'Book A must be adjacent to book I'.

1148 **T25 - Pooling** This task involves applying a pooling operation on a numerical  $N \times N$  grid. The pooling operation  
 1149 uses an  $n \times n$  sliding window ( $n < N$ ) that moves across the grid from left to right and top to bottom. The results from  
 1150 each window are then arranged based on their positions to create a new output matrix.

1151 **T26 - Reversi** In this game similar to Reversi, players take turns placing pieces on an  $n \times n$  grid. After placing a  
 1152 piece, any of the opponent's pieces located between two of the player's pieces (in the same row, column, or diagonal)  
 1153 will be flipped. The task is to determine the state of the board after rounds, starting from a given configuration.

1154 **T27 - Standard Sudoku** Given a partially filled Sudoku grid, the task is to fill the remaining empty cells with numbers  
 1155 between 1 and 9, ensuring that no number repeats in the same row, column, or  $3 \times 3$  subgrid.

1156 **T28 - Statistical Counting** Calculate the total score of a string by scanning it from left to right, where consecutive  
 1157 identical letters earn points (for example, two or more consecutive A's add 1 point, B's add 2 points, etc.). The task is to  
 1158 start with a score of 0 and return the final summing value.

1159 **T29 - String Deletion and Modification** The task is to transform a string by repeatedly applying a set of ordered  
 1160 string manipulation rules until no more changes are possible, where each rule modifies the string based on specific  
 1161 patterns or conditions present in the current string state. For example, a modification rule can be "If the string ends with  
 1162 'ba', replace it with 'ab'."

1163 **T30 - String Insertion** The task is to transform a string by scanning it from left to right and inserting specific  
 1164 characters after certain character patterns (e.g., each pattern WXYZ requires inserting W immediately after it occurs).  
 1165 All operations are performed simultaneously on the original string.

1166 **T31 - String Splitting** A dismantling engineer has old machines and can obtain machine parts through a set of  
 1167 predefined methods. By continuously cycling through these methods in a specific order, the engineer dismantles  
 1168 machines or combines parts to create new components, and the task is to determine the total number of parts and  
 1169 remaining machines after all possible cycles.

1170 **T32 - String Synthesis** Given an initial set of blocks and a set of synthesis rules that combine different types of blocks,  
 1171 the task is to determine the final block(s) after repeatedly applying these rules in order until no more combinations are  
 1172 possible.

1173 **T33 - Synthesis Decomposition** A farmer grows various crops and can exchange them for agricultural products.  
 1174 Using a set of methods, he can trade specific combinations of crops for products, following a cyclic pattern until no  
 1175 further exchanges are possible. The goal is to determine the synthesis result for each round.

1176 **T34 - Boolean Expressions** This task determines whether a randomly generated Boolean expression—built from the  
 1177 constants True and False and the operators and, or, and not—evaluates to true or false.

1178 **T35 - Causal Judgment** This task involves querying LLMs to read a brief scenario and predict the answer an average  
 1179 person would give to a causal question about it, including moral, intentional, or counterfactual aspects.

1180 **T36 - Date Understanding** This task involves querying LLMs to interpret a few sentences that reference dates and  
 1181 answer a related question (e.g., compute and return a specific date in MM/DD/YYYY format).

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 1183  
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1188 **T37 - Disambiguation QA** For a sentence containing a potentially ambiguous pronoun, the task is to decide whether its  
 1189 reference is genuinely unclear; if it is clear, identify the noun to which the pronoun refers.  
 1190

1191 **T38 - Dyck Languages** The task aims to complete a Dyck-4 string by providing the missing closing parentheses that  
 1192 properly balance the given prefix.  
 1193

1194 **T39 - Formal Fallacies** The task examines a set of statements and judges whether the informal argument that follows is  
 1195 deductively valid or commits a formal fallacy, with particular attention to negations.  
 1196

1197 **T40 - Geometric Shapes** The task aim to analyze a full SVG path description and identify the geometric figure that  
 1198 would be drawn.  
 1199

1200 **T41 - Hyperbaton** Given two sentences, choose which of two sentences follows the natural English ordering of  
 1201 adjectives.  
 1202

1203 **T42-T44 - Logical Deduction 3/5/7 Objects** Use spatial clues to determine the correct ordering of a set of objects  
 1204 (3/5/7 objects).  
 1205

1206 **T45 - Movie Recommendation** From four candidate films, select the one that best matches the preferences implied by  
 1207 the movies a user has already enjoyed.  
 1208

1209 **T46 - Multi-Step Arithmetic** Perform multi-step calculations involving addition, subtraction, multiplication, and  
 1210 division to obtain the correct result.  
 1211

1212 **T47 - Navigate** Follow a sequence of movement instructions and state whether the agent finishes at its starting point.  
 1213

1214 **T48 - Object Counting** Given a list of items and their quantities, count how many belong to a specified category.  
 1215

1216 **T49 - Penguins in a Table** Refer to a table of individual penguins and their attributes (possibly with extra information)  
 1217 to answer a question about one of those attributes.  
 1218

1219 **T50 - Reasoning about Colored Objects** Using the provided context, state the color of a particular object on a surface.  
 1220

1221 **T51 - Ruin Names** Make a single-character change to an artist, band, or movie name to create a humorous new meaning.  
 1222

1223 **T52 - Salient Translation Error Detection** Examine a German sentence and its English translation, and classify the  
 1224 main translation error present.  
 1225

1226 **T53 - Snarks** From a pair of nearly identical sentences, identify the one that is sarcastic.  
 1227

1228 **T54 - Sports Understanding** Judge whether a fabricated sports-related statement is plausible.  
 1229

1230 **T55 - Temporal Sequences** Given a timeline of a person's daily activities, identify a time slot when they could have  
 1231 performed another specified task.  
 1232

1233 **T56-T58 - Tracking Shuffled 3/5/7 Objects** Trace a set of objects (3/5/7 objects) through a series of pairwise swaps to  
 1234 determine their final positions.  
 1235

1236 **T59 - Web of Lies** Decide whether a Boolean function described in a word problem evaluates to true or false.  
 1237

1238 **T60 - Word Sorting** Arrange the provided words in standard alphabetical order.  
 1239

1240 **T61 - AB** Rewrite an A::B token string by exhaustively applying neighbor collision rules and return the final sequence.  
 1241

1242 **T62 - Acre** From example Bicket detector outcomes, decide whether a new object set turns the detector "on", "off", or  
 1243 is "undetermined".  
 1244

1245 **T63 - Advanced Geometry** Solve analytic geometry questions (e.g. angle, orthocentre, in-circle radius) given vertex  
 1246 coordinates.  
 1247

1248 **T64 - AIW** Answer small "Alice-in-Wonderland" social reasoning problems about siblings, friends, or colleagues.  
 1249

1250 **T65 - ARC\_1D** Infer the mapping rule that maps example 1D grids to output grids and apply it to a test grid.  
 1251

1252 **T66 - ARC\_AGI** Same as ARC\_1D but with rotations, mirrors and permutations on 2-D grids.  
 1253

1254 **T67 - Base Conversion** The task is to convert integers between arbitrary bases.  
 1255

1256 **T68 - Basic Arithmetic** The task is to evaluate the value of basic arithmetic expressions.  
 1257

1258 **T69 - BF** Based on outputs of example BF codes, the task is to output the contents of a new BF program.  
 1259

1242 **T70 - Binary Alternation** The task is to produce the minimum number of character swaps to make a binary string to be  
 1243 alternating, that is, no two adjacent characters are equal.

1244 **T71 - Binary Matrix** Given binary matrices, the task is to find the distance to the nearest 0 for each cell in the matrix.

1245 **T72 - Bitwise Arithmetic** The task is to compute results of expressions with mixed bitwise and arithmetic operators.

1246 **T73 - Caesar Cipher** The task is to decrypt a Caesar cipher text.

1247 **T74 - Calendar Arithmetic** Given a description of the calendar, answer a question by conducting arithmetic calculations  
 1248 such as adding or subtracting days / months / years or computing weekday differences.

1249 **T75 - Chain Sum** The task is to calculate simple arithmetic problem and output the answer.

1250 **T76 - Circuit Logic** Given a logic circuit with logical operators, the goal is to evaluate the output of given inputs.

1251 **T77 - Codeio** The task is to read and reason about task description and pseudocode programs and report outputs of the  
 1252 program given inputs.

1253 **T78 - Color Cube Rotation** After rotating a 3D colored cube, the task is to name the color on a queried face.

1254 **T79 - Complex Arithmetic** The task is to perform arithmetic with complex numbers and report answers.

1255 **T80 - Count Bits** Given a large number, the goal is to count the number of occurrence of 1 bits in the binary  
 1256 representation of this number.

1257 **T81 - Count Primes** The task is to count number of primes numbers within an interval.

1258 **T82 - Countdown** The task is to write an expression that can reach a target integer using given numbers and the four  
 1259 operations.

1260 **T83 - Course Schedule** Given a list of courses need to be taken and their prerequisites, the task is to determine if all  
 1261 courses can be finished.

1262 **T84 - Decimal Arithmetic** The task is to evaluate decimal expressions with given precision.

1263 **T85 - Dice** The task is to compute probabilities of rolling results in fair-dice experiments with various dices with  
 1264 different number of sides.

1265 **T86 - Emoji Mystery** The task is to deduce hidden sentences expressed with emoji symbols.

1266 **T87 - Family Relationships** The task is to answer kinship queries in family trees.

1267 **T88 - Figlet Font** The task is to read FIGlet banners and output the content as strings.

1268 **T89 - Fraction Simplification** The task is to simplify fractions to the lowest terms.

1269 **T90 - Futoshiki** The task is to fill in values to empty spaces of Futoshiki puzzles that have inequalities.

1270 **T91 - Game of Life** The task is to simulate Conway's Game-of-Life for k steps.

1271 **T92 - Game of Life Halting** The task is to decide whether a Game-of-Life configuration halts within k steps, that is,  
 1272 there are no cells alive.

1273 **T93 - GCD** The task is to compute greatest common divisors of numbers.

1274 **T94 - Graph Color** The task is to provide a coloring for this graph such that every vertex is not connected to a vertex  
 1275 of the same color.

1276 **T95 - Group Anagrams** Given a list of words, the task is to cluster words that are anagrams.

1277 **T96 - Intermediate Integration** Given an expression, the task is to calculate the indefinite integral.

1278 **T97 - Isomorphic Strings** The task is to decide if two strings can be isomorphic if the characters in one string can be  
 1279 replaced to get the second string.

1280 **T98 - Jugs** Given empty jugs with different sizes, the task is to give a plan of how to fill any of the available jugs with  
 1281 the target amount of water by filling, emptying, or pouring from a jug to another.

1282 **T99 - Knight Swap** The task is to swap two knights on a chessboard in the fewest moves.

1283 **T100 - Knights Knaves** The task is to determine who is a knight (truth-teller) or knave from their statements.

1296 **T101 - Largest Island** The task is to find max connected component size in a binary grid.  
 1297  
 1298 **T102 - LCM** The task is to find the Least Common Multiple (LCM) of numbers.  
 1299  
 1300 **T103 - Leg Counting** The task is to count how many legs there are in total when given a list of animals.  
 1301  
 1302 **T104 - Letter Jumble** For each word in a sentence, the letter may have been randomly shuffled. The task is to reconstruct original words from jumbled letters.  
 1303  
 1304 **T105 - List Functions** Given examples of how inputs are mapped to outputs, reason and use the same mapping to generate output given an input.  
 1305  
 1306 **T106 - Manipulate Matrix** Apply a sequence of matrix transformations to a matrix and output the result.  
 1307  
 1308 **T107 - Maze** Compute the shortest path length from start to goal in an maze.  
 1309  
 1310 **T108 - Modulo Grid** Identify the mathematical pattern which defines a grid, then use that pattern to fill in the question marks in this grid.  
 1311  
 1312 **T109 - Needle Haystack** The task is to locate a short pattern inside a longer string.  
 1313  
 1314 **T110 - Number Filtering** Given a list of numbers and a requirement, remove numbers not satisfying this requirement.  
 1315  
 1316 **T111 - Number Format** The task is to pick the largest/smallest number out of several options.  
 1317  
 1318 **T112 - Number Sequence** Predict the next term of integer sequences based on previous patterns.  
 1319  
 1320 **T113 - Number Sorting** The task is to sort number lists based on required order.  
 1321  
 1322 **T114 - Palindrome Generation** The task is, given a list of letters, to form a valid palindrome.  
 1323  
 1324 **T115 - Palindrome Partitioning** Given a string, the task is to find all ways to partition it such that every substring is a palindrome.  
 1325  
 1326 **T116 - Polynomial Equations** The task is to find real values of a variable in a polynomial equation.  
 1327  
 1328 **T117 - Polynomial Multiplication** The task is to calculate the result of multiplying two polynomials.  
 1329  
 1330 **T118 - Pool Matrix** The task is to perform max- or average-pooling on numeric matrices.  
 1331  
 1332 **T119 - Products** The task is to compute multiplications of numbers.  
 1333  
 1334 **T120 - Propositional Logic** Given a list of premises, the task is to infer a correct conclusion from the premise.  
 1335  
 1336 **T121 - Quantum Lock** There are some buttons, a light, and a number. The light will toggle between red and green whenever you press a button. Each button performs a mathematical operation to the number, but the operation may depend on the state of the light. You must press the shortest correct sequence of buttons to reach the target value.  
 1337  
 1338 **T122 - Ransom Note** Given two strings representing a ransom note and a magazine, determine if the ransom note can be constructed using the letters in the magazine  
 1339  
 1340 **T123 - Rearc** Find the common rule that maps input grids to output grids and apply the rule to predict corresponding output of a test input grid.  
 1341  
 1342 **T124 - Rectangle Count** The task is to count how many rectangles are present in an ASCII grid.  
 1343  
 1344 **T125 - Rotate Matrix** Given a square matrix, the task is to rotate it and output the rotated matrix.  
 1345  
 1346 **T126 - Rotten Oranges** You are given an  $n \times n$  grid where each cell can be empty cell, contain a fresh orange, or contain a rotten orange. Every minute, any fresh orange that is 4-directionally adjacent to a rotten orange becomes rotten. Your task is determine the minimum number of minutes that must elapse until no cell has a fresh orange.  
 1347  
 1348 **T127 - Rubiks Cube** Given a Rubik's cube, the task is to provide a solution to solve this cube using Singmaster notation.  
 1349  
 1350 **T128 - Rush Hour** Given a rush hour parking grid, the task is to give a plan of movements of cars to achieve required car positions.  
 1351  
 1352 **T129 - Self Reference** The task is to evaluate self-referential arithmetic expressions to output the number of possible solutions.  
 1353  
 1354 **T130 - Shortest Path** The task is to find the length of the shortest path in a grid.

1350 **T131 - Simple Equations** The task is to solve equations with one variable.  
 1351  
 1352 **T132 - Simple Geometry** Given polygon with different number of sides and all interior angles but one angle, the task is  
 1353 to calculate the remaining interior angle.  
 1354 **T133 - Simple Integration** The task is to find solution to indefinite integral problems with one variable.  
 1355  
 1356 **T134 - Sokoban** The task is to find a list of actions to solve a Sokoban level.  
 1357  
 1358 **T135 - Spell Backward** The task is to reverse input strings.  
 1359  
 1360 **T136 - Spiral Matrix** Given a matrix, the task is to generate a list of elements in spiral order, starting from the top-left  
 1361 element.  
 1362  
 1363 **T137 - String Manipulation** The task is to repeatedly transform a string according to a set of rules until no further  
 1364 transformations can be performed, or a state is repeated.  
 1365  
 1366 **T138 - Syllogism** Given some statements, answer the provided question by retrieving information from the statements.  
 1367  
 1368 **T139 - Time Intervals** The task is to compute durations between two times / dates with various formats and complexities.  
 1369  
 1370 **T140 - Tower of Hanoi** Output an optimal (or specified) move list to transfer disks between pegs to solve a tower of  
 1371 Hanoi problem.  
 1372  
 1373 **T141 - Tsumego** Choose the single correct Go move to capture or save stones.  
 1374  
 1375 **T142 - Word Ladder** Transform one word to another by single-letter changes using dictionary words.  
 1376  
 1377 **T143 - Word Sequence Reversal** Given a list of words, the task is to reverse order of words.  
 1378  
 1379 **T144 - Zebra Puzzles** Given some statements, solve the logic puzzle by gathering information from statements and  
 1380 deduce the answer of the question.

Table 6: The evaluated capabilities of all 144 tasks, classified as Execution, Planning, and Reasoning tasks. The classification is based on human experts' knowledge and also the classification in original datasets if it exists.

|      | Task                   | Math | Spatial | Logical | Order | Optimization | Search |
|------|------------------------|------|---------|---------|-------|--------------|--------|
| 1378 | 2048                   | ✓    | ✓       | ✓       | ✗     | ✗            | ✗      |
| 1379 | Light Puzzles          | ✗    | ✓       | ✗       | ✗     | ✗            | ✗      |
| 1380 | Mahjong                | ✗    | ✗       | ✗       | ✓     | ✗            | ✗      |
| 1381 | Matrix Transform.      | ✗    | ✓       | ✗       | ✗     | ✗            | ✗      |
| 1382 | New operator           | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1383 | Number Multiplying     | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1384 | Pattern Recognition    | ✗    | ✓       | ✗       | ✗     | ✗            | ✓      |
| 1385 | Pooling                | ✓    | ✓       | ✗       | ✗     | ✗            | ✗      |
| 1386 | Reversi                | ✗    | ✓       | ✗       | ✗     | ✗            | ✗      |
| 1387 | Statistical Counting   | ✓    | ✗       | ✗       | ✓     | ✗            | ✗      |
| 1388 | String Del. & Modi.    | ✗    | ✗       | ✓       | ✓     | ✗            | ✓      |
| 1389 | String Insertion       | ✗    | ✗       | ✓       | ✓     | ✗            | ✓      |
| 1390 | String Splitting       | ✗    | ✗       | ✓       | ✓     | ✗            | ✓      |
| 1391 | String Synthesis       | ✗    | ✗       | ✓       | ✓     | ✗            | ✓      |
| 1392 | Synthesis Decom.       | ✗    | ✗       | ✓       | ✓     | ✗            | ✓      |
| 1393 | Dyck Languages         | ✗    | ✗       | ✓       | ✓     | ✗            | ✗      |
| 1394 | Multi-Step Arithmetic  | ✓    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1395 | Navigate               | ✗    | ✓       | ✗       | ✓     | ✗            | ✗      |
| 1396 | Object Counting        | ✓    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1397 | Ruin Names             | ✗    | ✗       | ✓       | ✗     | ✗            | ✓      |
| 1398 | Tracking Shuffled Obj. | ✗    | ✓       | ✓       | ✓     | ✗            | ✗      |
| 1399 | Word Sorting           | ✗    | ✗       | ✗       | ✓     | ✗            | ✗      |
| 1400 | AB                     | ✗    | ✗       | ✓       | ✓     | ✗            | ✗      |
| 1401 | ARC_1D                 | ✗    | ✓       | ✗       | ✓     | ✗            | ✗      |
| 1402 | ARC_AGI                | ✗    | ✓       | ✗       | ✓     | ✗            | ✗      |
| 1403 | Base Conversion        | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
|      | Basic Arithmetic       | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |

Continued on next page

Table 6 (continued from previous page)

|      | Task                     | Math | Spatial | Logical | Order | Optimization | Search |
|------|--------------------------|------|---------|---------|-------|--------------|--------|
| 1404 | BF                       | ✗    | ✗       | ✓       | ✓     | ✗            | ✗      |
| 1405 | Binary Alternation       | ✗    | ✗       | ✓       | ✓     | ✗            | ✗      |
| 1406 | Binary Matrix            | ✓    | ✓       | ✗       | ✗     | ✗            | ✓      |
| 1407 | Bitwise Arithmetic       | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1408 | Caesar Cipher            | ✗    | ✗       | ✗       | ✓     | ✗            | ✗      |
| 1409 | Chain Sum                | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1410 | Codeio                   | ✗    | ✗       | ✓       | ✓     | ✗            | ✗      |
| 1411 | Color Cube Rotation      | ✗    | ✓       | ✗       | ✓     | ✗            | ✗      |
| 1412 | Complex Arithmetic       | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1413 | Count Bits               | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1414 | Count Primes             | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1415 | Decimal Arithmetic       | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1416 | Dice                     | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1417 | Fraction Simplification  | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1418 | GCD                      | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1419 | Group Anagrams           | ✗    | ✗       | ✗       | ✓     | ✗            | ✗      |
| 1420 | Intermediate Integration | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1421 | Isomorphic Strings       | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1422 | Largest Island           | ✗    | ✓       | ✗       | ✗     | ✓            | ✗      |
| 1423 | LCM                      | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1424 | Leg Counting             | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1425 | Letter Jumble            | ✗    | ✗       | ✗       | ✓     | ✗            | ✗      |
| 1426 | List Functions           | ✓    | ✗       | ✗       | ✓     | ✗            | ✗      |
| 1427 | Manipulate Matrix        | ✓    | ✓       | ✗       | ✗     | ✗            | ✗      |
| 1428 | Number Filtering         | ✓    | ✗       | ✗       | ✗     | ✗            | ✓      |
| 1429 | Number Format            | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1430 | Number Sorting           | ✗    | ✗       | ✗       | ✓     | ✗            | ✗      |
| 1431 | Palindrome Generation    | ✗    | ✗       | ✗       | ✓     | ✗            | ✗      |
| 1432 | Palindrome Partitioning  | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1433 | Poly. Equations          | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1434 | Poly. Multiplication     | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1435 | Pool Matrix              | ✓    | ✓       | ✗       | ✗     | ✗            | ✗      |
| 1436 | Products                 | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1437 | Rectangle Count          | ✓    | ✓       | ✗       | ✗     | ✗            | ✓      |
| 1438 | Rotate Matrix            | ✗    | ✓       | ✗       | ✗     | ✗            | ✗      |
| 1439 | Rotten Oranges           | ✗    | ✓       | ✗       | ✗     | ✓            | ✗      |
| 1440 | Simple Equations         | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1441 | Simple Geometry          | ✓    | ✓       | ✗       | ✗     | ✗            | ✗      |
| 1442 | Simple Integration       | ✓    | ✗       | ✗       | ✗     | ✓            | ✗      |
| 1443 | Spell Backward           | ✗    | ✗       | ✗       | ✓     | ✗            | ✗      |
| 1444 | Spiral Matrix            | ✗    | ✓       | ✗       | ✗     | ✗            | ✗      |
| 1445 | String Manipulation      | ✗    | ✗       | ✓       | ✓     | ✗            | ✗      |
| 1446 | Time Intervals           | ✓    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1447 | Word Seq. Reversal       | ✗    | ✗       | ✗       | ✓     | ✗            | ✗      |
| 1448 | Planning                 | ✗    | ✓       | ✓       | ✗     | ✓            | ✗      |
| 1449 |                          | ✗    | ✗       | ✓       | ✗     | ✓            | ✗      |
| 1450 |                          | ✓    | ✗       | ✗       | ✗     | ✓            | ✗      |
| 1451 |                          | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1452 |                          | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1453 |                          | ✗    | ✓       | ✗       | ✗     | ✗            | ✗      |
| 1454 |                          | ✓    | ✗       | ✗       | ✓     | ✓            | ✗      |
| 1455 |                          | ✗    | ✓       | ✗       | ✓     | ✗            | ✓      |
| 1456 |                          | ✗    | ✓       | ✓       | ✗     | ✗            | ✓      |
| 1457 |                          | ✓    | ✓       | ✗       | ✗     | ✗            | ✓      |

Continued on next page

Table 6 (continued from previous page)

|      | Task                        | Math | Spatial | Logical | Order | Optimization | Search |
|------|-----------------------------|------|---------|---------|-------|--------------|--------|
| 1458 | Permut. and Combina.        | ✗    | ✓       | ✓       | ✓     | ✗            | ✗      |
| 1459 | Standard Sudoku             | ✓    | ✓       | ✗       | ✗     | ✗            | ✓      |
| 1460 | Movie Recommendation        | ✗    | ✗       | ✗       | ✗     | ✓            | ✓      |
| 1461 | Temporal Sequences          | ✓    | ✗       | ✓       | ✓     | ✗            | ✗      |
| 1462 | Countdown                   | ✓    | ✗       | ✗       | ✓     | ✗            | ✗      |
| 1463 | Course Schedule             | ✗    | ✗       | ✓       | ✓     | ✗            | ✗      |
| 1464 | Futoshiki                   | ✓    | ✓       | ✓       | ✗     | ✗            | ✗      |
| 1465 | Graph Color                 | ✗    | ✓       | ✓       | ✗     | ✓            | ✗      |
| 1466 | Jugs                        | ✓    | ✗       | ✓       | ✓     | ✓            | ✗      |
| 1467 | Knight Swap                 | ✗    | ✓       | ✓       | ✗     | ✓            | ✗      |
| 1468 | Maze                        | ✗    | ✓       | ✓       | ✗     | ✓            | ✗      |
| 1469 | Modulo Grid                 | ✓    | ✓       | ✓       | ✗     | ✗            | ✗      |
| 1470 | Quantum Lock                | ✗    | ✗       | ✓       | ✓     | ✓            | ✗      |
| 1471 | Rubiks Cube                 | ✗    | ✓       | ✓       | ✓     | ✗            | ✗      |
| 1472 | Rush Hour                   | ✗    | ✓       | ✓       | ✗     | ✗            | ✗      |
| 1473 | Shortest Path               | ✓    | ✓       | ✓       | ✗     | ✓            | ✗      |
| 1474 | Sokoban                     | ✗    | ✓       | ✓       | ✓     | ✗            | ✗      |
| 1475 | Tower of Hanoi              | ✓    | ✗       | ✓       | ✓     | ✓            | ✗      |
| 1476 | Tsumego                     | ✗    | ✓       | ✓       | ✗     | ✓            | ✗      |
| 1477 | Word Ladder                 | ✗    | ✗       | ✓       | ✗     | ✗            | ✓      |
| 1478 |                             |      |         |         |       |              |        |
| 1479 | Reasoning                   | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1480 | Logical Deduction           | ✓    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1481 | GSM                         | ✓    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1482 | MATH-Count&Prob.            | ✓    | ✗       | ✓       | ✗     | ✗            | ✓      |
| 1483 | MATH-Geometry               | ✓    | ✓       | ✗       | ✗     | ✗            | ✗      |
| 1484 | Hyperbaton                  | ✗    | ✗       | ✗       | ✓     | ✗            | ✗      |
| 1485 | Logical Deduction           | ✗    | ✓       | ✓       | ✓     | ✗            | ✗      |
| 1486 | Penguins in a Table         | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1487 | Reasoning Colored Obj.      | ✗    | ✓       | ✓       | ✗     | ✗            | ✗      |
| 1488 | Salient Trans. Err. Detect. | ✗    | ✗       | ✗       | ✗     | ✗            | ✓      |
| 1489 | Snarks                      | ✗    | ✗       | ✗       | ✗     | ✗            | ✓      |
| 1490 | Sports Understanding        | ✗    | ✗       | ✓       | ✗     | ✗            | ✓      |
| 1491 | Web of Lies                 | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1492 | Acres                       | ✗    | ✗       | ✓       | ✓     | ✗            | ✗      |
| 1493 | Advanced Geometry           | ✓    | ✓       | ✗       | ✗     | ✗            | ✗      |
| 1494 | AIW                         | ✓    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1495 | Calendar Arithmetic         | ✓    | ✗       | ✗       | ✗     | ✗            | ✗      |
| 1496 | Circuit Logic               | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1497 | Emoji Mystery               | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1498 | Family Relationships        | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1499 | Figlet Font                 | ✗    | ✓       | ✗       | ✓     | ✗            | ✗      |
| 1500 | Game of Life                | ✗    | ✓       | ✓       | ✗     | ✗            | ✗      |
| 1501 | Game of Life Halting        | ✗    | ✓       | ✓       | ✗     | ✗            | ✗      |
| 1502 | Knights Knaves              | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1503 | Needle Haystack             | ✗    | ✗       | ✗       | ✗     | ✗            | ✓      |
| 1504 | Number Sequence             | ✓    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1505 | Propositional Logic         | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1506 | Ransom Note                 | ✗    | ✗       | ✓       | ✗     | ✗            | ✓      |
| 1507 | Rearc                       | ✗    | ✓       | ✗       | ✗     | ✗            | ✓      |
| 1508 | Self Reference              | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1509 | Syllogism                   | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |
| 1510 | Zebra Puzzles               | ✗    | ✗       | ✓       | ✗     | ✗            | ✗      |