# Ludax: A GPU-Accelerated Domain Specific Language for Board Games

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#### **Abstract**

Games have long been used as benchmarks and testing environments for research in artificial intelligence. A key step in supporting this research was the development of game description languages: frameworks that compile domain-specific code into playable and simulatable game environments, allowing researchers to generalize their algorithms and approaches across multiple games without having to manually implement each one. More recently, progress in reinforcement learning (RL) has been largely driven by advances in hardware acceleration. Libraries like JAX allow practitioners to take full advantage of cutting-edge computing hardware, often speeding up training and testing by orders of magnitude. Here, we present a synthesis of these strands of research: a domain-specific language for board games which automatically compiles into hardware-accelerated code. Our framework, Ludax, combines the generality of game description languages with the speed of modern parallel processing hardware and is designed to fit neatly into existing deep learning pipelines. We envision Ludax as a tool to help accelerate games research generally, from RL to cognitive science, by enabling rapid simulation and providing a flexible representation scheme. We present a detailed breakdown of Ludax's description language and technical notes on the compilation process, along with speed benchmarking and a demonstration of training RL agents. The Ludax framework, along with implementations of existing board games, is open-source and freely available.

#### 1 Introduction

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For the past 75 years, games have served as vital tests and benchmarks for artificial intelligence research. While many specific games have been completely solved [36] or optimized beyond the abilities of the strongest human players [5, 42], the general space of games remains a fertile ground for measuring improvements in reasoning, planning, and strategic thinking. A critical part of this progress, however, is the ability to test approaches and algorithms on a set of environments that are both diverse and computationally efficient. [41]

To help drive further games and learning research, we introduce Ludax: a domain-specific language for board games that compiles into GPU-accelerated code written in the JAX library [3]. Ludax draws on two main inspirations: (1) Ludii [32], a general purpose description language for board games capable of representing more than 1400 games from throughout history and around the world, and (2) PGX [26], a collection of optimized JAX-native implementations of classic board games and video games designed to facilitate rapid training and evaluation of modern reinforcement learning (RL) agents. Ludax presents a flexible and general-purpose game representation format that can be leveraged for efficient simulation and learning on modern computing hardware.

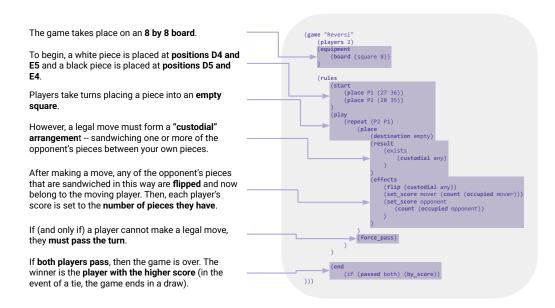


Figure 1: Natural language description of *Reversi* along with its corresponding translation into Ludax. Ludax uses "ludemic" syntax that represents high-level game components as separate program sections.

Ludax currently supports two-player, perfect-information, turn-based board games played by placing 36 pieces onto empty board cells. While this is a narrow class of games when compared to the full 37 breadth of human designs and the set of mechanics implemented by Ludii, it is still broad enough 38 to capture a wide range of existing games (e.g. Connect Four, Pente, Hex, ...) as well as many 39 unexplored *novel* games and variants that fall within that class. Further, Ludax is designed to be 40 easily expandable – like with Ludii, implementing new game mechanics in Ludax only requires 41 implementing new atomic components in the underlying description language. These components 42 can then be combined compositionally with existing elements of the language to produce an entirely 43 new range of possible games, instead of each game needing to be implemented separately. 44

Another design goal for Ludax is ease of use, both in terms of game design and experimentation.
The syntax of the description language is "ludemic" [32] – splitting game rules into clear sections
governing the game's setup, play mechanics, and end conditions. Like with Ludii, game programs
in Ludax resemble English descriptions of rules (see Figure 1). Further, by leveraging the structure
of the existing PGX library, environments instantiated in Ludax can be easily combined with existing
frameworks for GPU-accelerated search, reinforcement learning, or evolution [11, 45]. Ludax also
supports a basic web interface for interactive debugging, with the aim to provide support for online
interactive experiments in the future.

To our knowledge, Ludax is the first board game description language which compiles into GPUaccelerated code. In the following sections, we provide a detailed description of the language syntax, compilation process, and Ludax's expressive range. We also provide speed benchmarking compared to both Ludii and PGX, as well as an initial demonstration of training learned agents. Finally, we conclude with a discussion of potential use cases and future directions.

## 2 Related Work

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Game Description Languages: Game description languages have been used for many years and in a variety of domains. The Stanford GDL [28, 17, 39, 46] is among the most influential, helping to popularize research in general game playing [33] through its use in the International General Game Playing Competition [19, 18]. Other notable examples include VGDL [14, 37, 38] (primarily known from its use in the General Video Game AI framework [31]), RBG [25], Ludi [4], and its successor Ludii [32]. GDLs have also been used to describe the rules of card games [15] as well as to represent human goals in naturalistic simulated environments [9, 10]. Modern game description languages

have tended to move away from a basis in formal logic in favor of greater human usability, though there are benefits in efficiency gained by the use of regular languages [24].

**GPU-Accelerated Environments:** Recent years have seen a proliferation of learning environments 68 implemented in the JAX library or other frameworks that enable hardware (typically GPU) acceler-69 ation. Examples include single-agent and multi-agent physics simulators [16, 29, 1], ports of both 70 classic and recent reinforcement learning tasks [8, 27, 26, 30], combinatorial optimization problems 71 [2], multi-agent coordination problems [35], and driving simulators [21, 23]. While these efforts 72 have spurred significant progress and span a wide range of domains and task formulations, each of 73 them implement a fixed environment or set of environments. As such, they cannot easily be extended 74 to novel environments without first writing new hardware-accelerated code. We do also note the 75 existence of a number of description languages for other domains (e.g. probabilistic programming, 76 planning, single-player puzzles) that leverage JAX for efficient execution [6, 20, 13, 12]. 77

# 78 3 Description Language Details

Ludax's game description language draws heavily on the Ludii description language, particularly in its use of "ludemic" syntax that represents game rules in terms of high-level and easily-understandable components [32]. The complete grammar file and syntax details are available in the Supplemental Material.

#### 83 3.1 Equipment

The equipment section contains information about the physical components used by the game.

Currently, this only specifies the size and shape of the board (i.e. whether it is square, rectangular, hexagonal, or hexagonal-rectangular). The dimensions and shape of the board are used during compilation to help pre-compute certain game-relevant properties, such as the board indices corresponding to lines of specific lengths. In future versions of Ludax, the equipment section will also detail the different pieces used by each player if the game specifies more than one.

#### 90 3.2 Start Rules

The start section is an optional section that contains the rules for the game's setup. For most games, play begins on an empty board and the start section is omitted. In some games, such as *Reversi* (see Figure 1), pieces are placed in a particular arrangement at the start of play.

#### 94 3.3 Play Rules

Typically, the play rules of each game are the most involved, as they detail the core mechanics and 95 dynamics of the game. The play section is itself broken into one or more subsections called "play phases." Each phase has its own rules for player actions and turn-taking, as well as specific conditions for when to transition to another phase. Most games have only a single phase in which players 98 alternate turns until the game is over, specified with the repeat keyword. Some games include a once\_through phase that progresses through the turn order a single time before advancing to the 100 next phase. The sequence of player turns is specified independently for each phase. For instance, 101 Yavalax (Appendix Figure 4, bottom-left panel) begins with the first player making a single move 102 (i.e. (once\_through (P1) ...) ) before both players alternate taking two turns for the rest of the 103 game (i.e. (repeat (P2 P2 P1 P1) ...) ). 104

The core of each phase is a "play mechanic" that encodes the ways that players take their turns. In 105 the context of reinforcement learning, a play mechanic specifies both the action space (A) and the 106 transition function ( $\mathcal{T}: \mathcal{S} \times \mathcal{A} \to \mathcal{S}$ ). At a lower level, each play mechanic also defines a "legal action mask function" that returns whether each action is valid from the current game state. Currently, 108 Ludax supports only one kind of play mechanic: place. A place mechanic's primary argument is a 109 destination constraint which specifies where a piece may be placed on a given player's turn. For 110 many games, such as *Tic-Tac-Toe*, this is simply the set of empty board positions. For some games, 111 however, the destination constraint is more involved: in Connect Four (Appendix Figure 4, top-right 112 panel), legal actions are empty spaces that are on the bottom edge of the board or immediately above an occupied position (see subsection 3.5 for a discussion of how actions are represented more

generally in Ludax). Even further, some games have what we call result constraints which require 115 that a legal action results or doesn't result in a board with specific properties. Yavalax and Reversi 116 both use result constraints: the former forbids players from placing a piece that forms a line of five 117 or that forms only a single line of four, whereas the latter requires players to place a piece in a way 118 that "sandwiches" one or more of their opponent's pieces in a line. Finally, a play mechanic may 119 optionally specify one or more effects that modify the game state after the action is performed. 120 121 Effects are used to handle mechanics like capturing or flipping pieces, as well as updating each player's score (if the game uses score). Both *Reversi* and *Pente* (Appendix Figure 4, bottom-right 122 panel) use play effects to handle flipping and capturing pieces, respectively, with *Pente* also using the 123 score as an alternate winning condition. 124

Throughout this section, we have been referring to various properties of a game state and relationships between pieces / positions (e.g. whether pieces are "sandwiched," whether a line is formed, whether a piece is adjacent to another, ...). These are the lowest-level component's of Ludax's description language and are referred to collectively as masks, functions, and predicates. A mask takes in the current game state and returns a boolean array over each position on the board. Some masks, like occupied or edge, take additional grammatical arguments which might specify a particular player or region of the board. A function similarly takes in the current game state and returns a single non-negative integer. In Ludax's current form, line is probably the most commonly-used function—it returns the number of contiguous lines of a given player's pieces on the board, with a specified length and orientation. Lastly, a predicate maps from a game state to a single boolean truth value. Many predicates operate over the outputs of masks and functions, such as exists or equals, though some like mover\_is are computed directly from game states. Crucially, the outputs of masks, functions, and predicates can be combined compositionally using first-order logic (excluding quantification) to form more complicated expressions. So, the condition "if Player 2 makes a line of 4 in a row or a diagonal line of 3..." would be rendered as follows:

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(and (mover_is P2) (or (line 4) (line 3 orientation:diagonal)))
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Note that, for ease of use, Ludax automatically interprets the presence of a bare function inside a boolean operator as indicating a non-zero value. So, (line 4) is equivalent to (>= (line 4) 1).

#### 3.4 End Rules

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The last section of a game description in Ludax details the criteria that terminate a game. The end section contains one or more "end conditions" – these are applied *in order*, with the first condition to activate determining the ending behavior (i.e. which player wins or if the game ends in a draw). If none of the conditions activate, then the game continues. For instance, *Tic-Tac-Toe* includes both the end conditions (if (line 3) (mover win)) and (if (full\_board) (draw)), with the draw condition only triggering if the "three in a row condition" is not met. End conditions also frequently refer to a player's score, which is updated or set as a result of an action's effects (see above).

#### 3.5 Design Considerations

While Ludax draws heavily from the Ludii description language, there are some important dif-151 ferences which go beyond just changes in syntax. The first of these relates to how both systems 152 represent a game's action space. One of the design goals of Ludii is that game descriptions should 153 154 resemble as much as possible the rules in natural language. In Connect Four, for instance, players take a move by dropping a piece into one of seven columns of the board, at which point the piece 155 falls until it reaches the bottom or rests on another piece. Accordingly, the canonical representa-156 tion of Connect Four in Ludii features pieces that "Drop" into the "LastColumn" chosen by the 157 player (PGX implicitly represents the game in a similar way). As mentioned above, however, Ludax 158 represents the action space differently: players simply place a piece onto an empty board cell, with 159 actions that are not directly above an existing piece or the bottom of the board marked as illegal. Mechanically, the two implementations of *Connect Four* are identical – the difference lies in how they are encoded (especially to simulated players or reinforcement learning agents). The "column-based"

<sup>&</sup>lt;sup>1</sup>The adjacent mask is a special case – it takes *another* mask as an additional argument and returns the board positions adjacent to any of the active positions in the original mask.

representation has many advantages (it matches the physical properties of the game in real life and lowers the branching factor), but it is also game-specific. While Ludax also strives to represent game descriptions intuitively, we primarily aim to provide a unified representation format across games, such that general game-playing agents can more easily transfer knowledge and expertise from one game to another. As such, the size and form of the action space for any place-based game is determined only by the size and shape of the board. This choice is also partially motivated by the specifics of working with the JAX library (see Section 4) and has implications for benchmarking and downstream use-cases (see Section 6). 

## 4 Compiling Game Descriptions into Game Environments

In this section, we describe the high-level approach used to map from programs in the Ludax game description language to hardware-accelerated simulation environments. While Ludax specifically instantiates board game environments using the Lark Python library, the general approach is flexible enough to be used with different domains and parsing toolkits. Broadly speaking, Ludax operates by defining the leaves of the grammatical parse tree (i.e. individual masks, functions, and predicates) as atomic functions written in JAX, which are then dynamically composed from the bottom-up to form higher-level operators used by the environment class. Consider again the following game expression:

```
(and (mover_is P2) (or (line 4) (line 3 orientation:diagonal)))
```

During compilation, the leaf-level nodes (i.e. (mover\_is P2) and (line 4)) are converted into JAX functions which map from the current game state to (in this case) a boolean truth value, and those functions are then passed up the parse tree. Higher-level nodes, such as (and ...), receive the JAX functions corresponding to each of their children and return a *new* JAX function that also takes the game state as input and implements the appropriate operation (in this case, boolean conjunction). In pseudocode, using the Lark library's Transformer paradigm, this looks like the following:

```
def predicate_and(self, children):
    def predicate_fn(state):
        children_values = [child_fn(state) for child_fn in children]
        return all(children_values)

return predicate_fn
```

In actuality, both the "children functions" and the combined "predicate function" must be written to be compatible with JAX's vectorization scheme and just-in-time (JIT) compilation. This imposes a number of implementational constraints, most notably that the size and shape of all arrays must be fixed at compile time. This means, for instance, that the dimensions of the "legal action mask" (and, hence, the size of the action space in general) cannot change as the game progresses. In addition, values like the number of iterations in a loop or the positions of a lookup mask must essentially be "pre-specified." Crucially, however, values that are determined during *parsing* (such as the number of children for a given node, or the value of any arguments) can be safely passed into compiled JAX functions as static constants. This fact is what allows Ludax to create JAX functions *dynamically* that nonetheless obey the constraints of vectorization and JIT compilation. At the top of the parse tree, these composed JAX functions are ultimately used to define the behaviors that appear in the environment's step function, such as applying the player's action to the board and handling move effects.

We next discuss some of the specific optimizations used by Ludax. In general, these are not *global* optimizations: they apply only to certain compositions of game rules and mechanics. Our approach is to deploy these optimizations when they are available and to "fall back" on slower but more general solutions when they are not.

**Precomputation:** An important optimization used by the PGX library (and JAX environments more generally) is to express functions as batched matrix operations rather than iterative procedures. For instance, rather than checking for a line of pieces in *Tic-Tac-Toe* by starting at the position of the last move and scanning out in each direction (as Ludii's implementation does), PGX hard-codes the set of board indices that correspond to each possible line of three in the game (i.e. [[0, 1, 2], [0,

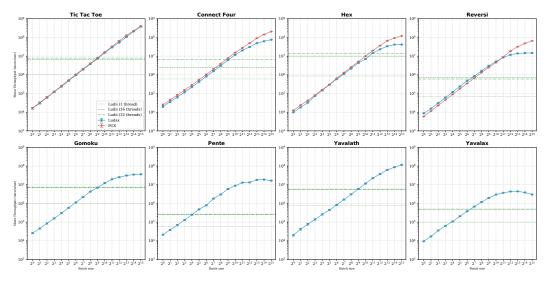


Figure 2: Average throughput (moves per second) on various exemplar games for Ludax, Ludii, and PGX. The top row of games are implemented in all three frameworks, while the bottom row of games are implemented only in Ludax and Ludii. Speeds for Ludax and PGX are reported for 500 episodes of various batch sizes on a workstation with a single NVIDIA 4090 GPU and 32 CPU cores, while speeds for Ludii are reported for parallel execution on the same workstation across 1, 16, and 32 threads. Error bars are standard deviations calculated over the 500 episodes.

3, 6], [0, 4, 7], ...]) and performs a single multi-dimensional index into the board array — if any of the of the board index triples all correspond to positions occupied by a single player, then the game is over. Ludax adopts and generalizes this approach: during parsing of line, for example, the line indices are computed with respect to the size and shape of the game board (i.e. rectangular, hexagonal, ...) as well as the length and orientation of the desired line (i.e. diagonal, vertical, ...). Again, because these values depend only on attributes that are determined during parsing, they can be passed into JAX functions as constants. Precomputation naturally causes a trade-off between compile-time and run-time efficiency. In our case, we opt to use precomputation whenever possible, though some masks and functions cannot be expressed this way.

**Dynamic State Attributes:** Different games require tracking different kinds of information about the current game state. Most obviously, some games track a score for each player while others do not. When Ludax compiles a game, it automatically extracts the attributes required to instantiate a game state and omits the others, thereby reducing the memory footprint of the entire state object. More importantly, Ludax also automatically adds intermediary computations to each call of the environment's step function that help speed up later mask, function, or predicate evaluations. For example, in *Hex*, the game ends when one player manages to connect two opposite sides of the board with a continuous path of their pieces. Naively, checking whether the edges of the board are linked requires the expensive step of computing the board's connected components after each move. However, *updating* the board's connected components as a result of placing a single piece can be done very efficiently (a technique used well in the PGX implementation). At compile time, Ludax determines whether a game makes use of a "connection" rule and modifies the step function to iteratively update and track the board's connected components if so, greatly speeding up later checks. In future extensions, this functionality will be used to accommodate games with atypical or computationally expensive rules without affecting the runtime of existing games.

### 5 Expressive Range

As mentioned above, Ludax currently supports a relatively narrow class of games: two-player, perfect-information board games played by placing, capturing, and flipping pieces. Both Ludii and PGX contain many games that Ludax does not: PGX includes implementations of *Backgammon*, *Chess*, *Shogi*, and *Go*, along with hidden-information and Atari-style games (though Ludax also

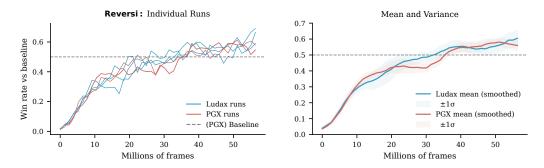


Figure 3: **Performance of reinforcement learning agents trained in the** Ludax **and** PGX **implementations of** *Reversi* **against the** PGX **baseline agent.** On the left, we plot the average winrate of the learned agents against the baseline over time and across three separate runs. On the right, we plot the average and variance of the winrates. Each run took roughly 3 hours to complete on a workstation with a single A100 GPU.

supports games that do not appear in PGX), while Ludii's description language can encode a vast array of board game styles and subgenres. Despite this, Ludax's description language remains quite expressive. In addition to simple m-n-k line completion games, Ludax supports complex and asymmetric winning conditions (e.g.  $mis\`ere$  variants, score-based victory), piece capturing and flipping, directional adjacency checks and restrictions, "custodial" mechanics, and games based on connecting arbitrary board regions. Ludax also supports regular rectangular and hexagonal boards of arbitrary sizes, as well as "hexagonal-rectangular" boards (e.g. as used in Hex). These components can then be combined compositionally to form a wide array of unique mechanics and dynamics. In addition, because Ludax is a general description language, implementing a single new game component expands the entire space of games in the framework. While the class of games representable in Ludax may at present be smaller than that of Ludii or other game description languages, it remains expansive.

### 6 Benchmarking

We benchmark the speed of Ludax on a set of 8 games, 4 of which are also implemented in both Ludii and PGX (allowing for a full comparison) and 4 of which are implemented only in Ludax and Ludii. Again, we emphasize that these 8 games are just *exemplars* of the class of games which Ludax supports, not an exhaustive list. A full description of each benchmark game is available in the Supplementary Material. We perform each of our benchmarking experiments on a workstation with a single NVIDIA 4090 GPU, 32 CPU cores, and 128GB of memory. In Figure 2 we plot the throughput (in steps per second) under a uniformly random action policy for each game environment against the batch size (log scale on both axes), with the standard deviation of throughputs across episodes as error bars. Ludii supports parallelization via multi-threading: we report throughput on the same workstation when parallelized on 1, 16, and 32 threads. Evaluations for Ludax and PGX were obtained by performing 100 warmup full-game episodes at the specified batch size, followed by measuring the speed over 500 episodes, with each evaluation taking at most a few minutes to complete. Evaluations for Ludii were obtained by running warmup episodes for 10 seconds, followed by measuring the speed over 30 seconds of episodes.<sup>2</sup>

Overall, Ludax achieves speeds that are competitive with state-of-the-art JAX environments. At small batch sizes, its throughput is similar to that of the PGX implementations. At larger batch sizes in more complicated games (i.e. *Hex* and *Reversi*), PGX takes a clear edge – though Ludax remains within an order of magnitude of PGX. The comparative "plateauing" of Ludax's speed at high batch sizes may be due to memory pressure – for instance, Ludax's implementation of *Hex* maintains both a board and the connected components for each game state, whereas the PGX implementation cleverly

<sup>&</sup>lt;sup>2</sup>We opted to measure speed for Ludax and PGX using a fixed number of episodes because JAX's compilation procedure makes it difficult to halt execution after a specific elapsed wall time.

combines both into a single array. This kind of optimization is of course theoretically implementable in Ludax as well, though again we emphasize the desiderata of avoiding *game-specific* solutions.

Ludax also outspeeds Ludii on 16 and 32 threads across all 8 games, achieving a maximum speedup of between ~3x (Hex) and ~55x (Pente). We note that there are factors that both advantage and disadvantage Ludax in this specific comparison against Ludii. One potential advantage for Ludax is its smaller representation space – implementations of basic mechanics in Ludii support a wider range of optional arguments and board types, with a corresponding increase in computational overhead (though see Section 4 for how this may be avoided). Conversely, Ludii's ability to use dynamically-sized data structures brings advantages that are particularly beneficial in uniformly random playouts, but would (partially) disappear in playouts using deep reinforcement learning. Specifically, where Ludax samples random actions from categorical probability distribution by masking over the entire board at each step, Ludii selects random actions simply by taking a random index from a list. Ludii also has optimized playout implementations tailored towards the many of the categories of games covered by Ludax [43], though these optimizations are also more difficult to apply in the context of deep learning.

## 7 Learned Agents

Finally, we demonstrate the feasibility of training reinforcement learning agents using the Ludax framework. We train our agent on the game *Reversi* (also known as *Othello*) using the AlphaZerostyle [42] training script from the PGX library<sup>3</sup> (making only slight modifications to accommodate minor differences between the Ludax and PGX APIs). We use the same ResNetV2 [22] network architecture and training hyperparameters as PGX (full details available in the Supplementary Material) and train three separate runs on a single A100 GPU. Each run lasted roughly 57 million frames and took roughly three hours to complete.

We compare the performance of agents trained in the Ludax and PGX environments against the baseline *Reversi* agent provided by the PGX library in Figure 3. Evaluations were performed by playing two batches of 1024 games (one with the learned agent as the first player and one as the second player), with actions sampled from the normalized output of the policy head at each step. We see that both learned agents achieve remarkably similar performances against the baseline, with little to no differences in learning speed or stability. While a more thorough, tournament-based evaluation would be necessary to properly rank the agents against each other, our objective is to demonstrate the general success of the training procedure and not to definitively defeat the baseline agent. Although the PGX implementation of the *Reversi* environment is slightly more efficient, this translated into only marginal improvements in overall runtime (about 1.5%) owing to the shared overhead of network forward passes and weight updates. Like PGX, Ludax offers a familiar API and an efficient set of implementations with which to train learned player agents.

#### 311 8 Limitations

Generality: As mentioned in Section 5, Ludax currently supports a smaller class of games than other comparable game description languages. While we aim to increase the range of games expressible in Ludax (see below), it will likely never match the full generality of Ludii. As such, other frameworks may be more appropriate for use-cases in which a broad range of games is more important than rapid simulation. Further, Ludax does not support genres other than board games (e.g. video games, card games, ...) – we leave the development of hardware accelerated description languages for such domains as an exciting area of future work.

**Efficiency:** Compared to bespoke JAX implementations of board games (such as in the PGX library), environments in Ludax have worse throughput – especially at larger batch sizes. While we deploy a number of optimizations to help close the efficiency gap when possible (see Section 4), there are ultimately unavoidable trade-offs between speed and generality. For the purpose of training or benchmarking single-task agents on existing games, hard-coded simulators are likely the superior choice. We note, however, that Ludax also provides a way for non-experts to design and benchmark on novel games without having to write any JAX code themselves.

<sup>&</sup>lt;sup>3</sup>https://github.com/sotetsuk/pgx/blob/main/examples/alphazero/train.py (used under Apache 2.0 license)

#### **Future Work** 9

The most obvious avenue of extension for Ludax is the implementation of additional game mechanics. 327 In particular, we aim to support games based on piece movement (e.g. Amazons), games with multiple 328 piece types (e.g. Checkers), and games with multiple distinct gameplay phases (e.g. Nine-Men's 329 Morris). Other generalizations, such as support for irregular board shapes, would also help expand 330 the space of games representable in Ludax. In addition, it's also very likely that the implementation 331 of specific gameplay elements could be further optimized for throughput and / or memory footprint. However, a balance must be struck between efficiency and generality: a less efficient solution which accommodates all valid games under the grammar is ultimately preferable to one which only applies 335 to a subset of games. Lastly, we aim to provide a more robust visual interface for Ludax, both for the purpose of facilitating human-subject research and the potential development of more "human-like" 336 artificial agents which process the game board at the pixel level and select actions spatially. 337

We are particularly excited about the potential application of Ludax to the study of automated game 338 design (or reward-guided program synthesis more generally [7] [44] [34]). Systems like GAVEL [47] 339 depend on both a broad representation space and rapid evaluation of novel games. The efficiency of 340 341 Ludax may make it possible to train a reinforcement learning agent from scratch as part of the inner loop of game evaluation, potentially unlocking a new range of computational features (e.g. learning 342 curves) that correlate with human notions of fun and engagement. Relatedly, Ludax may prove useful 343 to research on human behavior and play. Recent work has explored heuristic-based computational 344 models of human play on simple line completion games [48], and Ludax offers the possibility to both 345 accelerate computation and broaden the domain to a wider class of games. Finally, Ludax offers an avenue to extend recent research in general game playing (e.g. with large language models [40]) by providing a wide base of efficient game implementations that can in turn be leveraged for tree search algorithms or training world models.

#### **Conclusions** 10

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We introduce a novel framework for games research that combines the generality of game description 351 languages with the efficiency of modern hardware-accelerated learning environments. Our framework, 352 Ludax, represents a broad class of two-player board games and compiles directly into code in the 353 JAX Python library. Games in Ludax achieve speeds that are competitive with hand-crafted JAX 354 implementations and faster than the widely-used Ludii game description language, and Ludax 355 environments can easily be deployed in existing pipelines for deep reinforcement learning. Our 356 framework helps generalize and accelerate games research, with the potential to unlock entirely new 357 kinds of agents and systems. 358

#### **Broader Impact** 359

This paper presents a general framework with the goal of advancing reinforcement learning and 360 games research. While there are many potential societal consequences of such work in general, we 361 do not feel that any must be specifically highlighted here. Ludax does not use or reproduce any 362 copyrightable game material (i.e. art, specific expressions of rules, or game code). 363

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# **Appendices**

# 504 A Example Games and Syntax

Below we present the Ludax syntax for a small set of exemplar games (*Reversi*, *Connect Four*, *Yavalax*, and *Pente*) to help illustrate aspects of Ludax's syntax and structure.

```
(game "Reversi"
(players 2)
(equipment
                                                                                                        (game "Connect-Four"
(players 2)
(equipment
           (board (square 8))
                                                                                                                  (board (rectangle 6 7))
                                                                                                                  (play
(repeat (P1 P2)
           (start
                (place P1 (28 35))
(place P2 (27 36))
                                                                                                                             (place (destination (and empty (or
                 (repeat (P1 P2)
                                                                                                                                             (adjacent occupied direction:up)
                            (destination empty)
(result
(exists
                                       (custodial any)
                                                                                                                       (if (line 4) (mover win))
(if (full_board) (draw))
                                 (flip (custodial any))
                                  (set_score mover (count (occupied mover)))
(set_score opponent
        (count (occupied opponent))
                       (force_pass)
                 (if (passed both) (by_score))
)))
(game "Yavalax"
(players 2)
(equipment
                                                                                                            (players 2)
(equipment
(board (square 19))
           (board (square 13))
           (play
                                                                                                                  (play
                (once_through (P1)
                                                                                                                       (once_through (P1)
                      (place (destination empty))
                                                                                                                             (place (destination center))
                 (repeat (P2 P2 P1 P1)
                                                                                                                        (repeat (P2 P1)
                                                                                                                             (place
                                                                                                                                  ace
(destination empty)
(effects
(capture (custodial 2) increment_score:true)
                            (destination empty)
                                       (not (line 5))
(not (= (line 4) 1))
                                                                                                                        (if (line 5) (mover win))
                                                                                                                        (if (>= (score mover) 10) (mover win))
(if (full_board) (draw))
                (if (>= (line 4) 2) (mover win))
(if (full_board) (draw))
```

Figure 4: Ludax syntax for *Reversi* and *Connect Four* (classic board games), as well as *Yavalax* and *Pente* (modern board games).

### 507 B Ludax Grammar

Below we present the complete grammar specification for Ludax, using the syntax of the Lark Python library.

510 // -----

```
game: "(game" name players equipment rules ")"
511
512
    // ---Players---
513
    players: "(players" positive_int ")"
514
515
    // ---Equipment---
516
    equipment: "(equipment" board")"
    board: "(board" (board_square | board_rectangle | board_hexagon | board
518
        _hex_rectangle) ")"
519
    board_square: "(square" number ")"
520
    board_rectangle: "(rectangle" number number ")"
521
    board_hexagon: "(hexagon" number ")"
    board_hex_rectangle: "(hex_rectangle" number number ")"
523
    // ---Rules---
    rules: "(rules" start_rules? play_rules end_rules ")"
526
527
    // ---Start rules---
528
   start_rules: "(start" start_rule+ ")"
529
    start_rule: start_place
530
    start_place: "(place" player_reference pattern_arg ")"
531
   // ---Play rules---
533
   play_rules: "(play" play_phase+ ")"
534
   play_phase: phase_once_through | phase_repeat
535
   phase_once_through: "(once-through" play_mover_order play_mechanic ")"
536
    phase_repeat: "(repeat" play_mover_order play_mechanic ")"
537
    play_mover_order: "(" player_reference+ ")"
538
    play_mechanic: play_place force_pass?
    play_place: "(place" mover_reference? place_destination_constraint place
541
        _result_constraint? play_effects?")"
542
    force_pass: "(force_pass" ")"
543
544
    // ---Constraints---
545
    place_destination_constraint: "(destination" super_mask ")"
   place_result_constraint: "(result" super_predicate ")"
548
    // ---Effects---
549
    play_effects: "(effects" play_effect+ ")"
550
   play_effect: effect_capture
551
              | effect_flip
552
              | effect_increment_score
553
              | effect_set_score
554
555
    effect_capture: "(capture" super_mask mover_reference? increment_score_arg?
556
557
    effect_flip: "(flip" super_mask mover_reference? ")"
558
    effect_increment_score: "(increment_score" mover_reference function ")"
559
    effect_set_score: "(set_score" mover_reference function ")"
560
561
   // ---Functions---
    function: function_add
563
           | function_connected
564
            | function_constant
565
           | function count
566
           | function_line
567
           | function_multiply
568
           | function_score
569
```

```
| function_subtract
570
571
    function_add: "(add" function+ ")"
572
    function_connected: "(connected" multi_mask_arg mover_reference?
573
        direction_arg? ")"
574
   function_constant: positive_int
575
    function_count: "(count" super_mask ")"
    function_line: "(line" positive_int orientation_arg? exact_arg? ")"
    function_multiply: "(multiply" function+ ")"
    function_score: "(score" mover_reference ")"
579
   function_subtract: "(subtract" function function ")"
580
581
   // ---End rules---
582
    end_rules: "(end" end_rule+ ")"
    end_rule: "(if" super_predicate end_rule_result ")"
584
    ?end_rule_result: result_win | result_lose | result_draw | result_by_score
585
586
   // -- Result definitions --
587
   result_win: "(" mover_reference "win" ")"
588
   result_lose: "(" mover_reference "lose" ")"
589
   result_draw: "(" "draw" ")"
   result_by_score: "(" "by_score" ")"
592
   // -- Mask definitions --
593
   super_mask: mask | super_mask_and | super_mask_or | super_mask_not
594
    super_mask_and: "(and" super_mask+ ")"
595
    super_mask_or: "(or" super_mask+ ")"
596
    super_mask_not: "(not" super_mask ")"
597
598
    mask: mask_adjacent
599
       | mask_center
600
        | mask_corners
601
        | mask_custodial
602
        | mask_edge
603
        | mask_empty
604
        | mask_occupied
605
        | mask_pattern
607
        | mask_prev_move
608
   mask_adjacent: "(adjacent" super_mask direction_arg? ")"
609
   mask center: "center"
610
   mask_corners: "corners"
611
   mask_custodial: "(custodial" custodial_length_arg mover_reference?
612
        orientation_arg? ")"
613
   mask_edge: "(edge" edge ")"
614
   mask_empty: "empty"
615
   mask_occupied: "occupied" | "(occupied" mover_reference ")"
616
   mask_pattern: "(pattern" dimensions_arg pattern_arg rotate_arg? ")"
617
   mask_prev_move: "(prev_move" mover_reference ")"
618
619
    // "Multi-masks" are special keywords that are manually split into multiple
620
    // sub-masks at compile time. This is mostly useful for the "connected"
621
        function,
622
    // which expects a list of masks to check for connections between
623
    multi_mask: multi_mask_corners
624
             | multi_mask_edges
625
             | multi_mask_edges_no_corners
626
627
   multi_mask_corners: "corners"
```

```
multi_mask_edges: "edges"
629
   multi_mask_edges_no_corners: "edgesNoCorners"
630
631
   // ---Predicate definitions---
632
    super_predicate: predicate | super_predicate_and | super_predicate_or |
633
        super_predicate_not
634
    super_predicate_and: "(and" super_predicate+ ")"
635
    super_predicate_or: "(or" super_predicate+ ")"
636
    super_predicate_not: "(not" super_predicate ")"
637
638
   predicate: predicate_equals
639
            | predicate_exists
640
            | predicate_full_board
641
              predicate_function
642
              predicate_greater_equals
643
              predicate_less_equals
644
            | predicate_mover_is
645
            | predicate_passed
646
647
   predicate_equals: "(=" function+ ")"
648
   predicate_exists: "(exists" super_mask ")" // technically equivalent to (>=
649
         (count mask) 1)
   predicate_full_board: "(" "full_board" ")"
651
   predicate_function: function // special syntax which is equivalent to "(>=
652
        function 1)"
653
   predicate_greater_equals: "(>=" function function ")"
654
   predicate_less_equals: "(<=" function function ")"</pre>
655
   predicate_mover_is: "(mover_is" player_reference ")"
656
   predicate_passed: "(passed" (mover_reference | BOTH) ")"
657
    // Additional (potentially optional) arguments for predicates
659
    custodial_length_arg: ANY | positive_int
660
   dimensions_arg: "(" positive_int positive_int ")"
661
   direction_arg: "direction:" direction
662
   exact_arg: "exact:" boolean
663
   increment_score_arg: "increment_score:" boolean
   multi_mask_arg: multi_mask | "(" super_mask+ ")"
   orientation_arg: "orientation:" orientation
666
   pattern_arg: "(" positive_int+ ")"
667
   rotate_arg: "rotate:" boolean
668
669
   // General-purpose definitions
670
   ?number: SIGNED_NUMBER
671
   ?positive_int: /[0-9]+/
672
    ?boolean: TRUE | FALSE
673
    ?edge: TOP | BOTTOM | LEFT | RIGHT | TOP_LEFT | TOP_RIGHT | BOTTOM_LEFT |
674
675
        BOTTOM_RIGHT
    ?direction: UP | DOWN | LEFT | RIGHT | UP_LEFT | UP_RIGHT | DOWN_LEFT |
676
        DOWN_RIGHT | VERTICAL | HORIZONTAL | ORTHOGONAL | DIAGONAL |
677
       BACK_DIAGONAL | FORWARD_DIAGONAL | ANY
678
   ?orientation: VERTICAL | HORIZONTAL | ORTHOGONAL | DIAGONAL | BACK_DIAGONAL
679
         | FORWARD_DIAGONAL | ANY
680
    // -----
681
682
    ?player_reference: P1 | P2
683
   ?mover_reference: MOVER | OPPONENT
684
   name: STRING
685
   variable_name: /?[a-z][a-z0-9]*/
   id: /[a-zA-Z0-9_]+/
```

```
688
    // Constants
689
    TOP: "top"
690
    BOTTOM: "bottom"
691
    UP: "up"
692
    DOWN: "down"
693
    LEFT: "left"
694
    RIGHT: "right"
695
    TOP_LEFT: "top_left"
696
    TOP_RIGHT: "top_right"
697
    BOTTOM_LEFT: "bottom_left"
698
    BOTTOM_RIGHT: "bottom_right"
699
    UP_LEFT: "up_left"
700
    UP_RIGHT: "up_right"
701
    DOWN_LEFT: "down_left"
702
    DOWN_RIGHT: "down_right"
703
    VERTICAL: "vertical"
704
    HORIZONTAL: "horizontal"
705
    ORTHOGONAL: "orthogonal"
706
    DIAGONAL: "diagonal"
707
    BACK_DIAGONAL: "back_diagonal"
708
    FORWARD_DIAGONAL: "forward_diagonal"
709
    ANY: "any"
710
    TRUE: "true"
711
    FALSE: "false"
712
    MOVER: "mover"
713
    OPPONENT: "opponent"
714
    P1: "P1"
715
    P2: "P2"
716
    BOTH: "both"
717
```

# C Benchmark Game Descriptions

719

Below, we present natural language descriptions of the rules for each of the exemplar games analyzed in Section 6.

722 **Tic-Tac-Toe:** Players take turns placing a piece into an empty space on a square 3-by-3 board. If a player forms a line of three of their pieces in a row (either vertically, horizontally, or diagonally), they win. If the board is completely full but no lines have been formed, then the game ends in a draw.

Connect Four: Players take turns placing a piece into the top of one of the seven columns on a 6-by-7 board. The piece then "falls" until it rests on either the bottom of the board or another piece. A player can't place a piece into a column that is already "full." If a player forms a line of four of their pieces in a row (either vertically, horizontally, or diagonally), they win. If the board is completely full but no lines have been formed, then the game ends in a draw.

Hex: Players take turns placing a piece into an empty space on an 11-by-11 board composed of hexagonal tiles (forming a parallelogram, see visual depiction here). The objective for the first player is to form a continuous path of their pieces that connects the top edge of the board with the bottom edge, while the objective for the second player is to do the same but connect the left and right edges of the board. The first player to achieve their objective wins the game. Because of the geometric properties of the board, it's not possible for the game to end in a draw.

Reversi: The game takes place on a square 8-by-8 board. To begin, a white piece is placed at positions D4 and E5 and a black piece is placed at positions D5 and E4 (see visual depiction here). Players take turns placing a piece into an empty space such that a line of one or more of the opponent's pieces are "sandwiched" on either end by the player's pieces. This configuration is called a "custodial" arrangement of pieces. After placing a piece, any of the opponent's pieces which are in such a custodial arrangement are flipped and now belong to the player who just moved. It's possible for a

single move to form multiple custodial arrangements in different directions, in which case all of the relevant pieces are flipped. If a player cannot make a legal move, they must pass (and they cannot pass without making a move otherwise). If both players pass, then the game is over. The winner is determined by the player who has the largest number of pieces on the board at the end of the game (in the event of a tie, the game ends in a draw).

747 *Gomoku*: Players take turns placing a piece into an empty space on a square 15-by-15 board. If a
748 player forms a line of exactly five of their pieces in a row (either vertically, horizontally, or diagonally),
749 they win. However, forming a line of six or more does not count – the player must have at least one
750 line of exactly five. If the board is completely full but no lines of exactly five have been formed, then
751 the game ends in a draw.

**Pente:** Players take turns placing a piece into an empty space on a square 19-by-19 board. If a player forms a line of five of their pieces in a row (either vertically, horizontally, or diagonally), they win. In addition, if placing a piece causes a line of exactly two of the opponent's pieces to be put into a custodial arrangement, the two pieces are captured and removed from a board. Note that placing a piece *into* a custodial arrangement formed by the opponent does not result in any pieces being captured. A player who captures at least 10 of the opponent's pieces over the course of the game wins. In the variant of *Pente* implemented in Ludii and Ludax, the first player must make their first move into the exact center of the board.

Yavalath: Players take turns placing a piece into an empty space on a regular hexagonal board with a diameter of 9 spaces. If a player forms a line of four of their pieces in any direction (either diagonally or horizontally<sup>4</sup>), they win. However, if a player forms a line of three of their pieces in a row without also forming a line of four, they lose. If the board is completely full but no lines of four or three have been formed, then the game ends in a draw.

Yavalax: To begin, the first player places a piece into an empty space on a square 13-by-13 board. 765 Starting with Player 2, players then take turns placing two pieces into empty spaces on the board. 766 If a player forms at least two distinct lines of four of their pieces in any direction (either vertically, 767 horizontally, or diagonally), they win. However, a player may not place a piece into a space if doing 768 so would form a line of five pieces in any direction or if it would form exactly one line of four pieces 769 in any direction. Note that this restriction applies to a player's first move of their turn even if they 770 could form a second line of four pieces with their second move of the turn (and thus win). If the 771 board is completely full and neither player has formed at least two distinct lines of four pieces, then 772 the game ends in a draw. 773

# D Training Hyperparameters

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Below we provide the exact training hyperparameters used in the reinforcement learning experiments in Section 7. These are largely copied from the PGX implementation.

• Model architecture: Resnet V2 777 Number of channels: 128 778 • Number of layers: 6 779 780 • Self-play batch size: 1024 • Self-play simulations: 32 781 Self-play max steps: 256 782 Training batch size: 4096 783 • Learning rate: 0.001 784 • Evaluation frequency: 5 785 • Training iterations: 219 786

Note that each "iteration" consists of generating play data for 256 steps using the self-play batch size of 1024 (see [26]). We train the model for 219 iterations, which corresponds to  $256 \times 1024 \times 219 = 57409536$  (or roughly 57 million) steps in the environment.

<sup>&</sup>lt;sup>4</sup>Ludax assumes a canonical orientation for hexagonal boards in which the diameter stretches from left to right, though it is functionally equivalent to the orientation in which the diameter runs vertically)