
Co-rewarding: Stable Self-supervised RL for Eliciting Reasoning in Large Language Models

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Abstract

While reinforcement learning with verifiable rewards (RLVR) is effective to improve the reasoning ability of large language models (LLMs), its reliance on human-annotated labels leads to the scaling up dilemma, especially for complex tasks. Recent self-rewarding methods investigate a label-free alternative to unlock the reasoning capabilities of LLMs, yet they frequently encounter the non-negligible training collapse issue, as the single-view supervision signal easily forms the self-consistent illusion, yielding the reward hacking. Inspired by the success of self-supervised learning, we propose *Co-rewarding*, a novel self-supervised RL framework that improves training stability by seeking complementary supervision from another views. Specifically, we instantiate Co-rewarding in two ways: (1) *Co-rewarding-I* is a data-side instantiation that derives reward signals from contrastive agreement across semantically analogous questions; and (2) *Co-rewarding-II* is a model-side instantiation that maintains a slowly-updated reference teacher with pseudo labels to realize self-distillation. Intuitively, such instantiations introduce different levels of discrepancy to increase the difficulty of training collapse on trivial reasoning solutions. Empirically, Co-rewarding exhibits stable training across various setups, and outperforms other self-rewarding baselines by +3.31% improvements on average on multiple mathematical reasoning benchmarks, especially by +7.49% on Llama-3.2-3B-Instruct. Notably, Co-rewarding reaches or even surpasses RLVR with ground-truth (GT) label in several cases, such as a Pass@1 of 94.01% on GSM8K with Qwen3-8B-Base remarkably higher than GT. Our code is publicly available at <https://github.com/tmlr-group/Co-rewarding>.

1 Introduction

Large language models (LLMs) [1, 2, 3] have demonstrated remarkable general-purpose capabilities in a wide range of linguistic tasks [4, 5, 6, 7]. To further elicit their reasoning ability in complex scenarios, reinforcement learning with verifiable rewards (RLVR) [8, 9] is developed for post-training with externally verifiable signals like program execution results [10] or mathematical equivalence [8]. Despite the impressive improvement, the reliance on high-quality ground-truth (GT) labels of RLVR remains as a major bottleneck [11, 12] in the spirit of the scaling law, which subsequently motivates the emerging exploration of self-rewarding methods with unlabeled data [13, 14, 15].

One prominent line of such label-free methods leverages the internal signals (e.g., entropy [16, 17] and self-certainty [13]) to strengthen the confidence of the model in reasoning. Another critical line seeks the answer-level consensus [14, 18] to construct pseudo labels as reward basis. while effective

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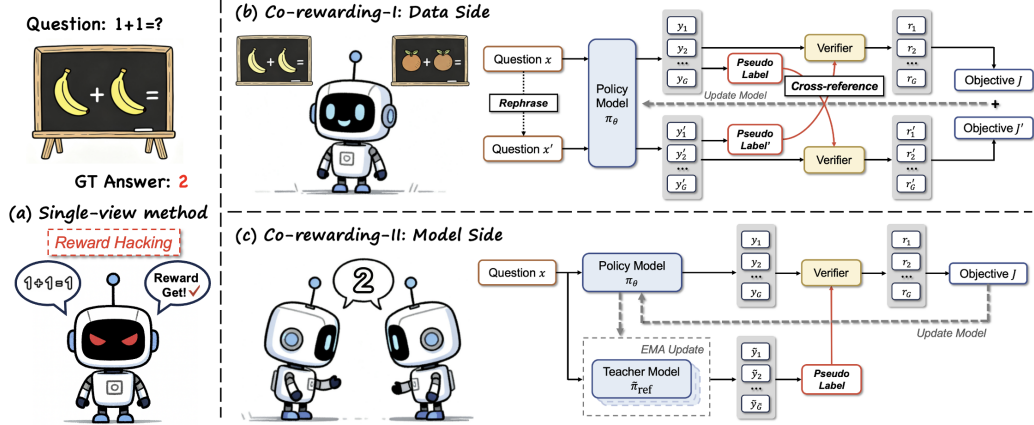


Figure 1: **Illustration of Co-rewarding framework:** Unlike single-view methods that rely only on internal reward signal on original question (a), Co-rewarding introduces complementary supervision. On the data side (b), paraphrased questions yield pseudo-labels for cross-reference. On the model side (c), teacher model isolated from current policy provides stabilized pseudo-labels for updates.

initially, these self-rewarding approaches frequently exhibit non-negligible training collapse [19] (indicated as Figure 3), which limits the scalability of such label-free training manners.

The collapse phenomenon stems from reward hacking [20] under self-consistent illusion: the reward signal is internally produced by the policy model from a single-view data perspective, which is easily trapped by trivial solutions along with training (see Figure 7). Specifically, for entropy- or certainty-based rewards, the policy model may concentrate probability mass on a small set of tokens and produce repetitive strings that minimize entropy or maximize self-certainty [19]. And for consensus-based rewards, the policy model can converge to a consistent yet incorrect answer that attains high consensus across rollouts [18]. Overall, the policy model continually reduces uncertainty without sustained gains in correctness, inflating the reward but eroding exploration and diversity. It ultimately collapses when a persistent hacking strategy emerges.

To this end, we introduce *Co-rewarding*, a self-supervised RL framework that seeks complementary supervision from another views, inspired by self-supervised learning [21, 22, 23]. Conceptually, one fundamental characteristic of self-rewarding methods lies on that supervision intertwined with current policy on single-view outputs, for which we propose to seek reasoning invariance across different views (see Figure 1). Specifically, we investigate two initiations of Co-rewarding: (1) *Co-rewarding-I*: a data-side initiation that constructs rewards via contrastive agreement across semantically analogous questions, each providing pseudo labels for the other; and (2) *Co-rewarding-II*: a model-side initiation that introduces an extra teacher with dynamically updated policy and provides stable pseudo-labels insulated from current online policy. By introducing cross-view supervision on data and decoupling the reward signal from the current policy, Co-rewarding effectively mitigates training collapse and yields stable self-supervised RL training. Experiments across multiple datasets validate the stability and superiority of Co-rewarding, compared to several recent baselines across several LLM families including Qwen3/2.5 and Llama. Notably, both Co-rewarding-I and -II reach or exceed training with ground-truth labels in several settings, such as achieving up to 94.01% Pass@1 on GSM8K.

2 Co-rewarding

2.1 Conceptual Philosophy: Invariance beyond the Single-view

At the core of self-rewarding methods lies a fundamental tension: the model derives supervisory signals from its own outputs, inevitably intertwining supervision with policy and risks collapse. True reasoning competence, however, cannot be reduced to the mere correctness of isolated answers. It should instead reflect invariance that extends beyond the single-view output for consistency. This calls for training signals that remain valid across different data views or persist throughout the temporal evolution of the model, providing a more reliable basis on which self-supervised RL can rely. In this aspect, stability arises from invariance that prevents reasoning against superficial variations in data and guides the model towards increasingly valid reasoning trajectories throughout training.

This philosophy yields our Co-rewarding framework, whose core idea is to ground self-supervised RL in invariance rather than the suspicious single-view feedback. We instantiate it in two orthogonal ways: by enforcing analogy-invariance on the data side (Co-rewarding-I) and by disentangling supervision through temporal invariance on the model side (Co-rewarding-II).

2.2 Two Initiations of Co-rewarding Framework

Co-rewarding-I: on the Data Side. Inspired by contrastive learning, such as SimCLR [21] and InfoNCE [24], where two views of the same data are encouraged to have similar representations, we hypothesize an analogy-invariance inductive property of LLMs in eliciting reasoning capacity: questions that share the same mathematical essence but differ in surface form (e.g., via paraphrasing, background substitution, or reformatting) should elicit the comparably valid and similar reasoning results. This forms the foundation for a self-referential training signal: contrastive agreement among different question variants can serve as an optimization proxy. Co-rewarding-I defines contrastive agreement as a principle that aligns model reasoning outputs, treating consistent inter-view agreement as a signal for valid inference. This complements single-view self-rewarding strategies by introducing a form of collective validity verification with broader input consideration.

Building upon the discussed contrastive agreement, we initiate our *Co-rewarding-I* as illustrated in Figure 1. Formally, its learning objective can be formulated based on GRPO:

$$\mathcal{J}_{\text{Co-rewarding-I}}(\theta) = \underbrace{\mathbb{E}_{x \in \mathcal{D}, \{y_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}(\cdot|x)} \mathcal{R}_{\theta}(\hat{A})}_{\mathcal{J}_{\text{original}}(\theta)} + \underbrace{\mathbb{E}_{x' \in \mathcal{D}', \{y'_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}(\cdot|x')}}_{\mathcal{J}_{\text{rephrased}}(\theta)} \mathcal{R}_{\theta}(\hat{A}'), \quad (1)$$

where the relative advantages are estimated by the *cross-refereed* supervision as follows:

$$\hat{A}_i = \frac{r(y'_v, y_i) - \text{mean}(\{r(y'_v, y_i)\}_{i=1}^G)}{\text{std}(\{r(y'_v, y_i)\}_{i=1}^G)}, \quad \hat{A}'_i = \frac{r(y_v, y'_i) - \text{mean}(\{r(y_v, y'_i)\}_{i=1}^G)}{\text{std}(\{r(y_v, y'_i)\}_{i=1}^G)}. \quad (2)$$

Specifically, given a set of original questions, we utilize the rephrased version that keeps the semantical equivalence for the model to respond, and then collect the self-generated pseudo-labels based on the majority voting mechanism [18] as follows to supervise learning on the counterparts,

$$y_v \leftarrow \arg \max_{y^*} \sum_{i=1}^G 1[\text{ans}(y_i) = \text{ans}(y^*)], \quad y'_v \leftarrow \arg \max_{y^*} \sum_{i=1}^G 1[\text{ans}(y'_i) = \text{ans}(y^*)]. \quad (3)$$

The overall pipeline can be viewed as a dual-path structure with cross-reference in the reward shaping process, it may also be compatible with other self-generated feedbacks [25] on the output-side information due to the generality of the core idea. While in the current version, we choose the majority voting mechanism in the implementation for the empirical effectiveness and simplicity.

We summarize the pseudo code of Co-rewarding-I in Algorithm 1. Our contrastive objective operates on self-generated reasoning answers, encouraging the model to align its reasoning results to different questions that share the similar semantic intent. Formally, for each input question, the signal of Co-rewarding-I increases when the model’s output is consistent with the majority answer obtained from its analogical counterparts, and decreases when it diverges. This contrastive agreement promotes semantic invariance, implicitly increasing the difficulty of reaching trivial solutions to obtain the reward (e.g., achieving the arbitrary answers but consistent on single input) by involving data-side analogy. We leave a more intuitive case study in the Appendix E.10 to present the rephrased questions.

Co-rewarding-II: on the Model Side. On the data side, our Co-rewarding-I provides complementary supervision by involving question analogy, while its pseudo-labels are still generated by the current online policy and may depend on rephrasing quality; consequently, supervision remains partially entangled with the policy. Inspired by self- or weakly supervised methods [26, 27] like the representative BYOL [22], DINO [23], and Co-teaching [28], which share the common intuition of introducing an auxiliary network to provide supervision beyond the current model, we initiate *Co-rewarding-II* from another view of complementary supervision: a model-side strategy that sources pseudo-labels from a teacher reference, which disentangle the self-supervision reward from the online policy. To avoid the heavy cost of adding and maintaining another LLM in training, Co-rewarding-II reuses the GRPO reference model as the teacher to generate the rollouts and produce pseudo-labels. In particular, the teacher is dynamically updated as an exponential moving average (EMA) of the student policy to ensure pseudo-label quality improving as the policy improves.

Intuitively, we illustrate *Co-rewarding-II* in Figure 1. Its learning objective can be formulated as:

$$\mathcal{J}_{\text{Co-rewarding-II}}^{(k)}(\theta) = \mathbb{E}_{x \in \mathcal{D}, \underbrace{\{y_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}^{(k)}(\cdot|x)}_{\text{policy student rollout}}, \underbrace{\{\tilde{y}_j\}_{j=1}^{\tilde{G}} \sim \tilde{\pi}_{\text{ref}}^{(k)}(\cdot|x)}_{\text{reference teacher rollout}}} \mathcal{R}_{\theta}(\hat{A}^{(k)}), \quad (4)$$

where $\{y_i\}_{i=1}^G$ are policy rollouts and $\{\tilde{y}_j\}_{j=1}^{\tilde{G}}$ are reference teacher rollouts at the k -th training step, and the estimated advantage $\mathcal{R}(\hat{A}^{(k)})$ is computed as follows:

$$\hat{A}_i^{(k)} = \frac{r(\tilde{y}_v^{(k)}, y_i) - \text{mean}(\{r(\tilde{y}_v^{(k)}, y_i)\}_{i=1}^G)}{\text{std}(\{r(\tilde{y}_v^{(k)}, y_i)\}_{i=1}^G)}, \quad \tilde{y}_v^{(k)} = \arg \max_{y^*} \sum_{j=1}^{\tilde{G}} \mathbf{1}[\text{ans}(\tilde{y}_j) = \text{ans}(y^*)], \quad (5)$$

where the pseudo label $\tilde{y}_v^{(k)}$ is obtained via majority voting from reference rollouts, and the reference model is updated via an EMA with the policy to play a role of a slowly updated teacher:

$$\tilde{\pi}_{\text{ref}}^{(k)} \leftarrow \alpha^{(k)} \cdot \tilde{\pi}_{\text{ref}}^{(k-1)} + (1 - \alpha^{(k)}) \cdot \pi_{\theta_{\text{old}}}^{(k)}, \quad \alpha^{(k)} = 1 - \frac{(\alpha_{\text{end}} - \alpha_{\text{start}})}{2} \left(1 + \cos \left(\frac{\pi k}{K} \right) \right) \quad (6)$$

where $\alpha^{(k)} \in (0, 1)$ is the EMA weight, updated according to a cosine annealing schedule from α_{start} to α_{end} , such that the teacher is updated rapidly at the beginning and progressively more slowly, thereby evolving smoothly and remaining temporally decoupled from the current online policy.

We summarize the pseudo code of Co-rewarding-II in Algorithm 2. This design can be interpreted as a kind of self-distillation, in which a slowly updated teacher supervises a faster-moving student. Such a paradigm breaks the single-step on-policy feedback loop inherent in existing self-rewarding methods [13, 17, 18], raises the cost of exploiting trivial low-entropy shortcuts or spurious consensus, and offers a stable reward source without introducing an additional LLM or optimizer. In this way, it effectively prevents training collapse by implicitly seeking a temporal invariance for true reasoning.

Remark 1. Overall, the two instantiations of Co-rewarding embody our core idea from different perspectives: I leverages data-side analogy-invariance to provide cross supervision, while II employs model-side self-distillation to stabilize learning. Together, they reflect that stable self-supervised reasoning elicitation can emerge from both the diversity of data perspectives and the disentanglement of supervision signals. Moreover, Co-rewarding offers a flexible framework, in which key components, such as pseudo-labeling strategies, data rephrasing techniques, teacher model update rules, and policy optimization, can be seamlessly substituted with other advanced approaches [9].

3 Experimental Results

Superior Performance of Co-rewarding over self-rewarding baselines. Table 1 and Table 4 report the experimental results trained on MATH and DAPO-14k, respectively. We observe that both Co-rewarding-I and Co-rewarding-II occupy more darker cells in the tables, demonstrating stronger performance than other self-rewarding SoTA baselines. Specifically, Co-rewarding-I achieves an average relative performance gain of +3.46% over the best baselines across three mathematical benchmarks and models in Table 1, while Co-rewarding-II achieves a larger average relative gain of +7.29% in Table 4. Additional results experimental are provided in Appendix E.

Surpassing GT-Reward in certain benchmarks. Surprisingly, we observe that both Co-rewarding-I and Co-rewarding-II outperform GT-Reward in certain cases. On GSM8K, they together achieve an average relative improvement of +2.94% over GT-Reward in Table 1, while Co-rewarding-II further delivers a larger gain of +5.44% in Table 4. Co-rewarding also shows advantages on the coding benchmark CRUX in several cases. Notably, Co-rewarding-II achieves a remarkably high Pass@1 of 94.01% with Qwen3-8B-Base on GSM8K. These findings highlight the potential of self-supervised RL to elicit stronger reasoning abilities, particularly with Co-rewarding mitigating training collapse.

Importance of stability for performance gain. As shown in Table 4, self-rewarding baselines exhibit noticeably limited performance gain in certain cases, such as Self-Certainty with Qwen3-4B-Base on GSM8K. Figure 2 further reflects this by showing that baselines improve quickly at the beginning but soon collapse on GSM8K and AMC, whereas Co-rewarding sustains steady progress. This collapse restricts the baselines to effective training on only a small portion of the data, preventing further

Table 1: **Main Results (%) of Co-rewarding and baselines trained on MATH.** Cell background colors indicate relative performance: darker colors denote better results within each model group.

Training Set: MATH	Mathematics			Code		Instruction	Multi-Task
Methods	MATH500	GSM8K	AMC	LiveCode	CRUX	IFEval	MMLU-Pro
<i>Qwen3-8B-Base</i>							
Before RL	72.4	27.82	20.93	23.41	54.75	50.89	52.92
- GT-Reward [8]	82.6	87.26	54.22	30.52	63.25	52.78	57.11
- Self-Certainty [13]	80.2	80.74	50.75	27.20	64.38	50.98	54.17
- Entropy [17]	80.2	87.19	49.54	29.38	62.00	51.81	54.86
- Majority-Voting [18]	79.8	89.76	49.09	30.52	63.38	51.80	56.93
- Co-rewarding-I (Ours)	81.2	93.70	51.20	30.81	66.00	55.79	59.95
- Co-rewarding-II (Ours)	80.8	92.42	53.46	30.23	62.83	60.70	57.50
<i>Qwen3-4B-Base</i>							
Before RL	71.2	26.15	21.08	11.00	38.88	46.43	47.23
- GT-Reward [8]	78.6	89.76	51.20	26.07	55.38	47.80	53.96
- Self-Certainty [13]	71.6	71.79	38.86	22.37	57.00	48.15	48.93
- Entropy [17]	77.0	88.10	47.44	25.59	52.88	50.44	49.90
- Majority-Voting [18]	77.4	90.07	45.33	26.54	57.50	48.78	54.35
- Co-rewarding-I (Ours)	78.8	91.28	46.08	26.64	56.50	50.35	53.26
- Co-rewarding-II (Ours)	78.0	88.86	45.93	26.25	55.00	51.30	53.88
<i>Llama-3.2-3B-Instruct</i>							
Before RL	39.2	65.73	10.54	9.86	25.37	57.32	31.14
- GT-Reward [8]	47.0	77.94	22.14	9.57	31.87	47.51	34.32
- Self-Certainty [13]	43.4	74.91	18.83	9.95	25.87	54.88	33.34
- Entropy [17]	43.4	66.19	20.18	11.66	24.62	54.70	33.52
- Majority-Voting [18]	46.8	78.77	20.48	11.00	31.25	47.96	33.18
- Co-rewarding-I (Ours)	50.2	79.45	23.80	11.28	29.88	48.89	33.77
- Co-rewarding-II (Ours)	49.8	79.30	22.59	10.80	30.63	49.90	33.61

improvements with continued training. These observations underscore the importance of avoiding training collapse in self-supervised RL to unlock further performance gains.

Each part contributes to Co-rewarding. Table 5 summarizes the ablations. For Co-rewarding-I, replacing cross supervision with training only on original or rephrased data yields comparable results, but both underperform the full method. This indicates that the quality of original and rephrased data is similar, while the cross supervision across views provides the crucial gains. For Co-rewarding-II, removing the EMA update of the reference teacher model causes clear degradation, highlighting the necessity of teacher updates for improving pseudo-label quality.

Co-rewarding alleviates collapse and provides stable self-supervised RL. We use 5,000 questions from the MATH test split as a validation set to monitor training process. Figure 3 shows the all three self-rewarding baselines collapse on both MATH and DAPO-14k. Co-rewarding-I remains stable on MATH but still collapses on DAPO-14k, suggesting that its stability depends on the property of training data. A plausible explanation is that MATH contains more questions with rich background descriptions than DAPO-14k, which makes rephrasing more effective in amplifying differences between original and rephrased data, thereby strengthening contrastive agreement. In contrast, Co-rewarding-II consistently maintains stability across datasets, as its design decouples supervision from the online policy and thus breaks the entanglement between supervision signals and the policy itself.

4 Conclusion

In this work, we introduced Co-rewarding, a self-supervised RL framework that elicits the reasoning capability of LLMs through complementary supervision. Unlike prior self-rewarding methods that entangle rewards with single-view outputs and risk collapse, Co-rewarding establishes stability by decoupling the reward signal from the current online policy with the single-view output. Specifically, Co-rewarding-I leverages contrastive agreement across semantically analogous questions, while Co-rewarding-II employs a dynamically updated teacher to provide insulated pseudo-labels. Together, these designs construct cross-referable reward signals without explicit labels, aligning RL with invariances in reasoning rather than the mere correctness of isolated outputs. We hope this work will inspire further exploration into self-supervised RL for reasoning to advance the development.

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A Preliminary

Problem Setups. Given a LLM π_θ parameterized by θ and a dataset $\mathcal{D}$ of question–answer pairs (x, a) , the model generates a response $y \sim \pi_\theta(\cdot | x)$ autoregressively. Let $y = (y_1, \dots, y_n)$, where each token is sampled as $y_t \sim \pi_\theta(\cdot | x, y_{<t})$ given the generated prefix $y_{<t}$. We consider the LLM outputs a stepbystep reasoning trace and a final answer. A verifiable reward function $r(a, y)$ compares the extracted answer $\text{ans}(y)$ with the ground truth a as follows:

$$r(a, y) = \begin{cases} 1 & \text{If } \text{ans}(y) \text{ is correct with answer } a, \\ 0 & \text{If } \text{ans}(y) \text{ is incorrect with answer } a. \end{cases} \quad (7)$$

Then, the general objective of training LLM for reasoning via RLVR [8, 9] can be formulated with the policy model π_θ as follows:

$$\max_{\pi_\theta} \mathbb{E}_{(x,a) \in \mathcal{D}, y \sim \pi_\theta(x)} [r(a, y) - \beta \cdot \text{KL}[\pi_\theta(y|x) || \pi_{\text{ref}}(y|x)]], \quad (8)$$

where π_{ref} is an initial reference policy, and β is a coefficient controlling the KL divergence to prevent excessive deviation from the reference model. Intuitively, the training target is to maximize the reward in passing specific reasoning questions while maintaining the general capability of LLM.

Group Relative Policy Optimization (GRPO). In practice, we adopt GRPO [8], a widely used and representative optimization method for objective Eq. (8) that estimates the advantage by normalizing the reward across multiple sampled outputs for the same question. Specifically, for a given question x , GRPO samples G outputs from the old policy π_{old} as $\{y_i\}_{i=1}^G \sim \pi_{\text{old}}(\cdot|x)$. It then computes a reward for each output y_i via a deterministic reward function, forming a group of rewards $\{r(a, y_i)\}_{i=1}^G$ to estimate the advantage $\hat{A}_i$ as follows:

$$\hat{A}_i = \frac{r(a, y_i) - \text{mean}(\{r(a, y_i)\}_{i=1}^G)}{\text{std}(\{r(a, y_i)\}_{i=1}^G)}. \quad (9)$$

Then, the target policy is optimized by maximizing the advantage while ensuring the policy model remains close to the reference policy:

$$\begin{aligned} \mathcal{J}_{\text{GRPO}}(\theta) = & \mathbb{E}_{(x,a) \in \mathcal{D}, \{y_i\}_{i=1}^G \sim \pi_{\text{old}}(\cdot|x)} \\ & \underbrace{\frac{1}{G} \sum_{i=1}^G \frac{1}{|y_i|} \sum_{t=1}^{|y_i|} \left(\min \left[c_{i,t}(\theta) \hat{A}_{i,t}, \text{clip}(c_{i,t}(\theta), 1 - \epsilon, 1 + \epsilon) \hat{A}_{i,t} \right] - \beta \mathbb{D}_{\text{KL}}(\pi_\theta || \pi_{\text{ref}}) \right)}_{\mathcal{R}_\theta(\hat{A})}, \end{aligned} \quad (10)$$

where

$$c_{i,t}(\theta) = \frac{\pi_\theta(y_{i,t}|x, y_{i,<t})}{\pi_{\text{old}}(y_{i,t}|x, y_{i,<t})}, \quad \mathbb{D}_{\text{KL}}(\pi_\theta || \pi_{\text{ref}}) = \frac{\pi_\theta(y_{i,t}|x, y_{i,<t})}{\pi_{\text{ref}}(y_{i,t}|x, y_{i,<t})} - \log \frac{\pi_{\text{ref}}(y_{i,t}|x, y_{i,<t})}{\pi_\theta(y_{i,t}|x, y_{i,<t})} - 1. \quad (11)$$

Note that the $\text{clip}(\cdot, 1 - \epsilon, 1 + \epsilon)$ in Eq. (10) is used to ensure that updates do not deviate excessively from the old policy by bounding the policy ratio between $1 - \epsilon$ and $1 + \epsilon$ in a risk function $\mathcal{R}(\hat{A})$. We also provide a comprehensive discussion on additional training variants for RLVR, such as DAPO [9] and Dr. GRPO [29], which we leave in Appendix B.

B Related Work

Large Language Model Reasoning. LLMs have shown impressive performance on vast tasks that require reasoning, including solving mathematical problems [30], writing code [10], and logical questions [31]. One of the key techniques that has improved LLM reasoning is Chain-of-Thought (CoT) prompting [32]. CoT encourages the model to generate intermediate reasoning steps before producing the final answer, which has been shown to enhance performance on tasks like arithmetic, commonsense reasoning, and symbolic reasoning. Subsequent work has extended CoT by integrating it with various strategies, including compositional generalization [33, 34] and employing structural reasoning approaches [35, 36, 37]. In addition, CoT serves as a fundamental framework for techniques like fine-tuning [38], argentic workflow [39], and paving the way for inference-time scaling [40].

Algorithm 1 *Co-rewarding-I*

```
1: Input: policy model  $\pi_\theta$ , learning rate  $\eta$ , training dataset  $\mathcal{D}$ , rephrased training dataset  $\mathcal{D}'$ , total iterations  $K$ .
2: Output: trained policy model  $\pi_\theta$ .
3: for all iteration  $k = 1, \dots, K$  do
4:   Sample mini-batch inputs  $\mathcal{B} \subseteq \mathcal{D}$  and  $\mathcal{B}' \subseteq \mathcal{D}'$ .
5:   for all input question  $x \in \mathcal{B}$  and  $x' \in \mathcal{B}'$  do
6:     Sample rollouts  $\{y_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}(\cdot | x)$ .
7:     Sample rollouts  $\{y'_i\}_{i=1}^{G'} \sim \pi_{\theta_{\text{old}}}(\cdot | x')$ .
8:     Obtain pseudo labels by Eq. (3).
9:     Estimate relative advantages by Eq. (2).
10:    Compute the objective by Eq. (1).
11:    Update  $\theta \leftarrow \theta - \eta \nabla_\theta \mathcal{J}_{\text{Co-rewarding-I}}(\theta)$ .
12:   end for
13: end for
```

Algorithm 2 *Co-rewarding-II*

```
1: Input: policy model  $\pi_\theta$ , learning rate  $\eta$ , training dataset  $\mathcal{D}$ , total iterations  $K$ .
2: Output: trained policy model  $\pi_\theta$ .
3: for iteration  $k = 1, \dots, K$  do
4:   Sample mini-batch  $\mathcal{B} \subseteq \mathcal{D}$ .
5:   for all  $x \in \mathcal{B}$  do
6:     Sample rollouts  $\{y_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}^{(k)}(\cdot | x)$ .
7:     Update the reference teacher by Eq. (6).
8:     Sample rollouts  $\{\tilde{y}_j\}_{j=1}^{\tilde{G}} \sim \tilde{\pi}_{\text{ref}}^{(k)}(\cdot | x)$ .
9:     Obtain pseudo label from  $\{\tilde{y}_j\}_{j=1}^{\tilde{G}}$  by Eq. (5).
10:    Estimate the relative advantage by Eq. (5).
11:    Compute the objective by Eq. (4).
12:    Update  $\theta \leftarrow \theta - \eta \nabla_\theta \mathcal{J}_{\text{Co-rewarding-II}}^{(k)}(\theta)$ .
13:   end for
14: end for
```

RL for Large Language Models. Several RL algorithms have been developed primarily for alignment tasks. Specifically, DPO [41], CPO [42], and their variants [43, 44, 45, 46, 47] rely on pairs of outputs labeled by human preference [48, 49]. In contrast, KTO [50] and BCO [51] require only a single binary label (like or dislike) for each output. Besides, the PRM [52, 53] and Step-KTO [54] offer step-by-step guidance by incorporating feedback at each reasoning step rather than focusing solely on the final outputs. Recently, the follow-up work of GRPO improves the optimization objective, *e.g.*, DAPO [9], Dr. GRPO [29], REINFORCE++ [55], CPPO [56], and GPG [57]. Another line of research generalizes GRPO to broader applications such as multimodal reasoning [58, 59, 57, 60, 61] and logical reasoning [31].

RL without External Reward. RL methods have shown promising scaling capabilities to enhance the reasoning abilities of LLMs [62], yet they are often limited by the availability of training data for reward signals [63, 64]. Notably, Wang et al. [65] demonstrate that RL can effectively bootstrap LLM reasoning with as little as a single training example, highlighting the potential to minimize or even eliminate reliance on external reward signals during training. Recent efforts leverage distinct strategies for reward assignment. For instance, SIRLC [66] and AZR [67] utilize an LLM-as-the-judge approach to assign rewards. In contrast, methods like SRT, TTRL, and their variants [18, 14, 68, 15] employ self-consistency [25] to generate pseudo-rewards, reducing dependence on external annotations. Meanwhile, INTUITOR, RLSC, and RENT [13, 69, 17] harness the internal confidence scores of LLMs as intrinsic reward signals. Additionally, EMPO and its variants [16, 70] promote reasoning by minimizing entropy during the reasoning process, further diversifying the approaches to incentivize robust LLM reasoning performance from internal signals.

C Pseudo Code of Co-rewarding

To intuitively present the pipeline of Co-rewarding, we summarize the pseudo codes of Co-rewarding-I and Co-rewarding-II in Algorithm 1 and Algorithm 2, respectively.

D Additional Experimental Details

D.1 Setups

Backbone Models and Baselines. We employ a diverse set of LLMs from different families and scales in our experiments, including the Qwen2.5 series (Qwen2.5-3B/7B) [3], the Qwen3 series (Qwen3-1.7B/4B/8B-Base) [71], and the Llama3 series (Llama-3.2-3B-Instruct) [72]. Beyond the vanilla GRPO that utilized the GT label for rewarding, we compare our Co-rewarding against several recent state-of-the-art (SoTA) self-reward reasoning approaches, denoted as Self-Certainty [13], Entropy [17] and Majority Voting [18]. The details of all baselines are summarized in Appendix D.2.

Implementation Details. We implement our algorithms based on the VeRL framework [73], and experiments are conducted on $4 \times \text{H100-80GB}$ GPUs. For our experiments, we totally use three training sets: MATH [74] (7,500 questions), DAPO-14k [9] (en-version of DAPO-Math-17k, about 14.1k questions), and OpenRS [75] (7,000 questions). During RL training, we use a global batch size of 128, set the number of rollouts to $G = \tilde{G} = 8$ per question for both I and II, and adopt AdamW with a learning rate of 3×10^{-6} . In Co-rewarding-I, question rephrasing is performed by the open-source Qwen3-32B model. In Co-rewarding-II, the EMA weight is scheduled from $\alpha_{\text{start}} = 0.99$ to $\alpha_{\text{end}} = 0.9999$ using cosine annealing. More implementation details are reported in Appendix D.3.

Evaluation Details. To provide a comprehensive evaluation of model capabilities, we utilize a diverse set of benchmarks spanning mathematical reasoning, code generation, instruction-following, and general multi-task abilities. Specifically: (1) Mathematical reasoning: MATH500 [53], GSM8K [76], and AMC [77]. (2) Code generation: LiveCodeBench [78] release_v6 and CRUX [79]. (3) Instruction-following and multi-task abilities: IFEval [80] and MMLU-Pro [81]. Additional evaluation details are provided in Appendix D.4.

D.2 Details of Baselines

We compare our proposed Co-rewarding-I and II against GT-reward and several recent state-of-the-art (SoTA) self-reward approaches:

- **GT-Reward** [8]: Originally introduced by DeepSeek-R1 [62], GT-Reward supervises training using ground-truth (GT) answers, determining whether model rollouts are correct or not, to guide RL optimization.
- **Self-Certainty** [13]: This method maximizes *self-certainty*, defined as the KL-divergence between the uniform distribution and the model’s decoding distribution, serving as reward to encourage more confident predictions.
- **Entropy** [17]: This method minimizes the entropy of the model’s rollout distribution, using negative entropy as reward to maximize model confidence.
- **Majority-Voting** [18]: By generating multiple rollouts per question, Majority-Voting selects the most frequent answer as a pseudo-label to supervise training.

For all methods, we adopt the widely used GRPO as the policy optimization algorithm.

D.3 More Implementation Details

The detailed training configurations are summarized in Table 2, and all baseline methods are trained under the same setup for fairness. For the training system prompt, we adopt the official default prompt provided by VeRL³, shown below:

Let’s think step by step and output the final answer within \boxed{ }.

³<https://github.com/volcengine/verl>

Table 2: Detailed training settings.

Settings	Co-rewarding-I	Co-rewarding-II
Batch Size	128	128
Max Prompt Length	512	512
Max Response Length	3072	3072
Train Steps	170-220	300-330
Learning Rate	3e-6	3e-6
# Policy Rollout G	8	8
# Reference Rollout $\tilde{G}$	-	8
Clip Ratio	0.2	0.2
Warmup Style	Cosine	Cosine
Warmup Steps Ratio	0.1	0.1
KL Loss Coefficient	0.005	0.001
Optimizer	AdamW ($\beta_1 = 0.9, \beta_2 = 0.999, \epsilon = 10^{-8}$)	
Training Temperature	1.0	1.0
Evaluation Temperature	0.8	0.8
EMA α_{start}	-	0.99
EMA α_{end}	-	0.9999

In addition, the semantically analogical questions used in Co-rewarding-I are generated by Qwen3-32B through a rewriting prompt. The exact rewriting instruction is provided as follows:

```

You are given a math problem. Please rewrite it using different wording and a
different real-world scenario, while keeping the underlying mathematical
meaning and answer exactly the same.

Guidelines:
1. Do not change the math logic or the final answer.
2. Use different words and a new context to make it look like a different problem.
3. Avoid copying phrases or sentence structures from the original.
4. Make sure the rewritten question is natural, clear, and solvable.
5. Output ONLY between the following markers, and strictly in this format (no extra
explanation):

### RESULT_START
ORIGINAL:
<original question>
REWRITE:
<rewritten question>
### RESULT_END

```

D.4 More Evaluation Details

We conduct the evaluation across a diverse set of benchmarks, spanning mathematical reasoning, code generation, instruction-following, and general multi-task abilities. Specifically: (1) Mathematical reasoning: We evaluate on MATH500 [53], GSM8K [76], and AMC [77]. For MATH500 and GSM8K, we report pass@1 accuracy using the lighteval library⁴. For AMC, we use the ttrl⁵ library and report avg@8 as the metric. (2) Code generation: We assess coding ability using LiveCodeBench [78] release_v6 and CRUX [79]. LiveCodeBench is evaluated with its official evaluation library⁶, and CRUX is evaluated via the ZeroEval library⁷; for both datasets, we report pass@1 accuracy. (3) Instruction-following and multi-task abilities: We evaluate on IFEval [80] and

⁴<https://github.com/huggingface/lighteval>

⁵https://github.com/ruixin31/Spurious_Rewards/tree/main/code/ttrl

⁶<https://github.com/LiveCodeBench/LiveCodeBench>

⁷<https://github.com/WildEval/ZeroEval>

Table 3: **Statistics and usages of datasets** used in our experiments.

Dataset Name	# Data Size	Usage
MATH-Train [74]	7,500	Training Set
MATH-Test [74]	5,000	Validation Set
DAPO-14k [9]	14,109	Training Set
Open-RS [75]	7,000	Training Set
MATH500 [53]	500	Evaluation Benchmark
GSM8K [76]	1,319	Evaluation Benchmark
AMC [77]	83	Evaluation Benchmark
LiveCodeBench [78]	1,055	Evaluation Benchmark
CRUX [79]	800	Evaluation Benchmark
MMLU-Pro [81]	12,032	Evaluation Benchmark
IFEval [80]	541	Evaluation Benchmark

Table 4: **Main Results (%) of Co-rewarding and baselines trained on DAPO-14k**. Cell background colors indicate relative performance: darker colors denote better results within each model group. Additional Results of Qwen3-8B-Base trained on OpenRS refer to Table 7.

Training Set: DAPO-14k	Mathematics			Code		Instruction	Multi-Task
Methods	MATH500	GSM8K	AMC	LiveCode	CRUX	IFEval	MMLU-Pro
<i>Qwen3-8B-Base</i>							
Before RL	72.4	27.82	20.93	23.41	54.75	50.89	52.92
- GT-Reward [8]	86.6	87.19	61.75	30.52	63.75	53.11	60.27
- Self-Certainty [13]	82.0	77.63	49.85	27.77	60.75	50.58	54.24
- Entropy [17]	79.4	80.82	45.48	30.14	62.00	51.56	54.57
- Majority-Voting [18]	78.6	91.66	50.00	30.33	61.62	51.54	55.65
- Co-rewarding-I (Ours)	78.4	88.02	51.20	29.38	62.50	50.17	55.39
- Co-rewarding-II (Ours)	80.6	94.01	54.37	31.66	67.12	53.31	59.83
<i>Qwen3-4B-Base</i>							
Before RL	71.2	26.15	21.08	11.00	38.88	46.43	47.23
- GT-Reward [8]	83.6	85.14	52.86	18.58	56.88	47.70	55.35
- Self-Certainty [13]	68.4	44.81	35.39	25.88	50.12	45.58	48.84
- Entropy [17]	76.6	82.79	43.37	26.35	50.75	48.20	50.22
- Majority-Voting [18]	73.4	64.06	40.81	26.16	53.00	48.91	51.06
- Co-rewarding-I (Ours)	73.8	75.89	43.83	26.25	50.12	46.84	51.51
- Co-rewarding-II (Ours)	77.8	91.89	48.49	26.64	54.87	48.90	52.83
<i>Llama-3.2-3B-Instruct</i>							
Before RL	39.2	65.73	10.54	9.86	25.37	57.32	31.14
- GT-Reward [8]	49.4	78.17	25.90	10.33	31.37	53.10	33.83
- Self-Certainty [13]	42.4	74.71	17.32	11.18	28.38	54.50	33.51
- Entropy [17]	44.0	65.85	17.32	9.95	25.00	55.78	31.95
- Majority-Voting [18]	42.8	70.96	17.62	10.14	29.50	54.07	32.95
- Co-rewarding-I (Ours)	46.0	70.58	20.93	9.57	27.25	53.04	32.61
- Co-rewarding-II (Ours)	49.8	78.62	19.73	10.43	32.25	51.92	34.46

MMLU-Pro [81], using the `lm-evaluation-harness` library⁸ for both. Overall, we summarize the statistics of the datasets used in this paper in Table 3.

E Additional Experimental Results

E.1 Further Analysis

Code generalization with preserved general performance. Although trained solely on math-oriented datasets, the models show improvements on coding benchmarks, suggesting a cross-domain

⁸<https://github.com/EleutherAI/lm-evaluation-harness>

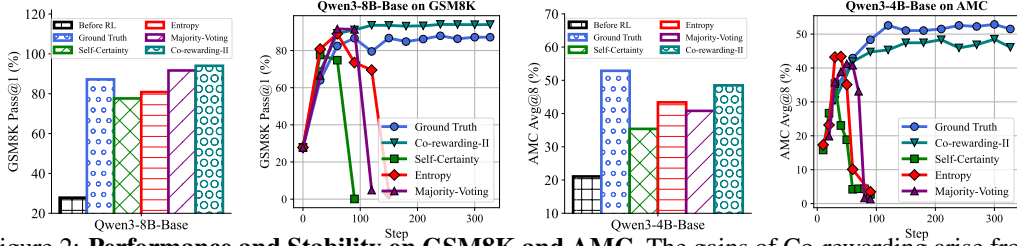


Figure 2: **Performance and Stability on GSM8K and AMC.** The gains of Co-rewarding arise from its training stability, which supports continuous improvements throughout learning.

Table 5: **Ablation study of Co-rewarding.** For Co-rewarding-I, ablations train only on original or rephrased data. For Co-rewarding-II, ablation removes EMA updates of the reference teacher.

Methods	MATH500	GSM8K	AMC	LiveCode	CRUX	IFEval	MMLU-Pro
<i>Qwen3-8B-Base</i>							
Co-rewarding-I	81.2	93.70	51.20	30.81	66.00	55.79	59.95
- Majority-Voting w/ Original	79.8	89.76	49.09	30.52	63.38	51.80	56.93
- Majority-Voting w/ Rephrased	79.2	91.51	50.75	31.66	60.38	52.24	57.26
Co-rewarding-II	80.6	94.01	54.37	31.66	67.12	53.31	59.83
- w/o Updating Reference	78.0	88.40	51.66	30.62	63.75	52.48	58.01
<i>Llama-3.2-3B-Instruct</i>							
Co-rewarding-I	50.2	79.45	23.80	11.28	29.88	48.89	33.77
- Majority-Voting w/ Original	46.8	78.77	20.48	11.00	31.25	47.96	33.18
- Majority-Voting w/ Rephrased	44.0	78.85	21.23	10.04	17.25	47.84	33.72
Co-rewarding-II	49.8	78.62	19.73	10.43	32.25	51.92	34.46
- w/o Updating Reference	45.0	76.72	17.92	10.05	30.63	51.33	33.94

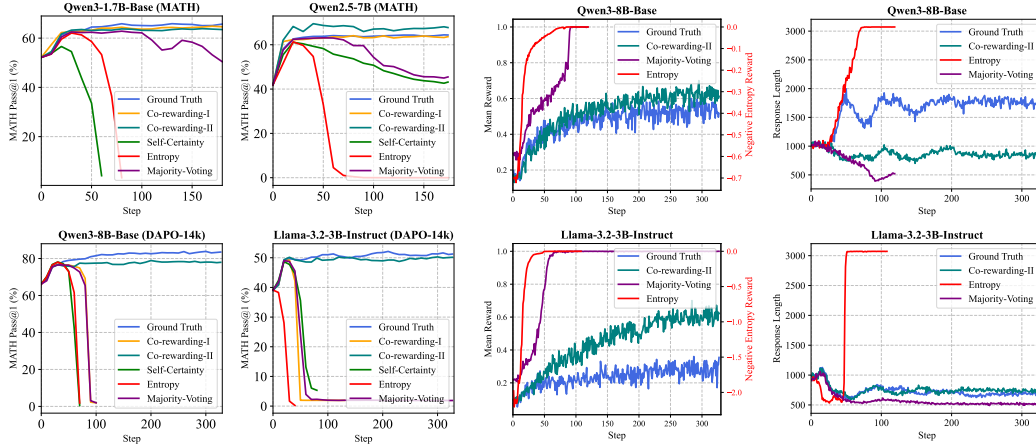


Figure 3: **Performance curves comparison** on validation set. *Top:* Qwen3-1.7B-Base and Qwen2.5-7B trained on the MATH set. *Bottom:* Qwen3-8B-Base and Llama-3.2-3B-Instruct trained on the DAPO-14k set. Figure 4: **Reward (left) and response length (right)** of Qwen3-8B-Base and Llama-3.2-3B-Instruct trained on DAPO-14k. Due to different reward scale from other methods, Entropy reward is plotted on the right y -axis of left panels, where the reward is the negative entropy.

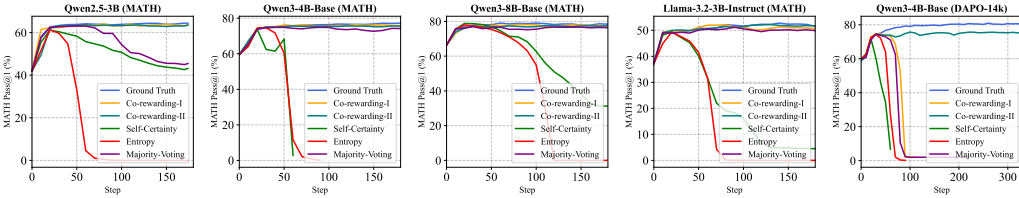


Figure 5: **Performance curves on validation set.** *Left to Right:* {Qwen2.5-3B, Qwen3-4B-Base, Qwen3-8B-Base, Llama-3.2-3B-Instruct} trained on MATH, Qwen3-4B-Base trained on DAPO-14k.

generalization from math to code in self-supervised reasoning elicitation. Moreover, Co-rewarding preserves general instruction-following and multi-task ability on MMLU-Pro and IFEval. As shown

Table 6: **Supplement Results (%) of Co-rewarding and baselines trained on MATH.** Cell background colors: darker colors denote better results within each model group.

Training Set: MATH	Mathematics			Code		Instruction	Multi-Task
Methods	MATH500	GSM8K	AMC	LiveCode	CRUX	IFEval	MMLU-Pro
<i>Qwen2.5-3B</i>							
Before RL	53.6	19.48	10.69	9.95	18.50	29.83	32.50
- GT-Reward [8]	65.4	82.18	32.98	13.93	32.12	33.66	36.74
- Self-Certainty [13]	64.2	80.52	28.92	10.90	29.00	32.22	33.88
- Entropy [17]	63.2	80.44	29.67	9.05	29.00	32.94	35.35
- Majority-Voting [18]	64.6	82.41	33.13	14.03	36.38	35.19	35.50
- Co-rewarding-I (Ours)	65.4	84.53	30.57	16.40	36.88	33.86	36.38
- Co-rewarding-II (Ours)	65.2	81.72	32.38	22.25	40.25	32.74	30.79
<i>Qwen2.5-7B</i>							
Before RL	69.4	24.71	15.81	3.79	26.38	38.19	44.76
- GT-Reward [8]	76.4	88.02	45.63	15.92	45.12	41.49	41.12
- Self-Certainty [13]	72.8	84.31	38.55	12.04	54.12	37.24	43.30
- Entropy [17]	72.2	81.43	39.61	16.49	51.88	40.33	42.79
- Majority-Voting [18]	74.4	84.53	40.96	15.45	51.00	38.60	43.35
- Co-rewarding-I (Ours)	74.6	89.61	41.27	15.73	55.58	42.86	40.51
- Co-rewarding-II (Ours)	73.6	89.31	42.77	8.25	47.50	41.82	37.45
<i>Qwen3-1.7B-Base</i>							
Before RL	57.0	19.56	8.43	4.45	7.50	33.65	33.00
- GT-Reward [8]	69.6	81.57	35.54	13.74	35.25	36.16	39.12
- Self-Certainty [13]	58.2	40.25	23.04	9.86	18.00	32.96	35.13
- Entropy [17]	63.6	71.79	31.63	13.74	31.37	35.37	36.67
- Majority-Voting [18]	65.2	81.57	34.78	13.08	34.25	35.45	36.00
- Co-rewarding-I (Ours)	67.6	83.01	32.22	13.50	32.38	35.56	35.53
- Co-rewarding-II (Ours)	66.2	80.89	33.28	14.40	32.88	36.94	37.59

Table 7: **Supplement Results (%) of Co-rewarding and baselines trained on OpenRS.** Cell background colors: darker colors denote better results within each model group.

Training Set: Open-RS	Mathematics			Code		Instruction	Multi-Task
Methods	MATH500	GSM8K	AMC	LiveCode	CRUX	IFEval	MMLU-Pro
<i>Qwen3-8B-Base</i>							
Before RL	72.40	27.82	20.93	23.41	54.75	50.89	52.92
- GT-Reward [8]	80.20	89.76	54.97	39.00	63.00	52.94	55.49
- Self-Certainty [13]	82.60	85.22	50.00	37.00	64.62	52.12	56.03
- Entropy [17]	80.60	87.41	48.95	38.00	61.25	52.53	56.80
- Majority-Voting [18]	78.00	84.23	51.96	36.75	58.00	51.13	54.92
- Co-rewarding-I (Ours)	78.20	92.65	50.60	28.91	63.12	53.11	57.21
- Co-rewarding-II (Ours)	80.00	90.90	53.01	39.75	62.75	52.92	56.55
<i>Qwen3-4B-Base</i>							
Before RL	71.20	26.15	21.08	11.00	38.88	46.43	47.23
- GT-Reward [8]	78.80	85.22	49.55	33.50	55.12	46.41	50.12
- Self-Certainty [13]	73.20	33.43	35.84	32.50	49.50	46.47	48.24
- Entropy [17]	76.80	87.57	42.62	35.00	53.87	47.61	52.42
- Majority-Voting [18]	76.00	64.14	44.58	32.25	50.25	46.35	48.75
- Co-rewarding-I (Ours)	72.80	83.93	39.41	26.54	53.25	48.11	50.82
- Co-rewarding-II (Ours)	76.60	89.23	42.32	34.00	51.50	48.45	51.80

in Table 8, Co-rewarding-II outperforms other self-rewarding baselines in 12 of 14 MMLU-Pro categories, demonstrating that its gains do not come at the expense of broader general-domain performance. More detailed results of MMLU-Pro and IFEval refer to Appendix E.8 and E.9.

Co-rewarding attempts to balance exploration-exploitation. Figure 4 shows reward and response length curves. Entropy and Majority-Voting quickly reach the highest reward, indicating reward hacking rather than genuine reasoning improvement. In contrast, GT-Reward and Co-rewarding

Table 8: **Detailed performance of MMLU-Pro with Qwen3-8B-Base** trained on DAPO-14k. More results refer to Appendix E.8.

<i>MMLU-Pro (Qwen3-8B-Base)</i>							
Methods	biology	business	chemistry	computer sci.	economics	health	history
- GT-Reward	77.96	70.85	60.42	61.95	71.33	59.79	51.44
- Self-Certainty	75.73	58.05	50.53	56.83	69.31	54.77	50.40
- Entropy	74.76	59.70	51.33	56.10	67.90	55.87	48.04
- Majority-Voting	75.32	61.47	54.24	58.29	69.67	58.20	49.34
- Co-rewarding-I	76.85	61.22	53.45	59.02	66.82	55.62	48.29
- Co-rewarding-II	76.71	68.69	64.58	61.71	68.25	56.85	51.71
Methods	law	math	other	philosophy	physics	psychology	engineering
- GT-Reward	31.52	73.28	56.28	52.71	61.97	67.30	46.14
- Self-Certainty	30.43	63.06	51.63	46.29	51.73	66.42	41.07
- Entropy	28.97	63.96	51.51	48.90	53.04	66.80	42.32
- Majority-Voting	31.16	64.62	52.27	48.90	53.27	66.92	40.97
- Co-rewarding-I	30.34	66.17	51.73	48.90	55.19	66.42	39.63
- Co-rewarding-II	31.16	72.17	52.49	52.10	63.21	68.17	47.16

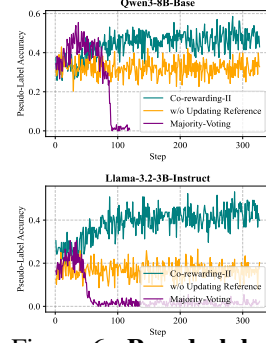


Figure 6: **Pseudo label accuracy comparison.**

exhibit smoother, gradually increasing rewards, reflecting stable training. The response length curves further illustrate this difference: GT-Reward lengthens responses to explore correct reasoning paths; Majority-Voting collapses to short outputs, restricting exploration; and Entropy collapses its probability mass onto a small set of tokens, repeatedly generating them until truncation. Co-rewarding instead maintains moderate response lengths throughout training, suggesting a balanced exploration–exploitation trade-off. Additional curves for other LLMs are provided in Appendix E.3.

EMA is essential in Co-rewarding-II for improving pseudo-label quality. Figure 6 compares pseudo-label accuracy across Co-rewarding-II, “w/o Updating Reference”, and Majority-Voting. Co-rewarding-II steadily improves accuracy as training progresses, while “w/o Updating Reference” remains nearly flat, underscoring the role of EMA updates in allowing the teacher to co-evolve with the policy and generate higher-quality pseudo labels. By contrast, Majority-Voting briefly improves but then collapses to near zero, evidencing reward hacking through consistent yet incorrect outputs.

Case Study of the model reasoning with different learning methods. Figure 7 provides a concrete example to illustrate the qualitative difference between self-rewarding baselines and our Co-rewarding. Majority-Voting exhibits reward hacking by boxing an incorrect answer “0” to pursue consensus, even though the reasoning steps are correct. Entropy produces repetitive outputs as its decoding probability distribution collapses onto a narrow set of tokens during entropy minimization. In contrast, Co-rewarding generates coherent step-by-step reasoning and correctly boxes the final answer, showing its capacity to overcome reward hacking and elicit genuine reasoning.

E.2 More Results on Other Training Sets and LLMs

Table 6 reports additional results of Qwen2.5-3B and Qwen3-1.7B-Base and Qwen3-4B-Base trained on MATH, while Table 7 extends the experiments of Qwen3-8B-Base and Qwen3-4B-Base to another training set OpenRS [75]. It can be observed that Co-rewarding occupies relatively darker areas. Across models and training sets, Co-rewarding-I and II achieve an average relative improvement of +2.23% on GSM8K, with notably high pass@1 scores of 92.65% and 90.90% for Qwen3-8B-Base trained on OpenRS, respectively. Moreover, thanks to its stability, Co-rewarding-II delivers more reliable gains than self-rewarding baselines, which occasionally suffer lower performance on certain models or benchmarks, e.g., Self-Certainty on Qwen3-1.7B-Base in Table 6 or Majority-Voting on Qwen3-4B-Base in Table 7. These results further demonstrate the effectiveness of Co-rewarding.

E.3 More Curves of Reward, Response Length and Pseudo Label Accuracy

Figure 8 supplements the reward and response curves of Qwen3-4B-Base trained on DAPO-14k. The trends are consistent with Qwen3-8B-Base and Llama-3.2-3B-Instruct in Figure 4: Majority-Voting and Entropy rapidly increase rewards at early stage and quickly peak, a clear sign of reward hacking. In contrast, GT-Reward and Co-rewarding-II exhibit smoother, steadily rising rewards, indicating genuine learning of reasoning ability. Moreover, Co-rewarding-II maintains moderate response lengths on Qwen3-4B-Base, further demonstrating its generality in balancing the exploration–exploitation trade-off during reasoning training, which is a core principle of RL [82].

Additionally, the right panel of Figure 8 presents the pseudo-label accuracy of Qwen3-4B-Base, showing trends consistent with Qwen3-8B-Base and Llama-3.2-3B-Instruct in Figure 6. As training

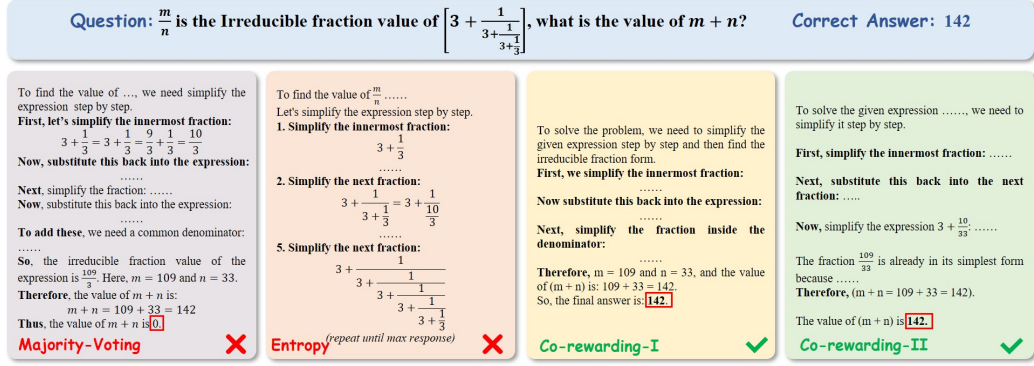


Figure 7: **Case study:** An example comparing the generations from *Majority-Voting*, *Entropy*, and our proposed *Co-rewarding*. The results clearly reveal the reward hacking behavior exhibited by Majority-Voting and Entropy, while ours generate the correct answer. Full results refer to Appendix E.11.

Table 9: **Performance (%) of test-time training (TTT).** Since self-supervised methods are label-free, they can be leveraged during inference for test-time training to further enhance performance.

LLMs	Methods	AMC							
		avg@8	pass@8	avg@16	pass@16	avg@32	pass@32	avg@64	pass@64
<i>Qwen2.5-7B</i>	Before-TTT	15.81	46.99	17.55	66.27	16.34	74.70	17.32	75.90
	Self-Certainty	41.57	74.70	39.23	74.70	39.68	78.31	39.95	87.95
	Entropy	38.70	56.63	39.76	68.67	39.57	79.52	39.34	81.93
	Majority-Voting	43.67	63.86	43.67	67.47	43.49	78.31	44.35	85.54
	Co-rewarding-I	44.88	60.24	45.33	60.24	45.44	71.08	45.76	73.49
	Co-rewarding-II	43.22	69.88	41.34	75.90	40.36	78.31	41.64	87.95
<i>Qwen3-8B-Base</i>	Before-TTT	20.93	61.45	21.31	73.49	19.58	79.52	20.97	86.75
	Self-Certainty	49.85	78.31	50.68	78.31	50.41	84.34	49.55	89.16
	Entropy	48.64	74.70	49.92	80.72	49.96	87.95	50.23	89.16
	Majority-Voting	50.90	73.49	50.00	72.29	50.60	80.72	51.36	85.54
	Co-rewarding-I	52.86	68.67	53.46	74.70	53.24	81.93	53.58	84.34
	Co-rewarding-II	48.64	72.29	48.19	73.49	50.19	83.13	49.28	91.57

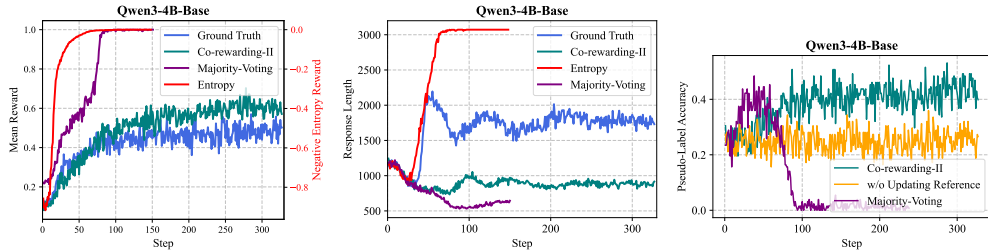


Figure 8: **Curves of reward (Left), response length (Middle), and pseudo label accuracy (Right) of Qwen3-4B-Base** trained on DAPO-14k. Entropy reward is plotted on the right y -axis due to its different reward scale. Note that entropy minimization is to maximizing the negative entropy.

progresses, Co-rewarding-II steadily improves pseudo-label accuracy, while “w/o Updating Reference” remains around 25%. Majority-Voting briefly increases accuracy but soon collapses to zero, clearly indicating reward hacking. This highlights our design philosophy of pairing a fast policy student with a slowly updated teacher, which decouples supervision from the online policy while enabling the teacher to co-evolve with the student, thereby sustaining improvements in pseudo-label quality.

E.4 More Results of Validation Performance Curves

As a supplement to Figure 3, Figure 5 presents validation performance curves for Qwen2.5-3B, Qwen3-4B-Base, Qwen3-8B-Base, Llama3.2-3B-Instruct trained on MATH, as well as Qwen3-4B-Base trained on DAPO-14k. Self-Certainty and Entropy collapse rapidly across all settings, as their supervision signals are tied to internal confidence or entropy and are easily exploited. Majority-Voting also collapses in several cases, reflecting that sampling pseudo labels from outputs cannot prevent

Table 10: **Detailed MMLU-Pro performance on Qwen3-4B-Base and Llama-3.2-3B-Instruct trained on DAPO-14k.** Results are reported for each of the 14 categories in MMLU-Pro.

<i>Qwen3-4B-Base</i>							
Methods	biology	business	chemistry	computer sci.	economics	engineering	health
- GT-Reward	73.50	63.49	59.71	56.34	65.05	42.93	50.86
- Self-Certainty	71.41	54.37	45.93	50.73	63.27	35.91	50.12
- Entropy	70.99	56.02	50.44	48.29	63.15	34.37	48.41
- Majority-Voting	70.43	55.77	52.83	53.41	62.79	38.09	50.61
- Co-rewarding-I	73.92	59.82	50.71	54.15	64.93	41.49	49.76
- Co-rewarding-II	72.66	59.95	55.65	53.41	64.10	39.73	50.61
Methods	history	law	math	other	philosophy	physics	psychology
- GT-Reward	44.88	26.34	69.80	48.81	44.69	57.04	65.79
- Self-Certainty	39.63	24.43	59.44	43.94	40.08	47.04	59.65
- Entropy	40.68	26.43	60.99	45.13	43.69	50.89	61.90
- Majority-Voting	40.94	23.43	64.17	43.39	44.09	50.73	63.66
- Co-rewarding-I	40.94	23.25	63.73	44.91	42.69	50.58	60.78
- Co-rewarding-II	42.26	24.79	67.58	44.59	41.88	54.19	62.91
<i>Llama3.2-3B-Instruct</i>							
Methods	biology	business	chemistry	computer sci.	economics	engineering	health
- GT-Reward	54.81	36.25	25.18	33.41	42.65	21.57	39.36
- Self-Certainty	55.23	32.95	27.21	31.95	42.77	20.54	39.12
- Entropy	52.86	31.05	23.94	32.93	41.71	20.43	38.02
- Majority-Voting	56.07	32.95	22.79	30.98	44.19	18.99	39.61
- Co-rewarding-I	51.88	34.22	22.88	34.88	44.67	19.09	38.63
- Co-rewarding-II	56.21	34.35	27.03	35.61	43.01	19.92	40.34
Methods	history	law	math	other	philosophy	physics	psychology
- GT-Reward	30.18	22.71	34.20	34.74	32.06	28.33	50.38
- Self-Certainty	30.45	24.98	33.38	31.60	29.86	28.56	50.50
- Entropy	33.86	21.89	32.35	33.01	29.46	24.25	47.50
- Majority-Voting	32.02	25.25	34.35	34.20	29.86	24.79	48.25
- Co-rewarding-I	33.86	23.25	32.12	33.01	31.86	25.40	48.75
- Co-rewarding-II	32.28	24.34	35.83	36.26	33.27	28.18	49.12

hacking. By contrast, Co-rewarding-I maintains stability across MATH-trained models through data-side contrastive agreement, while Co-rewarding-II consistently provides stability across all models and datasets by disentangling supervision with a slowly updated teacher, making hacking substantially harder and optimization more reliable.

E.5 Results of Test-time Training (TTT)

Thanks to the label-free nature of self-supervised methods, which do not require GT labels, they are naturally compatible with test-time training (TTT), enabling further refinement of the model during inference. Table 9 reports the TTT results on the challenging competition-level benchmark AMC across Co-rewarding and other self-rewarding baselines. We observe that Co-rewarding matches or even surpasses existing methods, achieving the best results on 11 out of 18 metrics. These findings broaden the applicability of self-supervised RL: beyond post-training for reasoning elicitation, it can also be leveraged at inference time to further improve performance on specific benchmarks.

E.6 More Results of Benchmark Performance Curves

As a supplement to Figure 3 and Figure 2, Figure 9 presents performance curves on MATH500, GSM8K, AMC, and CRUX with Qwen3-8B-Base, Qwen3-4B-Base, and Llama-3.2-3B-Instruct. Consistent with earlier findings, Self-Certainty, Entropy, and Majority-Voting rapidly collapse across benchmarks and models, while Co-rewarding-II and GT-Reward sustain continued and stable improvements. These results underscore the link between performance and training stability: stable training enables models to continue improving by effectively learning knowledge from more data.

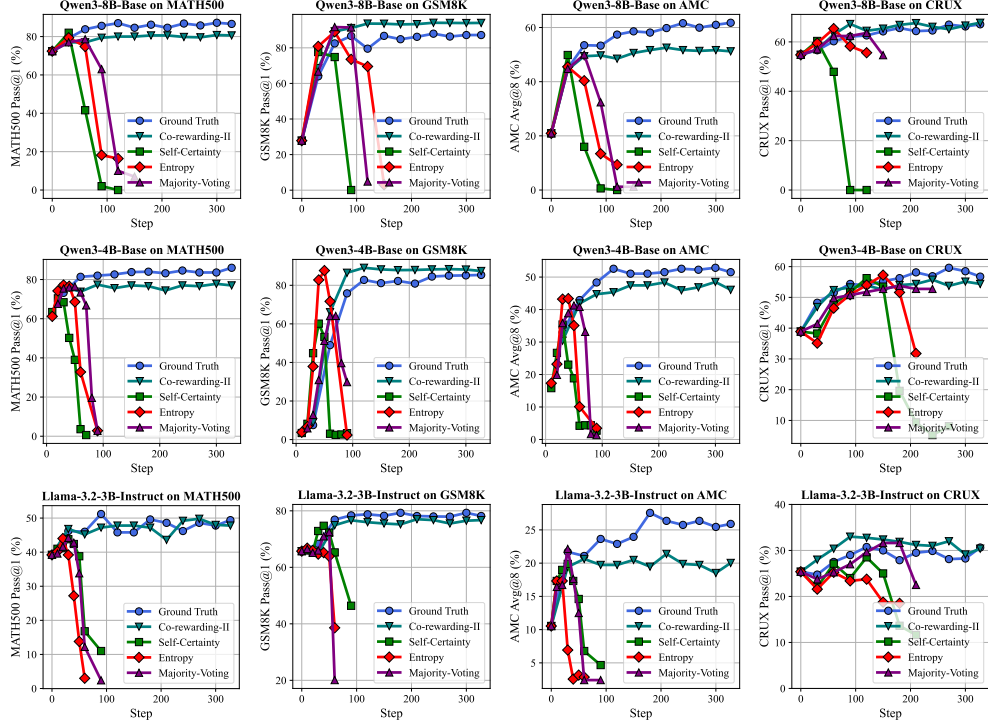


Figure 9: Performance curves on benchmarks of MATH500, GSM8K, AMC and CRUX across Qwen3-8B-Base, Qwen3-4B-Base, and Llama-3.2-3B-Instruct trained on DAPO-14k.

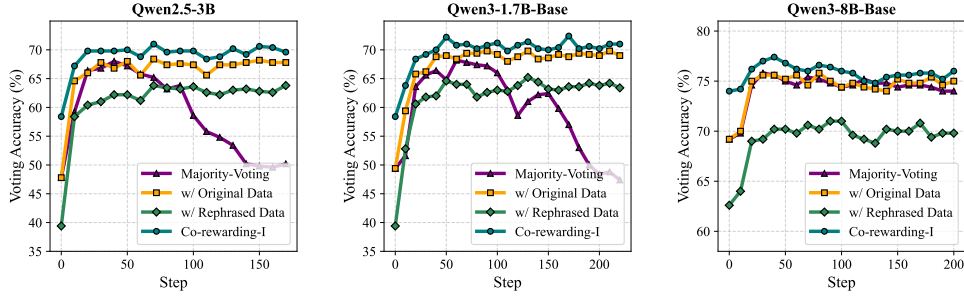


Figure 10: Curves of voting accuracy of Majority-Voting, Co-rewarding-I and its ablations with Qwen2.5-3B, Qwen3-1.7B-Base and Qwen3-8B-Base trained on MATH.

E.7 Voting Accuracy Analysis of Co-rewarding-I

To demonstrate the stability and efficiency of Co-rewarding-I, we compare its voting accuracy against that of Majority-Voting in Figure 10 and Figure 11. These experiments are conducted on Qwen2.5-3B, Qwen3-1.7B-Base and Qwen3-8B-Base models, all trained on the MATH dataset. Across all settings, the Majority-Voting method exhibits reward hacking, where its performance sharply declines after reaching an early peak, particularly on more difficult questions (levels 2 to 5). In contrast, Co-Rewarding-I maintains a stable voting accuracy on both original and rephrased data. Ultimately, it achieves the highest overall voting accuracy across all models and dataset configurations.

E.8 More Results of MMLU-Pro Evaluation

As a complement to Table 8, Table 10 and Table 11 report detailed MMLU-Pro results for models trained on DAPO-14k and OpenRS, respectively. We observe that Co-rewarding consistently preserves

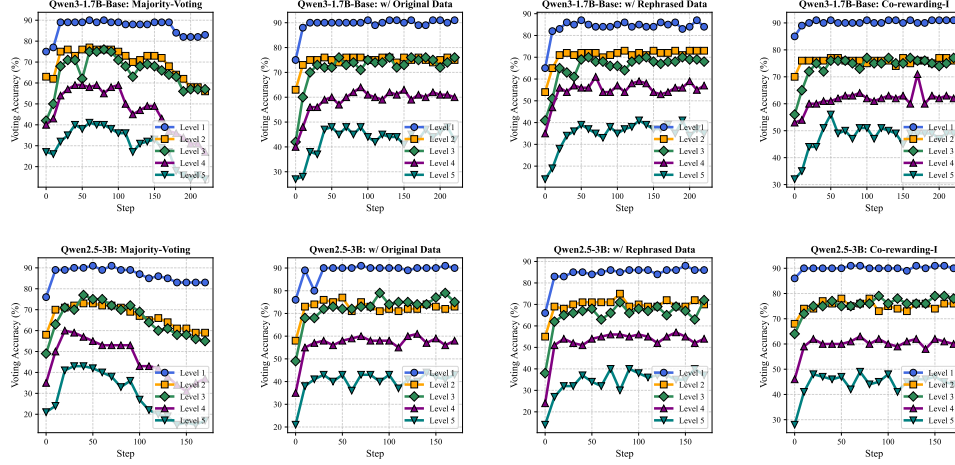


Figure 11: Voting accuracy of Majority-Voting, Co-rewarding-I and its ablated variants across different difficulty levels of questions *Top*: Qwen3-1.7B-Base. *Bottom*: Qwen2.5-3B.

general-domain performance across diverse subjects, indicating that though trained on math-oriented datasets, its improvements do not come at the cost of broader capabilities from other domains.

E.9 More Results of IFEval Evaluation

The aim of IFEval is used to evaluate the instruction-following ability of LLMs. In Table 1, Table 4, Table 6 and Table 7, we report average IFEval performance due to space constraints. Specifically, the evaluation of IFEval includes four metrics: {prompt_level_strict_acc, inst_level_strict_acc, prompt_level_loose_acc and inst_level_loose_acc}, which apply different levels of answer matching. As a supplement, complete results are provided in Table 12, Table 13, and Table 14. The results show that Co-rewarding not only preserves the inherent instruction-following ability of base models but also often surpasses GT-Reward across multiple models. This further confirms that Co-rewarding’s gains on mathematical and coding benchmarks are achieved without sacrificing general-domain instruction-following ability.

E.10 Original Questions vs. Rephrased Questions

To provide an intuitive illustration, we present several examples of original questions with their rephrased versions in Table 15. We observe that such rephrasings are reasonable and effective, as they preserve the same underlying mathematical essence while presenting the problems in a substantially different surface form. This reflects the high quality of our rephrased data and forms the basis of Co-rewarding-I: by leveraging contrastive agreement across data-invariant variants, the model is encouraged to elicit more robust reasoning ability.

E.11 Complete Case Study

As a supplement to Figure 7, we present the complete generation outputs of this case study. The full outputs clearly reveal the reward hacking behaviors of existing self-rewarding baselines. Self-Certainty and Entropy fall into repetitive outputs—for example, Self-Certainty repeatedly generates “Understanding,” and Entropy repeatedly produces “Simplify the next fraction” until truncated at the maximum length. This arises because their decoding probability mass collapses onto a small subset of tokens, leading the model to loop over them. Majority-Voting shows another form of reward hacking by boxing an incorrect answer “0” to maximize consensus across rollouts and thereby secure the highest reward. In contrast, both Co-rewarding and GT-Reward yield the correct final answer along with a coherent step-by-step reasoning path.

Table 11: **Detailed MMLU-Pro performance on Qwen3-8B-Base and Qwen3-4B-Base trained on OpenRS.** Results are reported for each of 14 categories in MMLU-Pro.

<i>Qwen3-4B-Base</i>							
Methods	biology	business	chemistry	computer sci.	economics	engineering	health
- GT-Reward	70.99	59.82	52.30	54.63	65.05	39.01	51.22
- Self-Certainty	69.87	54.50	44.08	49.27	63.63	37.36	50.24
- Entropy	70.71	58.68	49.03	51.22	63.39	37.46	49.63
- Majority-Voting	69.60	55.77	47.17	53.17	63.39	36.02	48.78
- Co-rewarding-I	69.04	55.39	47.79	53.41	63.86	38.39	50.61
- Co-rewarding-II	70.85	58.81	53.27	53.90	66.11	37.15	52.81
Methods	history	law	math	other	philosophy	physics	psychology
- GT-Reward	39.63	24.98	65.58	47.84	40.68	54.50	62.53
- Self-Certainty	39.63	24.25	58.11	46.65	40.88	46.42	61.40
- Entropy	39.90	22.16	62.18	45.02	43.09	50.19	59.90
- Majority-Voting	40.68	22.52	60.25	46.10	41.08	48.42	60.65
- Co-rewarding-I	40.68	24.25	62.18	44.37	44.49	49.58	61.65
- Co-rewarding-II	41.21	25.89	64.91	45.24	39.28	52.27	59.40
<i>Qwen3-8B-Base</i>							
Methods	biology	business	chemistry	computer sci.	economics	engineering	health
- GT-Reward	74.76	63.24	55.48	63.17	68.96	41.38	57.09
- Self-Certainty	75.03	63.62	53.62	55.61	68.96	39.83	57.09
- Entropy	75.73	64.39	54.51	58.29	65.05	41.69	55.87
- Majority-Voting	76.15	60.20	54.15	56.34	69.91	38.91	55.75
- Co-rewarding-I	76.43	65.78	57.07	62.20	69.43	43.14	56.60
- Co-rewarding-II	76.84	64.25	54.68	62.43	68.12	42.00	58.06
Methods	history	law	math	other	philosophy	physics	psychology
- GT-Reward	50.92	30.25	67.58	52.49	51.10	57.20	67.67
- Self-Certainty	49.34	28.88	68.02	51.62	52.10	56.89	66.42
- Entropy	50.39	30.43	65.28	51.41	47.09	54.50	66.67
- Majority-Voting	48.03	28.88	63.43	53.68	48.10	52.50	64.66
- Co-rewarding-I	50.13	29.97	68.54	52.92	50.70	56.66	65.54
- Co-rewarding-II	51.44	30.06	65.80	51.51	52.10	57.58	65.78

Question:

$\frac{m}{n}$ is the Irreducible fraction value of

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$$

, what is the value of $m + n$?

Co-rewarding-I Answer:

To solve the problem, we need to simplify the given expression step by step and then find the irreducible fraction form. The expression is:

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}.$$

First, we simplify the innermost fraction:

$$3 + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{10}{3}.$$

Now substitute this back into the expression:

$$3 + \frac{1}{3 + \frac{1}{\frac{10}{3}}} = 3 + \frac{1}{3 + \frac{3}{10}}.$$

Table 12: **Detailed IFEval Performance on Qwen2.5-3B/7B, Qwen3-1.7B/4B/8B-Base and Llama-3.2-3B-Instruct traid on MATH.** Results are reported for loose and strick settings respectively.

Methods	IFEval									
	Average	Prompt Strict	Prompt Loose	Inst. Strict	Inst. Loose	Average	Prompt Strict	Prompt Loose	Inst. Strict	Inst. Loose
<i>Qwen2.5-3B</i>					<i>Qwen2.5-7B</i>					
Before RL	29.83	22.55	27.17	31.89	37.70	38.19	29.57	34.57	41.85	46.76
- GT-Reward	33.66	25.51	31.42	35.85	41.85	41.49	31.79	39.56	43.65	50.96
- Self-Certainty	32.22	24.40	29.76	34.65	40.05	37.24	28.47	34.38	40.05	46.04
- Entropy	32.94	24.77	30.50	35.13	41.37	40.33	30.13	37.87	43.29	50.00
- Majority-Voting	35.19	26.25	32.72	37.53	44.24	38.60	29.21	35.86	41.61	47.72
- Co-rewarding-I	33.86	23.84	31.61	36.09	43.88	41.73	32.35	39.37	44.48	50.72
- Co-rewarding-II	32.74	23.29	29.02	36.33	42.33	41.82	31.79	40.29	43.88	51.31
<i>Qwen3-1.7B-Base</i>					<i>Qwen3-4B-Base</i>					
Before RL	33.65	25.69	30.86	36.45	41.60	46.43	36.04	44.18	48.68	56.83
- GT-Reward	36.16	27.35	31.79	40.64	44.84	47.80	37.34	46.77	49.40	57.67
- Self-Certainty	32.96	24.58	29.20	36.69	41.36	48.15	39.37	46.76	49.52	56.95
- Entropy	35.37	26.61	31.42	39.44	44.00	50.44	40.67	48.61	52.52	59.07
- Majority-Voting	35.45	26.06	32.16	38.72	48.84	48.78	37.89	47.50	50.36	59.65
- Co-rewarding-I	35.56	27.91	31.23	39.32	43.76	50.35	40.67	49.35	51.56	59.83
- Co-rewarding-II	36.94	27.17	33.64	40.05	46.88	51.30	41.40	49.54	53.12	61.15
<i>Qwen3-8B-Base</i>					<i>Llama3-2-Instruct</i>					
Before RL	50.32	40.11	50.27	51.07	59.83	57.32	46.77	55.27	60.19	67.03
- GT-Reward	52.78	41.96	51.76	54.44	62.95	47.41	37.34	42.88	52.52	57.31
- Self-Certainty	50.98	39.74	49.54	52.88	61.75	54.88	43.81	52.68	58.15	64.87
- Entropy	51.81	40.67	51.20	52.76	62.59	54.70	43.81	52.68	57.67	64.63
- Majority-Voting	51.80	39.74	51.02	53.60	62.83	47.96	37.34	43.44	52.88	58.18
- Co-rewarding-I	55.79	43.99	57.11	55.63	66.42	49.14	39.37	45.66	53.12	58.39
- Co-rewarding-II	60.70	55.64	65.59	56.00	65.59	49.90	39.93	45.66	54.68	59.35

Table 13: **Detailed IFEval performance on Qwen3-4B/8B-Base and Llama-3.2-3B-Instruct traid on DAPO-14k.** Results are reported for loose and strict settings in IFEval, respectively.

Methods	IFEval				
	Average	Prompt Strict	Prompt Loose	Inst. Strict	Inst. Loose
<i>Qwen3-4B-Base</i>					
Before RL	46.43	36.04	44.18	48.68	56.83
- GT-Reward	47.70	37.52	45.84	49.76	57.67
- Self-Certainty	45.58	35.67	43.99	47.84	54.80
- Entropy	48.20	37.71	46.58	50.48	58.03
- Majority-Voting	48.91	39.19	47.69	50.24	58.51
- Co-rewarding-I	46.84	36.41	45.66	48.80	56.47
- Co-rewarding-II	48.90	39.56	46.21	51.44	58.39
<i>Qwen3-8B-Base</i>					
Before RL	50.32	40.11	50.27	51.07	59.83
- GT-Reward	53.11	41.59	52.13	54.56	64.15
- Self-Certainty	50.58	41.04	49.54	51.68	60.07
- Entropy	51.56	41.59	49.91	53.48	61.27
- Majority-Voting	51.54	41.22	51.02	52.64	61.27
- Co-rewarding-I	50.17	40.67	48.24	52.16	59.59
- Co-rewarding-II	53.31	41.40	53.23	54.20	64.39
<i>Llama3.2-3B-Instruct</i>					
Before RL	57.32	46.77	55.27	60.19	67.03
- GT-Reward	53.10	42.33	49.91	57.19	62.95
- Self-Certainty	54.50	44.55	51.76	58.03	63.67
- Entropy	55.78	45.29	53.23	59.11	65.47
- Majority-Voting	54.07	42.33	52.50	56.83	64.63
- Co-rewarding-I	53.04	42.33	51.02	55.76	63.07
- Co-rewarding-II	51.92	41.59	48.24	56.00	61.87

Next, simplify the fraction inside the denominator:

$$3 + \frac{3}{10} = \frac{30}{10} + \frac{3}{10} = \frac{33}{10}.$$

Now substitute this back into the expression:

Table 14: **Detailed IFEval Performance on Qwen3-8B/4B-Base trained on Open-RS.** Results are reported for loose and strict settings in IFEval, respectively.

Methods	IFEval									
	Average	Prompt Strict	Prompt Loose	Inst. Strict	Inst. Loose	Average	Prompt Strict	Prompt Loose	Inst. Strict	Inst. Loose
	<i>Qwen3-8B-Base</i>					<i>Qwen3-4B-Base</i>				
Before RL	50.32	40.11	50.27	51.07	59.83	46.43	36.04	44.18	48.68	56.83
- GT-Reward	52.53	41.59	51.02	54.56	62.95	47.80	37.34	46.77	49.40	57.67
- Self-Certainty	52.12	41.59	50.83	53.72	62.35	46.47	35.86	44.73	48.56	56.71
- Entropy	52.94	43.25	51.94	53.72	62.83	46.41	36.97	44.73	48.68	55.28
- Majority-Voting	51.13	40.67	49.35	53.36	61.15	46.35	36.41	44.18	48.80	56.00
- Co-rewarding-I	53.11	41.40	53.05	53.95	64.02	46.83	35.86	46.77	47.84	56.83
- Co-rewarding-II	52.92	42.14	52.50	54.08	62.95	48.45	38.26	46.77	50.24	58.51

Table 15: **Case studies:** Original vs. Rephrased Questions

Original Question	Rephrased Question
Sam is hired for a 20-day period. On days that he works, he earns \$60. For each day that he does not work, \$30 is subtracted from his earnings. At the end of the 20-day period, he received \$660. How many days did he not work?	A contractor agrees to a job lasting 20 days. For every day the job is completed on time, the contractor earns \$60. However, for each day the work is delayed, a fine of \$30 is applied. After the 20-day period, the contractor's total earnings are \$660. How many days was the job delayed?
Karen drove continuously from 9:40 a.m. until 1:20 p.m. of the same day and covered a distance of 165 miles. What was her average speed in miles per hour?	A traveler set off at 9:40 a.m. and reached their destination at 1:20 p.m. the same day after traveling a total of 165 miles. What was their average speed during the trip in miles per hour?
Solve for x : $\frac{1}{2} + \frac{1}{x} = \frac{5}{6}$.	A tank is partially filled by two different pipes. One pipe fills half the tank in an hour, and together with another pipe, they fill five-sixths of the tank in the same time. If the second pipe alone fills $\frac{1}{x}$ of the tank in an hour, find the value of x .

$$3 + \frac{1}{\frac{33}{10}} = 3 + \frac{10}{33}.$$

To add these, we need a common denominator:

$$3 + \frac{10}{33} = \frac{99}{33} + \frac{10}{33} = \frac{109}{33}.$$

The fraction $\frac{109}{33}$ is already in its simplest form because 109 and 33 have no common factors other than 1. Therefore, $m = 109$ and $n = 33$, and the value of $m + n$ is:

$$109 + 33 = 142.$$

So, the final answer is:

$$\boxed{142}.$$

Co-rewarding-II Answer:

To solve the given expression $3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$, we need to simplify it step by step.

First, simplify the innermost fraction:

$$3 + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{10}{3}.$$

Next, substitute this back into the next fraction:

$$3 + \frac{1}{3 + \frac{1}{\frac{10}{3}}} = 3 + \frac{1}{3 + \frac{3}{10}} = 3 + \frac{1}{\frac{30}{10} + \frac{3}{10}} = 3 + \frac{1}{\frac{33}{10}} = 3 + \frac{10}{33}.$$

Now, simplify the expression $3 + \frac{10}{33}$:

$$3 + \frac{10}{33} = \frac{99}{33} + \frac{10}{33} = \frac{109}{33}.$$

The fraction $\frac{109}{33}$ is already in its simplest form because 109 and 33 are coprime (they have no common factors other than 1).

Thus, the irreducible fraction value of the given expression is $\frac{109}{33}$, where $m = 109$ and $n = 33$. Therefore, $m + n = 109 + 33 = 142$.

The value of $m + n$ is 142.

GT-Reward Answer:

Alright, I have this problem:

The expression is:

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$$

And I need to find the irreducible fraction $\frac{m}{n}$ that equals this expression, then determine $m + n$.

First, I need to simplify the expression step by step. It looks like a continued fraction, so I'll start by simplifying the innermost part and work my way out.

Let's denote the entire expression as:

$$E = 3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$$

****Step 1: Simplify the innermost fraction****

Start with the innermost fraction:

$$3 + \frac{1}{3}$$

Calculate this:

$$3 + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{10}{3}$$

So now, the expression becomes:

$$E = 3 + \frac{1}{3 + \frac{1}{\frac{10}{3}}}$$

Wait, I need to simplify $\frac{1}{\frac{10}{3}}$:

$$\frac{1}{\frac{10}{3}} = \frac{3}{10}$$

Now, the expression is:

$$E = 3 + \frac{1}{3 + \frac{3}{10}}$$

****Step 2: Simplify the next fraction****

Now, simplify $3 + \frac{3}{10}$:

$$3 + \frac{3}{10} = \frac{30}{10} + \frac{3}{10} = \frac{33}{10}$$

So, the expression now is:

$$E = 3 + \frac{1}{\frac{33}{10}}$$

Simplify $\frac{1}{\frac{33}{10}}$:

$$\frac{1}{\frac{33}{10}} = \frac{10}{33}$$

Now, the expression is:

$$E = 3 + \frac{10}{33}$$

****Step 3: Combine the terms****

Convert 3 to a fraction with denominator 33:

$$3 = \frac{99}{33}$$

Now, add the fractions:

$$E = \frac{99}{33} + \frac{10}{33} = \frac{109}{33}$$

So, the expression simplifies to $\frac{109}{33}$.

****Step 4: Ensure the fraction is irreducible****

Check if 109 and 33 have any common divisors. 109 is a prime number (I think), and 33 is 3×11 . Since 109 isn't divisible by 3 or 11, the fraction $\frac{109}{33}$ is already in its irreducible form.

****Step 5: Find $m + n$ ****

Here, $m = 109$ and $n = 33$, so:

$$m + n = 109 + 33 = 142$$

Wait, let's double-check the calculations to ensure no mistakes were made.

****Verification:****

Starting over:

$$E = 3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$$

Innermost:

$$3 + \frac{1}{3} = \frac{10}{3}$$

Next layer:

$$3 + \frac{1}{\frac{10}{3}} = 3 + \frac{3}{10} = \frac{33}{10}$$

Outermost:

$$3 + \frac{1}{\frac{33}{10}} = 3 + \frac{10}{33} = \frac{99}{33} + \frac{10}{33} = \frac{109}{33}$$

Yes, it checks out. Therefore, $m + n = 109 + 33 = 142$.

142

Majority-Voting Answer:

To find the value of $\frac{m}{n}$ as the irreducible fraction of the expression $3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$, we need to simplify the expression step by step.

First, let's simplify the innermost fraction:

$$3 + \frac{1}{3} = 3 + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{10}{3}$$

Now, substitute this back into the expression:

$$3 + \frac{1}{3 + \frac{1}{\frac{10}{3}}} = 3 + \frac{1}{3 + \frac{3}{10}}$$

Next, simplify the fraction $3 + \frac{3}{10}$:

$$3 + \frac{3}{10} = \frac{30}{10} + \frac{3}{10} = \frac{33}{10}$$

Now, substitute this back into the expression:

$$3 + \frac{1}{\frac{33}{10}} = 3 + \frac{10}{33}$$

To add these, we need a common denominator:

$$3 + \frac{10}{33} = \frac{99}{33} + \frac{10}{33} = \frac{109}{33}$$

So, the irreducible fraction value of the expression is $\frac{109}{33}$. Here, $m = 109$ and $n = 33$. Therefore, the value of $m + n$ is:

$$m + n = 109 + 33 = 142$$

Thus, the value of $m + n$ is 0.

Self-Certainty Answer:

Alright, I have this math problem to solve:

****Problem:**** I need to solve a math problem. The problem is about understanding how to solve math problems. I need to understand how to solve this problem so I can understand more about math.

Understanding the Problem

First, I need to understand what the problem is. I need to understand math so I can understand more about numbers and how to solve problems with numbers. Understanding math will help me understand more about how to understand problems.

Understanding Math Problems

After understanding the problem, I need to understand more about math problems. Understanding math problems will help me understand how to understand these problems. Understanding these problems will help me understand more about how to understand math.

Understanding Mathematical Concepts

After understanding math problems, I need to understand more about mathematical concepts. Understanding mathematical concepts will help me understand more about math. Understanding these concepts will help me understand more about how math helps me understand numbers.

Understanding Mathematical Operations

After understanding mathematical concepts, I need to understand more about mathematical operations. Understanding mathematical operations will help me understand more about how to understand these operations. Understanding these operations will help me understand more about how these operations help me understand math.

Understanding Mathematical Operations in Math

After understanding mathematical operations, I need to understand

Understanding Mathematical Methods

After understanding mathematical operations, I need to understand more about mathematical methods.

Understanding How Mathematical Methods Help

.....

Understanding How These Methods Help Me Understand Math

After understanding more about how these methods help me, I need to understand

Understanding How These Methods Help Me Understand Math Problems

After understanding how these methods help me understand math, I need to understand

Understanding How These Methods Help Me Solve Math Problems

After understanding how these methods help me understand these math problems, I need to understand

Understanding How These Methods Help Me Solve These Problems

After understanding how these methods help me solve these problems, I need to understand more about

Entropy Answer:

To find the value of $\frac{m}{n}$ as the irreducible fraction of the expression:

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$$

Let's simplify the expression step by step.

1. **Simplify the innermost fraction:**

$$3 + \frac{1}{3}$$

$$3 + \frac{1}{3} = \frac{3 \times 3}{3} + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{9+1}{3} = \frac{10}{3}$$

2. **Simplify the next fraction:**

$$3 + \frac{1}{3 + \frac{1}{3}} = 3 + \frac{1}{\frac{10}{3}}$$

$$3 + \frac{1}{\frac{10}{3}} = 3 + \frac{1 \times 3}{10} = 3 + \frac{3}{10} = \frac{3 \times 10}{10} + \frac{3}{10} = \frac{30}{10} + \frac{3}{10} = \frac{30+3}{10} = \frac{33}{10}$$

3. **Simplify the next fraction:**

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}} = 3 + \frac{1}{\frac{33}{10}}$$

$$3 + \frac{1}{\frac{33}{10}} = 3 + \frac{1 \times 10}{33} = 3 + \frac{10}{33} = \frac{3 \times 33}{33} + \frac{10}{33} = \frac{99}{33} + \frac{10}{33} = \frac{99+10}{33} = \frac{109}{33}$$

4. **Simplify the next fraction:**

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}} = 3 + \frac{1}{\frac{109}{33}}$$

$$3 + \frac{1}{\frac{109}{33}} = 3 + \frac{1 \times 33}{109} = 3 + \frac{33}{109} = \frac{3 \times 109}{109} + \frac{33}{109} = \frac{327}{109} + \frac{33}{109} = \frac{327+33}{109} = \frac{360}{109}$$

5. **Simplify the next fraction:**

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}}} = 3 + \frac{1}{\frac{360}{109}}$$

$$3 + \frac{1}{\frac{360}{109}} = 3 + \frac{1 \times 109}{360} = 3 + \frac{109}{360} = \frac{3 \times 360}{360} + \frac{109}{360} = \frac{1080}{360} + \frac{109}{360} = \frac{1080+109}{360} = \frac{1189}{360}$$

6. **Simplify the next fraction:**

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}}}} = 3 + \frac{1}{\frac{1189}{360}}$$

.....
7. **Simplify the next fraction:**

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}}}} = 3 + \frac{1}{\frac{3927}{1189}}$$

.....
8. **Simplify the next fraction:**

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}}}} = 3 + \frac{1}{\frac{12970}{3927}}$$

.....
9. **Simplify the next fraction:**

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}}}}} = 3 + \frac{1}{\frac{42837}{12970}}$$

.....
10. **Simplify the next fraction:**

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}}}}} = 3 + \frac{1}{\frac{141481}{42837}}$$

.....
11. **Simplify the next fraction:**

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}}}}}}} = 3 + \frac{1}{\frac{467280}{141481}}$$

.....
12. **Simplify the next fraction:**

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}}}}}}} = 3 + \frac{1}{\frac{1543321}{467280}}$$

.....
13. **Simplify the next fraction:**

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}}}}}}} = 3 + \frac{1}{\frac{5097243}{1543321}}$$

$$3 + \frac{1}{\frac{5097243}{1543321}} = 3 +$$