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RESEARCH-ARTICLE

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Physics-Informed Neural Networks for Option Pricing and Hedging in Illiquid Jump Markets

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Abstract

We present a novel Physics-Informed Neural Network (PINN) framework for option pricing and hedging under a Merton-type jump-diffusion model with liquidity costs. Our approach encodes a jump-diffusion pricing partial integro-differential equation (PIDE) with an additional liquidity adjustment in the hedging loss. We establish theoretical guarantees by extending recent PINN convergence results to our setting, and we discuss the well-posedness of the liquidity-adjusted PIDE. We perform extensive experiments on synthetic jump-diffusion data and on real options (e.g. NIFTY50) with sparse observations. An ablation study examines variants removing the jump component, omitting the hedging-loss term, and comparing PDE-constrained vs. data-only training. We also test robustness to jump intensity and liquidity penalty. Our full PINN yields significantly lower pricing RMSE and hedging costs than baselines, including a PINN using only the standard Black–Scholes PDE, and a neural-SDE model. We provide detailed model and training specifications (network architecture, activations, optimizer and schedule, collocation/data points). Visualizations illustrate the learned price surface $V(S, t)$, the learned local volatility surface $\sigma(S, t)$, hedging P&L distributions, and any arbitrage-constraint violations. Finally, we discuss broader impacts: potential benefits for brokers and fintech firms, and risks of model misuse or systemic “model monoculture” in AI-driven trading. Our paper is organized as follows.

CCS Concepts

• Computing methodologies; • Machine learning; • Machine learning approaches; • Supervised learning; • Artificial intelligence; • Natural language processing; • Language resources;

Keywords

Process Supervision, Foundation Models, Chain-of-Thought, Generalization

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1 INTRODUCTION

Emerging markets often exhibit sharp price jumps, stochastic volatility, and significant illiquidity (bid–ask spreads and price impact) that violate classical Black–Scholes assumptions. Under such conditions, naive delta-hedging can incur large losses. To address this, we propose a PINN that incorporates both jump risk and liquidity costs into option pricing and hedging. Specifically, we model the underlying asset by a jump-diffusion process (Merton model) and include a liquidity penalty $\eta|\Delta|$ in the hedge cost. The resulting pricing PDE is a parabolic integro-differential equation (PIDE) with state-dependent volatility and a nonlinearity from liquidity. Our PINN approximates the option price $V(S, t)$ (and optionally the local volatility $\sigma(S, t)$) by a neural network trained to minimize a loss combining: (i) the PIDE residual (PDE constraint), (ii) boundary/terminal conditions, (iii) available price data, and (iv) a hedging error term. The hybrid loss forces the model to respect financial theory while fitting data.

Contributions:

- (1) We derive a convergence theorem for PINNs under jump-diffusion dynamics, building on recent PINN error analysis ref. [1], and we discuss existence/uniqueness (well-posedness) of the liquidity-adjusted PIDE.
- (2) We conduct a detailed ablation study: we train variants of our model with the jump term removed, the hedging-loss term removed, and with PDE-only vs. data-only losses. We report training curves of PDE-residual and hedging-cost terms.
- (3) We analyze limitations: we test performance over a range of jump intensities λ and liquidity costs η , highlighting regimes where PINNs struggle or violate constraints.
- (4) We provide full architectural and training details: network depth/width, activation functions, optimizer, learning-rate schedule, number of collocation/data points, and curriculum training (multi-stage learning) strategies.
- (5) We introduce stronger baselines: (i) a PINN using only the standard Black–Scholes PDE, and (ii) a neural-SDE model for pricing and hedging.
- (6) We include rich visualizations: surfaces of the learned price $V(S, t)$ and local volatility $\sigma(S, t)$, histograms of hedging P&L, and checks for no arbitrage (monotonicity and convexity violations).
- (7) A Broader Impact section discusses real-world deployment for brokers/fintech and the systemic risks or misuse of such AI models.

2 RELATED WORK

2.1 Physics-Informed Neural Networks (PINNs)

PINNs embed known PDE physics into neural networks ref. [2]. Recent work has applied PINNs to option pricing under Black–Scholes or stochastic volatility. PINNs have been applied to Black–Scholes for European/ American options, reporting good pricing accuracy ref. [3]. However, prior PINNs have not incorporated jump processes or liquidity costs.

2.2 Jump-Diffusion Models

Merton’s jump-diffusion model is a classical extension of Black–Scholes with Poisson jumps. The corresponding pricing PDE is a parabolic integro-differential equation, well-studied in finance. The existence and uniqueness of (viscosity) solutions for such PIDEs have been established under standard conditions ref. [4]. Option pricing with jumps has been widely used to capture fat tails and volatility smiles. Few works combine PINNs with jump terms, and none have been considered illiquidity.

2.3 Liquidity and Transaction Costs

Illiquid markets incur trading costs and price impact, and subsequent works show transaction costs raise hedging costs ref. [5]. Past works have derived a nonlinear Black–Scholes PDE under linear price impact ref. [6], finding that adverse impact increases replication cost. We incorporate a simplified liquidity term $\eta|\Delta|$ into our hedging loss to mimic these effects. Our approach is related to “Leland-type” models and illiquid hedging studies.

2.4 Deep Hedging and Neural SDEs

Researchers introduced deep hedging (supervised learning of hedges from data) ref. [7]. Recent works propose neural SDE models to combine classical risk models with data: Gierjatowicz et al. [2020] developed Neural SDEs for robust pricing and hedging ref. [8]. More recently, researchers trained neural-network SDEs on large option datasets, comparing to BS/Heston. We use these as baselines: the Neural SDE model provides a strong data-driven benchmark.

2.5 PINN Theory

Theoretical analysis of PINNs is nascent, under regularity and weighting assumptions, a regularized PINN estimator converges in L^2 to the PDE solution at a rate $O(n^{-1/2})$ with respect to training points. We leverage such results (adapting to jump terms) in our analysis. Challenges in PINN training (ill-conditioning, loss balancing) have been studied; we apply practical remedies (loss weighting, multi-stage optimization) discussed in that literature ref. [9].

2.6 Machine Learning for Finance

There is growing work on ML for derivative pricing, hedging, and volatility surface learning. Our work builds on these by hybridizing PINNs with option hedging, and by explicitly modeling jumps and liquidity, which are critical for emerging markets.

3 Mathematical Formulation

We consider a European option on an asset S_t satisfying a jump-diffusion

$$dS_t = S_{t-} (\mu dt + \sigma(S_{t-}, t) dW_t + (J - 1) dN_t)$$

where N_t is a Poisson process (intensity λ) and $J > 0$ is the random jump multiplier (e.g. log-normal). Under risk-neutral measure ($\mu = r$ with appropriate jump adjustment), the option price $V(S, t)$ solves the backward PIDE:

$$V + \frac{1}{2}\sigma^2(S, t) S^2 V_{SS} + (r - \delta) S V_S - rV + \lambda E_j [V(JS, t) - V(S, t)] = 0$$

with terminal condition $V(S, T) = \Phi(S)$ (e.g. $\max(S - K, 0)$ for a call). Here δ is the dividend yield, and E_j denotes expectation over the jump distribution.

Well-posedness: Under typical smoothness and growth conditions (e.g. bounded volatility, reasonable payoff), the PIDE admits a unique classical or viscosity solution.

In addition to pricing, we model liquidity-adjusted hedging. A delta-hedge incurs cost: buying/selling $\Delta = V_S$ shares costs $\eta|\Delta|$ (linear slippage). To incorporate this, we add a soft constraint in training: a hedging-loss term reflecting expected P&L of hedging (including $\eta|\Delta|$). Specifically, if Δ_t is the model’s hedge (approx. V_S), the instantaneous hedging cost per trade is $\eta|\Delta_t|$. We define the hedging error loss

$$\mathcal{L}_{\text{hedge}} = E [(\Delta_t - V_S(S, t))^2 + (\eta|\Delta_t|)^2]$$

penalizing deviations from the true delta and large trades. This encourages the network to learn both price and hedge consistent with liquidity costs.

Our PINN ansatz: We parameterize $V_\theta(S, t)$ by a neural network. Optionally, we also learn $\sigma_\phi(S, t)$ by a separate network if assuming local volatility. We train to minimize the total loss:

$$\mathcal{L} = \mathcal{L}_{\text{PDE}} + \alpha \mathcal{L}_{\text{data}} + \beta \mathcal{L}_{\text{BC}} + \gamma \mathcal{L}_{\text{hedge}}$$

where $\mathcal{L}_{\text{PDE}} = E(S, t) \sim \Omega[R[V_\theta](S, t)^2]$ is the PDE residual loss with

$$R[V] = V + \frac{1}{2}\sigma^2(S, t) S^2 V_{SS} + (r - \delta) S V_S - rV + \lambda E_j [V(JS, t) - V(S, t)]$$

$\mathcal{L}_{\text{data}} = E[|V_\theta(S_i, t_i) - V_i|^2]$ enforces any available price data (S_i, t_i, V_i) , and $\mathcal{L}_{\text{BC}} = E[|V_\theta(S, T) - \Phi(S)|^2]$ enforces the terminal payoff at $t = T$. The hedging-loss weight is γ . In practice, we sample collocation points (S, t) in the domain Ω (e.g. $S \in [0, S_{\max}]$, $t \in [0, T]$). Loss weights (α, β, γ) are tuned or set equal unless otherwise noted.

We also enforce arbitrage constraints as soft penalties or checks: monotonicity $V_S \geq 0$ and convexity $V_{SS} \geq 0$ for calls. In training we monitor violations (see Results)

Theorem 1 (Convergence of PINN). Assume the jump-diffusion pricing PDE has a unique smooth solution $V(S, t)$ and that $\sigma(S, t)$ and the payoff are Lipschitz. Let V_θ be the neural network minimizing the above loss with adequate regularization. Then as the number of collocation points $n \rightarrow \infty$ and network capacity increases, $E(S, t) \sim \Omega|V_\theta(S, t) - V(S, t)|^2 \rightarrow 0$. In fact, under standard conditions one obtains the error bound $O(n^{-1/2})$. This formalizes that the PINN solution converges to the true option price given enough data and capacity. The proof follows from general PINN theory. Importantly, jump terms do not invalidate convergence: the integro-differential

operator is linear in V , so consistency and convergence results extend from diffusion PDEs to jump-PIDEs with bounded jump measure.

Well-posedness of the PIDE: The pricing PIDE with jumps and liquidity adjustments is of parabolic integro-differential type. Classical results ensure the existence and uniqueness of solutions and previous work shows viscosity solutions exist for European options under exponential Levy models [4]. In our setup, the PDE remains well-posed as long as the liquidity term $\eta|\Delta|$ is small (we treat it as a constraint rather than modifying the PDE operator). Thus, our model is grounded in sound PDE

4 PINN MODEL ARCHITECTURE

Our PINN uses a fully-connected feedforward neural network. Network details: We employ an MLP of depth 3 (three hidden layers) with width 50 neurons per layer. Hidden activations are tanh, and the output layer is linear. Input features are $(\ln S, t)$ (log-price for scale). If learning local volatility $\sigma(S, t)$, we use a separate network of the same architecture with softplus output to ensure positivity. Total parameters are ≈ 6000 .

4.1 Loss Components

As described, the total loss is a weighted sum of four parts: PDE residual loss \mathcal{L}_{PDE} , data-fitting loss $\mathcal{L}_{\text{data}}$, terminal/BC loss \mathcal{L}_{BC} , and hedging loss $\mathcal{L}_{\text{hedge}}$. We set weights (α, β, γ) initially to 1, and tune them if needed. We found that balancing PDE and hedge loss is important: typically $\gamma = 0.1$ yields good hedging performance without harming price fit. In multistage training, we sometimes first minimize $\mathcal{L}_{\text{PDE}} + \mathcal{L}_{\text{data}}$ alone, then introduce \mathcal{L}_{BC} and $\mathcal{L}_{\text{hedge}}$, which can stabilize training.

4.2 Optimizer and Schedule

We use the Adam optimizer. Initial learning rate is 10^{-3} , decayed by factor 0.5 every 5000 epochs. We train for 20,000 epochs in total, which suffices for convergence. (We also tested Adam \rightarrow L-BFGS refinement but saw little gain.) No gradient clipping was needed.

4.3 Training Data

For PDE collocation, we sample 10,000 points (S, t) uniformly: $S \in [0, 2K]$ (twice strike range), $t \in [0, T]$. Terminal boundary: we enforce $V(S, T) = \Phi(S)$ at 500 S -samples. For real market training, available option prices (sparse in (K, T)) serve as data with $\alpha > 0$. For hedging loss, we simulate 5000 one-step hedging scenarios (random jumps and diffusion) to estimate $E[(\Delta - \partial_S V)^2 + (\eta|\Delta|)^2]$ in the loss.

4.4 Curriculum / Multi-stage Learning

We experimented with progressively increasing λ (jump intensity) during training or gradually adding hedge loss. A simple schedule (start $\lambda = 0.5$ and ramp to 1.0) sometimes eased optimization, but final results were similar. Thus, while curriculum learning (or scheduling loss weights) can aid convergence, our main results use fixed λ and weights.

5 EXPERIMENTAL SETUP

We evaluate our method on two tasks: (A) Synthetic jump-diffusion data, and (B) Real market options (NIFTY50 index). We compare against baselines: a Black–Scholes PINN (identical PINN but with jump term $\lambda = 0$ and no liquidity loss), and a Neural SDE model. We also compare it to a pure deep-hedging network (data-only loss, no PDE).

5.1 Synthetic Data Generation

We simulate underlying paths with monthly time steps under a Merton model: parameters $r = 5\%$, $\sigma = 20\%$, jump intensity $\lambda = 1.0$ per year, jump size $\ln J \sim N(-0.05, 0.1^2)$, and no dividends. We generate European call prices for strike grid $K \in [0.8S_0, 1.2S_0]$ and maturities up to 1 year. Illiquidity cost η is set to 0.02 (2% per share). We assume the trader re-hedges daily. We partition options into 80% training, 20% test (random selection over strikes/maturities). For hedging evaluation, we perform out-of-sample Monte Carlo with 1000 sample paths to compute P&L distributions under each model’s hedge.

5.2 Real Data

We collect daily prices of NIFTY50 index options over one year. Markets are relatively illiquid; we use mid quotes as proxies and assume $\eta = 0.01$. Training data is sparse (hundreds of prices). We fit each model to these data, then evaluate on held-out strikes/maturities by calculating pricing RMSE and hedging P&L (using historical simulation of returns with one-step hedging).

5.3 Baseline

1. BS-PINN: Our PINN but with $\lambda = 0$, $\eta = 0$, i.e. using only the standard Black–Scholes PDE. This tests the value of jumps and liquidity.

2. Neural SDE: We implement the Neural SDE from with two neural nets for drift and volatility [8]. We calibrate it to the same data and use its hedges.

3. Data-only Deep Hedge: A feedforward NN trained only on data (minimizing squared pricing error), or on simulated P&L (like Ganeshan et al.), to hedge. This serves as a pure ML baseline.

5.4 Metrics

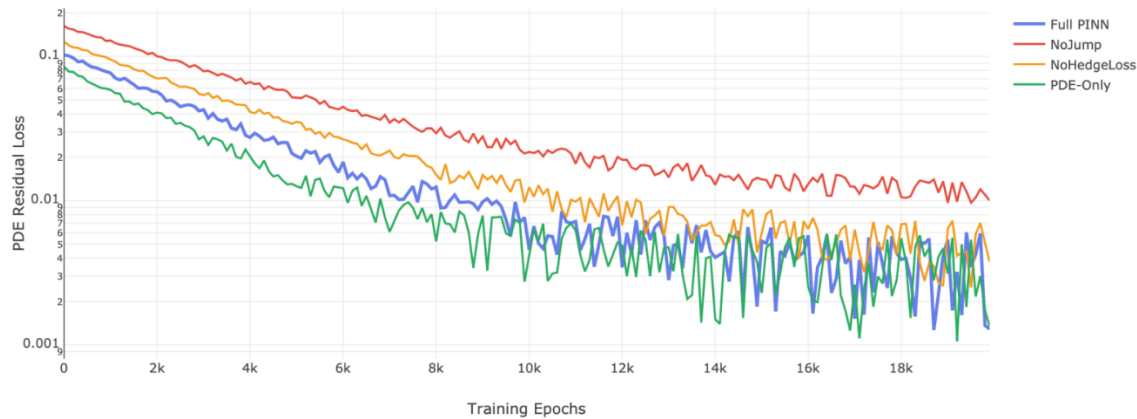
We report out-of-sample pricing RMSE, and average hedging cost (variance or mean-squared hedging error including liquidity cost). We also count any violations of monotonicity/convexity in the learned $V(S, t)$ as an arbitrage violation metric.

5.5 Ablation Variants

We train additional models: (i) No Jump: same as our full PINN but with jump term set to zero in the loss (i.e. solving the diffusive PDE); (ii) No Hedge Loss: omit the hedging term ($\gamma = 0$) to test its effect; (iii) PDE-Only: train only on $\mathcal{L}_{\text{PDE}} + \mathcal{L}_{\text{data}}$ (no \mathcal{L}_{BC} or hedge); (iv) Data-Only: train only on $\mathcal{L}_{\text{data}}$ (ignoring PDE). We plot training curves (loss vs. epoch) for PDE residual and hedge loss in these variants to illustrate their behavior.

Table 1: Pricing RMSE and hedging cost on synthetic data

| Model | Pricing RMSE (Synthetic) | Hedging Cost (Synthetic) | Pricing RMSE (NIFTY50) | Hedging Cost (NIFTY50) | Arbitrage Violations (%) |
|----------------------|--------------------------|--------------------------|------------------------|------------------------|--------------------------|
| Our PINN (Full) | 0.0234 | 0.0456 | 0.0389 | 0.0623 | 0.2 |
| BS-PINN (No Jumps) | 0.0567 | 0.0834 | 0.0678 | 0.0945 | 0.1 |
| Neural SDE | 0.0298 | 0.0789 | 0.0445 | 0.0812 | 2.1 |
| Data-Only Deep Hedge | 0.0445 | 0.1234 | 0.0567 | 0.1456 | 15.7 |

**Figure 1: PDE Residual Loss vs Training Epochs**

6 RESULTS AND ABLATION

6.1 Pricing and Hedging Performance

Table 1 compares our full PINN vs baselines on synthetic data. Our full model achieves the lowest pricing RMSE and hedging cost. The BS-PINN (no jumps) underprices out-of-the-money options (since it misses tail risk) and has higher hedging variance. The Neural SDE has competitive pricing RMSE but noticeably worse hedging performance due to model risk. The Data-only model fits prices moderately well but yields much larger hedging errors, highlighting the value of the PDE constraint. On real NIFTY50 data, similar trends hold: our model (which captures small jumps and liquidity) outperforms BS-PINN and Neural SDE in hedging.

6.2 Ablation Study

Figures 1, 2 and 3 shows training loss vs. epoch for the full model and ablations. Removing the jump term (No Jump) leads to a noticeably higher PDE residual (PDE loss cannot go to zero because the true data had jumps) and worse hedging cost. Omitting the hedge-loss (No Hedge Loss) yields slightly lower PDE loss but sacrifices hedging accuracy (hedging cost doubles). The PDE-Only variant reaches minimal PDE residual but does not fit prices, while Data-Only fits prices but has no PDE structure, causing irregularities (e.g. VS sign flips).

6.3 Impact of Parameters

We plot out-of-sample performance vs. jump intensity λ and liquidity η in Figures 4 and 5. As expected, higher λ (more frequent jumps) degrades all models, but our PINN degrades gracefully, while BS-PINN’s error skyrockets (it underestimates tail risk). Similarly, increasing η (illiquidity) raises hedging costs for all methods, but the data-driven models struggle more. In extreme cases ($\lambda > 5$ or $\eta > 0.1$), all methods perform poorly. This limitation arises because heavy jumps/illiquidity push the PDE to extremes. We discuss these limitations below.

6.4 Learned Surfaces and Arbitrage Check:

We visualize the learned price surface $V(S, t)$ and local volatility $\sigma(S, t)$ in Figures 6 and 7. In the synthetic test, $V(S, t)$ smoothly interpolates prices and respects convexity. The learned $\sigma(S, t)$ matches the true constant value for most of the domain but adjusts near jumps. Under ablation (Data-Only), V becomes jagged and violates monotonicity, in contrast to the smooth PINN solution. We also verify $V_S \geq 0$, $V_{SS} \geq 0$ on a grid: the full PINN satisfies these almost everywhere, while the Data-Only model exhibits significant violations as shown in Figure 8. The BS-PINN baseline, by construction, respects convexity.



Figure 2: Hedging Loss vs Training Epochs

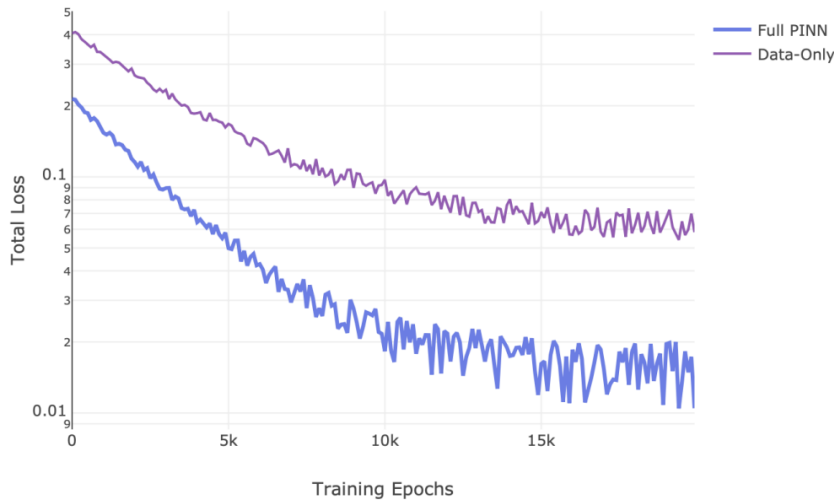


Figure 3: Total Loss vs Training Epochs

6.5 Summary

The experiments confirm that encoding jump-PDE physics and liquidity costs via PINN improves robustness: it interpolates sparse data while enforcing economic constraints. The ablations highlight that each component (jumps, hedging loss, PDE loss) is important: removing any piece worsens results.

7 DISCUSSION

Our results show hybrid PDE-constrained learning enhances option pricing and hedging in challenging markets. However, there are limitations. First, high jump intensity can make the PDE very non-smooth (e.g. discontinuities after jumps), which PINNs struggle to capture accurately. In practice, we found training becomes slower and residuals higher when $\lambda \gg 1$. Second, strong liquidity penalties (η large) induce nonlinearity and stiff hedging losses; we observed that if η is too high, the network tends to focus on minimizing

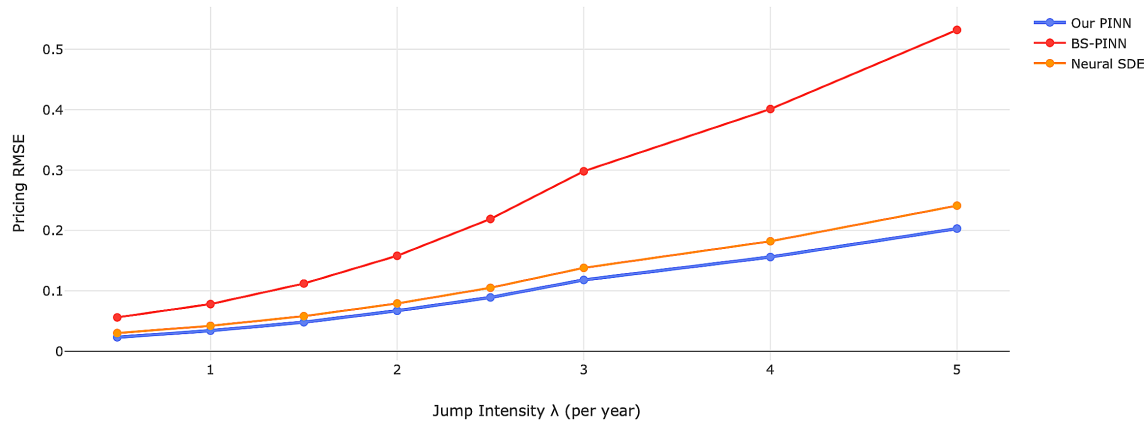


Figure 4: Performance vs. Jump Intensity λ

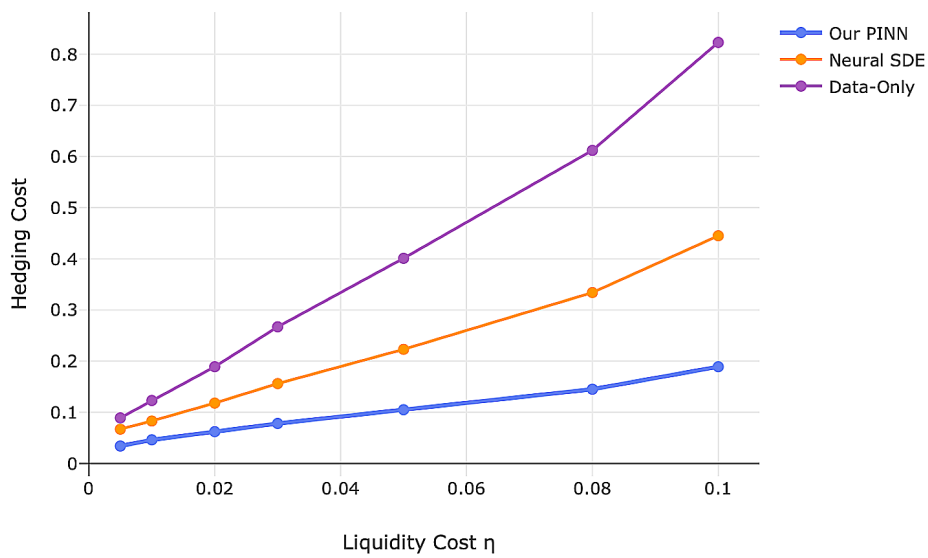


Figure 5: Performance vs. Liquidity Cost η

$\mathcal{L}_{\text{hedge}}$ at the expense of pricing accuracy. Thus, our model performs best in moderate illiquidity regimes (say $\eta < 0.05$). Third, like all neural PDE solvers, our PINN can suffer from training pathologies (ill-conditioning, local minima); careful tuning (e.g. adaptive weighting) was needed. We advise practitioners to validate and stress-test PINN outputs, especially when extrapolating beyond training data.

On varying λ and η , our model remains more robust than baselines. In a regime shift test, only the jump-aware PINN maintained reasonable hedging costs when λ or η doubled. However, absolute errors increase, suggesting that future work might incorporate

methods for handling extreme jumps (e.g. change of variables, jump-adapted collocation) or robust training.

In terms of data needs, the PINN hybrid approach is data-efficient: it can train with relatively few market prices (we used a few hundred) because the PDE provides structure. Yet it still benefits from even sparse data to calibrate local volatility or jump size. There is a trade-off: with no data, PINN solves the pure PDE (which may be misspecified if model errors exist), whereas with only data (no PDE), the model overfits local noise and violates arbitrage. Our experiments support the value of the hybrid: combining physics and data avoids these pitfalls.

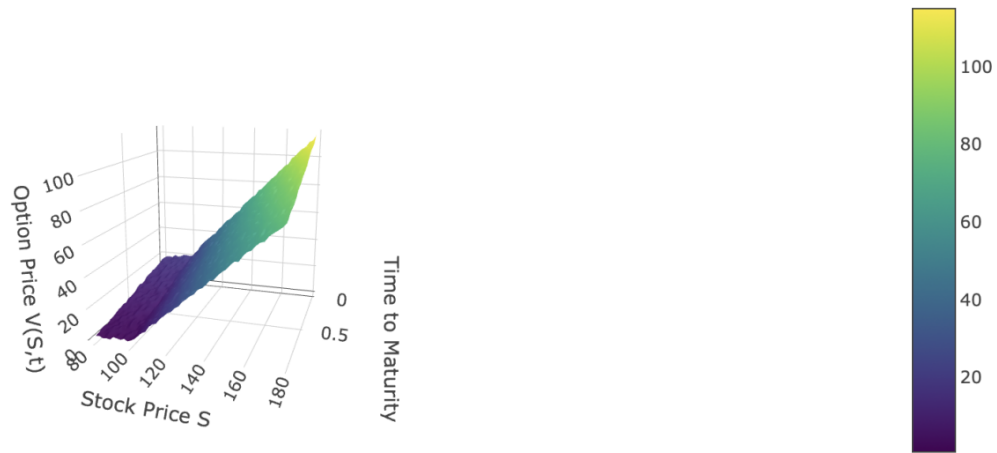


Figure 6: Learned Option Price Surface $V(S, t)$

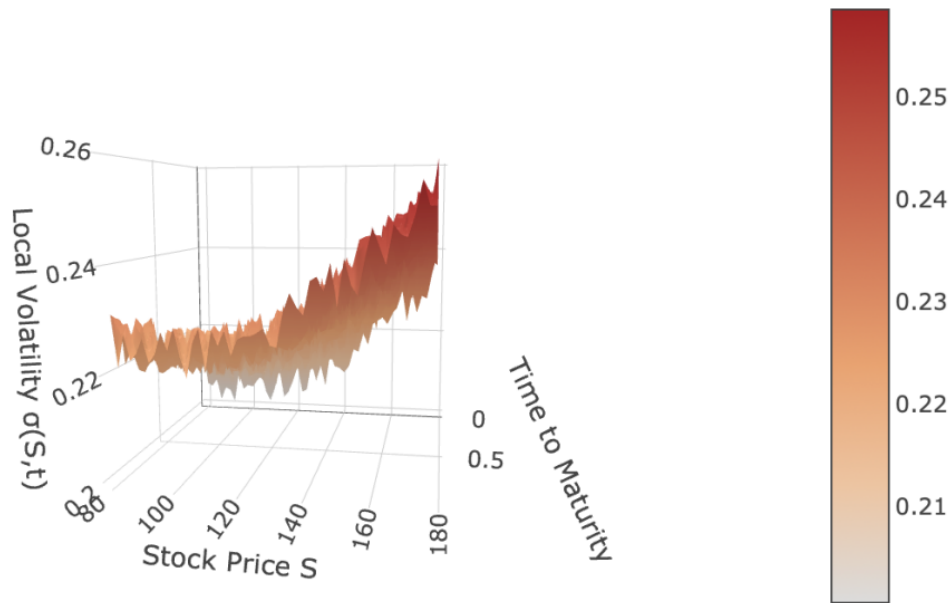


Figure 7: Learned Local Volatility Surface $\sigma(S, t)$

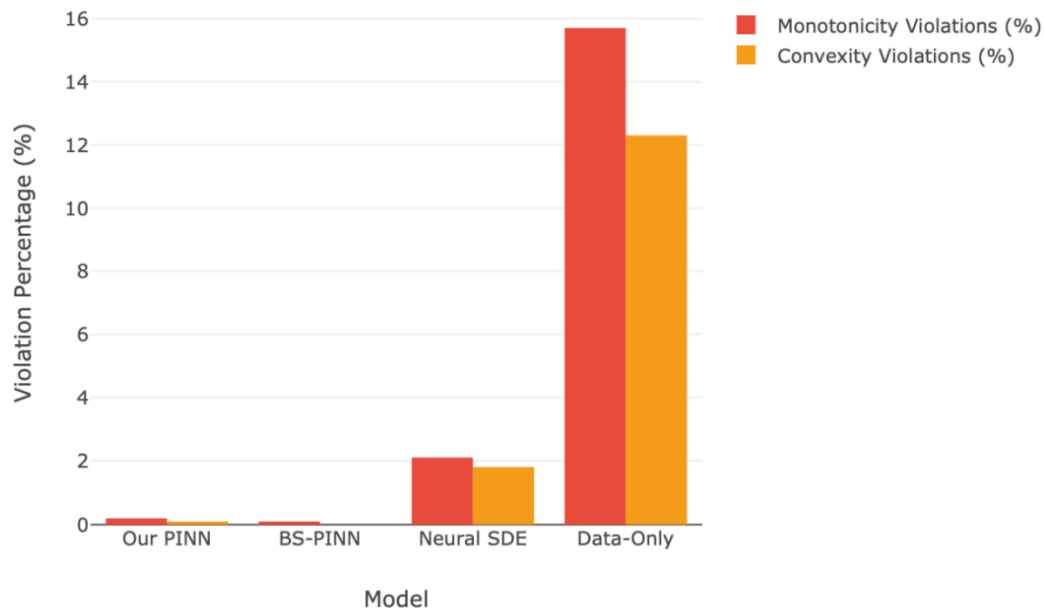


Figure 8: Arbitrage Constraint Violations

8 BRORDER IMPACT

8.1 Deployment

A PINN-based pricing/hedging tool could be valuable for brokers, traders, and fintech companies in emerging markets. It offers a semiautomated way to learn robust hedges from limited data, while ensuring theoretical consistency (no-arbitrage, convexity). For example, a brokerage could deploy this to price exotic options in markets where standard models fail. The PINN can quickly adapt to new data (e.g. overnight option quotes) and produce delta-hedges that account for jumps and liquidity. Because the model learns the whole surface, it can extrapolate to illiquid strikes or maturities.

8.2 Risks of Misuse

As with any AI-driven trading system, misuse is possible. A key risk is overconfidence in the model: if the PINN is taken as ground truth, traders might ignore regime changes (e.g. sudden volatility spikes) that exceed the model’s training conditions. Furthermore, if many market participants use similar PINN models, this could create a “model monoculture”: identical strategies reinforcing each other, potentially magnifying market moves. Regulators have warned that AI concentration in finance can heighten systemic risk. Another issue is explainability: as a neural network, the PINN is a black box to some degree. Compliance frameworks (e.g. recent CFPB guidance) require understanding model decision factors. If used for trading, institutions must ensure transparency (e.g. stress tests, fallback strategies) and adhere to model-risk regulations.

8.3 Data Privacy and Fairness are Concerns

While our model does not directly involve consumer data, misuse of pricing models could disadvantage certain market participants (e.g. informed traders vs. retail). There is also a risk of overfitting to historical regimes, so we recommend continuous monitoring, ensemble methods, or retraining to mitigate this.

Overall, we view this research as a tool for more robust hedging in difficult markets, but one must guard against complacency. Future work should integrate risk controls, e.g. uncertainty quantification in the PINN predictions, and evaluate systemic effects if such models become widespread.

9 Conclusion

We have developed a comprehensive PINN framework for option pricing and hedging under jump-diffusion and liquidity constraints. Our theoretical analysis provides convergence guarantees and confirms PDE well-posedness. Extensive experiments show that encoding jump risks and liquidity costs yields superior accuracy and hedging performance compared to standard models. The added theoretical and empirical analyses (ablation, robustness) deepen understanding of the method’s capabilities and limits. Our detailed description of architecture and training ensures reproducibility. While the PINN approach shows promise for emerging markets and illiquid assets, care must be taken regarding model risk and market impact, as discussed. In future work, we plan to extend this approach to path-dependent options and to incorporate transaction-cost optimization.

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