Assembly Path Planning via Variable Lifting and Physics Simulation

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***Abstract*—Effective assembly path planning plays a crucial role in various applications. However, the planning often encounter challenges due to narrow passages, where parts can easily get stuck or face difficulty in navigating through. From an insight that feasible initialization can play a significant role in path planning, we propose an initialization scheme using variable lifting. In this approach, the positions of the static part are treated as adjustable variables, allowing us to effectively widen narrow passages and facilitating the feasible initialization. Furthermore, physics simulation-based path refinement is proposed to efficiently resolve the problem while maintaining the minimal penetration during the solution process. Several examples are implemented to demonstrate how the proposed framework addresses challenging assembly path planning problems.**

I. Introduction

Path planning is a fundamental and important issue in robotic assembly tasks. Despite its significance, state-of-the-art implementations of assembly using robot [1, 2, 3] are predominantly relying on simple path planning. This limited scope is due to the fact that the scenarios addressed mainly involve simple peg-in-hole, gear insertion, and etc. In such cases, extracting the inverse path of the parts is relatively straightforward and intuitive. However, as soon as more intricate geometries come into play, the assembly path planning problem often becomes more complex, giving rise to a *narrow passage* issue [4]. Exploring a constrained configuration space becomes necessary to ensure a valid assembly without any unwanted penetrations, which poses a significant challenge.

Although there is no complete solution yet, assembly path planning problem has been addressed in a variety of fields, including graphics, computational design, and robotics [5]. Some traditional works employ grid decomposition to search assembly path [6], which is quite expensive and exhaustive. Following with the development of sampling-based algorithms in robotics [7, 8, 9], assembly planning using them have been developed [10, 11]. However, sampling over narrow passage is still a critical issue.

Recently, [4] presented a planning method for a diverse assembly dataset based on the idea of assembly-by-disassembly and physics simulation. Starting from the given assembled state, they verify actions via physics simulation that can produce meaningful separation and then queue them sequentially. We also utilize physics simulation; however, we find the entire motion plan of a part at once. This approach allows for cases where the assembled configuration is not specified and also provides better scalability in terms of planning the overall motion of the robot, including grasp planning and the utilization of additional constraints.

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A key insight of our methodology is the importance of feasible initialization. As also mentioned in well-known library [12], path planning involving contacts can easily encounter difficulties without proper initialization. The challenge arises from the fact that contact features are typically well-defined only in situations with minimal penetration. In the context of planning the entire path with unspecified assembled state, initializing a feasible path becomes highly challenging. To address this issue, we propose a solution that involves decomposing the geometry of the static part (i.e., which remains stationary during the assembly process) then treat them as a movable entity, incorporating its state as variables in the problem we aim to solve. As a result, we can always guarantee the initial solution to be feasible and deal with contact constraints in a well-conditioned manner, allowing us to quickly find solutions to problems like challenging dish-rack assembly or toggle clasp.

II. Method

A. Problem Formulation

By representing a trajectory as a set of waypoints, robotic path planning for assembly can be formulated as an optimization problem:

$$\min_X \sum_{k=0}^{N-1} \text{dist}(X_k, X_{k+1}) + o(X_N) \quad \text{s.t. } f(X) \geq 0 \qquad (1)$$

where N is the number of waypoints, $X = \{X_1, \cdots, X_N\}$ is the set of pose of the workpiece (i.e., the part to be assembled by the robot) at the waypoints, X_0 is the initial pose, $o(X_N)$ is the cost term for achieving the desired assembled state, and f is for gap function that measures the distance between a workpiece and a static part (i.e., the part that is fixed during the assembly). Note that the path length cost in (1) can be replaced with other types of constraint.

B. Variable Lifting and Initialization

Our key insight is that starting from a feasible initial path and minimizing penetration during the solution process often leads to better solutions, as it avoids getting stuck in multiple conflicting constraints. To overcome this issue, we propose

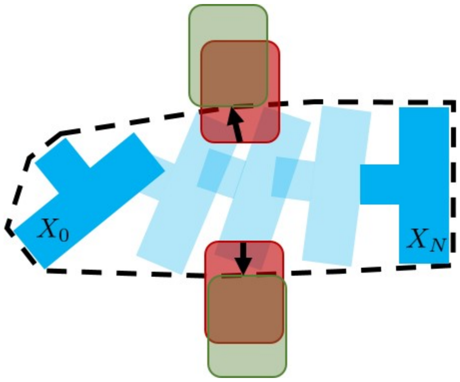

Fig. 1: Illustrative example of our initialization process: The blue objects represent the initial solution of the workpiece, while the black dotted line represents its convex hull. The red object indicates the true location of the static part, and the green object represents the initialized position of the static part after applying variable lifting.

the utilization of variable lifting. By considering the state of the static part as an adjustable parameter, we can increase the flexibility in finding a feasible initial solution.

In the variable lifting process, we first decompose the static part into m convex shapes. Then, we consider the positions of each decomposed static part as a new variable, denoted as $x_s \in \mathbb{R}^{3m}$, which is actually must be equivalent to their constant real positions denoted as $p_s \in \mathbb{R}^{3m}$. Then the modified version of (1) can be written as

$$\begin{aligned} \min_{X, x_s} \ & \sum_{k=0}^{N-1} \text{dist}(X_k, X_{k+1}) + o(X_N) \\ \text{s.t.} \ & f(X, x_s) \geq 0 \quad x_s = p_s \end{aligned} \tag{2}$$

Then the initial solution of (2) is obtained as follow: First, we obtain the convex hull of the workpiece geometry in the initial trajectory. Next, we conduct a collision check between this convex hull and each decomposed static parts. If there is a penetration, we push the part along the corresponding normal vector to resolve the penetration. The above process easily ensures the feasibility of the initial solution thanks to the lifted variable x_s. Fig. 1 illustrates an example of our initialization process.

C. Path Refinement using Physics Simulation

While equation (2) can be solved using a nonlinear problem solver (such as sequential quadratic programming), we leverage the concept of physics simulation to address it. In our approach, during each timestep of the simulation, the decomposed static parts undergo a small incremental movement towards their actual positions denoted as p_s. The distance they move at each timestep is determined by a tuning parameter, representing the maximum allowable penetration that the solver can accommodate during the solution process.

In the simulation, the cost function can be treated as a potential function. For instance, the path length cost can be represented as a chain of objects interconnected by rotational and positional springs. Then each timestep, the simulation

Algorithm 1 Assembly Path Planning

Initialize X, x_s by the variable lifting (Sec. II-B)
Determine the incremental movement distance δ
while not converge **do**
    Incrementally update $x_s \to x_s + \Delta x_s$ based on δ
    Obtain $\hat{v}$ from physics simulation (3)
    Update X using $\hat{v}$
end while

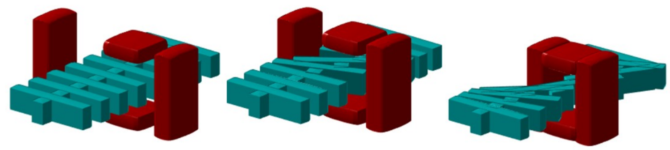

Fig. 2: Snapshots of path refinement using physics simulation for toggle clasp scenario.

solver solves the following problem:

$$\text{Solve } A\hat{v} = b + J^T\lambda \quad \text{s.t.} \quad J\hat{v} + f \geq 0 \tag{3}$$

where $\hat{v}$ is the representative velocity for the time step, A and b involves the (approximated) Hessian and gradient of the potential function, respectively, λ is the contact force acting on the workpiece, and J is the Jacobian of f. Computation of f and J is performed based on the framework proposed in [13]. Also we employ SubADMM [14] to solve (3) as it can efficiently handle numerous constraints. After solving $\hat{v}$ from (3), we use it to update X.

Our physics simulation-based method does not prioritize the strict minimization of cost through perfect convergence. Instead, our priority lies in achieving the equality of x_s and p_s while maintaining minimal penetration throughout the solution process. This approach also shares certain similarities with the trust-region method, as we aim to limit the range within which linearization and contact modeling can be effectively applied. However, our method offers the flexibility to easily adjust the level of trust in the solution through the lifted variable x_s.

The overall algorithm is summarized in Algorithm 1. In essence, our approach combines the variable lifting technique on the static part state to generate an initial feasible path, and employs a simulation-based refinement method to continually ensure feasibility and enforce the static part variables. By adopting this combined approach, we successfully overcome the difficulties associated with narrow feasible regions in robotic assembly path planning.

III. Examples

Here we utilize variable lifting and simulation-based path refinement techniques to perform assembly path planning in three specific examples.

A. Toggle Clasp

Toggle clasps are widely used and popular for their convenient and quick locking mechanism, making them a common choice in various applications, such as jewelry and sewing

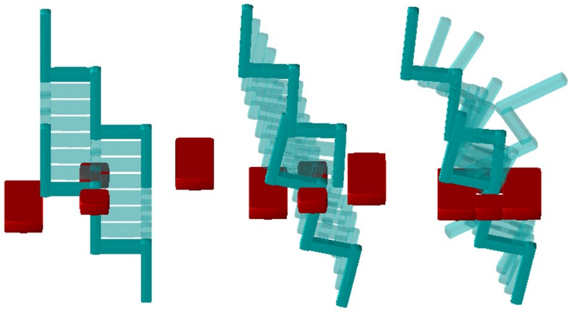

Fig. 3: Snapshots of path refinement using physics simulation for bent stick insertion scenario.

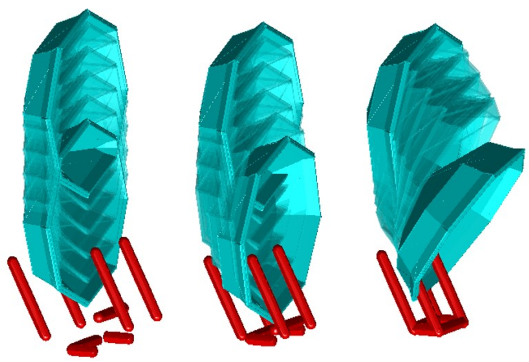

Fig. 4: Snapshots of path refinement using physics simulation for dish assembly scenario.

projects. The mechanism is designed to secure two components together by passing through a narrow passage, ensuring a secure closure. Due to this narrow passage, the path planning of the toggle clasp is challenging. To guide the planning, we define the terminal cost as the distance to a reference pose on the opposite side of the static part and perform path planning using our framework. As a result, the assembly path that effectively navigate through the narrow gap was obtained, as shown in Fig. 2.

B. Bent Stick Insertion

Assembling a complex and intricately bent dynamic part through a narrow passage is a challenging task, as without proper planning, it is prone to getting stuck in the gap. In our approach, we initiate the planning process by widening the narrow gap and then gradually narrowing it using simulations. This method allowed us to effectively obtain a collision-free assembly path.

C. Dish Assembly

Inserting a dish into a narrow gap of a drying rack is also challenging for a manipulator, especially when the dish is thick. It is particularly challenging since it is difficult to determine a feasible goal position. To address this, we define the terminal cost $o(X_N)$ as the distance to an approximate reference pose located at the center of the rack, facing horizontal direction. By employing our proposed initialization scheme and refinement process, feasible final position of the plate placed on the drying rack, along with a feasible path could be achieved.

IV. Conclusions

We present an assembly path planning framework specifically designed to address the challenges posed by narrow passages. The framework leverages variable lifting and simulation-based path refinement to effectively navigate through constrained spaces. The effectiveness of the framework is demonstrated through various examples.

However, there are certain limitations of the proposed framework. One limitation is the potential for intermittent infeasibility between waypoints. Additionally, during the path refinement process of returning static parts to their original positions, it is possible to derive paths that do not achieve the primary objective of assembling parts. To address these limitations, future research directions can focus on exploring continuous collision detection techniques and incorporating sampling-based initialization methods.

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