Stabilization of T-S Fuzzy Impulsive Systems via A Step-Function Method

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***Abstract*—In this paper, a step-function method is used to analyze the stability of *Takagi-Sugeno* (T-S) fuzzy impulsive systems. In the step-function method, the Lyapunov-like function does not need to be monotonically decreasing or continuous in each impulsive interval, which widens the stability analysis method of Lyapunov-like to a certain extent. By combining with the mixed impulse, the mixed impulse method of single-step step-function and the mixed impulse method of multi-steps step-function are constructed, and several relatively conservative and sufficient stability conditions of T-S fuzzy impulsive systems are obtained. In addition, the upper bound of impulsive interval is estimated, which is larger than that of other methods. Finally, numerical simulation results are used to verify the effectiveness of the proposed method.**

***Index Terms*—step-function, Lyapunov-like function, stability, T-S fuzzy, impulsive systems**

I. Introduction

As an extremely important hybrid system, impulsive system is a special system used to describe the suddenly change of the state of the system at some time points. For example, in the ideal small ball collision problem [1], the process of the small ball before contact and collision with the ground can be described by a continuous differential equation. However, at the moment of contact and collision with the ground, its motion state will suddenly be changed. At this time, At this point, the process needs to be described by a difference equation. So, the ideal small ball collision motioncan be considered as an impulsive system model. It not only has the characteristics of continuous dynamics, but also has the characteristics of discrete ones. In addition, impulsive phenomenon is common in many fields of human society, including satellite communication [2], environmentology [3], finance [4] and so on. On the other hand, the impulsive controller has the characteristics of simple design and low cost, so impulsive control methods are widely used to stabilize complex systems. In recent years, many control methods have been proposed, including impulsive control [5]–[8], intermittent control [9]–[11], adaptive control [12]–[14], sliding mode control [15]–[17], and so on. In [6], by combining impulsive control with Lyapunov stability principle, Yang *et al* deduced the stability synchronization criteria for a class of neural networks with time-varying delays. In [9], You *et al.* achieved finite-time stabilization for uncertain systems by designing an aperiodic intermittent control strategy. In addition, numerical examples were used to verify the effectiveness of the method. In [14], in order to improve the control performance and stability of the system, Zheng *et al* constructed an adaptive sliding mode neural control method. Also, numerical cases were used to prove the feasibility of the method. In [16], Tong *et al* guaranteed the stabilization of nonlinear systems with delay and disturbance by using sliding mode control. Besides, numerical examples were used to prove the effectiveness of the method.

A known nonlinear system can be replaced by many fuzzy IF-THEN rules to obtain a T-S fuzzy model [18], which makes the original system has rich linear dynamic. Its substitution principle is fuzzy logic. Owing to the fact that fuzzy IF-THEN rules have the characteristics of linear local dynamics, it can transform the nonlinear part of the system into an equivalent linear part which is expected to be used to analyse more complex nonlinear systems. In recent years, the research on the stability of T-S fuzzy impulsive systems has attracted great attention, and many results [19]–[30] have been emerged. Most of the works focuses on the analysis of the stability of T-S fuzzy impulsive systems by combining the Lyapunov-like function. The stability analysis of nonlinear system with time delay was studied by constructing the co-positive Lyapunov-like function in [19]. In [29], the authors

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constructed a Lyapunov function related to event-triggered to realize the stabilization of T-S fuzzy systems. A fuzzy feedback controller and a Lyapunov – Krasovskii functional are designed in [30], which is used to analyze the stability of t-s fuzzy systems. However, the disadvantage is that the traditional Lyapunov stability analysis method requires that the Lyapunov-like function is continuous in each impulsive interval, and its value also decreases with the increase of time, which will make the obtained stability conditions become relatively conservative. In order to remove this situation, the authors carries out the research in [1], and an innovative method is proposed. The key is to construct a step-function on a Lyapunov-like function, which converges to zero with the increase of time. Therefore, in this new stability analysis method, the Lyapunov-like function need not be monotonically decreasing, and can also be discontinuous in some special cases. In [22], Although the authors added the step function to the analysis method, they did not consider two types of impulses combination situation. Generally speaking, the state of the system may suddenly become larger or smaller, so these two types of impulses should exist, which is more in line with the real life.

Inspired by [1] and [22], by combining with the mixed impulse, we come up with the mixed impulse method of single-step step-function and the mixed impulse method of multi-steps step-function. Its main idea is to treat each impulsive interval as a step. Therefore, on this basis, and we also constructed multi-steps step-function. In addition, the proposed method is applied to stabilize T-S fuzzy impulsive systems. Also, the upper bound of impulsive interval is estimated and compares it with other methods.

This paragraph contains the outline of the remaining part of this paper. Section II mainly elaborates the establishment of T-S fuzzy impulsive system and some mathematical knowledge used. Section III mainly introduces the construction principle of the mixed impulse method of multi-steps function and the stability analysis conditions obtained by using it. In section IV, the proposed method is exerted on T-S fuzzy impulsive system, and several relatively broad stability analysis conditions are obtained. In section V, two numerical simulations are used to verify the feasibility of the proposed method. Finally, Section VI makes a conclusion of this paper.

II. Preliminaries

The following is a classical nonlinear impulsive system

$$\begin{cases} \dot{x}(t) = h(t, x(t)), & t \in (\tau_{k-1}, \tau_k], \\ \varDelta x(\tau_k) = \varUpsilon_k(x(\tau_k)), & t = \tau_k, \\ x(t_0^+) = x_0, \end{cases} \tag{1}$$

where $k \in N^*$, $N^* = 1, 2, \cdots$, $x(t) \in R^n$ is the state vector, and $h \in C(R^+ \times R^n, R^n)$, $R^+ = [0, +\infty)$, which is a nonlinear function. $\varDelta x(\tau_k) = x(\tau_k^+) - x(\tau_k^-)$ represents the sudden change of the state of system (1) at the time instants τ_k , τ_k represents the impulsive time instants satisfing $0 \leqslant \tau_1 < \tau_2 < \cdots < \tau_k < \tau_{k+1} < \cdots < \tau_\infty$, in which $\tau_\infty = \lim_{k\to\infty} \tau_k$. It should be noted that τ_∞ could be finite, also could be infinite. Suppose that $x(t)$ is a left continuous function at impulsive time instants τ_k, that is, $x(\tau_k) = x(\tau_k^-)$, and $x^* = 0$ is the equilibrium point, that is, $g(t, 0) = \varUpsilon_k(0) = 0$, for all $t \geqslant t_0$.

The impulsive system can be described by fuzzy IF-THEN rules, in which the latter represents the transformation of the nonlinear part of the system into an equivalent linear part. So, the T-S fuzzy impulsive system with r fuzzy rules can be written as follows:

Rule i:
IF $\eth_1(t)$ is $\varDelta_{i1}$ and $\eth_2(t)$ is $\varDelta_{i2}$ and, $\cdots$, and $\eth_q(t)$ is $\varDelta_{iq}$, THEN

$$\begin{cases} \dot{x}(t) = \varLambda_i x(t), t \in (\tau_{k-1}, \tau_k], \\ x(\tau_k^+) = [1 + R_{ik}] x(\tau_k), \end{cases} \tag{2}$$

where $k \in N^*$, $\varDelta_{ij}\,(i = 1, 2, \cdots, r;\ j = 1, 2, \cdots, q)$ represent the fuzzy set, $\varLambda_i$ are constant matrices, $\eth_1(t) \vee \eth_q(t)$ are the vector of the premise variables, r is the number of fuzzy IF-THEN rules, and the scalar R_{ik} represent impulsive intensity, in which $i = 1, 2, \cdots, r$.

Therefore, system (1) can be rewritten as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} g_i(\eth(t)) \varLambda_i x(t), t \in (\tau_{k-1}, \tau_k], \\ x(\tau_k^+) = \sum_{i=1}^{r} g_i(\eth(\tau_k)) [1 + R_{ik}] x(\tau_k), \end{cases} \tag{3}$$

where $g_i(\eth(t)) = \frac{\psi_i(\eth(t))}{\sum_{i=1}^{r} \psi_i(\eth(t))}$, $\psi_i(\eth(t)) = \prod_{j=1}^{q} \varDelta_{ij}(\eth_j(t))$. In addition, $g_i(\eth(t)) \geqslant 0$, and $\sum_{i=1}^{r} g_i(\eth(t)) = 1$, for all $t \geqslant t_0$.

Definition 1 ([22]): The following three types of functions are defined:

(i) $\mu \in \mathcal{K}$, *if* $\mu \in C([0, c), R^+)$ *is strictly monotonic increasing, and* $\mu(0) = 0$*;*
(ii) $\mu \in \mathcal{K}_\infty$, *if* $\mu \in \mathcal{K}$ *, and* μ *is an unbounded function;*
(iii) $\mu \in \mathcal{K}L$, *if* $\mu \in C([0, c) \times R^+, R^+)$*, and with regard to each immobilizing* b*,* $\mu(a, b) \in \mathcal{K}$*, with regard to each immobilizing* a*,* $\mu(a, b)$ *is decreasing. Furthermore,* $\mu(a, b)$ *tends to 0 as* $b \to \infty$*.*

Definition 2 ([1]): The origin of system (1) is *Globally Uniformly Attractively Stable* (GUAS) if there exists a function $\beta(s, t) \in \mathcal{K}L$ such that every solution $x(t, t_0, x_0)$ of system (1) satisfies that $\|x(t, t_0, x_0)\| \leqslant \beta(\|x_0\|, t - t_0)$, for almost all $t \geqslant t_0$.

III. Theoretical analysis and Main results

In this section, a stability analysis method for impulsive systems called step-function method is introduced. The section is divided into two parts: single-step step-function method and multi-steps step-function method. Moreover, several circumstances of two methods and the corresponding stability analysis theorems are given.

A. Single-step step-function method

Inspired by the measurement equation in [1], the following theoretical results and their proofs are obtained.

Theorem 1: Let $x(t) = x(t, t_0, x_0)$ is the solution of impulsive system (1). If there exists a function $V : R^n \to R^+$, which satisfies the local Lipschitz condition such that the step-function $U(t)$ is defined as

$$U(t) = \begin{cases} sup\ V(x(t)), & t \in [\tau_0, \tau_1], \\ sup\ V(x(t)), & t \in (\tau_{k-1}, \tau_k], \ k > 1, \\ 0, & t \geqslant \tau_\infty, \end{cases} \tag{4}$$

and $U(t)$ satisfies the following conditions:

(i) $U(\tau_1) \leqslant \delta(V(x_0))$, *in which the function* $\delta \in \mathcal{K}$;

(ii) *as time* t *increases, the step-function* $U(t)$ *decreases, and* $\lim\limits_{t \to \tau_\infty} U(t) = 0$.

Therefore, the origin of system (1) is GUAS, and in addition, if $\tau_\infty = \infty$, the origin of system (1) is Globally Asymptotically Stable (GAS).

Proof 1: $V(x)$ is a positive definite function, there must be Π_1, $\Pi_2 \in \mathcal{K}_\infty$ such that

$$\Pi_1(\|x\|) \leqslant V(x) \leqslant \Pi_2(\|x\|). \tag{5}$$

Then, it can be obtained that

$$U(\tau_0) = U(\tau_1) \leqslant \delta(V(x_0)) \leqslant \delta(\Pi_2 \|x_0\|). \tag{6}$$

Furthermore, the comparison function $\beta_\theta(U(\tau_1), t - \tau_0)$ is constructed as follows:

$$\beta_\theta(U(\tau_1), t - \tau_0) = \begin{cases} U(\tau_1), & \tau_0 \leqslant t \leqslant \tau_1, \\ \frac{U(\tau_k) - U(\tau_{k-1})}{\tau_k - \tau_{k-1}}(t - \tau_{k-1}) + U(\tau_{k-1}) \\ \tau_{k-1} \leqslant t \leqslant \tau_k, \\ 0, & t \geqslant \tau_\infty. \end{cases} \tag{7}$$

Due to the function β_θ is continuous in the whole time domain, and the function $U(\tau_1) e^{\tau_0 - t} \geqslant 0$, we have

$$\beta_\theta(U(\tau_1), t - \tau_0) \leqslant \bar{\beta}(U(\tau_1), t - \tau_0), \tag{8}$$

where $\bar{\beta}(U(\tau_1), t - \tau_0) \triangleq \beta_\theta(U(\tau_1), t - \tau_0) + U(\tau_1) e^{\tau_0 - t}$.

Obviously, the function $\bar{\beta}(U(\tau_1), t - \tau_0)$ is continuous in the whole time domain. With regard to each immobilizing $t - \tau_0$, the function $\bar{\beta}(U(\tau_1), t - \tau_0) \in \mathcal{K}$, and with regard to each immobilizing $U(\tau_1)$, the function $\bar{\beta}(U(\tau_1), t - \tau_0)$ tends to 0 as $t \to \tau_\infty$. So, it can be obtained that the function $\bar{\beta}(U(\tau_1), t - \tau_0) \in \mathcal{KL}$. Based on the above analysis, the following result can be obtained

$$\begin{gathered} \Pi_1(\|x(t)\|) \leqslant V(x(t)) \leqslant U(t) \leqslant \\ \beta_\theta(U(\tau_1), t - \tau_0) \leqslant \bar{\beta}(U(\tau_1), t - \tau_0). \end{gathered} \tag{9}$$

which implies that

$$\|x(t)\| \leqslant \Pi_1^{-1} \bar{\beta}(\delta(\Pi_2(\|x_0\|)), t - \tau_0). \tag{10}$$

Obviously, the function $\Pi_1^{-1} \bar{\beta}(\delta(\Pi_2(\|x_0\|)), t - \tau_0) \in \mathcal{K}L$.

So, the proof is completed.

Remark 1: for all $k > 1$, and $t \in (\tau_{k-1}, \tau_k]$, the following the special case can be considered

$$V\left(x\left(\tau_{k-1}^+\right)\right) \geqslant V(x(t)), 1 < k \leqslant \lambda,$$

then it can be obtained that $V\left(x\left(\tau_{k-1}^+\right)\right) \geqslant \sup\limits_{t \in (\tau_{k-1}, \tau_k]} V(x(t))$,

$$V(x(\tau_{k-1})) \geqslant V(x(t)), k \geqslant \lambda + 1,$$

then it can be obtained that $V(x(\tau_{k-1})) \geqslant \sup\limits_{t \in (\tau_{k-1}, \tau_k]} V(x(t))$,
in which $\lambda > 1$, and $\lambda \in N^*$.

Due to the above results combine two different types of impulses in [22], it is called the mixed impulse.

For the above case, the following corollary can be obtained.

Corollary 1: (The mixed impulse method of single-step step-function): Suppose that there exists a function $V : R^n \to R^+$, which satisfies the local Lipschitz condition such that the step-function $U(t)$ defined as

$$U(t) = \begin{cases} \sup\limits_{t \in [\tau_0, \tau_1]} V(x(t)), & t \in [\tau_0, \tau_1], \\ V\left(x\left(\tau_{k-1}^+\right)\right), & t \in (\tau_{k-1}, \tau_k], 1 < k \leqslant \lambda, \\ V(x(\tau_{k-1})), & t \in (\tau_{k-1}, \tau_k], k \geqslant \lambda + 1, \\ 0, & t > \tau_\infty, \end{cases} \tag{11}$$

and $U(t)$ satisfies the following conditions:

(i) $U(\tau_1) \leqslant \delta(V(x_0))$, *in which the function* $\delta \in \mathcal{K}$;

(ii) *as time* t *increases, the step-function* $U(t)$ *decreases, and* $\lim\limits_{t \to \tau_\infty} U(t) = 0$;

(iii) $U(t) \geqslant V(x(t))$, *with regard to each impulsive interval* $t \in (\tau_{k-1}, \tau_k], k > 1$, $\lambda > 1$, *and* $\lambda \in N^*$.

Then, the origin of system (1) is GUAS, and in addition, if $\tau_\infty = \infty$, the origin of system (1) is GAS.

Remark 2: Let $\lambda = 1$, (11) becomes the case of single-step step-function for stabilizing impulse in [22]. And let $\lambda \to \infty$, (11) becomes the case of single-step step-function for destabilizing impulse in [22].

As shown in Figure 1, it does not require that the function $V(x(t))$ is monotonically decreasing at each impulsive interval, the function $U(t)$ is above the function $V(x(t))$, and monotonically tends to 0.

B. Multi-steps step-function method

In this subsection, motivated by the single-step step-function, we extend it to the case of multi-steps step function. On this basis, several relatively conservative stability analysis conditions for system (1) are obtained.

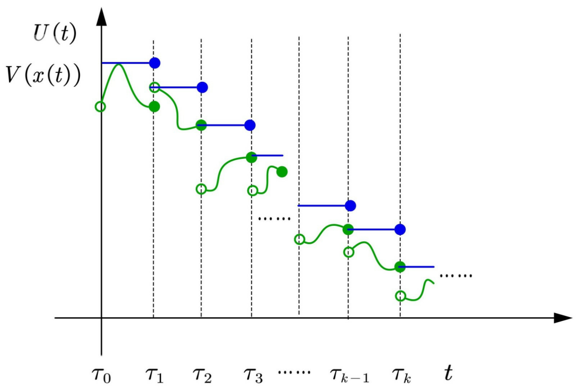

Fig. 1. Diagram of Corollary 1 with $\lambda = 2$, $V(x(t))$ (green) and $U(t)$ (blue).

Theorem 2: Suppose that there exists a function $V : R^n \to R^+$, which satisfies the local Lipschitz condition, and an integer $n\,(n \geqslant 2)$ such that the step-function $U\,(t)$ defined as

$$U\,(t) = \begin{cases} sup\ V\,(x\,(t))\,, & t \in [\tau_0, \tau_n]\,, \\ sup\ V\,(x\,(t))\,, & t \in \left(\tau_{nk}, \tau_{n(k+1)}\right]\,,\ k \geqslant 1, \\ 0, & t > \tau_\infty, \end{cases} \tag{12}$$

and $U\,(t)$ satisfies the following conditions:

(i) $U\,(\tau_1) \leqslant \delta\,(V\,(x_0))$, *in which the function* $\delta \in \mathcal{K}$*;*
(ii) *as time t increases, the step-function* $U\,(t)$ *decreases, and* $\lim\limits_{t \to \tau_\infty} U\,(t) = 0$.

Then, the point of system (1) is GUAS, and in addition, if $\tau_\infty = \infty$, the origin of system (1) is GAS.

Proof 2: Similarly, the comparison function is constructed as follow:

$$\beta_\theta\,(U\,(\tau_n)\,, t - \tau_0) = \begin{cases} U\,(\tau_n) & \tau_0 \leqslant t \leqslant \tau_n, \\ \frac{U(\tau_{nk}) - U\left(\tau_{n(k-1)}\right)}{\tau_{nk} - \tau_{n(k-1)}}\left(t - \tau_{n(k-1)}\right) & \\ +U\left(\tau_{n(k-1)}\right), & \tau_{n(k-1)} \leqslant t \leqslant \tau_{nk}, \\ 0, & t \geqslant \tau_\infty. \end{cases} \tag{13}$$

Due to the process of proving Theorem 2 is like that of Theorem 1, the proof is omitted.

Remark 3: It can be easily obtained that the multi-steps step-function method can reduce to the case of singl-step step-function method when $n = 1$. By comparing with Theorem 1, Theorem 2 can deal with the case of $V\,(x\,(t))$ that is discontinuous in the impulsive interval.

Similarly, Corollary 2 can be obtained by Theorem 2.

Corollary 2: (The mixed impulse method of mutli-steps step-function): Suppose that there exists a function $V : R^n \to R^+$, which satisfies the local Lipschitz condition, integer $n\,(n \geqslant 2)$, $\lambda = n$, $\lambda > 1$, and $\lambda \in N^*$ such that the step-function $U\,(t)$ defined as:

$$U\,(t) = \begin{cases} \sup\limits_{t \in [\tau_0, \tau_n]} V\,(x\,(t))\ , & t \in [\tau_0, \tau_n]\,, \\ V\left(x\left(\tau_{k-1}^+\right)\right), & t \in \left(\tau_{n(k-1)}, \tau_{nk}\right]\,, 1 \leqslant k \leqslant \lambda, \\ V\,(x\,(\tau_{k-1}))\,, & t \in \left(\tau_{n(k-1)}, \tau_{nk}\right]\,, k \geqslant \lambda + 1, \\ 0, & t > \tau_\infty, \end{cases} \tag{14}$$

(i) $U\,(\tau_1) \leqslant \delta\,(V\,(x_0))$, in which the function $\delta \in \mathcal{K}$;
(ii) as time t increases, the step-function $U\,(t)$ decreasing, and $\lim\limits_{t \to \tau_\infty} U\,(t) = 0$;
(iii) $U(t) \geqslant V\,(x\,(t))$, with regard to each impulsive interval $t \in (\tau_{k-1}, \tau_k]\,, k > 1$, $\lambda > 1$ and $\lambda \in N^*$.

Then, the origin of system (1) is GUAS, and in addition, if $\tau_\infty = \infty$, the origin of system (1) is GAS.

Remark 4:

Let $\lambda = 1$, (14) becomes the case of multi-steps step-function for stabilizing impulse in [22]. Also, let $\lambda \to \infty$, (14) becomes the case of multi-steps step-function for destabilizing impulse in [22].

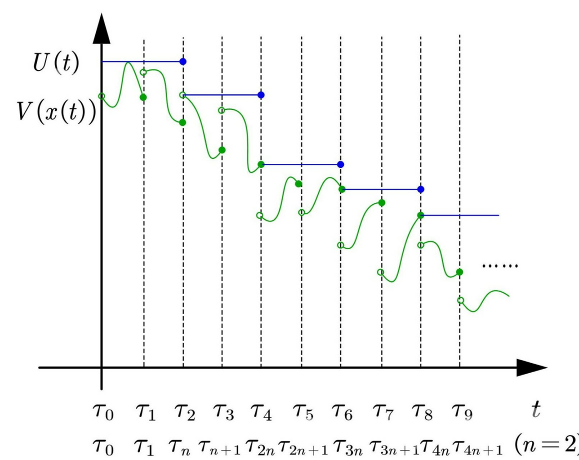

Fig. 2. Diagram of Corollary 2 with $\lambda = n = 2$, $V(x(t))$ (green) and $U(t)$ (blue).

Figure 2 shows the principle diagram of Corollary 2. It also does not need that the function $V\,(x\,(t))$ is monotonically decreasing in each impulsive interval. In addition, Corollary 2 can deal with the case that the function $V\,(x\,(t))$ is discontinuous.

On the basis of Theorem 2, the following several stability analysis conditions can be obtained.

Theorem 3: Suppose that exists a function $V : R^n \to R^+$, which satisfies the local Lipschitz condition, an integer $n\,(n \geqslant 2)$, and scalars ϱ, ω_k such that the following conditions are satisfied:

(i) $D^+V\,(x\,(t)) \leqslant \varrho V\,(x\,(t))\,, t \in (\tau_{k\ -1}, \tau_k]\,,\ k \geqslant 1$;
(ii) $V\left(x\left(\tau_k^+\right)\right) \leqslant \omega_k V\,(x\,(\tau_k))\,, k \geqslant 1$;
(iii) $\xi = \max\limits_{i=1,2,\cdots,n} \left\{ \sup\limits_{t \in (\tau_{i-1}, \tau_i]} \left[\left(\prod\limits_{j=0}^{i-1} \omega_j \right) e^{\varrho(t - \tau_0)} \right] \right\} < \infty$;

(iv) $\max\limits_{i=n(k-1)+1,\cdots,nk}\left\{\sup\limits_{t\in(\tau_{i-1},\tau_i]}\left[\left(\prod\limits_{j=n(k-1)}^{i-1}\omega_j\right)\times e^{\varrho\left(t-\tau_{n(k-1)}\right)}\right]\right\}\leqslant\phi<1,\ k>1.$

Then, the origin of system (1) is GUAS, and in addition, if $\tau_\infty=\infty$, the origin of system (1) is GAS.

Proof 3: The goal now is to prove that Theorem 2 satisfies these conditions. By observing the first three conditions in Theorem 3, it can be concluded that

(1) when $t\in[\tau_0,\tau_1]$,

$$V(x(t))\leqslant V(x_0)e^{\varrho(t-\tau_0)},V(x(\tau_1))\leqslant V(x_0)e^{\varrho(\tau_1-\tau_0)},$$

(2) when $t\in(\tau_1,\tau_2]$,

$$\begin{aligned}V(x(t))&\leqslant V\left(x\left(\tau_1^+\right)\right)e^{\varrho(t-\tau_1)}\\&\leqslant\omega_1V(x(\tau_1))e^{\varrho(t-\tau_1)}\\&\leqslant\omega_1V(x_0)e^{\varrho(t-\tau_0)},\end{aligned}$$

(3) when $t\in(\tau_2,\tau_3]$,

$$\begin{aligned}V(x(t))&\leqslant V\left(x\left(\tau_2^+\right)\right)e^{\varrho(t-\tau_2)}\\&\leqslant\omega_2V(x(\tau_2))e^{\varrho(t-\tau_2)}\\&\leqslant\omega_2\omega_1V(x_0)e^{\varrho(t-\tau_0)},\end{aligned}$$

(4) when $t\in(\tau_{n-1},\tau_n]$,

$$\begin{aligned}V(x(t))&\leqslant V\left(x\left(\tau_{n-1}^+\right)\right)e^{\varrho(t-\tau_{n-1})}\\&\leqslant\omega_{n-1}V(x(\tau_{n-1}))e^{\varrho(t-\tau_{n-1})}\\&\leqslant\omega_1\omega_2\cdots\omega_{n-1}V(x_0)e^{\varrho(t-\tau_0)}.\end{aligned}$$

Hence, when $t\in[\tau_0,\tau_n]$,

$$\begin{aligned}U(t)&=\sup_{t\in[\tau_0,\tau_n]}V(x(t))\\&\leqslant V(x_0)\times\max_{i=1,2\cdots n}\left\{\sup_{t\in(\tau_{i-1},\tau_i]}\left[\left(\prod_{j=0}^{i-1}\omega_j\right)\right]e^{\varrho(t-\tau_0)}\right\}\\&=\xi V(x_0),\end{aligned}$$

the above proves that (i) in Theorem 2 is satisfied.

Similarly, by observing the fourth condition of Theorem 3, it can be concluded that

When $t\in\left(\tau_{n(k-1)},\tau_{nk}\right]$, $k>1$,

$$\begin{aligned}U(t)&=\sup_{t\in\left(\tau_{n(k-1)},\tau_{nk}\right]}V(x(t))\\&\leqslant\max_{i=n(k-1)+1,\cdots,nk}\left\{\sup_{t\in(\tau_{i-1},\tau_i]}V(x(t))\right\}\\&\leqslant V\left(x\left(\tau_{n(k-1)}\right)\right)\\&\quad\times\max_{i=n(k-1)+1,\cdots,nk}\left\{\sup_{t\in(\tau_{i-1},\tau_i]}\left[\left(\prod_{j=n(k-1)}^{i-1}\omega_j\right)\times e^{\varrho\left(t-\tau_{n(k-1)}\right)}\right]\right\}\\&\leqslant\phi V\left(x\left(\tau_{n(k-1)}\right)\right).\end{aligned}$$

Therefore, when $k\geqslant1$, it can be concluded that

$$\begin{aligned}U(\tau_{nk})-U\left(\tau_{n(k-1)}\right)&=\sup_{t\in\left(\tau_{n(k-1)},\tau_{nk}\right]}V(x(t))-\sup_{t\in\left(\tau_{n(k-2)},\tau_{n(k-1)}\right]}V(x(t))\\&\leqslant\phi V\left(x\left(\tau_{n(k-1)}\right)\right)-\sup_{t\in\left(\tau_{n(k-2)},\tau_{n(k-1)}\right]}V(x(t))\\&\leqslant(\phi-1)V\left(x\left(\tau_{n(k-1)}\right)\right)\\&\leqslant0.\end{aligned}$$

Furthermore, when $t\in\left(\tau_{n(k-1)},\tau_{nk}\right]$, and then it can be obtained that

$$\begin{aligned}U(t)&\leqslant\phi V\left(\tau_{n(k-1)}\right)\\&\leqslant\phi^2V\left(\tau_{n(k-2)}\right)\\&\cdots\\&\leqslant\xi\phi^{k-1}V(x_0),\end{aligned}$$

as $k\to\infty$, we have $\lim_{t\to\tau_\infty}U(t)=0$.

So, the above proves that (ii) in Theorem 2 is satisfied. the proof is completed.

On the basis of Corollary 2, Corollary 3 is obtained.

Corollary 3: suppose that $x(t)=x(t,t_0,x_0)$ is the solution of impulsive system (1). If there is a positive definite function $V:R^n\to R^+$, which satisfies the local Lipschitz condition, the integers $n\,(n\geqslant2)$, $\lambda=n$, the scalars $\varrho>0$, ω_k, $\lambda>1$, and $\lambda\in N^*$ such that the following conditions are satisfied:

(i) $D^+V(x(t))\leqslant-\varrho V(x(t)),t\in(\tau_{k-1},\tau_k],\ 1\leqslant k\leqslant\lambda$;

(ii) $D^+V(x(t))\leqslant\varrho V(x(t)),t\in(\tau_{k-1},\tau_k],\ k\geqslant\lambda+1$;

(iii) $V\left(x\left(\tau_k^+\right)\right)\leqslant\omega_kV(x(\tau_k)),\ k\geqslant1$;

(iv) $\xi=\max\limits_{i=1,2,\cdots,n}\left\{\left[\left(\prod\limits_{j=0}^{i-1}\omega_j\right)\right]\right\}<\infty,\ k=1$;

(v) $\max\limits_{i=n(k-1)+1,\cdots,nk}\left\{\left[\left(\prod\limits_{j=n(k-1)}^{i-1}\omega_j\right)e^{-\varrho\left(\tau_{i-1}-\tau_{n(k-1)}\right)}\right]\right\}\leqslant1,\ 1<k\leqslant\lambda$;

(vi) $\max\limits_{i=n(k-1)+1,\cdots,nk}\left\{\left[\left(\prod\limits_{j=n(k-1)}^{i-1}\omega_j\right)e^{\varrho\left(\tau_i-\tau_{n(k-1)}\right)}\right]\right\}\leqslant1,\ k\geqslant\lambda+1$;

(vii) $\omega_{n(k-1)}\omega_{n(k-1)-1}\cdots\omega_{n(k-2)+1}e^{-\varrho\left(\tau_{n(k-1)}-\tau_{n(k-2)}\right)}\leqslant\phi<1,\ 1<k\leqslant\lambda$;

(viii) $\omega_{n(k-1)-1}\omega_{n(k-1)-2}\cdots\omega_{n(k-2)}e^{\varrho\left(\tau_{n(k-1)}-\tau_{n(k-2)}\right)}\leqslant\phi<1,\ k\geqslant\lambda+1$;

(ix) $\omega_{n\lambda-1}\omega_{n\lambda-2}\cdots\omega_{n(\lambda-1)+1}e^{-\varrho\left(\tau_{n\lambda}-\tau_{n(\lambda-1)}\right)}\leqslant\phi<1.$

Then, the origin of system (1) is GUAS, and in addition, if $\tau_\infty=\infty$, the origin of system (1) is GAS.

Proof 4: To prove Corollary 3, it is only necessary to prove that the all conditions in Corollary 2 are satisfied in Corollary

3. And in order to achieve the goal, the step-function $U(t)$ is defined as follow:

$$U(t)=\begin{cases}\underset{t\in[\tau_0,\tau_n]}{sup}V(x(t)), & t\in[\tau_0,\tau_n],\\ V\left(x\left(\tau_{k-1}^{+}\right)\right), & t\in\left(\tau_{n(k-1)},\tau_{nk}\right],1\leqslant k\leqslant\lambda,\\ V\left(x\left(\tau_{k-1}\right)\right), & t\in\left(\tau_{n(k-1)},\tau_{nk}\right],k\geqslant\lambda+1,\\ 0, & t>\tau_{\infty}.\end{cases}$$

Similarly, by observing the first three conditions in Corollary 3, the following results can be concluded
(1) when $t\in[\tau_0,\tau_1]$,

$$V(x(t))\leqslant V(x_0)e^{-\varrho(t-\tau_0)},$$

(2) when $t\in(\tau_1,\tau_2]$,

$$\begin{aligned}V(x(t))&\leqslant V\left(x\left(\tau_1^{+}\right)\right)e^{-\varrho(t-\tau_1)}\\&\leqslant\omega_1V(x(\tau_1))e^{-\varrho(t-\tau_1)}\\&\leqslant\omega_1V(x_0)e^{-\varrho(\tau_1-\tau_0)},\end{aligned}$$

(3) when $t\in(\tau_2,\tau_3]$,

$$\begin{aligned}V(x(t))&\leqslant V\left(x\left(\tau_2^{+}\right)\right)e^{-\varrho(t-\tau_2)}\\&\leqslant\omega_2V(x(\tau_2))e^{-\varrho(t-\tau_2)}\\&\leqslant\omega_2\omega_1V(x_0)e^{-\varrho(\tau_2-\tau_0)},\end{aligned}$$

(4) when $t\in(\tau_{n-1},\tau_n]$,

$$\begin{aligned}V(x(t))&\leqslant V\left(x\left(\tau_{n-1}^{+}\right)\right)e^{-\varrho(t-\tau_{n-1})}\\&\leqslant\omega_{n-1}V(x(\tau_{n-1}))e^{-\varrho(t-\tau_{n-1})}\\&\leqslant\omega_1\omega_2\cdots\omega_{n-1}V(x_0)e^{-\varrho(\tau_{n-1}-\tau_0)}.\end{aligned}$$

Hence, when $t\in[\tau_0,\tau_n]$,

$$\begin{aligned}U(t)&=\underset{t\in[\tau_0,\tau_n]}{sup}V(x(t))\\&\leqslant V(x_0)\times\max_{i=1,2\cdots n}\left\{\left[\left(\prod_{j=0}^{i-1}\omega_j\right)\right]e^{-\varrho(\tau_{i-1}-\tau_0)}\right\}\\&=\xi V(x_0),\end{aligned}$$

the above proves that the first condition of Corollary 2 is satisfied.

Besides, when $t\in\left(\tau_{n(k-1)},\tau_{n(k-1)+1}\right],1<k\leqslant\lambda$,

$$V(x(t))\leqslant V\left(x\left(\tau_{n(k-1)}^{+}\right)\right)e^{-\varrho\left(t-\tau_{n(k-1)}\right)}.$$

The following results can be obtained by mathematical induction

(1) when $t\in\left(\tau_{n(k-1)+1},\tau_{n(k-1)+2}\right],1<k\leqslant\lambda$,

$$\begin{aligned}V(x(t))&\leqslant V\left(x\left(\tau_{n(k-1)+1}^{+}\right)\right)e^{-\varrho\left(t-\tau_{n(k-1)+1}\right)}\\&\leqslant\omega_{n(k-1)+1}V\left(x\left(\tau_{n(k-1)+1}\right)\right)e^{-\varrho\left(t-\tau_{n(k-1)+1}\right)}\\&\leqslant\omega_{n(k-1)+1}V\left(x\left(\tau_{n(k-1)}^{+}\right)\right)\times e^{-\varrho\left(\tau_{n(k-1)+1}-\tau_{n(k-1)}\right)},\end{aligned}$$

(2) when $t\in\left(\tau_{n(k-1)+2},\tau_{n(k-1)+3}\right],1<k\leqslant\lambda$,

$$\begin{aligned}V(x(t))&\leqslant V\left(x\left(\tau_{n(k-1)+2}^{+}\right)\right)e^{-\varrho\left(t-\tau_{n(k-1)+2}\right)}\\&\leqslant\omega_{n(k-1)+2}V\left(x\left(\tau_{n(k-1)+2}\right)\right)e^{-\varrho\left(t-\tau_{n(k-1)+2}\right)}\\&\leqslant\omega_{n(k-1)+2}\omega_{n(k-1)+1}V\left(x\left(\tau_{n(k-1)}^{+}\right)\right)\\&\quad\times e^{-\varrho\left(\tau_{n(k-1)+2}-\tau_{n(k-1)}\right)},\end{aligned}$$

therefore, when $t\in(\tau_{nk-1},\tau_{nk}],1<k\leqslant\lambda$,

$$\begin{aligned}V(x(t))&\leqslant V\left(x\left(\tau_{nk-1}^{+}\right)\right)e^{-\varrho(t-\tau_{nk-1})}\\&\leqslant\omega_{nk-1}V(x(\tau_{nk-1}))e^{-\varrho(t-\tau_{nk-1})}\\&\leqslant\omega_{n(k-1)+1}\cdots\omega_{nk-1}V\left(x\left(\tau_{n(k-1)}^{+}\right)\right)\times e^{-\varrho\left(\tau_{nk-1}-\tau_{n(k-1)}\right)}\\&\leqslant\phi V\left(x\left(\tau_{n(k-1)}^{+}\right)\right)\\&\leqslant V\left(x\left(\tau_{n(k-1)}^{+}\right)\right)\\&=U(t).\end{aligned}$$

To get the fifth condition of Corollary 3, when $t\in\left(\tau_{n(k-1)},\tau_{nk}\right],1<k\leqslant\lambda$,

$$\begin{aligned}V(x(t))&\leqslant\max_{i=n(k-1)+1,\cdots,nk}\left\{\underset{t\in(\tau_{i-1},\tau_i]}{sup}V(x(t))\right\}\\&\leqslant V\left(x\left(\tau_{n(k-1)}^{+}\right)\right)\\&\quad\times\max_{i=n(k-1)+1,\cdots,nk}\left\{\left[\left(\prod_{j=n(k-1)}^{i-1}\omega_j\right)\right.\right.\\&\quad\left.\left.\times e^{-\varrho\left(\tau_{i-1}-\tau_{n(k-1)}\right)}\right]\right\}\\&\leqslant V\left(x\left(\tau_{n(k-1)}^{+}\right)\right)\\&=U(t).\end{aligned}$$

Similarly, when $t\in\left(\tau_{n(k-1)},\tau_{nk}\right],k\geqslant\lambda+1$, the following result can be obtained

$$\begin{aligned}V(x(t))&\leqslant\max_{i=n(k-1)+1,\cdots,nk}\left\{\underset{t\in\left(\tau_{i-1},\tau_i\right]}{sup}V(x(t))\right\}\\&\leqslant V\left(x\left(\tau_{n(k-1)}\right)\right)\\&\quad\times\max_{i=n(k-1)+1,\cdots,nk}\left\{\left[\left(\prod_{j=n(k-1)}^{i-1}\omega_j\right)e^{\varrho\left(\tau_i-\tau_{n(k-1)}\right)}\right]\right\}\\&\leqslant V\left(x\left(\tau_{n(k-1)}\right)\right)\\&=U(t).\end{aligned}$$

Thus, the third condition in Corollary 2 is satisfied.

Furthermore, when $t \in \left(\tau_{n(k-1)}, \tau_{nk}\right], 1 \leqslant k \leqslant \lambda$,

$$\begin{aligned} U\left(\tau_{nk}\right) - U\left(\tau_{n(k-1)}\right) &= V\left(x\left(\tau_{n(k-1)}^{+}\right)\right) - V\left(x\left(\tau_{n(k-2)}^{+}\right)\right) \\ &\leqslant \omega_{n(k-1)} V\left(x\left(\tau_{n(k-1)}\right)\right) \\ &- V\left(x\left(\tau_{n(k-2)}^{+}\right)\right) \\ &\leqslant \omega_{n(k-1)}\omega_{n(k-1)-1}\ldots\omega_{n(k-2)+1} \\ &\times V\left(x\left(\tau_{n(k-2)}^{+}\right)\right) \\ &\times e^{-\varrho\left(\tau_{n(k-1)}-\tau_{n(k-2)}\right)} \\ &\quad - V\left(x\left(\tau_{n(k-2)}^{+}\right)\right) \\ &\leqslant (1-\phi) V\left(x\left(\tau_{n(k-2)}^{+}\right)\right) \\ &\leqslant (1-\phi) U\left(\tau_{n(k-1)}\right) \\ &\leqslant 0. \end{aligned}$$

Similarly, when $t \in \left(\tau_{n(k-1)}, \tau_{nk}\right], k \geqslant \lambda+1$,

$$\begin{aligned} U\left(\tau_{nk}\right) - U\left(\tau_{n(k-1)}\right) &= V\left(x\left(\tau_{n(k-1)}\right)\right) - V\left(x\left(\tau_{n(k-2)}\right)\right) \\ &\leqslant V\left(x\left(\tau_{n(k-1)-1}^{+}\right)\right) e^{\varrho\left(\tau_{n(k-1)}-\tau_{n(k-1)-1}\right)} \\ &- V\left(x\left(\tau_{n(k-2)}\right)\right) \\ &\leqslant \omega_{n(k-1)-1} V\left(x\left(\tau_{n(k-1)-1}\right)\right) \\ &\times e^{\varrho\left(\tau_{n(k-1)}-\tau_{n(k-2)}\right)} - V\left(x\left(\tau_{n(k-2)}\right)\right) \\ &\leqslant \omega_{n(k-1)-1}\omega_{n(k-1)-2}\ldots\omega_{n(k-2)} V\left(x\left(\tau_{n(k-2)}\right)\right) \\ &\times e^{\varrho\left(\tau_{n(k-1)}-\tau_{n(k-2)}\right)} - V\left(x\left(\tau_{n(k-2)}\right)\right) \\ &\leqslant (1-\phi) V\left(x\left(\tau_{n(k-2)}\right)\right) \\ &\leqslant (1-\phi) U\left(\tau_{n(k-1)}\right) \\ &\leqslant 0. \end{aligned}$$

In addition, we need to prove the following result

$$\begin{aligned} \underset{t\in\left(\tau_{\lambda n}, \tau_{n(\lambda+1)}\right]}{U(t)} - \underset{t\in\left(\tau_{n(\lambda-1)}, \tau_{\lambda n}\right]}{U(t)} &= V\left(x\left(\tau_{n\lambda}\right)\right) - V\left(x\left(\tau_{n(\lambda-1)}^{+}\right)\right) \\ &\leqslant V\left(x\left(\tau_{n\lambda-1}^{+}\right)\right) e^{-\varrho\left(\tau_{n\lambda}-\tau_{n\lambda-1}\right)} \\ &- V\left(x\left(\tau_{n(\lambda-1)}^{+}\right)\right) \\ &\leqslant \omega_{n\lambda-1} V\left(x\left(\tau_{n\lambda-1}\right)\right) \\ &\times e^{-\varrho\left(\tau_{n\lambda}-\tau_{n\lambda-1}\right)} - V\left(x\left(\tau_{n(\lambda-1)}^{+}\right)\right) \\ &\leqslant \omega_{n\lambda-1} V\left(x\left(\tau_{n\lambda-2}^{+}\right)\right) e^{-\varrho\left(\tau_{n\lambda}-\tau_{n\lambda-2}\right)} \\ &- V\left(x\left(\tau_{n(\lambda-1)}^{+}\right)\right) \\ &\leqslant \omega_{n\lambda-1}\omega_{n\lambda-2}\cdots\omega_{n(\lambda-1)+1} \\ &\times V\left(x\left(\tau_{n(\lambda-1)}^{+}\right)\right) e^{-\varrho\left(\tau_{n\lambda}-\tau_{n(\lambda-1)}\right)} \\ &- V\left(x\left(\tau_{n(\lambda-1)}^{+}\right)\right) \\ &\leqslant (\phi-1) V\left(x\left(\tau_{n(\lambda-1)}^{+}\right)\right) \\ &\leqslant (\phi-1) U\left(\tau_{n\lambda}\right) \\ &\leqslant 0. \end{aligned}$$

Hence, when $t \in \left(\tau_{n(k-1)}, \tau_{nk}\right]$, the following result can be obtained

$$\begin{aligned} U(t) &= U\left(\tau_{nk}\right) \\ &\leqslant \phi U\left(\tau_{n(k-1)}\right) \\ &\leqslant \phi^2 U\left(\tau_{n(k-2)}\right) \\ &\ldots \\ &\leqslant \xi\phi^{k-1} V\left(x_0\right), \end{aligned}$$

as $k \to \infty$, $\lim_{t\to\tau_\infty} U(t) = 0$.

Therefore, all the conditions for Corollary 3 are satisfied by Corollary 2, the proof is completed.

IV. Step-function method for T-S fuzzy impulsive systems

In this section, we will apply the theoretical analysis results of the previous section to fuzzy impulsive systems to obtain several relatively broad and adequate stability conditions for T-S fuzzy impulsive systems.

Theorem 4: Suppose that there is a positive definite symmetric matrix Q, integer $n\,(n \geqslant 2)$, and the scalars ϱ, ξ, ϕ, $\omega_k\,(k = 1, 2, \cdots)$ satisfy the following conditions:

(i) $Q\Lambda_i + \Lambda_i^T Q \leqslant \varrho Q, i = 1, 2, \cdots, r$;

(ii) $\left[1 + \sum_{i=1}^{r} g_i\left(\eth\left(\tau_k\right)\right) R_{ik}\right]^2 \leqslant \omega_k, k = 1, 2, 3, \cdots$;

(iii) $\xi = \max_{i=1,2,\cdots,n}\left\{\sup_{t\in\left(\tau_{k-1},\tau_k\right)}\left[\left(\prod_{j=0}^{i-1}\omega_j\right) e^{\varrho\left(t-\tau_0\right)}\right]\right\} < \infty$;

(iv) $\max_{i=n(k-1)+1,\cdots,nk}\left\{\sup_{t\in\left(\tau_{i-1},\tau_i\right]}\left[\left(\prod_{j=n(k-1)}^{i-1}\omega_j\right) \times e^{\varrho\left(t-\tau_{n(k-1)}\right)}\right]\right\} \leqslant \phi < 1,\ k \geqslant 1.$

Then, the origin of system (3) is GUAS, and in addition, if $\tau_\infty = \infty$, the origin of system (3) is GAS.

Proof 5: The following function $V(x(t))$ is defined as follows:

$$V(x(t)) = x^T(t)\, Q x(t).$$

When $t \in \left(\tau_{(k-1)}, \tau_k\right]$, it can be concluded that

$$\begin{aligned} \dot{V}(x(t)) &= \sum_{i=1}^{r} g_i\left(\eth(t)\right) x^T(t)\left[Q\Lambda_i + \Lambda_i^T Q - \varrho Q\right] x(s) \\ &+ \varrho V(x(t)) \\ &\leqslant \varrho V(x(t)). \end{aligned}$$

then, it can be concluded that

$$V(x(t)) \leqslant V\left(x\left(\tau_{k-1}^{+}\right)\right) e^{\varrho\left(t-\tau_{k-1}\right)}, t \in \left(\tau_{k-1}, \tau_k\right].$$

Furthermore, when $t = \tau_k$,, it can be concluded that

$$
\begin{aligned}
V\left(x\left(\tau_k^+\right)\right) &= x^T\left(\tau_k^+\right) Q x\left(\tau_k^+\right) \\
&= \left[x\left(\tau_k\right) + \sum_{i=1}^{r} g_i\left(\eth\left(\tau_k\right)\right) R_{ik} x\left(\tau_k\right)\right]^T Q \\
&\quad \times \left[x\left(\tau_k\right) + \sum_{i=1}^{r} g_i\left(\eth\left(\tau_k\right)\right) R_{ik} x\left(\tau_k\right)\right] \\
&= \left[1 + \sum_{i=1}^{r} g_i\left(\eth\left(\tau_k\right)\right) R_{ik}\right]^2 x^T\left(\tau_k\right) Q x\left(\tau_k\right) \\
&\leqslant \omega_k V\left(x\left(\tau_k\right)\right).
\end{aligned}
$$

The proof of the remaining two conditions are the same as in the previous proof, so they are omitted here.

Let $V\left(x\left(t\right)\right) = x^T\left(t\right) Q x\left(t\right)$ in Corollary 3, then Corollary 4 can be obtained.

Corollary 4: Suppose that there is a positive definite symmetric matrix Q, the integers $n\left(n \geqslant 2\right)$, $\lambda = n$, and the scalars $\varrho > 0,\ \xi > 0,\ \phi > 0,\ \omega_k\left(k = 1, 2, \cdots\right)$ satisfy the following conditions:

(i) $Q\Lambda_i + \Lambda_i^T Q \leqslant \varrho Q,\ i = 1, 2, \cdots r$;

(ii) $\left[1 + \sum\limits_{i=1}^{r} g_i\left(\eth\left(\tau_k\right)\right) R_{ik}\right]^2 \leqslant \omega_k,\ k = 1, 2, \cdots$;

(iii) $\xi = \max\limits_{i=1,2,\cdots,n}\left\{\left[\left(\prod\limits_{j=0}^{i-1}\omega_j\right)\right]\right\} < \infty,\ k = 1$;

(iv) $\max\limits_{i=n(k-1)+1,\cdots,nk}\left\{\left[\left(\prod\limits_{j=n(k-1)}^{i-1}\omega_j\right) e^{-\varrho\left(\tau_{i-1}-\tau_{n(k-1)}\right)}\right.\right.$ $\leqslant 1,\ 1 < k \leqslant \lambda$;

(v) $\max\limits_{i=n(k-1)+1,\cdots,nk}\left\{\left[\left(\prod\limits_{j=n(k-1)}^{i-1}\omega_j\right) e^{\varrho\left(\tau_i-\tau_{n(k-1)}\right)}\right]\right\}$ $1,\ k \geqslant \lambda + 1$;

(vi) $\omega_{n(k-1)}\omega_{n(k-1)-1}\cdots\omega_{n(k-2)+1}e^{-\varrho\left(\tau_{n(k-1)}-\tau_{n(k-2)}\right)} \leqslant \phi < 1,\ 1 < k \leqslant \lambda$;

(vii) $\omega_{n(k-1)-1}\omega_{n(k-1)-2}\cdots\omega_{n(k-2)}e^{\varrho\left(\tau_{n(k-1)}-\tau_{n(k-2)}\right)} \leqslant \phi < 1,\ k \geqslant \lambda + 1$;

(viii) $\omega_{n\lambda-1}\omega_{n\lambda-2}\cdots\omega_{n(\lambda-1)+1}e^{-\varrho\left(\tau_{n\lambda}-\tau_{n(\lambda-1)}\right)} \leqslant \phi < 1$.

Then, the origin of system (3) is GUAS, and in addition, if $\tau_\infty = \infty$, the origin of system (3) is GAS.

To use numerical examples to prove the feasibility of the proposed method, it is easier to verify the stability of equidistant impulsive systems in the case of $n = 2$, then Corollary 5 is obtained.

Corollary 5: Let $n = 2$ in system (3) and $\tau_k - \tau_{k-1} \equiv \iota,\ \iota > 0,\ k = 1, 2, \cdots$. Suppose that there exists a positive definite symmetric matrix Q, and the scalars $\lambda = n,\ \varrho > 0,\ \xi > 0,\ \phi > 0,\ \omega_k\left(k = 1, 2, \cdots\right)$ satisfy the following conditions:

(i) $Q\Lambda_i + \Lambda_i^T Q \leqslant \varrho Q,\ i = 1, 2, \cdots r$;

(ii) $\left[1 + \sum\limits_{i=1}^{r} g_i\left(\vartheta\left(\tau_k\right)\right) R_{ik}\right]^2 \leqslant \omega_k,\ k = 1, 2, \cdots$;

(iii) $\max\left\{\omega_{2k-2},\ \omega_{2k-1}\omega_{2k-2}e^{-\varrho\iota}\right\} \leqslant 1,\ 1 < k \leqslant \lambda$;

(iv) $\max\left\{\omega_{2k-2}e^{\varrho\iota}, \omega_{2k-1}\omega_{2k-2}e^{2\varrho\iota}\right\} \leqslant 1,\ k \geqslant \lambda + 1$;

(v) $\omega_{2k-3}\omega_{2k-2}e^{-2\varrho\iota} \leqslant \phi < 1,\ 1 < k \leqslant \lambda$;

(vi) $\omega_{2k-4}\omega_{2k-3}e^{2\varrho\iota} \leqslant \phi < 1,\ k \geqslant \lambda + 1$;

(vii) $\omega_{2\lambda-1}e^{-2\varrho\iota} \leqslant \phi < 1$.

Then, the origin of system (3) is GUAS, and in addition, if $\tau_\infty = \infty$, the origin of system (3) is GAS.

V. Numerical Examples

In this section, two numerical examples are used to verify the effectiveness of the proposed theories.

Example 1: Rössler's system can be described by the following set of differential equations [31]

$$
\begin{cases}
\dot{x}_1 = -x_2 - x_3, \\
\dot{x}_2 = x_1 + \nu_1 x_2, \\
\dot{x}_3 = \nu_2 x_1 - \varpi\left(\mathrm{x}_1\right) x_3,
\end{cases}
\tag{15}
$$

with the nonlinear function $\varpi\left(x_1\right) = \nu_3 - x_1$, in which $\nu_1 = 0.34, \nu_2 = 0.4, \nu_3 = 4.5$. Figure 3 shows the Rössler's system appears chaotic phenomenon with the initial condition $x\left(0\right) = \left[1, 0.6, -0.2\right]^T$.

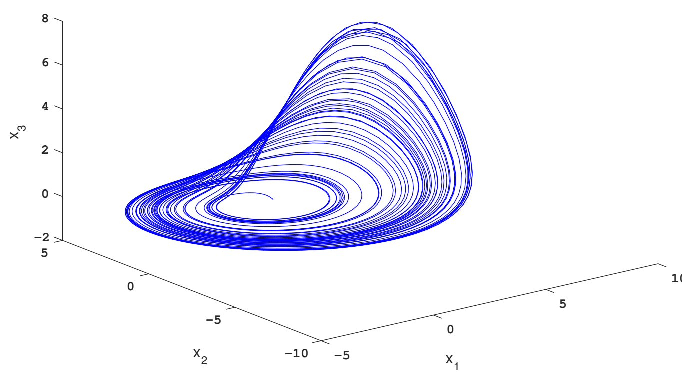

Fig. 3. The Rössler's system appears chaotic phenomenon with the initial condition $x\left(0\right) = \left[1, 0.6, -0.2\right]^T$.

By using fuzzy IF-THEN rules, system (15) can be replaced by the following T-S fuzzy impulsive model:

Rule 1:
IF $x_1\left(t\right)$ is Δ_1,
THEN

$$
\begin{cases}
\dot{x}\left(t\right) = \Lambda_1 x\left(t\right), t \in \left(\tau_{k-1}, \tau_k\right], \\
x\left(\tau_k^+\right) = \left[1 + R_{1k}\right] x\left(\tau_k\right),
\end{cases}
$$

Rule 2:
IF $x_1\left(t\right)$ is Δ_2,
THEN

$$
\begin{cases}
\dot{x}\left(t\right) = \Lambda_2 x\left(t\right), t \in \left(\tau_{k-1}, \tau_k\right], \\
x\left(\tau_k^+\right) = \left[1 + R_{2k}\right] x\left(\tau_k\right),
\end{cases}
$$

in which $x\left(t\right) = \left[x_1\left(t\right), x_2\left(t\right), x_3\left(t\right)\right]^T$, and the membership functions of fuzzy sets Δ_1 and Δ_2 are as follow:

$$
\Lambda_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & \nu_1 & 0 \\ \nu_2 & 0 & -\zeta \end{bmatrix},
$$

$$\Lambda_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & \nu_1 & 0 \\ \nu_2 & 0 & \zeta \end{bmatrix},$$

$$\Delta_1 (x_1 (t)) = \frac{1}{2} \left[1 - \frac{\varpi (x_1 (t))}{\zeta_1} \right],$$

and

$$\Delta_2 (x_1 (t)) = \frac{1}{2} \left[1 + \frac{\varpi (x_1 (t))}{\zeta_1} \right],$$

where $\zeta_1 = 10$.

Hence, system (15) can be rewritten as follow:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} g_i (\eth (t)) \Lambda_i x (t), t \in (\tau_{k-1}, \tau_k], \\ x(\tau_k^+) = \sum_{i=1}^{2} g_i (\eth (\tau_k)) [1 + R_{ik}] x (\tau_k), \end{cases} \tag{16}$$

in which $k \in N^*, g_i (\eth (t)) = \frac{\psi_i(\eth(t))}{\sum_{i=1}^{2} \psi_i(\eth(t))}$, and $\psi_i (x_1 (t)) = \Delta_i (x_1 (t))$.

Let $\tau_k - \tau_{k-1} \equiv \iota$, $R_{1k} = R_{2k} = 0.05, 1 < k \leqslant 2; R_{1k} = R_{2k} = -0.99, k \geqslant 3, Q = I$, then the parameters in Corollary 5 are $\varrho = 20.0180, w_k = 1.1025, 1 < k \leqslant \lambda; w_k = 0.0001, k \geqslant \lambda + 1$, respectively. Furthermore, by using Corollary 5, the upper bound of impulsive interval ι is estimated. Table I lists the upper bound results of impulsive interval obtained by Corollary 5 and other methods, which is larger than the these results.

TABLE I
COMPARISON OF THE UPPER BOUND OF IMPULSIVE INTERVAL ι BETWEEN THE DIFFERENT METHODS

Corollary 5	[26]	[23]
0.4601	0.2301	0.1186

In addition, setting $\iota = 0.3$, it is easy to obtain that Corollary 5 is feasible if $0.0016 \leqslant \phi < 1$. Hence, the origin of system (15) is stable, the time response curves of the controlled Rössler's system are shown in Figure 4, where the initial condition is set to $x(0) = [-3, -0.7, 1.5]^T$.

VI. CONCLUSIONS

In this paper, a new stability analysis method for T-S fuzzy impulsive systems is proposed, which are the mixed impulse method of single-step step-function and the mixed impulse method of multi-steps step-function. In the proposed method, the Lyapunov-like function does not need to be monotonically decreasing or continuous in each impulsive interval, which widens the stability analysis method of Lyapunov-like to a certain extent. In addition, the upper bound of impulsive interval is estimated. Compared with other methods, the result of the proposed method , which is larger than the these results. However, the proposed method does not consider the delay, which makes the method has some limitations. So, our subsequent work will combine the delay with multi-steps step-function method to obtain a new stability analysis method for T-S fuzzy impulsive systems.

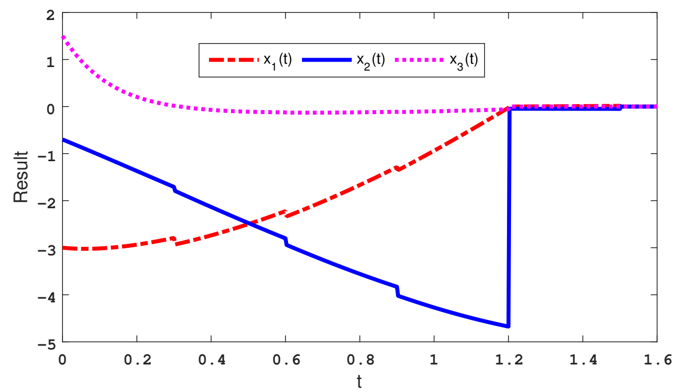

Fig. 4. The time response curves of the controlled Rössler's system with the proposed method.

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