TokenSkip: Controllable Chain-of-Thought Compression in LLMs

Anonymous ACL submission

Abstract

Chain-of-Thought (CoT) has been proven effective in enhancing the reasoning capabilities of large language models (LLMs). Recent advancements, such as OpenAI's o1 and DeepSeek-R1, suggest that scaling up the length of CoT sequences during inference could further boost LLM reasoning performance. However, due to the autoregressive nature of LLM decoding, longer CoT outputs lead to a linear increase in inference latency, adversely affecting user experience, particularly when the CoT exceeds 10,000 tokens. To address this limitation, we analyze the semantic importance of tokens within CoT outputs and reveal that their contributions to reasoning vary. Building on this insight, we propose `TokenSkip`, a simple yet effective approach that enables LLMs to selectively skip less important tokens, allowing for controllable CoT compression. Extensive experiments across various models and tasks demonstrate the effectiveness of `TokenSkip` in reducing CoT token usage while preserving strong reasoning performance. Notably, when applied to Qwen2.5-14B-Instruct, `TokenSkip` reduces reasoning tokens by 40% (from 313 to 181) on GSM8K, with less than a 0.4% performance drop[1].

1 Introduction

Chain-of-Thought (CoT) prompting (Nye et al., 2021; Wei et al., 2022; Kojima et al., 2022) has emerged as a cornerstone strategy for enhancing Large Language Models (LLMs) in complex reasoning tasks. By eliciting step-by-step inference, CoT enables LLMs to decompose intricate problems into manageable subtasks, thereby improving their problem-solving performance (Yao et al., 2023; Wang et al., 2023; Zhou et al., 2023; Shinn et al., 2023). Recent advancements, such as OpenAI's o1 (OpenAI et al., 2024) and DeepSeek-R1 (DeepSeek-AI et al., 2025), further demonstrate that scaling up CoT lengths from hundreds to thousands of reasoning steps could continuously improve LLM reasoning. These breakthroughs have underscored CoT's potential to advance LLM capabilities, expanding the boundaries of AI-driven problem-solving.

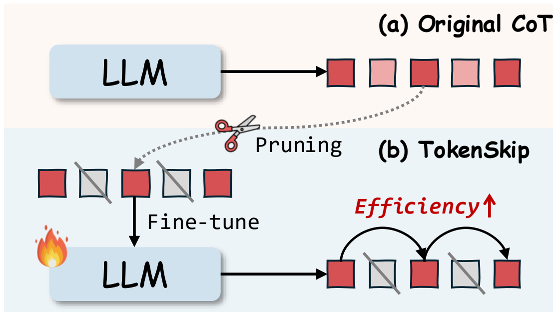

Figure 1: In contrast to vanilla CoT that generates all reasoning tokens sequentially, `TokenSkip` enables LLMs to *skip* tokens with less semantic importance (*e.g.*,) and learn shortcuts between critical reasoning tokens, facilitating controllable CoT compression.

Despite its effectiveness, the increased length of CoT sequences introduces substantial computational overhead. Due to the autoregressive nature of LLM decoding, longer CoT outputs lead to proportional increases in both inference latency and memory footprints of key-value cache. Additionally, the quadratic computational cost of attention layers further exacerbates this burden. These issues become particularly pronounced when CoT sequences extend into thousands of reasoning steps, resulting in significant computational costs and prolonged response times. While prior research has explored methods for selectively skipping reasoning steps (Ding et al., 2024; Liu et al., 2024), recent findings (Jin et al., 2024; Merrill and Sabharwal, 2024) suggest that such reductions may conflict with test-time scaling (OpenAI, 2024; Snell et al., 2025), ultimately impairing LLM reasoning per-

[1]All of our codes and checkpoints will be released to facilitate future research.

formance. Therefore, striking an optimal balance between CoT efficiency and reasoning accuracy remains a critical open challenge.

In this work, we delve into CoT efficiency and seek the answer to an important question: *"Does every token in the CoT output contribute equally to deriving the answer?"* We empirically analyze the semantic importance of tokens within CoT outputs and reveal that their contributions to the reasoning performance vary, as depicted in Figure 2. Building on this insight, we introduce `TokenSkip`, a simple yet effective approach that enables LLMs to *skip* less important tokens within CoT sequences and learn shortcuts between critical reasoning tokens, thereby allowing for controllable CoT compression with adjustable ratios. Specifically, as shown in Figure 1, `TokenSkip` constructs compressed CoT training data with various compression ratios, by pruning unimportance tokens from original LLM CoT trajectories. Then, it conducts a general supervised fine-tuning process on target LLMs with this training data, facilitating LLMs to automatically trim redundant tokens during reasoning.

We conduct extensive experiments across various models, including LLaMA-3.1-8B-Instruct and the Qwen2.5-Instruct series, using two widely recognized math reasoning benchmarks: GSM8K and MATH-500. The results validate the effectiveness of `TokenSkip` in compressing CoT outputs while maintaining robust reasoning performance. Notably, Qwen2.5-14B-Instruct exhibits almost **NO** performance drop (less than 0.4%) with a $\mathbf{40}\%$ reduction in token usage on GSM8K. On the challenging MATH-500 dataset, LLaMA-3.1-8B-Instruct effectively reduces CoT token usage by $\mathbf{30}\%$ with a performance decline of less than 4%, resulting in a $\mathbf{1.4}\times$ inference speedup. Further analysis underscores the coherence of `TokenSkip` in specified compression ratios and its potential scalability with stronger compression techniques.

`TokenSkip` is distinguished by its low training cost. For Qwen2.5-14B-Instruct, `TokenSkip` fine-tunes only 0.2% of the model's parameters using LoRA. The size of the compressed CoT training data is no larger than that of the original training set, with 7,473 examples in GSM8K and 7,500 in MATH. The training is completed in approximately 2 hours for the 7B model and 2.5 hours for the 14B model on two 3090 GPUs. These characteristics make `TokenSkip` an efficient and reproducible approach, suitable for use in efficient and cost-effective LLM deployment.

To sum up, our key contributions are:

1. To the best of our knowledge, this work is the *first* to investigate the potential of enhancing CoT efficiency through *token skipping*, inspired by the varying semantic importance of tokens in CoT trajectories of LLMs.
2. We introduce `TokenSkip`, a simple yet effective approach that enables LLMs to skip redundant tokens within CoTs and learn shortcuts between critical tokens, facilitating CoT compression with adjustable ratios.
3. Our experiments validate the effectiveness of `TokenSkip`. When applied to Qwen2.5-14B-Instruct, `TokenSkip` reduces reasoning tokens by 40% (from 313 to 181) on GSM8K, with less than a 0.4% performance drop.

2 Background and Preliminaries

In this section, we discuss the relevant research background and present preliminary studies on token efficiency in CoT sequences, exploring its impact on the reasoning performance of LLMs.

2.1 Token Importance

We first investigate a critical research question to CoT efficiency: *"Does every token in the CoT output contribute equally to deriving the answer?"* In other words, we would like to know if there is any token redundancy in CoT sequences that could be eliminated to improve CoT efficiency.

Token redundancy has been recognized as a longstanding and fundamental issue in LLM efficiency (Hou et al., 2022; Zhang et al., 2023; Lin et al., 2024; Chen et al., 2024). Recently, it has garnered intensive research attention in prompt compression (Li et al., 2023; Jiang et al., 2023; Pan et al., 2024), which focuses on removing redundant tokens from input prompt to reduce API token usage. To address this issue, Selective Context (Li et al., 2023) proposed to measure the importance of tokens in a piece of text based on the semantic confidence of LLMs:

$$I_1\left(x_i\right) = -\log P\left(x_i \mid \boldsymbol{x}_{<i}; \boldsymbol{\theta}_{\mathcal{M}_L}\right), \qquad (1)$$

where $\boldsymbol{x} = \{x_i\}_{i=1}^{n}$ is the given text, x_i denotes a token, and $\mathcal{M}_L$ denotes the LLM used to compute the confidence of each token. Intuitively, such measurement could be seamlessly applied to CoT tokens generated by LLMs. We show an example of this measurement in Figure 2.

Problem: Marcus is half of Leo's age and five years younger than Deanna. Deanna is 26. How old is Leo?

Chain-of-Thought: Let 's break it down step by step : 1. Deanna is 26 years old . 2. Marcus is five years younger than Deanna , so Marcus is 26 - 5 = 21 years old . 3. Marcus is half of Leo 's age , so Leo 's age is twice Marcus 's age . 4. Since Marcus is 21 , Leo 's age is 2 x 21 = 42 . (Selective Context)

Chain-of-Thought: Let 's break it down step by step : 1. Deanna is 26 years old . 2. Marcus is five years younger than Deanna , so Marcus is 26 - 5 = 21 years old . 3. Marcus is half of Leo 's age , so Leo 's age is twice Marcus 's age . 4. Since Marcus is 21 , Leo 's age is 2 x 21 = 42 . (LLMLingua-2)

Final Answer: 42.

Figure 2: Visualization of token importance within a CoT sequence, with darker colors indicating higher values. This figure compares two token importance measurements: Selective Context and LLMLingua-2.

Revovering the Compressed Chain-of-Thought

Compressed CoT: break down Deanna 26 Marcus five younger 26 - 5 21 Marcus half Leo's age twice Marcus Marcus 21, Leo's age 2 x 21 = 42.

Recovered Compressed CoT: Let's break it down step by step. Deanna is 26 years old. Marcus is five years younger than Deanna: M = D - 5. Marcus's age: M = 26 - 5 = 21. Marcus is half of Leo's age: M = L / 2. Leo is twice Marcus's age: L = 2M. Leo's age: L = 2 x 21 = 42.

Figure 3: Recovering the compressed CoT for GSM8K math word problem using LLaMA-3.1-8B-Instruct.

Despite its simplicity, LLMLingua-2 (Pan et al., 2024) argued that there exist two major limitations in the aforementioned measurement that hinder the compression performance. Firstly, as shown in Figure 2, the intrinsic nature of LLM perplexity leads to lower importance measures (i.e., higher confidence) for tokens at the end of the sentence. Such position dependency impacts the factual importance measurement of each token. Furthermore, the unidirectional attention mechanism in causal LMs may fail to capture all essential information needed for token importance within the text.

To tackle these limitations, LLMLingua-2 introduced utilizing a bidirectional BERT-like LM (Devlin et al., 2019) for token importance measurement. It utilizes GPT-4 (OpenAI, 2023) to label each token as "*important*" or not and trains the bidirectional LM with a token classification objective. The token importance is measured by the predicted probability of each token:

$$I_2(x_i) = P\left(x_i \mid \boldsymbol{x}_{\leq n}; \boldsymbol{\theta}_{\mathcal{M}_B}\right), \quad (2)$$

where $\mathcal{M}_B$ denotes the bidirectional LM.

In this study, we apply LLMLingua-2 as the token importance measurement to LLM CoT outputs. Similar to plain text, we observe that the semantic importance of tokens within CoT outputs varies, as shown in Figure 2. For instance, mathematical equations tend to have a greater contribution to the final answer, consistent with recent research (Ma et al., 2024). In contrast, semantic connectors such as "*so*" and "*since*" generally contribute less. These findings highlight the token redundancy in LLM CoT outputs and the substantial potential to enhance CoT efficiency by trimming this redundancy.

2.2 CoT Recovery

We further explore the following research question: *"Are LLMs capable of restoring the CoT process from compressed outputs?"* The answer is yes. As shown in Figure 3 and detailed in Appendix A, examples restored from compressed CoTs using LLaMA-3.1-8B-Instruct demonstrate that LLMs could effectively comprehend the semantic information encoded in the compressed CoT and restore the CoT process. This capability ensures that the interpretability of compressed CoTs is maintained. Additionally, when required by users, the complete CoT process can be recovered and presented.

In summary, the empirical analysis above underscores the potential of trimming redundant tokens to enhance CoT efficiency, as well as the ability of LLMs to restore CoT from compressed outputs. However, enabling LLMs to autonomously skip redundant CoT tokens and identify shortcuts between critical reasoning tokens presents a non-trivial challenge. To the best of our knowledge, this work is the *first* to explore CoT compression through *token skipping*. In the following sections, we present our proposed methodology in detail.

3 TokenSkip

We introduce TokenSkip, a simple yet effective approach that enables LLMs to skip less important tokens, enabling controllable CoT compression with adjustable ratios. This section demonstrates the details of our methodology, including token pruning (§3.1), training (§3.2), and inference (§3.3).

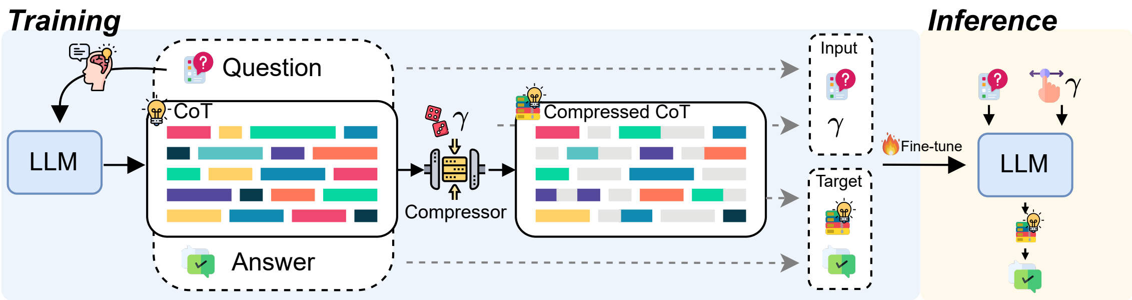

Figure 4: Illustration of `TokenSkip`. During the training phase, `TokenSkip` first generates CoT trajectories from the target LLM. These CoTs are then compressed to a specified ratio, γ, based on the semantic importance of tokens. `TokenSkip` fine-tunes the target LLM using compressed CoTs, enabling controllable CoT inference at the desired γ.

3.1 Token Pruning

The key insight behind `TokenSkip` is that "*each reasoning token contributes differently to deriving the answer.*" To enhance CoT efficiency, we propose to trim redundant tokens from LLM CoT outputs and fine-tune LLMs using these trimmed CoT trajectories. The token pruning process is guided by the concept of *token importance*, as detailed in Section 2.1.

Specifically, given a target LLM $\mathcal{M}$, one of its CoT trajectories $\boldsymbol{c} = \{c_i\}_{i=1}^{m}$, and a desired compression ratio $\gamma \in [0, 1]$, `TokenSkip` first calculates the semantic importance of each CoT token $I(c)$, as defined in Eq (2). The tokens are then ranked in descending order based on their importance values. Next, the γ-th percentile of these importance values is computed, representing the threshold for token pruning:

$$I_\gamma = \text{np.percentile}\left(\left[I\left(c_1\right), .., I\left(c_m\right)\right], \gamma\right). \quad (3)$$

Finally, CoT tokens with an importance value greater than or equal to I_γ are retained in the compressed CoT trajectory:

$$\widetilde{\boldsymbol{c}} = \left\{c_i \mid I\left(c_i\right) \geq I_\gamma\right\}, 1 \leq i \leq m. \quad (4)$$

3.2 Training

Given a training dataset $\mathcal{D}$ with N samples and a target LLM $\mathcal{M}$, we first obtain N CoT trajectories with $\mathcal{M}$. Then, we filter out trajectories with incorrect answers to ensure the high quality of training data. For the remaining CoT trajectories, we prune each CoT with a randomly selected compression ratio γ, as demonstrated in Section 3.1. For each $\langle$question, compressed CoT, answer$\rangle$, we inserted the compression ratio γ after the question. Finally, each training sample is formatted as follows:

$$\mathcal{Q}\ [\text{EOS}]\ \gamma\ [\text{EOS}]\ \text{Compressed CoT}\ \mathcal{A},$$

where $\langle \mathcal{Q}, \mathcal{A} \rangle$ indicates the $\langle$question, answer$\rangle$ pair. Formally, given a question $\boldsymbol{x}$, compression ratio γ, and the output sequence $\boldsymbol{y} = \{y_i\}_{i=1}^{l}$, which includes the compressed CoT $\widetilde{\boldsymbol{c}}$ and the answer $\boldsymbol{a}$, we fine-tunes the target LLM $\mathcal{M}$, enabling it to perform chain-of-thought in a compressed pattern by minimizing

$$\mathcal{L} = \sum_{i=1}^{l} \log P\left(y_i \mid \boldsymbol{x}, \gamma, \boldsymbol{y}_{<i}; \boldsymbol{\theta}_{\mathcal{M}}\right), \quad (5)$$

where $\boldsymbol{y} = \{\widetilde{c}_1, \cdots, \widetilde{c}_{m'}, a_1, \cdots, a_t\}$. Note that the compression is performed solely on CoT sequences, and we keep the answer $\boldsymbol{a} = \{a_i\}_{i=1}^{t}$ unchanged. To preserve LLMs' reasoning capabilities, we also include a portion of the original CoT trajectories in the training data, with γ set to 1.

3.3 Inference

The inference of `TokenSkip` follows autoregressive decoding. Compared to original CoT outputs that may contain redundancy, `TokenSkip` facilitates LLMs to skip *unimportant* tokens during the chain-of-thought process, thereby enhancing reasoning efficiency. Formally, given a question $\boldsymbol{x}$ and the compression ratio γ, the input prompt of `TokenSkip` follows the same format adopted in fine-tuning, which is $\mathcal{Q}\ [\text{EOS}]\ \gamma\ [\text{EOS}]$. The LLM $\mathcal{M}$ sequentially predicts the output sequence $\hat{\boldsymbol{y}}$:

$$\hat{\boldsymbol{y}} = \arg\max_{\boldsymbol{y}^*} \sum_{j=1}^{l'} \log P\left(y_j \mid \boldsymbol{x}, \gamma, \boldsymbol{y}_{<j}; \boldsymbol{\theta}_{\mathcal{M}}\right),$$

where $\hat{\boldsymbol{y}} = \{\hat{c}_1, \cdots, \hat{c}_{m''}, \hat{a}_1, \cdots, \hat{a}_{t'}\}$ denotes the output sequence, which includes CoT tokens $\hat{\boldsymbol{c}}$ and the answer $\hat{\boldsymbol{a}}$. We illustrate the training and inference process of `TokenSkip` in Figure 4.

4 Experiments

4.1 Experimental Setup

Models and Datasets We primarily evaluate our method using LLaMA-3.1-8B-Instruct (Dubey et al., 2024) and Qwen2.5-Instruct series (Yang et al., 2024). The evaluation leverages two widely-used math reasoning benchmarks: GSM8K (Cobbe et al., 2021) and MATH (Hendrycks et al., 2021). For training, we use the respective training sets from both datasets. Regarding the MATH dataset, due to the computation cost, we assess our method on a subset, MATH-500, which is identical to the test set used in Lightman et al. (2024). The subset comprises 500 representative problems, and we find that its evaluation yields results comparable to those from the full dataset.

Implementation Details We utilize LLMLingua-2 (Pan et al., 2024) as the token importance metric to generate our compressed CoT training data. The compression ratio γ is randomly selected from $\{0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$ for each training sample. We adopt LoRA (Hu et al., 2022), an efficient and reproducible approach that has been widely verified as effective in LLM fine-tuning, to train our models. The rank r is set to 8, and the scaling parameter α is set to 16. `TokenSkip` is characterized by its low training cost, with training taking $\sim$2 hours for the 7B model and $\sim$2.5 hours for the 14B model on 3090 GPUs. During inference, the maximum number of tokens `max_len` is set to 512 for GSM8K and 1024 for MATH[2]. All experiments are conducted using Pytorch 2.1.0 on 2$\times$NVIDIA GeForce RTX 3090 GPU (24GB) with CUDA 12.1, and an Intel(R) Xeon(R) Platinum 8370C CPU with 32 cores. We include more implementation details in Appendix B.1.

Baselines In our main experiments, we compare `TokenSkip` to two commonly used length control baselines: **1) Prompt-based Reduction.** In this approach, we instruct the LLM to reduce a fixed proportion of output tokens in the CoT process. Specifically, we append a prompt such as "*Please reduce 50% of the words in your Chain-of-Thought process.*" to the input instruction. **2) Truncation.** This method involves brute-force length truncation, where the maximum number of output tokens is restricted, compressing the CoT output to a fixed ratio. These baselines are referred to as `Prompt` and `Truncation` in Table 1, respectively.

[2]Since many samples reach the maximum length when testing `TokenSkip` on MATH-500, we adjust its length budget to `max_len`$\times\gamma$, with no adjustment for GSM8K.

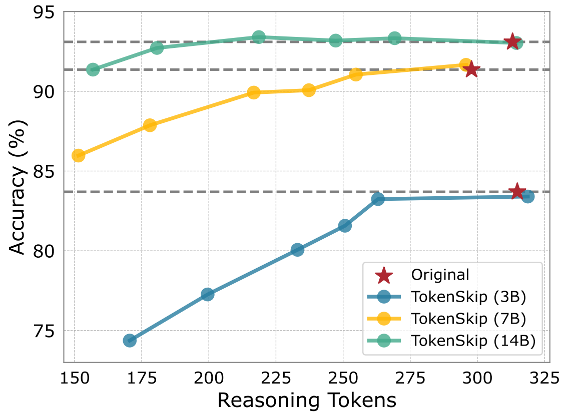

Figure 5: Compression performance of `TokenSkip` on Qwen2.5-Instruct models. Qwen2.5-14B-Instruct shows almost **no** performance drop with **40**% token trimming.

Evaluation Metrics We evaluate `TokenSkip` using three widely used metrics: accuracy, the number of CoT tokens, and inference latency per sample. Model performance is assessed using scripts from `DeepSeek-Math`[3]. Greedy decoding is employed to generate the outputs from the target LLM. Inference latency is measured on a single NVIDIA 3090 GPU with a batch size of 1. In addition to these metrics, we report the actual compression ratio of the CoTs to assess whether the compression aligns with the specified ratio.

4.2 Main Results

The performance of `TokenSkip` on GSM8K using the Qwen2.5-Instruct series[4] is illustrated in Figure 5. As the model scale increases, there is less performance degradation at higher compression ratios, indicating that larger LLMs are better at identifying shortcuts between critical reasoning tokens, enabling more efficient CoT generation. Notably, Qwen2.5-14B-Instruct exhibits almost **NO** performance drop (less than 0.4%) with **40**% token trimming. Even at a compression ratio of 0.5, the model maintains strong reasoning capabilities, with only 2% performance degradation. These results highlight the substantial potential of `TokenSkip` to reduce CoT token usage and accelerate reasoning in large-scale LLMs. Due to computational constraints, experiments with larger models are not conducted and are left for future exploration.

We further compare `TokenSkip` with two widely

[3]`https://github.com/deepseek-ai/DeepSeek-Math`

[4]For detailed results, please refer to Appendix B.2.

Methods	Ratio	GSM8K				MATH-500			
		Accuracy ↑	Tokens ↓	Latency (s) ↓	*Act*Ratio	Accuracy ↑	Tokens ↓	Latency (s) ↓	*Act*Ratio
`Original`	-	86.2$_{(0.0\downarrow)}$	213.17	5.96$_{1.0\times}$	-	48.6$_{(0.0\downarrow)}$	502.60	16.37$_{1.0\times}$	-
`Prompt`	0.9	84.1$_{(2.1\downarrow)}$	226.37	6.12$_{1.0\times}$	1.06	48.6$_{(0.0\downarrow)}$	468.04	15.39$_{1.1\times}$	0.93
	0.7	84.9$_{(1.3\downarrow)}$	209.39	5.51$_{1.1\times}$	0.98	48.4$_{(0.4\downarrow)}$	472.13	15.55$_{1.1\times}$	0.94
	0.5	83.7$_{(2.5\downarrow)}$	188.82	4.97$_{1.2\times}$	0.89	47.8$_{(0.4\downarrow)}$	471.11	15.48$_{1.1\times}$	0.94
`Truncation`	0.9	70.2$_{(26.0\downarrow)}$	202.06	5.29$_{1.1\times}$	0.95	47.8$_{(0.8\downarrow)}$	440.33	14.56$_{1.1\times}$	0.88
	0.7	25.9$_{(60.3\downarrow)}$	149.99	3.97$_{1.5\times}$	0.70	45.0$_{(3.6\downarrow)}$	386.89	12.85$_{1.3\times}$	0.77
	0.5	7.0$_{(79.2\downarrow)}$	103.69	2.95$_{2.0\times}$	0.49	27.4$_{(21.2\downarrow)}$	283.70	9.40$_{1.7\times}$	0.56
`TokenSkip`	1.0	86.7$_{(0.5\uparrow)}$	213.60	5.98$_{1.0\times}$	1.00	48.2$_{(0.4\downarrow)}$	504.79	16.43$_{1.0\times}$	1.00
	0.9	86.1$_{(0.1\downarrow)}$	198.01	5.65$_{1.1\times}$	0.93	47.8$_{(0.8\downarrow)}$	448.31	15.26$_{1.1\times}$	0.89
	0.8	84.3$_{(1.9\downarrow)}$	169.89	5.13$_{1.2\times}$	0.80	47.3$_{(1.3\downarrow)}$	398.94	13.39$_{1.2\times}$	0.79
	0.7	82.5$_{(3.7\downarrow)}$	150.12	4.36$_{1.4\times}$	0.70	46.7$_{(1.9\downarrow)}$	349.13	11.55$_{1.4\times}$	0.69
	0.6	81.1$_{(5.1\downarrow)}$	129.38	3.81$_{1.6\times}$	0.61	42.0$_{(6.6\downarrow)}$	318.36	10.58$_{1.6\times}$	0.63
	0.5	78.2$_{(8.0\downarrow)}$	113.05	3.40$_{1.8\times}$	0.53	40.2$_{(8.4\downarrow)}$	292.17	9.67$_{1.7\times}$	0.58

Table 1: Experimental results of `TokenSkip` on LLaMA-3.1-8B-Instruct. We report accuracy, average CoT token count (Tokens), average latency per sample, and actual compression ratio (*Act*Ratio) for comparison.

used length control baselines — `prompt`-based reduction and `truncation`. The experimental results are presented in Table 1. As shown, `prompt`-based reduction fails to achieve the specified compression ratio, with the actual ratio exceeding 0.89 even when the target is set to 0.5. While `truncation` adheres to the specified ratio, it results in significant degradation in reasoning performance. Specifically, at a compression ratio of 0.5, `truncation` causes a 79% accuracy drop on GSM8K and a 21% drop on MATH-500. In contrast, `TokenSkip` ensures adherence to the specified compression ratio (see Figure 6) while preserving strong reasoning capabilities. Notably, `TokenSkip` achieves an actual compression ratio of **0.53** on GSM8K with only a 10% performance drop, resulting in a **1.8**× speedup in average latency. On the challenging MATH-500 dataset, `TokenSkip` effectively reduces CoT token usage by **30**% with a performance drop of less than 4%. These results validate the effectiveness of `TokenSkip`.

4.3 Analysis

Compression Ratio In our main results, we focus on compression ratios greater than 0.5. To further investigate the performance of `TokenSkip` at lower compression ratios, we train an additional variant, denoted as `More Ratio`, with extra compression ratios of 0.3 and 0.4. As shown in Figure 6, the ratio adherence of models largely degrades at these lower ratios. We attribute this decline to the excessive trimming of reasoning tokens, which likely causes a loss of critical information in the completions, hindering the effective training of

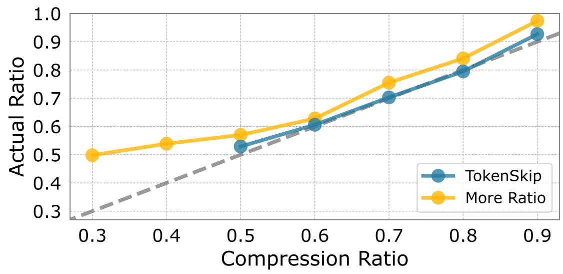

Figure 6: Comparison of ratio adherence across different compression ratio settings. The experimental results are obtained with LLaMA-3.1-8B-Instruct on GSM8K.

LLMs to learn CoT compression. Furthermore, we observe that the overall adherence of `More Ratio` is not as good as `TokenSkip` with the default settings, which further supports our hypothesis.

Importance Metric Figure 7 presents a performance comparison of `TokenSkip` across different token importance metrics. In addition to the metrics discussed in Section 2.1, we include `GPT-4o`[5] as a strong token importance metric for comparison. Specifically, for a given CoT trajectory, we prompt `GPT-4o` to trim redundant tokens according to a specified compression ratio, without adding any additional tokens. Additionally, we ask `GPT-4o` to suggest the *optimal* compression format of the CoT trajectory, referred to as `GPT-4o-Optimal` in Figure 7. We incorporate all training data generated by `GPT-4o` to train a variant of `TokenSkip`. We use the "`[optimal]`" token to prompt the model, obtaining the results of `GPT-4o-Optimal`.

As illustrated in Figure 7, `TokenSkip` utilizing

[5]We use the `gpt-4o-2024-08-06` version for experiments.

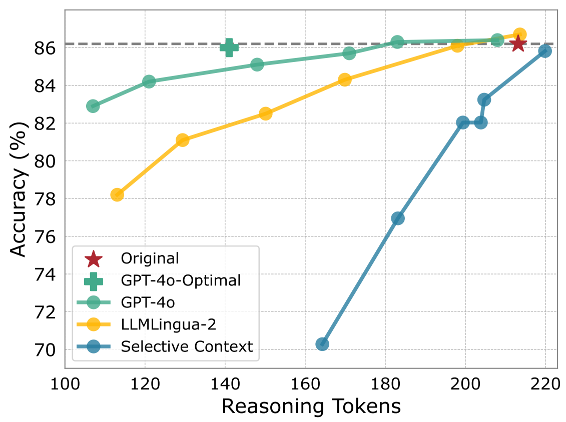

Figure 7: Performance comparison of TokenSkip using different token importance metrics, evaluated with LLaMA-3.1-8B-Instruct on GSM8K.

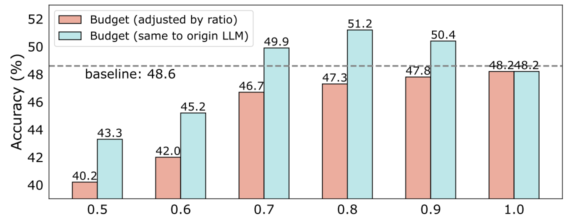

Figure 8: Performance comparison of TokenSkip with varying maximum length constraints, evaluated with LLaMA-3.1-8B-Instruct on the MATH-500 dataset.

LLMLingua-2 (Pan et al., 2024) outperforms the variant with Selective Context (Li et al., 2023), which aligns with our demonstrations in Section 2.1. Additionally, incorporating GPT-4o for token importance measurement further enhances compression performance, suggesting that a more robust CoT compressor could improve TokenSkip even further. However, the API costs associated with GPT-4o make it impractical for processing large datasets. In contrast, LLMLingua-2, which includes a BERT-size model, offers a cost-effective and efficient alternative for training TokenSkip. Furthermore, GPT-4o-Optimal achieves a better balance between reasoning accuracy and CoT token reduction, emphasizing the potential of flexible compression ratios in CoT generation — an avenue we plan to explore in future work.

Length Budget As outlined in Section 4.1, we adjust the maximum length budget to max_len$\times\gamma$ when evaluating TokenSkip on MATH-500, ensuring a fair comparison of compression ratios. However, this brute-force length truncation inevitably impacts the reasoning performance of LLMs, as LLMs are unable to complete the full generation. In this analysis, we explore whether LLMs can "*think*" more effectively using a compressed CoT format. Specifically, we evaluate TokenSkip under the same length budget as the original LLM (e.g., 1024 for MATH-500). The experimental results, shown in Figure 8, demonstrate a significant performance improvement of TokenSkip under this length budget, compared to those adjusted by compression ratios. Notably, with compression ratios of 0.7, 0.8, and 0.9, TokenSkip outperforms the original LLM, yielding an absolute performance increase of 1.3 to 2.6 points. These findings highlight TokenSkip's potential to enhance the reasoning capabilities of LLMs within the same length budget.

Case Study Figure 9 presents several examples of TokenSkip, derived from the test sets of GSM8K and MATH-500. These examples clearly illustrate that TokenSkip allows LLMs to learn shortcuts between critical reasoning tokens, rather than generating shorter CoTs from scratch. For instance, in the first case, TokenSkip facilitates LLaMA-3.1-8B-Instruct to skip semantic connectors such as "*of*" and "*the*", as well as expressions that contribute minimally to the reasoning, such as the first sentence. Notably, we observe that numeric values and mathematical equations are prioritized for retention in most cases. This finding aligns with recent research (Ma et al., 2024), which suggests that mathematical expressions may contribute more significantly to reasoning than CoT in natural language. Furthermore, we find that TokenSkip does not reduce the number of reasoning steps but instead trims redundant tokens within those steps.

5 Related Work

Efficient CoT While Chain-of-Thought (CoT) enhances task performance by simulating human-like reasoning patterns, its reasoning steps introduce significant computational overhead. As a result, researchers have sought methods to reduce this overhead while retaining the benefits of CoT. One intuitive approach is to simplify, skip (Marconato et al., 2024; Ding et al., 2024; Liu et al., 2024), or generate thinking steps in parallel (Ning et al., 2023) to improve efficiency. Another strategy involves compressing reasoning steps into continuous latent representations (Goyal et al., 2024; Deng et al., 2024; Hao et al., 2024; Cheng and Van Durme, 2024), allowing LLMs to reason without explicitly generating discrete word tokens. To minimize the generation of redundant natural lan-

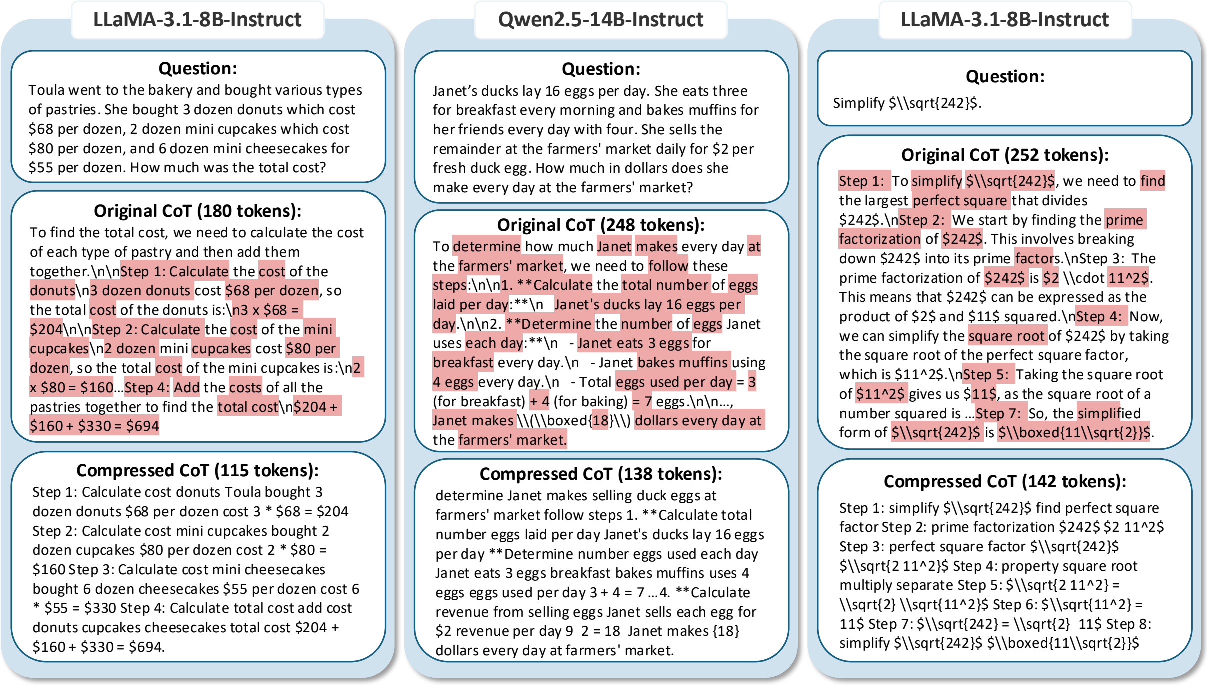

Figure 9: Three CoT compression examples from `TokenSkip`. For each sample, we list the question, original CoT outputs from corresponding LLMs, and the compressed CoT by `TokenSkip`. The tokens that appear in both the original CoT and the compressed CoT are highlighted in red.

guage information that has minimal impact on reasoning, Hu et al. (2023) implements structured syntax and symbols, while Han et al. (2024) guides token consumption through dynamic token budget estimation. Similarly, Kang et al. (2024) prompts larger LLMs (i.e., `GPT-4`) to directly compress CoT, then fine-tunes LLMs to reason using these compressed CoTs. In contrast, this work focuses on pruning CoT tokens based on their semantic importance. Additionally, `TokenSkip` leverages a small LM for token pruning, significantly reducing computational overhead.

Prompt Compression As LLMs advance in their zero-shot capabilities, the growing demand for complex instructions and long-context prompts has led to substantial computational and memory challenges in processing lengthy inputs. To address this bottleneck, researchers have explored various prompt compression techniques. One intuitive approach involves using a lightweight LM to generate more concise, semantically similar prompts (Chuang et al., 2024). However, given that explicit natural language representations often contain redundant information, some researchers have turned to implicit continuous tokens to represent task prompts (Wingate et al., 2022; Mu et al., 2024) and long-context inputs (Chevalier et al., 2023; Ge et al., 2024; Mohtashami and Jaggi, 2023). Other approaches focus on directly compressing input prompts by filtering and retaining high-informative tokens (Li et al., 2023; Jiang et al., 2023; Pan et al., 2024). For instance, Selective Context uses the perplexity of LLMs to measure token importance and removes tokens deemed less important. LLMLingua-2 (Pan et al., 2024) introduces a small bidirectional language model for token importance measurement and trains this LM with `GPT-4` compression data, which serves as the token importance metric in this work.

6 Conclusion

This work introduces `TokenSkip`, a simple yet effective approach for controllable Chain-of-Thought (CoT) compression. `TokenSkip` is built upon the semantic importance of CoT tokens — By selectively skipping less important tokens while preserving critical ones, `TokenSkip` enables LLMs to generate compressed CoTs with adjustable ratios, thereby striking an expected balance between reasoning efficiency and accuracy. Extensive experiments across various LLMs and tasks validate the effectiveness of `TokenSkip`. We hope our investigations in *token skipping* will offer valuable insights for advancing efficient CoT research and inspire future studies in this area.

Limitations

Due to computational constraints, experiments with larger LLMs, such as Qwen2.5-32B-Instruct and Qwen2.5-72B-Instruct, were not conducted. We believe that TokenSkip could achieve a more favorable trade-off between reasoning performance and CoT token usage on these models. Additionally, the token importance measurement used in our study, derived from the LLMLingua-2 compressor (Pan et al., 2024), was not specifically trained on mathematical data. This limitation may affect the compression effectiveness, as the model is not optimized for handling numerical tokens and mathematical expressions. Furthermore, experiments with long-CoT LLMs, such as QwQ-32B-Preview, were also excluded due to computational constraints. We plan to explore these aspects in future work, as we anticipate that TokenSkip 's potential can be further realized in these contexts.

Ethics Statement

The datasets used in our experiment are publicly released and labeled through interaction with humans in English. In this process, user privacy is protected, and no personal information is contained in the dataset. The scientific artifacts that we used are available for research with permissive licenses. And the use of these artifacts in this paper is consistent with their intended use. Therefore, we believe that our research work meets the ethics of ACL.

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Appendix

A CoT Recovery

In this section, we provide the detailed prompt for our recovery experiments, which is illustrated in Figure 10. Besides, we present the CoT recovery result from `GPT-4o` (OpenAI, 2023) in Figure 11. The recovered results demonstrate that `GPT-4o` could understand the compressed CoT content and correctly restore the original CoT process.

Input Prompt for CoT Recovery

Could you please recover the following compressed Chain-of-Thought output of a mathematical question to its original full content?

The question is:
Marcus is half of Leo's age and five years younger than Deanna. Deanna is 26. How old is Leo?

The compressed Chain-of-Thought:
break down Deanna 26 Marcus five years younger 26 - 5 21 Marcus half Leo's age Leo twice Marcus's age Marcus 21, Leo's age 2 x 21 = 42.

Original Full Chain-of-Thought:

Figure 10: Input prompt for LLaMA-3.1-8B-Instruct designed to recover the compressed CoT from a GSM8K math problem.

Revovering the Compressed Chain-of-Thought

Compressed CoT: break down Deanna 26 Marcus five younger 26 - 5 21 Marcus half Leo's age twice Marcus Marcus 21, Leo's age 2 x 21 = 42.

Recovered CoT: 1. We know that Deanna is 26 years old. 2. Marcus is five years younger than Deanna. So, Marcus's age is $26 - 5 = 21$. 3. Marcus is also half of Leo's age, which means Leo's age is twice Marcus's age. 4. Since Marcus is 21 years old, Leo's age is $2 \times 21 = 42$. So, Leo is 42 years old.

Figure 11: Recovering the compressed CoT for GSM8K math word problem using GPT-4o.

B Experimental Details

B.1 Implementation Details

We utilize LLMLingua-2 (Pan et al., 2024) as the token importance metric to generate our compressed CoT training data. The compression ratio γ is randomly selected from $\{0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$ for each training sample. We adopt LoRA (Hu et al., 2022) to train our models. The rank r is set to 8, and the scaling parameter α is set to 16. We train the models for 3 epochs on both datasets. The peak learning rate is set to 5e-5, following a cosine decay schedule. We use AdamW (Loshchilov and Hutter, 2019) for optimization, with a warmup ratio of 0.1. We implement our training process using the `LLaMA-Factory` (Zheng et al., 2024) library. Inference for both our method and all baselines is performed using the Huggingface transformers package. During inference, the maximum number of tokens `max_len` is set to 512 for GSM8K and 1024 for MATH. All experiments are conducted using Pytorch 2.1.0 on 2×NVIDIA GeForce RTX 3090 GPU (24GB) with CUDA 12.1, and an Intel(R) Xeon(R) Platinum 8370C CPU with 32 cores.

B.2 Detailed Results with Qwen

We provide detailed experimental results of the Qwen2.5-Instruct series evaluated on GSM8K in Table 2. As the model scale increases, there is less performance degradation at higher compression ratios, indicating that larger LLMs are better at identifying shortcuts between critical reasoning tokens, enabling more efficient CoT generation.

Scale	**Methods**	**Ratio**	Accuracy	Tokens	*Act*Ratio
3B	`Original`	-	83.7$_{(0.0\downarrow)}$	314.87	-
	`TokenSkip`	1.0	83.4$_{(0.3\downarrow)}$	318.79	1.00
		0.9	83.2$_{(0.5\downarrow)}$	262.99	0.83
		0.8	81.6$_{(2.1\downarrow)}$	250.71	0.79
		0.7	80.1$_{(3.6\downarrow)}$	233.03	0.73
		0.6	77.3$_{(6.4\downarrow)}$	199.55	0.63
		0.5	74.4$_{(9.3\downarrow)}$	170.55	0.54
7B	`Original`	-	91.4$_{(0.0\downarrow)}$	297.83	-
	`TokenSkip`	1.0	91.7$_{(0.3\uparrow)}$	295.78	1.00
		0.9	91.1$_{(0.3\downarrow)}$	254.77	0.86
		0.8	90.1$_{(1.3\downarrow)}$	237.27	0.80
		0.7	89.9$_{(1.5\downarrow)}$	216.73	0.73
		0.6	87.9$_{(3.5\downarrow)}$	178.07	0.60
		0.5	86.0$_{(5.4\downarrow)}$	151.44	0.51
14B	`Original`	-	93.1$_{(0.0\downarrow)}$	313.11	-
	`TokenSkip`	1.0	93.0$_{(0.1\downarrow)}$	314.55	1.00
		0.9	93.3$_{(0.2\uparrow)}$	269.22	0.86
		0.8	93.2$_{(0.1\uparrow)}$	247.24	0.79
		0.7	93.4$_{(0.3\uparrow)}$	218.62	0.70
		0.6	92.7$_{(0.4\downarrow)}$	180.68	0.57
		0.5	91.4$_{(1.7\downarrow)}$	156.85	0.50

Table 2: Experimental results on the Qwen2.5-Instruct series. We report accuracy, average CoT token count, and actual compression ratio (*Act*Ratio) for comparison.